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## New (Quark) Flavors

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Some possible characteristics of the new quark suggested by the discovery of T(9.4) are surveyed. An inverse scattering approach to the interquark potential is summarized.

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## INTRODUCTION

The Moderators of today's sessions have persuaded me that there is social value in making a public display of my ignorance of the properties of the next quark flavor. Before doing so I shall briefly recall the well-known flavors of quarks and leptons and summarize the immediate experimental motivation for speculating on the nature of a fifth quark flavor: the  $\Upsilon$  family of new heavy particles. Next I will survey a number of possible assignments for the putative fifth quark and indicate very briefly the consequences of each. I then turn to the problem of determining the force that binds quarkonium, and outline a novel application of the inverse scattering formalism. The strengths and weaknesses of this new approach are indicated.

## II. WHY MORE FLAVORS?

As Dr. Meshkov has indicated in his provocative introductory remarks,<sup>1</sup> our present theories do not deal with the questions of how many flavors of quarks and leptons exist, and of the mass spectrum of new flavors. At the moment, the primary impetus to consider new flavors comes from experiment. Beyond the well-established quarks:  $u$ ,  $d$ ,  $s$ , and  $c$  (each in three colors) and leptons:  $\nu_e$ ,  $e$ ,  $\nu_\mu$ ,  $\mu$ , there are now very strong indications<sup>2</sup> for a new massive lepton  $\tau$  ( $1.8 \text{ GeV}/c^2$ ) with a companion neutrino  $\nu_\tau$ . Amid the euphoria attending QCD and unified theories of the weak and electromagnetic interactions, I admit my concern at the proliferation of "fundamental" fields. A conservative count<sup>3</sup> includes 12 quarks, 4 leptons, 8 colored gluons, 1 photon, 3 intermediate bosons ( $W^+$ ,  $W^-$ ,  $Z^0$ ), 1 Higgs scalar, and as a future consideration 1 graviton, for a total of 30, with the end not in sight!

The strongest suggestion<sup>4</sup> of a fifth quark comes from the discovery<sup>5</sup> by the Columbia-Fermilab-Stony Brook Collaboration of the upsilon family of massive vector mesons. These were found in measurements of the reaction

$$p + N \rightarrow (\mu^+ \mu^-) + \text{anything}$$

with 400 GeV/c incident protons. The most recent published data are shown in Fig. 1. Two prominent peaks are seen near 9.4 and 10.0 GeV/c<sup>2</sup>. The data after subtraction of a smooth background are shown in Fig. 2. The curves represent two- (dashed line) and three-peak (solid line) fits to the data which correspond to the resonance parameters given in Table I. At least two resonances are observed, and their natural widths are consistent with zero.

Although I do not have time to discuss all the possible explanations of these structures, it is important to remark that many hypotheses are consistent with what is known experimentally. The favorite, in light of the charmonium experience,<sup>6</sup> is to regard T(9.4 GeV/c<sup>2</sup>) and T'(10.0) as the  $1^3S_1$  and  $2^3S_1$  states of a new (Q $\bar{Q}$ ) quarkonium family. It is this idea that we now pursue.

### III. WHAT IS THE NEW FLAVOR?

If T signals the existence of a new quark, what is it? How does it participate in the weak interactions? What are the attributes of flavored (Q $\bar{q}$ ) states? Unlike our recent adventure with charm, wherein a single definite and attractive conjecture<sup>7</sup> could be elaborated<sup>8</sup> and confronted with experimental data, there is not a pressing theoretical need for a particular new quark. Several assignments have already been discussed in the context of specific gauge theories.<sup>9</sup> Here I shall catalog a few possibilities in terms of charged-current structures.

If the new quark has charge  $e_Q = -1/3$  (b-quark):

1. Last year's favorite assignment was  $\begin{pmatrix} u \\ b \end{pmatrix}_R$ , which provided a natural explanation for the celebrated high- $\gamma$  anomaly in  $\bar{\nu}N$  scattering.<sup>10</sup> Although recent data from CERN<sup>11</sup> and Fermilab<sup>12</sup> speak against the spectacular effect first reported, a systematic difference between low-energy and high-energy results persists.<sup>13</sup> Whether that is to be understood as an experimental disagreement, an approach to scaling, "asymptotic freedom" violations of scaling, or new-flavor production is not entirely clear. Consequently I regard this assignment as unlikely but not impossible.

2. The assignment  $\begin{pmatrix} c \\ b \end{pmatrix}_R$  would imply charmed particles in decay products and a negligible effect upon neutrino physics. The lifetime of the flavored hadrons, estimated in a free quark model to be  $\tau \sim 10^{-15}$  sec., might permit the detection of extremely short tracks at extremely high energies.

3. Coupling one new quark to another, as in  $\begin{pmatrix} t \\ b \end{pmatrix}_{R,L}$ , leads to interesting alternatives. If  $M_b > M_t$ , where is the  $(t\bar{t})$  family? It should have been seen more prominently than  $T$  in the CFS experiment, so this ordering is ruled out. If  $M_b < M_t$ , the possibility of stable hadrons arises.<sup>14</sup> Depending upon mixing angles, the lifetime of the least massive flavored hadron could range from  $10^{-15}$  sec.  $\lesssim \tau \lesssim \infty$ .

