Fermi National Accelerator Laboratory

FERMILAB-Pub-77/96-THY
October 1977

Vibrational States in a Heavy Quark Bound System

R. C. GILES
Laboratory for Nuclear Science and Department of Physics,
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

AND

S.-H. H. TYE
Fermi National Accelerator Laboratory
Batavia, Illinois 60510

ABSTRACT

There are numerous speculations on the existence of heavy quarks Q . In the context of the quark-confining string model, the $(Q\bar{Q})$ bound system is predicted to exhibit, below the continuum threshold, vibrational levels in addition to the states expected from the charmonium picture. This is applied to the recent discovery of the upsilon (9.4).

Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration

The success of non-relativistic linear potential models[1] in describing the low-lying Ψ spectroscopy suggests that the physics of strong confining forces is particularly evident in the structure of bound states of heavy non-relativistic quark-antiquark pairs. If, as many theorists have speculated, further heavy quarks beyond the charmed one exist, the study of the spectrum of their charmonium-like bound states should be particularly illuminating. The recent discovery of the "Upsilon" enhancement[2] in $\mu^+\mu^-$ production around 9.4 GeV at Fermilab suggests, though certainly does not yet demand, such an interpretation.

Eichten and Gottfried[3] have argued that, as the heavy quark mass increases, the number of $^3S_1$ bound states lying below the continuum of heavy quark - light quark mesons also increases. Such bound states have no Okubo-Zweig-Iizuku (OZI) allowed decays, and may be expected to be seen as narrow resonances in e^+e^- annihilation with a rich set of radiative and hadronic transitions.

Our purpose in this note is to point out that more general excitations than radial excitations of the quarks in a linear potential may be expected to be observable below threshold. We refer to these as "vibrational" modes. Elsewhere[4] we have discussed such modes as they arise in the quark-confining string model.[5] The necessity of their existence may be inferred from somewhat more model independent considerations. The linear Regge trajectories of light hadrons, and the successes of linear potential models in describing charmonium together with the approximate inverse relation of the slope of the linear potential and the Regge slope, $k \sim 1/2\pi\alpha'$, may be most easily understood if strong confining forces tend to produce string-like configurations. In the context of a theory of (confined) color quarks, one might imagine that a string arises from a thin stretched-out tube of color electric flux between the quark and the anti-quark.[6] A straight string between heavy quarks immediately produces a linear potential. Such a picture cannot be relativistically invariant unless the energy-momentum localized on the string is allowed to be associated with transverse degrees of freedom. (There is no such thing as a relativistic rigid body.) The vibrational modes to which we have referred are the quantized excitations of such modes in the non-relativistic approximation.

The quark-confining string model is a minimal realization of these ideas and consists of the embedding of two-dimensional quantum chromodynamics into four-dimensional Minkowski space. It is a relativistically invariant, gauge-invariant field theory model with explicit quark confinement. The meson bound state equation has been derived in the non-relativistic limit.[4] The $Q\bar{Q}$ effective potential $V_n(r)$ has the form

$$V_n(r) = krU_n \tag{1}$$

where

$$U_n{}^2 = 1 + \frac{2n\pi}{k\left[(r-2d)^2+4d^2\right]} \tag{2}$$

and

$$d = \frac{kr^2}{4}\left\{kr+2MU_n\Big/\sqrt{2U_n{}^2-1}\right\}^{-1} \quad .$$

k is the strength of the linear potential and measures the quark-gluon coupling. d is a correction due to the finiteness of the quark mass ($d \to 0$ as $M \to \infty$).[5] n is the vibrational quantum number. We note that for $n=0$, $V_0(r) = kr$. Relativistic corrections and spin effects have been neglected. Taking the value from ref. 4, $k = 0.21$ GeV^2, the binding energy E of the first three $^3S_1$ states ($n=0$) and the first vibrational state ($n=1$) are plotted in Fig. 1 as a function of the quark mass M_Q. We may crudely estimate the continuum as follows. The mass of the $Q\bar{q}$ bound system

$$F = M_Q + m_q + E_p - \frac{3}{4}\,\delta \tag{3}$$

where the non-relativistic binding energy E_p is independent of M_Q (for $M_Q >> m_q$, the dynamics of the bound system is determined essentially by the light quark.) $m_q \sim 0$. δ is the total triplet-singlet splitting. Using the mass differences $m_{D*} - m_D$ and $m_{K*} - m_K$, and taking the strange quark mass to be roughly $0.3 \sim 0.4$ GeV, we parametrize δ to be $\frac{0.16}{m_Q}$. Using $m_D - M_c = E_p - \frac{0.12}{M_c}$, where $M_c = 1.154$ GeV, we obtain the curve for the $Q\bar{q} + \bar{Q}q$ continuum

$$\text{threshold energy} = 2\left(E_p - \frac{0.12}{M_Q}\right) \quad . \tag{4}$$

