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MASS GENERATION IN A NORMAL-PRODUCT FORMULATION

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For a model of spontaneously broken supersymmetry there has been established previously an infrared anomalous Ward-identity. Here we interpret its occurrence in physical terms, namely as mass generation and indicate how this is to be used as a general method of describing that phenomenon within the framework of normal-products.

In [1] the renormalization properties of the simplest model which exhibits spontaneous breakdown of supersymmetry [2] has been studied. The model consists of three scalar superfields, subject (in addition to supersymmetry) to a continuous symmetry R:

$$\delta_{\mathrm{R}}\phi_k = (2n_k - \theta^\alpha(\partial/\partial\theta^\alpha))\phi_k, \quad k=0,1,2, n_k = 1,0,1,$$

a discrete symmetry I:

$$\phi_0 \to \phi_0, \quad \phi_{1,2} \to -\phi_{1,2},$$

and parity invariance. After the shift $F_0 \to F_0 + f$ one finds in tree approximation the Lagrangian (in components)

$$\mathcal{L} = \mathcal{L}_{\mathrm{kin}} - m(A_1F_2 + A_2F_1 - \tfrac{1}{2}\psi_1\psi_2) - \tfrac{1}{16}gf(A_1^2 + \bar{A}_1^2) - \tfrac{1}{8}g(F_0A_1^2 + 2F_1A_0A_1 - \tfrac{1}{2}A_0\psi_1\psi_2 - A_1\psi_0\psi_1) + \text{c.c.}, \tag{1}$$

at $\langle A_0\rangle = 0$, which implies a mass splitting in the multiplet ϕ_1 (breakdown of supersymmetry) while ϕ_0 stays massless. In order to renormalize according to BPHZL (see ref. in [3]) an auxiliary mass term has been added $\mathcal{L}_M = M(n-1)(A_0F_0 - \tfrac{1}{2}\psi_0\psi_0) + \text{c.c.}$, which then necessarily implies masslessness of the fields A_0 and ψ_0 for all orders. Due to Goldstone's theorem this is certainly a reasonable assumption for ψ_0 (the Goldstone field) but there is no reason why A_0 should remain massless in higher orders. And, indeed, in [1] anomalous Ward-identities (WI's) have been proved:

$$W_\alpha\Gamma = uN_{5/2}^{5/2}[U_\alpha]\cdot\Gamma, \qquad U_\alpha = \psi_{0\alpha}\bar{A}_0 + \text{corrections} = W_\alpha(A_0\bar{A}_0 + \text{corrections}), \tag{2}$$

U_α in the one-loop approximation being exactly the variation of a mass term for the field A_0. In [1] such a term has been discarded as a counterterm in Γ_{eff} because one wanted to preserve infrared finiteness, which can, arguing heuristically, be nevertheless maintained if one performs for any $A_0\bar{A}_0$ line the spring insertion with this counterterm. And clearly, if its coefficient is properly chosen, one can remove the anomaly. In order to realize this rigorously we proceed as follows. We add to the Lagrangian (1) a mass term for A_0: $-\mu^2A_0\bar{A}_0$ i.e. break explicitly the supersymmetry. Using Appendix C of [1] we establish a broken supersymmetry WI to all orders which reads exactly like eq. (2), but now $u = -\mathrm{i}\mu^2 + u'$ (u' loop contributions), the only difference being that already in the tree approximation u differs from zero: $u = -\mathrm{i}\mu^2$ if $\Gamma_{A_0\bar{A}_0}(p=0, s=1) = -\mathrm{i}\mu^2$ (normalization condition).

Assuming that we have done this up to, let's say, nth order we now sum the perturbation series for vertex functions up to this order and impose in addition:

$$u = 0. \tag{3}$$

We obtain in this way an equation for μ^2; the solution of which determines μ^2 as a function of the other parameters of the theory (m, f, g and the coefficients appearing in the effective action) and $\hbar$. As an example:

tree approximation: $u' = 0$ i.e., eq. (3) $\Rightarrow \mu^2 = 0$;

1-loop approximation: $u' = 2f^2(\Gamma_{\psi_{0\alpha}\psi_0{}^\alpha\bar{A}_0F_0} - \Gamma_{\psi_{0\alpha}\psi_0{}^\alpha\bar{A}_0\bar{F}_0}) \Rightarrow \mu^2 = -\mathrm{i}u' > 0$.

(For the computation of u' one can consult [1] since it is independent of the mass of A_0).

With this value for μ^2 the vertex functions now fulfill the desired supersymmetry WI's $W_\alpha\Gamma = 0$.

In the one-loop approximation our result agrees

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with computations via the effective potential [4], but in addition exhibits quite explicitly how supersymmetry is realized which cannot so easily be seen in the effective potential approach.

We would like to mention furthermore that mass generation in this model fits well into the popular way of understanding it [5]: the effective potential in tree approximation

$$V=\sum_{k=0}^{2} F_k \bar{F}_k = (f+\tfrac{1}{8}gA_1^2)(f+\tfrac{1}{8}g\bar{A}_1^2) + (mA_2+\tfrac{1}{4}gA_0A_1)(m\bar{A}_2+\tfrac{1}{4}g\bar{A}_0\bar{A}_1),$$

carries an additional symmetry: $\delta A_0 = \lambda$, $\delta A_1 = 0$, $\delta A_2 = -\lambda(g/4m)A_1$, i.e. A_0 is a pseudo-Goldstone boson.

It is clear that the above procedure is of quite a general applicability and opens a way of calculating masses or mass sum rules with normal-product methods. Work along these lines is presently being undertaken.

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