4. Weak currents mediated by new intermediate bosons would allow assignments such as  $\begin{pmatrix} u \\ b \end{pmatrix}_{L'}$ , accompanied in the lepton sector by  $\begin{pmatrix} \nu \\ M^- \end{pmatrix}_{L'}$ , for example. With the implied new leptons, detectable short tracks are likely. Quarks which are weak isosinglets may also be imagined.

If the new quark has  $e_Q = +2/3$  (t-quark):

1. The assignments  $\begin{pmatrix} t \\ b \end{pmatrix}_{R,L}$  would lead to stable particles if  $M_t < M_b$ . If  $M_t > M_b$ , we must ask whether a  $(b\bar{b})$  quarkonium family could have been overlooked in the mass region between 4 and 10  $\text{GeV}/c^2$ . Given the smaller branching ratio into lepton pairs for charge =  $-1/3$ , I am not prepared to say this is impossible. I view it as unlikely, however.

2. A low- $y$  anomaly in  $\nu N$  scattering is implied by the assignment  $\begin{pmatrix} t \\ d \end{pmatrix}_R$ . At energies near the threshold, the effect would be an excess of events at medium values of  $y$ . The flavored particles would decay into ordinary hadrons, and would likely be too ephemeral to be observed as short tracks.

3. New weak currents are of course possible for this case as well.

#### Exotica:

Exotic quark charges, such as  $e_Q = +5/3, -4/3, \dots$  would yield spectacular signals at  $e^+e^-$  storage rings. We may also imagine stable quarks of any charge, or quarks which are distinguished by unconventional color properties.<sup>15</sup>

Compared to what lies ahead, finding charm experimentally was easy!

#### IV. APPROACHES TO THE INTERQUARK POTENTIAL

Let us now turn to the strong interaction properties of the new quarks. We hope<sup>16</sup> and believe<sup>17</sup> that the ideas of nonrelativistic quantum mechanics can fruitfully be applied to quarkonium systems. What is the interaction that binds heavy quarks together? A theoretical problem on which much work has been done is to solve QCD and compute the interquark potential.<sup>18</sup> This program is still underway.<sup>19</sup> Much remains to be learned experimentally about the T family. It is likely that the existence of additional narrow states can be settled at Fermilab, but the detailed spectroscopy and study of radiative transitions awaits new  $e^+e^-$  machines. At the level of phenomenology, a wide variety of studies has been undertaken. These include explicit potential models,<sup>17</sup> inferences from scaling rules,<sup>20</sup> general results from nonrelativistic quantum mechanics,<sup>21</sup> and general consequences of analyticity.<sup>22</sup>

Today I want to report on a new phenomenological approach, based on an approximate solution of the inverse scattering problem for quarkonium systems. I will merely state results, because a complete description has been given elsewhere.<sup>23</sup>

The inverse scattering program of Gel'fand, Levitan, Marchenko, and others<sup>24</sup> is basically a dispersion theory for the Schrödinger wavefunction. In one space dimension, a reflectionless potential<sup>25</sup> which supports N bound states (at  $E_n = -\kappa_1^2, -\kappa_2^2, \dots, -\kappa_N^2$ ) is given by

$$V(x) = -2 \frac{d^2}{dx^2} \log(\text{Det } A) \quad , \quad (1)$$

where

$$A_{mn} = \delta_{mn} + \frac{c_m e^{-\kappa_m x} c_n e^{-\kappa_n x}}{\kappa_m + \kappa_n} \quad . \quad (2)$$

It is determined in general by the  $2N$  parameters  $\kappa_1, \dots, \kappa_N$  and  $c_1, \dots, c_N$ . The Schrödinger wavefunctions are themselves given by

$$\psi_n(x) = - \frac{1}{c_n e^{-\kappa_n x}} \frac{\text{Det } A^{(n)}}{\text{Det } A} \quad , \quad (3)$$

where  $A^{(n)}$  is the matrix obtained by replacing the  $n$ -th column of  $A$  by its derivative. For the special case of a symmetric potential,

$$V(x) = V(-x) \quad , \quad (4)$$

the  $c_n$ 's can be eliminated<sup>26</sup> in favor of the bound-state pole positions

$$c_n^{2/2\kappa_n} = \prod_{m \neq n}^N \left| \frac{\kappa_m + \kappa_n}{\kappa_m - \kappa_n} \right| \quad . \quad (5)$$

Hence a symmetric one-dimensional reflectionless potential which supports  $N$  bound states can be reconstructed from the  $N$  pole positions.

Our hope is that this formalism will provide us with a reliable local approximation to a confining potential (which supports an infinite number of discrete levels). We investigate whether a suitable approximation can be obtained using a small number of bound states. To do so, we study several examples numerically.