(See ref. 3 for a more detailed consideration of the threshold.) We note that at $M_Q > 4$ GeV, the first vibrational level in the

e^+e^- channel is below threshold. In the Ψ spectroscopy, the first two vibrational levels in the e^+e^- channel are calculated to be at 4.0 and 4.4 GeV. Both of them are above the threshold and hence their identification is inconclusive.

For the quark mass $M_Q = 4.47$ GeV, the spectroscopy is exhibited in Fig. 2. The spin effects will split the levels. They are not included here. The triplet-singlet splittings and the tensor forces are under study.[7] The spin-orbit splittings are proportional[4] to M_Q^{-3} and hence are very small compared to that in the Ψ spectroscopy (i.e., a typical spin-orbit term is of the order of one MeV.)

Recent data[8] on the Upsilon seems to indicate a larger splitting between the resolved states than that between the ground state and the first radial excitation in a linear potential of slope $k \sim 0.21$. It is still quite possible that there is an unresolved state between those observed. If not, the linear potential model with slope 0.21 is wrong.[9] Though one is free, at a "phenomenological" level to adjust k accordingly, to do so without reason represents a loss rather than gain in understanding. In the quark-confing string model, k is determined by the product of the quadratic Casimir operator of the quark representation and the square of the color coupling. k will change substantially only if the heavy quarks are in an exotic (for quarks) respresentation of $SU(3)_C$ (eq. the $\underline{6}$) or if there are large dynamical changes in α_{color} with mass scale.

If the observed state at 10.0 GeV is the first radial excitation, the linear potential[1] requires $k = 0.41\ \mathrm{GeV}^2$ with $M = 4.33$ GeV. The spectrum is shown in Fig. 3. The increase in k also pushes the $(Q\bar{q} + \bar{Q}q)$ continuum up so that the first vibrational level is probably still below the threshold. Extrapolating from the Ψ spectroscopy, we expect the vibrational state to show up as a relatively narrow resonance in the e^+e^- channel even if it is a couple hundred MeV above the continuum threshold.

Such a large variation of k is inconsistent with the quark-confining string model as it stands. It will be important to search for an extra Υ state between the 9.4 and 10.0 GeV states. However, we note that although k increases from 0.21 to $0.41\ \mathrm{GeV}^2$ as M increases from 1.2 to 4.3 GeV, the effective dimensionless coupling

$$\alpha_{\mathrm{color}} \propto \frac{k}{M^2}$$

decreases as M increases.

A typical characteristic of the vibrational levels in the e^+e^- channel is their smaller leptonic width in comparison to those of the radial excitations. Qualitatively, this arises from the presence of a vibrational excitation between the quark and the antiquark, which adds a repulsive piece to the effective potential, thus decreasing the wave-function at the origin. Vibrational levels can be expected to decay to the P states via radiative transitions or can decay via OZI forbidden channels.

REFERENCES

[1] E. Eichten, *et al.*, Phys. Rev. Lett. **34**, 369; **36**, 500 (1976).

[2] S.W. Herb, *et al.*, Phys. Rev. Lett. **39**, 252 (1977).

[3] E. Eichten and K. Gottfried, Phys. Lett. **66B**, 286 (1977).

[4] R.C. Giles and S.-H.H. Tye, Phys. Rev. Lett. **37**, 1175 (1976), Phys. Rev. (to be published).

[5] S.-H.H. Tye, Phys. Rev. **D13**, 3416 (1976).

[6] This has been discussed in the context of MIT bag model for charmonium by K. Johnson.

[7] Y.J. Ng and S.-H.H. Tye, Phys. Rev. (to be published).

[8] W.R.Innes, *et al.*, FERMILAB-Pub-77/86-EXP.