We first consider the harmonic oscillator potential

$$V(x) = x^2 \quad , \quad (6)$$

which has bound states at

$$E_n = 2n - 1 \quad , \quad n = 1, 2, \dots, \quad (7)$$

for a reduced mass  $\mu = 1/2$ . To define the pole positions of the bound states in our reconstructed reflectionless potential, we must choose a zero of energy:

$$\kappa_n = [2\mu(E_0 - E_n)]^{1/2} \quad . \quad (8)$$

The parameter  $E_0$  is restricted to the interval  $E_N < E_0 < E_{N+1}$  for the  $N$ -bound-state reconstruction. We find that excellent results follow from the choice

$$E_0 = \frac{1}{2}(E_N + E_{N+1}) \quad . \quad (9)$$

The  $N = 1, 2, 3, 4, 5$  approximations to the harmonic oscillator potential (6) are shown in Fig. 3(a) - (e). The true potential is reproduced closely, up to the classical turning point of the last level included, after only a few bound states. I also show in Fig. 3(f) - (j) the resulting approximations to the Schrödinger wavefunctions. These may be compared with the exact wavefunctions in Fig. 3(k). The agreement is impressive.

Similar studies of the linear potential

$$V(x) = x \quad (10)$$



and the infinite square well

$$V(x) = \begin{cases} 0, & |x| < \pi/2 \\ \infty, & |x| > \pi/2 \end{cases} \quad (11)$$

are shown in Figs. 4 and 5. The results encourage the belief that this technique will lead to useful representations of a confining potential in the region of space probed by the low-lying levels.

The s-wave reduced radial equation in 3 space dimensions,

$$-\frac{u''(r)}{2\mu} + [V(r) - E]u(r) = 0 \quad , \quad (12)$$

has the same form as the 1-dimensional Schrödinger equation, but is supplemented by the boundary condition

$$u(0) = 0 \quad . \quad (13)$$

Therefore, only odd-parity one-dimensional solutions are admissible s-wave solutions in three dimensions. In the charmonium family,  $\psi(3.095)$  and  $\psi'(3.684)$  correspond to the  $n = 2$  and  $n = 4$  levels of the analogous 1-dimensional problem. We may use their positions to specify the parameters  $\kappa_2$  and  $\kappa_4$ . The remaining parameters  $\kappa_1$  and  $\kappa_3$  are fixed by means of eq. (3) to reproduce the wavefunctions at the origin, which are measured<sup>27</sup> by the leptonic widths of  $\psi$  and  $\psi'$ :

$$\Gamma(\mathcal{V} \rightarrow e^+e^-) = 16 \pi \alpha^2 e_Q^2 |\Psi(0)|^2 / M_{\mathcal{V}}^2 \quad . \quad (14)$$

Again we have found, through the study of simple examples, that reasonable approximations are reconstructed from the positions and leptonic widths of two states.

Our procedure, then, is to construct reflectionless potentials for which<sup>28</sup>

$$M(\psi) = 3.095 \text{ GeV}/c^2$$

$$M(\psi') = 3.684 \text{ GeV}/c^2$$

$$\Gamma(\psi \rightarrow e^+e^-) = \underline{4.8} \pm 0.6 \text{ keV}$$

$$\Gamma(\psi' \rightarrow e^+e^-) = \underline{2.1} \pm 0.3 \text{ keV} \quad .$$

We encounter two sources of ambiguity: the zero of energy ( $E_0$ ) discussed above, and the mass  $m_c$  of the charmed quark. We choose  $m_c = 1.1, 1.2, 1.3, 1.4, 1.5 \text{ GeV}/c^2$  and  $E_0 = 3.75, 3.8, 3.85, 3.9 \text{ GeV}$ . The twenty resulting reflectionless potentials are shown in Fig. 6. All of these reproduce--by construction--the positions and leptonic widths of  $\psi$  and  $\psi'$ . The diversity of the potential shapes is noteworthy. To choose among these potentials, we may examine their implications for other observables. I shall cite but two examples.

First, we may solve the p-wave Schrödinger equation in the reconstructed potentials for the position of the  $2^3P(\chi_c)$  levels. These are indicated as the dashed lines on the left sides of the potentials in Fig. 6. Contours of the resulting  $\chi_c$  masses in terms of the parameters  $E_0$  and  $m_c$  are shown in Fig. 7. Experiment<sup>29</sup> favors the lower right-hand half of the plot.

Next, we may use these potentials to predict the properties of the T family. The only quantity now measured is the T - T' splitting, for which the predictions are shown as contours in Fig. 8. Again, experiment<sup>5</sup> favors the lower right-hand half of the display. Additional distinctions between the potentials are discussed at length in Ref. 23.

Let us conclude by assaying the strengths and weaknesses of this new approach to the interquark potential. It requires no assumptions on the behavior of the potential at  $r = 0$  or  $r = \infty$ , but supplies no clues to the nature of the confining force. It provides a systematic display of the possible potential shapes, in the range of  $r$  relevant to the observed properties of the charmonium family. At the same time, because this intermediate range of  $r$  is least accessible to theoretical conjectures, the significance of inferences which can be drawn from the reconstructed potentials is not a priori clear. Only experimental properties of the  $^3S_1$  states are used to determine the potentials. However, we are not guaranteed a pleasingly shaped monotonic potential. One important benefit of the exercise is that it illustrates the ambiguities inherent in extrapolating from the  $\psi$  family to the  $T$  family. Finally, I regard the inverse method as extremely promising when it can be applied to the  $T$  family, in which three or four narrow  $^3S_1$  levels are to be expected.<sup>30</sup>

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TABLE I

Resonance Fit Parameters<sup>a</sup>

		2 peak	3 peak	
T	$M_1$	$9.41 \pm 0.013$	$9.40 \pm 0.013$	$\text{GeV}/c^2$
	$B \frac{d\sigma}{dy} \Big _{y=0}$	$0.18 \pm 0.01$	$0.18 \pm 0.01$	pb
T'	$M_2$	$10.06 \pm 0.03$	$10.01 \pm 0.04$	$\text{GeV}/c^2$
	$B \frac{d\sigma}{dy} \Big _{y=0}$	$0.069 \pm 0.006$	$0.065 \pm 0.007$	pb
T''	$M_3$	---	$10.40 \pm 0.12$	$\text{GeV}/c^2$
	$B \frac{d\sigma}{dy} \Big _{y=0}$	---	$0.011 \pm 0.007$	pb
$\chi^2/\text{DF}$		19.3/18	14.2/16	

<sup>a</sup>From Innes, et al., Ref. 5. Errors are statistical only.

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## FIGURE CAPTIONS

- Fig. 1: Dimuon spectrum above 6 GeV, from Innes, et al., Ref. 5.
- Fig. 2: Excess of the data over an exponential fit to the continuum. Errors shown are statistical only. The solid curve is the 3-peak fit; the dashed curve is the 2-peak fit.
- Fig. 3: Approximate reconstruction of the harmonic oscillator potential (a) - (b):  $N=1,2,3,4,5$  approximations to the potential. The true potential is shown for comparison; (f) - (j): wavefunctions obtained in the  $N=1,2,3,4,5$  approximations; (k): Exact wavefunctions.
- Fig. 4: Approximate reconstruction of the linear potential. See the caption to Fig. 3.
- Fig. 5: Approximate reconstruction of the infinite square-well potential. See the caption to Fig. 3.
- Fig. 6: Interquark potentials reconstructed from the masses and leptonic widths of  $\psi(3.095)$  and  $\psi'(3.684)$ . The levels of charmonium are indicated on the left-hand side of each graph. Those of the upsilon family are shown on the right-hand side of each graph. The solid lines denote  $^3S_1$  levels; dashed lines indicate the  $2^3P_J$  levels. The twenty potentials depicted correspond to the choices  $E_0 = 3.75, 3.8, 3.85, 3.9$  GeV and  $m_c = 1.1, 1.2, 1.3, 1.4, 1.5$  GeV/ $c^2$ .
- Fig. 7: Contours of the predicted mass of the  $2^3P_J(\chi_c)$  level of the charmonium system as functions of the parameters  $E_0$  and  $m_c$ .
- Fig. 8: Contours of the predicted  $T-T'$  level splitting as functions of the parameters  $E_0$  and  $m_c$ .