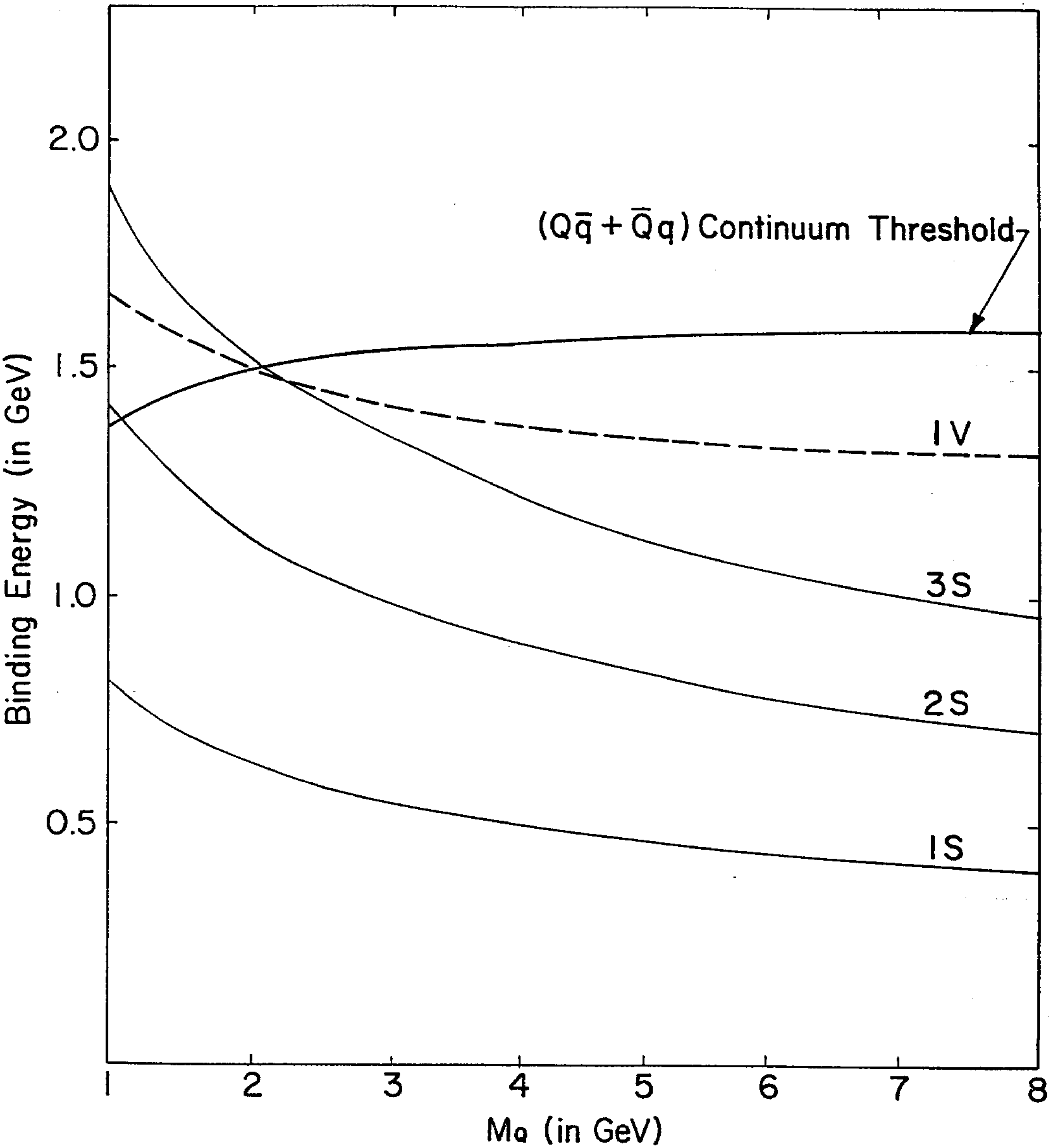
[9] For an alternative explanation, see, e.g., C. Quigg and J. Rosner, FERMILAB-Pub-77/82-THY.

FIGURE CAPTIONS

Fig. 1 $Q\bar{Q}$ binding energies as a function of quark mass. The S levels are calculated from (1) with n=0. The V level is calculated from (1) and (2) with n=1. The onset of the $Q\bar{q} + \bar{Q}q$ continuum is obtained from (4). This is a crude estimate. $k = 0.21\ \mathrm{GeV}^2$.

Fig. 2 This spectroscopy is calculated using $k = 0.21\ \mathrm{GeV}^2$ and $M_Q = 4.47$ GeV. Spin effects will split each level, but are not included. The solid lines are levels expected in the charmonium picture. The dashed lines are vibrational levels. The arrow indicates the $(Q\bar{q} + \bar{Q}q)$ continuum threshold.