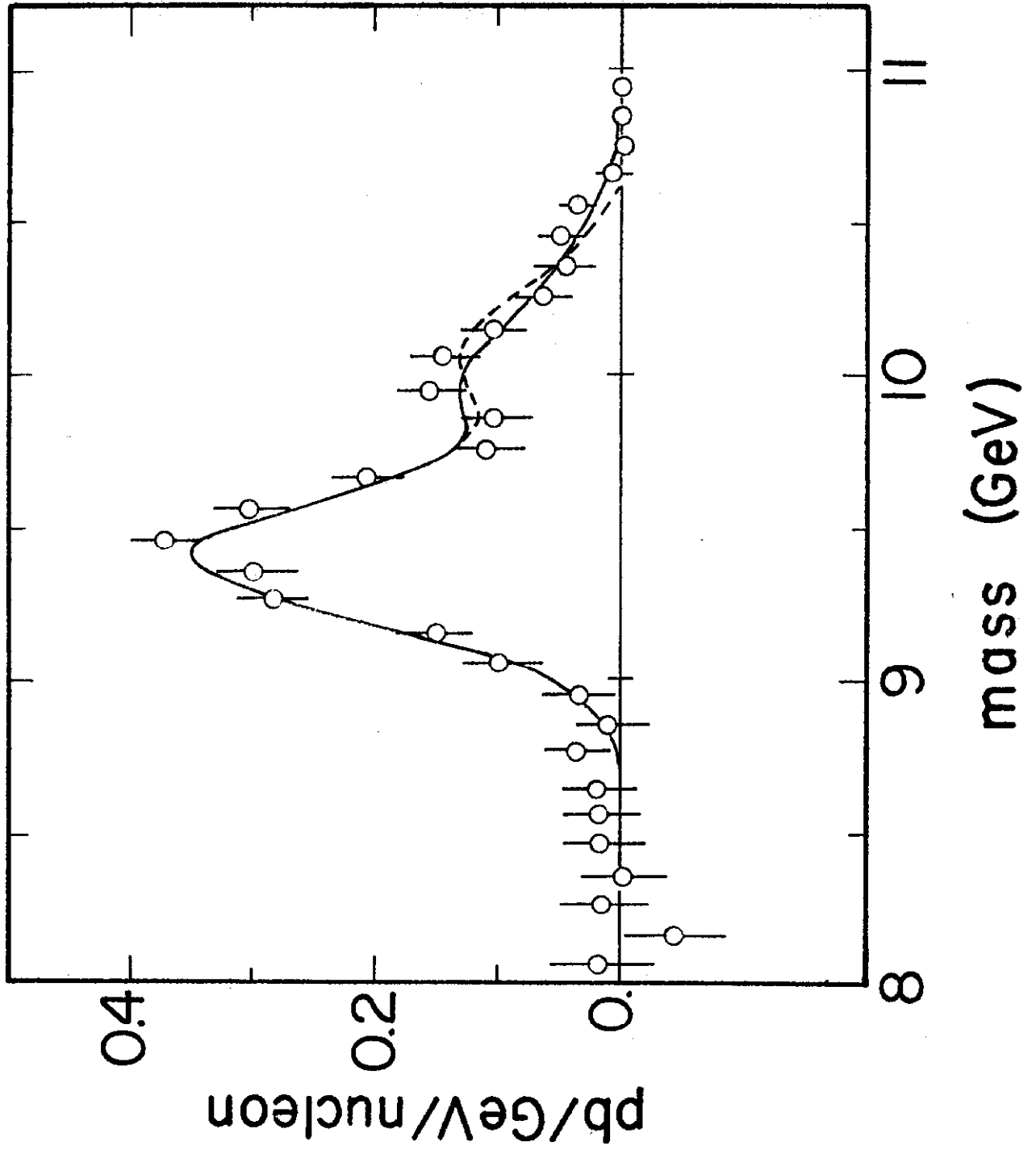


Fig. 2

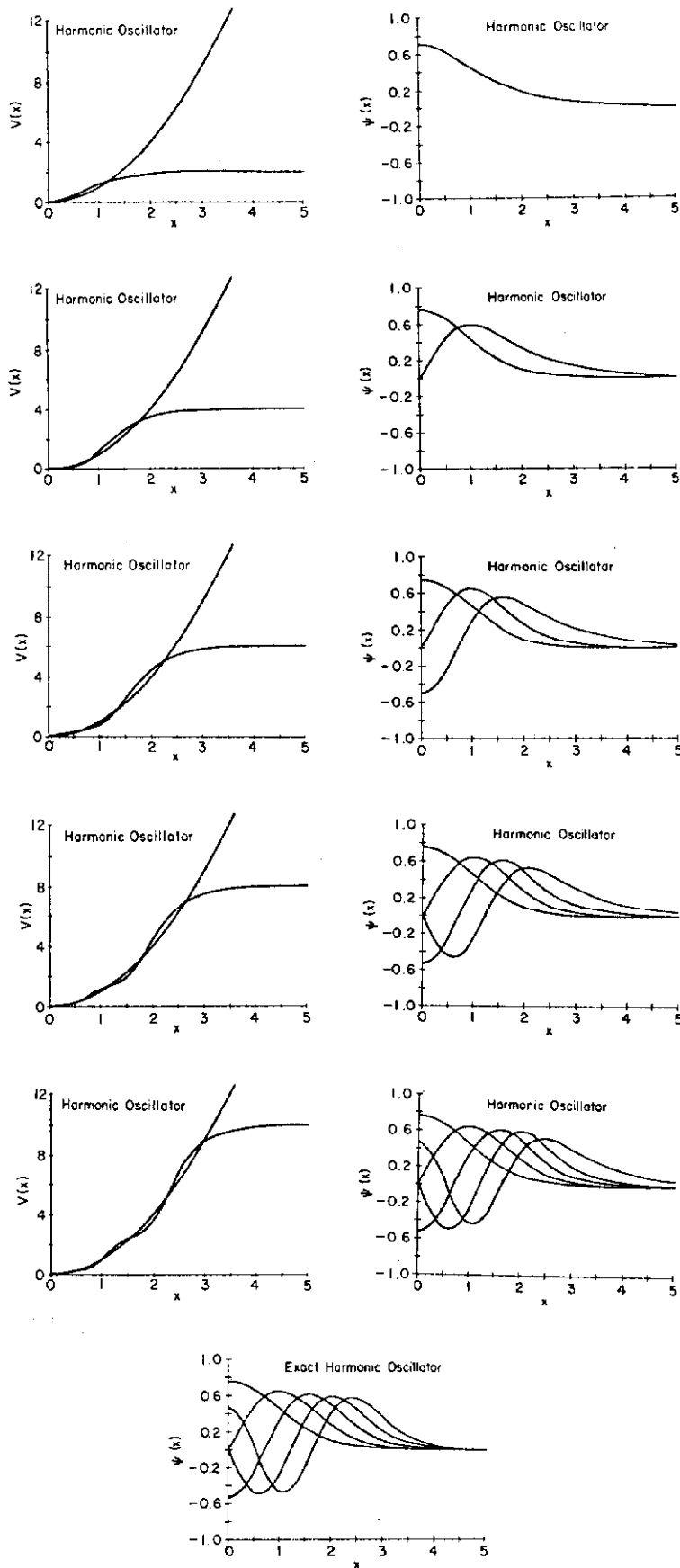