Fig. 3 This spectroscopy is calculated using $k = 0.41\ \mathrm{GeV}^2$ and $M_Q = 4.33$ GeV. The dashed lines are vibrational levels. Only the lowest D state is included.



$(Q\bar{q} + \bar{Q}q)$ Continuum Threshold
1V
3S
2S
1S
2.0
1.5
1.0
0.5
Binding Energy (in GeV)
1
2
3
4
5
6
7
8
M_Q (in GeV)

Fig. 1

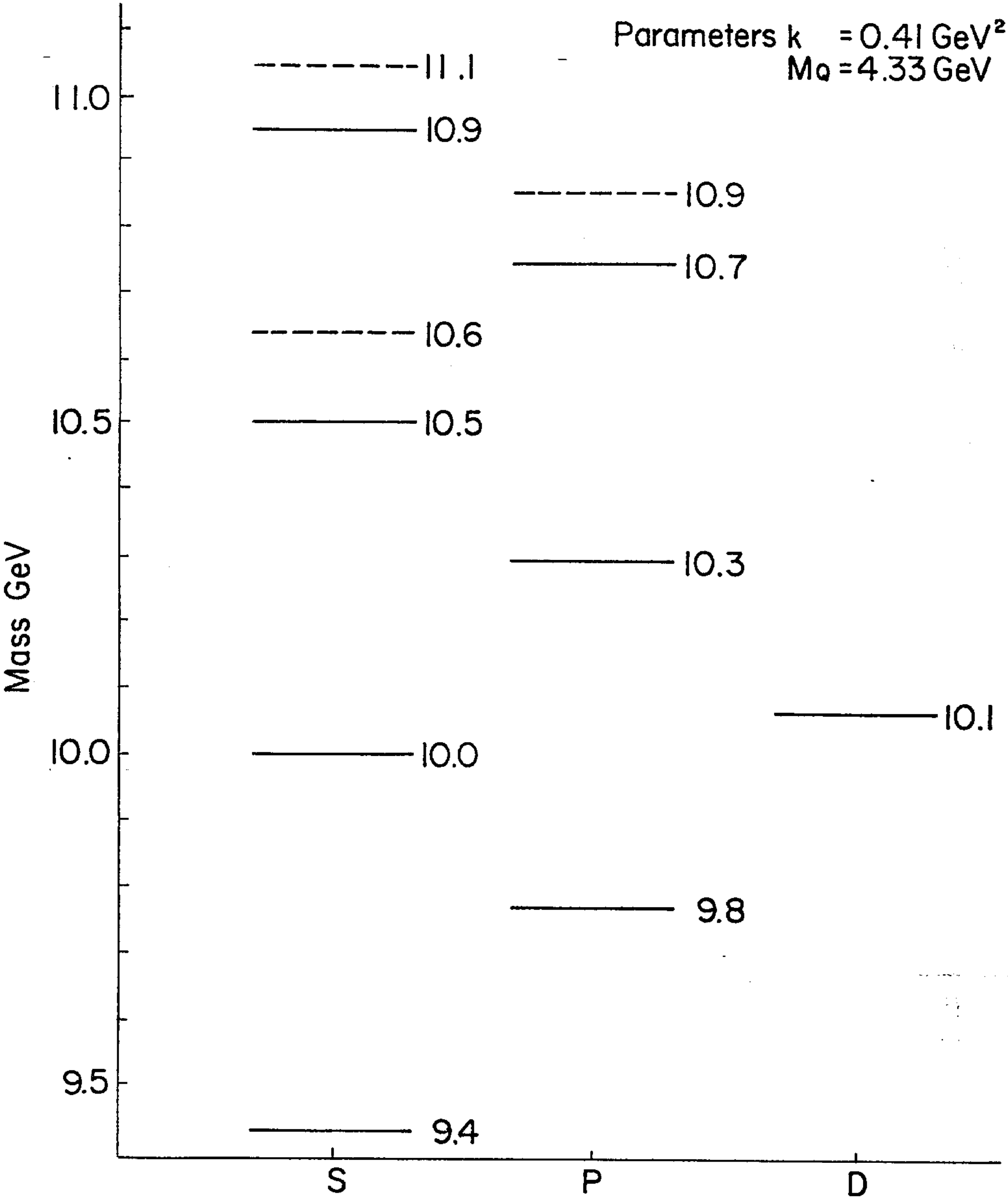