Fig. 3

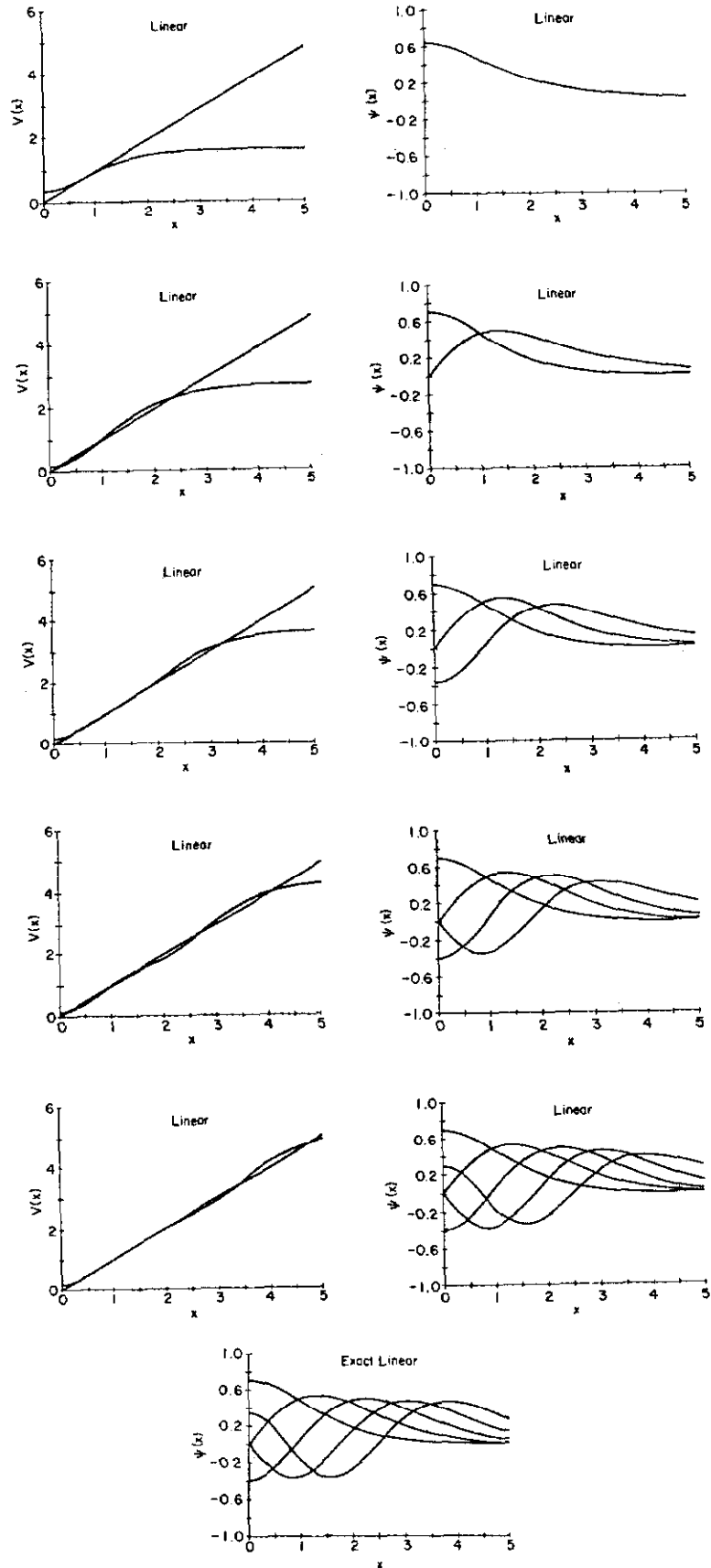


Fig. 4

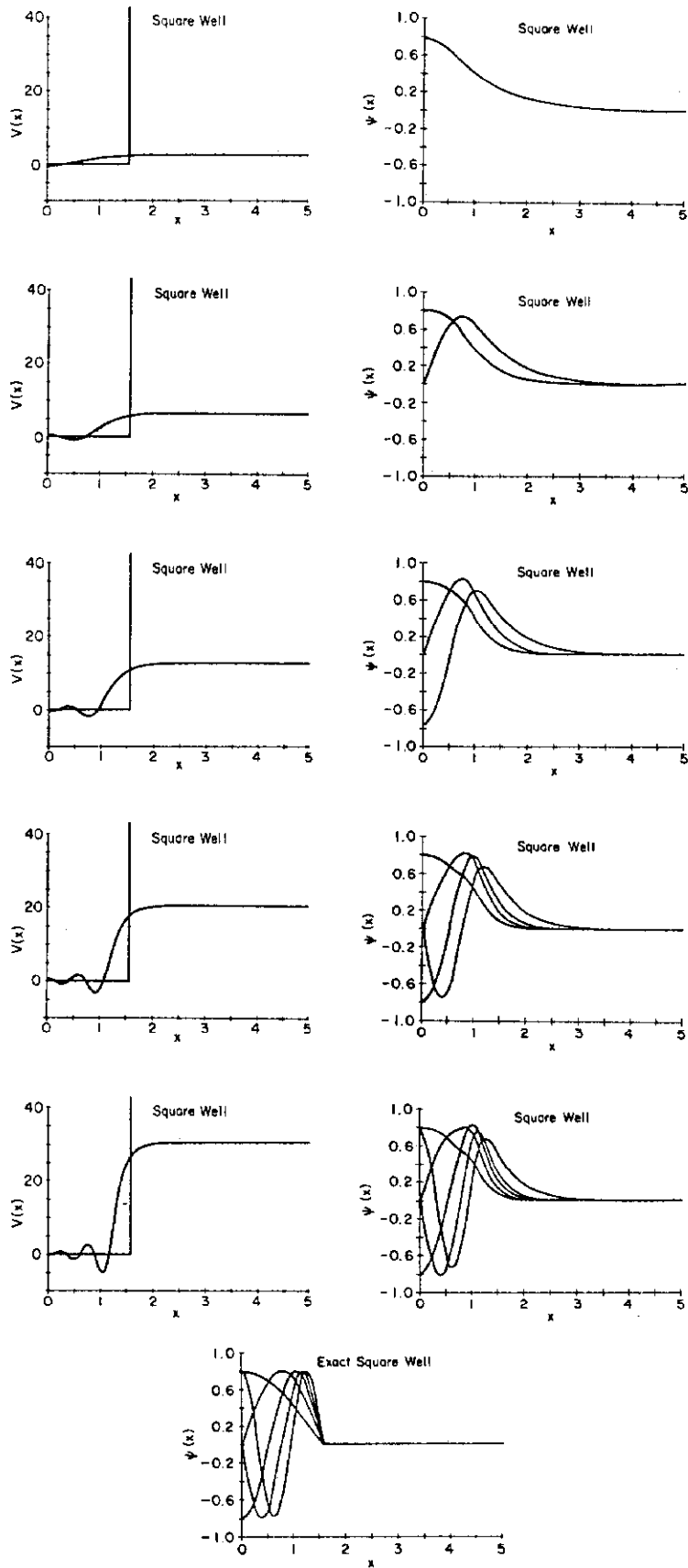