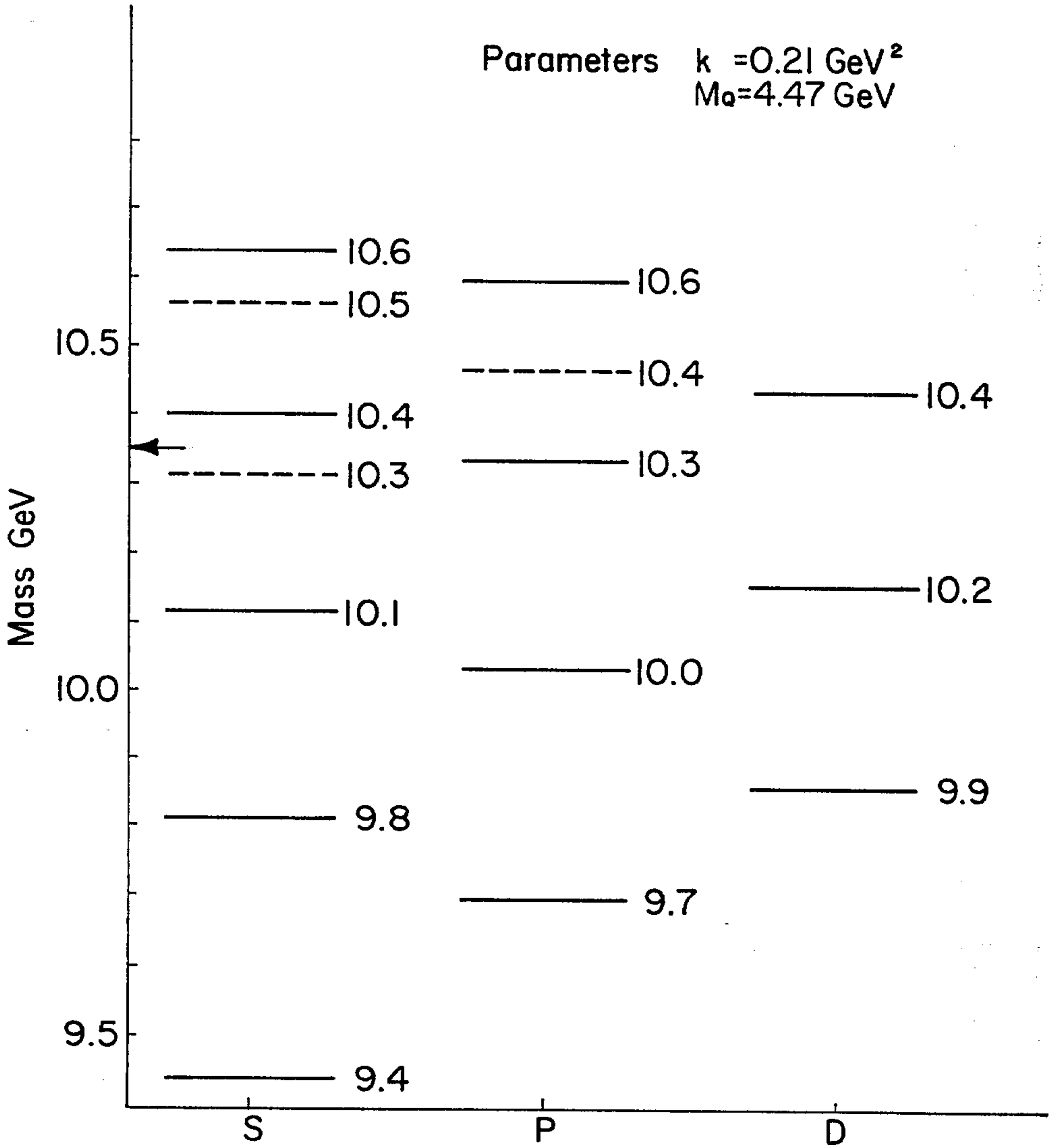

Parameters k = 0.41 GeV²
MQ = 4.33 GeV
Mass GeV
11.0
10.5
10.0
9.5
11.1
10.9
10.6
10.5
10.0
9.4
10.9
10.7
10.3
9.8
10.1
S
P
D

Fig. 3



Parameters k =0.21 GeV^2
M$_Q$=4.47 GeV
Mass GeV
10.5
10.0
9.5
10.6
10.5
10.4
10.3
10.1
9.8
9.4
10.6
10.4
10.3
10.0
9.7
10.4
10.2
9.9
S
P
D

Fig. 2