Fig. 5

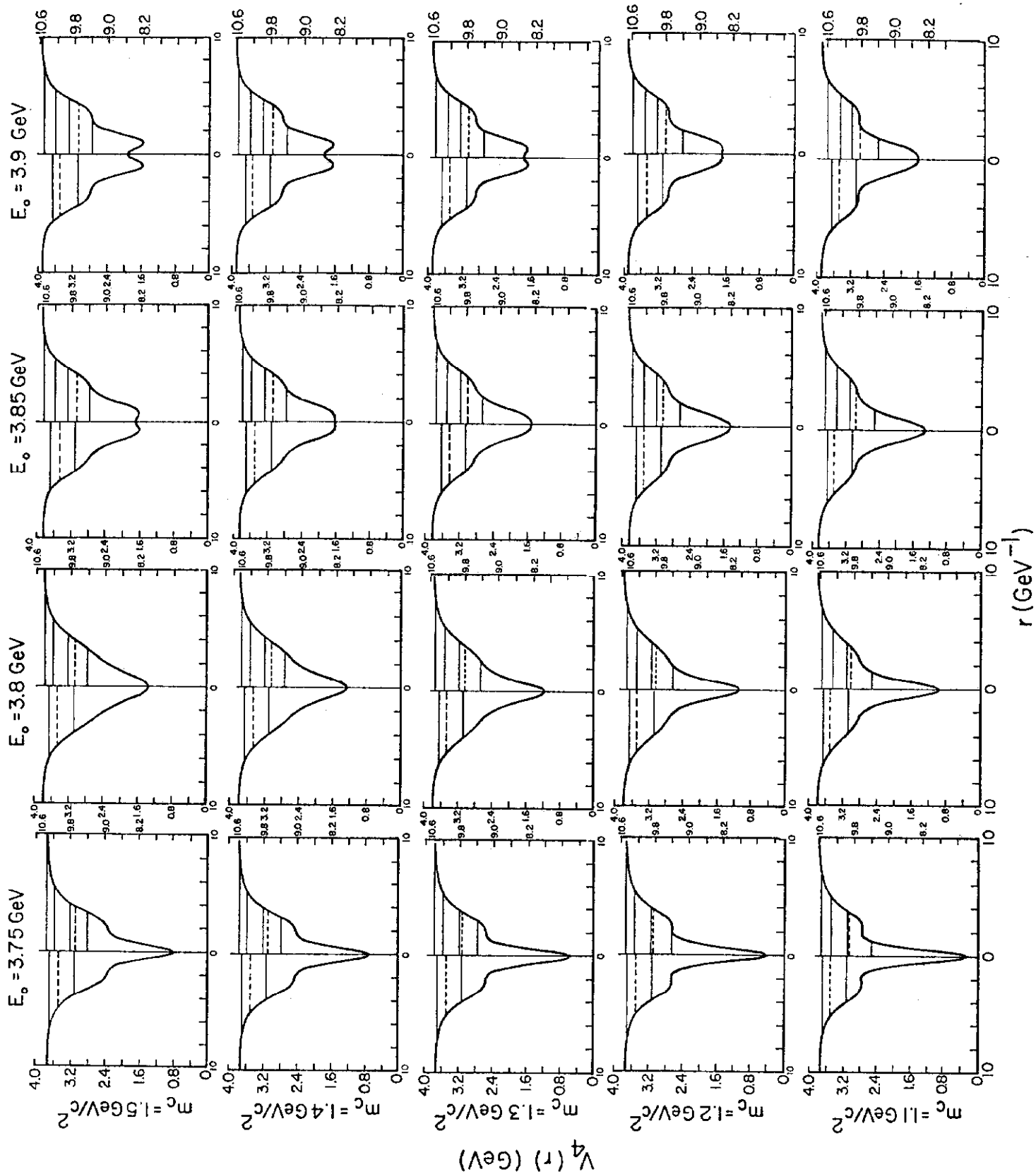


Fig. 6

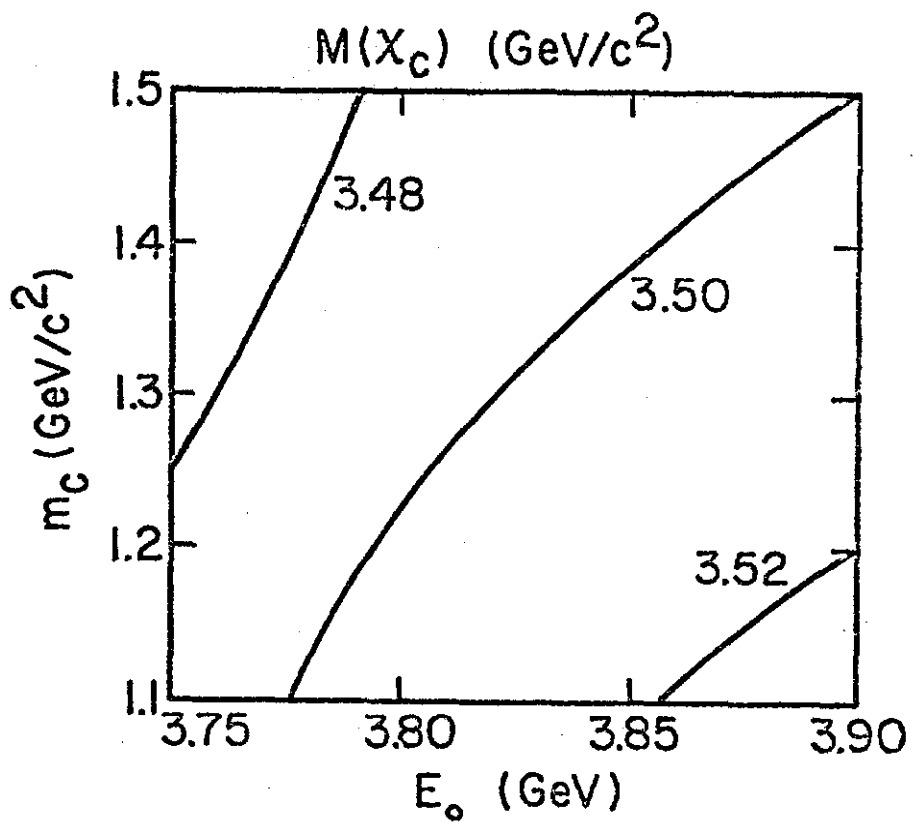


Fig. 7

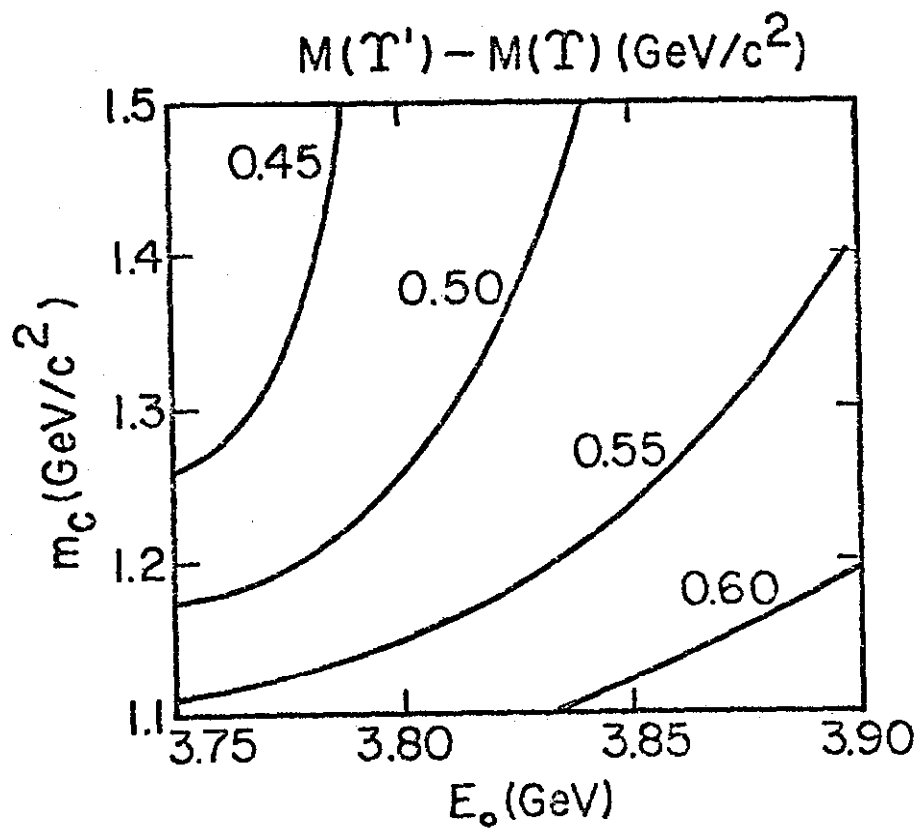


Fig. 8