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A Dynamical Model for Mixing of Axial Vector (Q) Mesons

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ABSTRACT

The 45° mixing angle experimentally observed for the Q states with one eigenstate decoupled from the $K^{*}\pi$ channel and the other decoupled from $K\rho$ is shown to arise naturally in a simple model where the mixing originates from coupling through the decay channels and SU(3) breaking is introduced by kinematic phase space factors.

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A recent analysis of the experimental properties of the Q mesons suggests a mixing of SU(3) eigenstates with a 45° mixing angle with one of the eigenstates decaying only to $K^{*}\pi$ and not to $K\rho$ and vice versa for the other state.[1] The purpose of this note is to point out that this kind of mixing arises naturally in a simple model suggested by the author at the 1966 Berkeley Conference.[2,3] The model was not considered seriously at the time because of an argument based on $SU(6)_W$ which showed that this dynamical mechanism could not produce any mixing.[3] This $SU(6)_W$ argument is now known to be invalid[4] and the experimental data on the Q mesons suggest exactly the mixing predicted by the model. Since the application to the Q system was never published, we present the essential features here.

We denote the strange members of the A_1 and B octets by Q_A and Q_B respectively. The dominant decay modes $K^{*}\pi$ and ρK are allowed for both Q_A and Q_B states. In the limit of SU(3) symmetry, conserved "parities" G_u and G_v analogous to G parity can be defined by replacing isospin by U spin or V spin in the definition of G parity. The neutral and charged Q's are eigenstates of G_u and G_v respectively. However, the charged ρ and π mesons are not eigenstates of either of these parities, just as the K mesons are not eigenstates of G parity. Thus there is no selection rule forbidding $K^{*}\pi$ and ρK final states for either of these decays. If the Q_A and Q_B are produced coherently in some experiment, they contribute coherently to the ρK and $K^{*}\pi$ final states.[3]

If SU(3) is broken, G_u and G_v parities are not conserved. There can then be mixing, analogous to $\omega\phi$ mixing, between the Q_A and Q_B states, even though G parity remains conserved and prevents mixing of the corresponding non-strange states. However, there is no ideal mixing angle determined by quark masses, as in the $\omega\phi$ case, because the Q_A and Q_B have the same quark constituents and are not mixed by a mass term. Some other SU(3) breaking mechanism is needed to produce the observed mixing.

Consider the decay of the mixed states

$$|Q_1\rangle = \cos\theta|Q_A\rangle + \sin\theta|Q_B\rangle \tag{1a}$$

$$|Q_2\rangle = -\sin\theta|Q_A\rangle + \cos\theta|Q_B\rangle \quad , \tag{1b}$$

where θ is the mixing angle.

For the $K^*\pi$ and ρK decay modes the branching ratio is unity in the SU(3) limit except for differences in kinematic (phase space) factors for the two final states. However, because the two octets have opposite charge conjugation behavior, the A_1-octet decay is described with F-coupling and the B-octet decay with D-coupling. The relative phases of the $K\rho$ and $K^*\pi$ decay amplitudes are thus opposite for the two cases

$$\langle K\rho|Q_A\rangle = -\langle K^*\pi|Q_A\rangle \tag{2a}$$

$$\langle K\rho|Q_B\rangle = \langle K^*\pi|Q_B\rangle \quad , \tag{2b}$$

the decay amplitudes for the mixed states (1) are then

$$<K^*\pi | Q_1> \; = \; \cos\,\theta <K^*\pi | Q_A> + \sin\theta <K^*\pi | Q_B> \tag{3a}$$

$$<K\rho | Q_1> \; = \; -\cos\,\theta <K^*\pi | Q_A> + \sin\,\theta <K^*\pi | Q_B> \tag{3b}$$

$$<K^*\pi | Q_2> \; = \; -\sin\,\theta <K^*\pi | Q_A> + \cos\theta <K^*\pi | Q_B> \tag{3c}$$

$$<K\rho | Q_2> \; = \; \sin\,\theta <K^*\pi | Q_A> + \cos\,\theta <K^*\pi | Q_B> \quad . \tag{3d}$$

Eqs. (3) show that for any mixing with a real phase, the effect for one eigenstate is to enhance the $K^*\pi$ decay mode and suppress the $K\rho$, and vice versa for the orthogonal eigenstate. For $\theta = 45^{\circ}$, we obtain

$$\frac{|<K\rho | Q_1>|^2}{|<K^*\pi | Q_1>|^2} = \frac{|<K^*\pi | Q_2>|^2}{|<K\rho | Q_2>|^2} = \frac{|<K^*\pi | Q_A> - <K^*\pi | Q_B>|^2}{|<K^*\pi | Q_A> + <K^*\pi | Q_B>|^2} . \tag{4a}$$

Thus Q_1 is decoupled from $K\rho$ and Q_2 is decoupled from $K^*\pi$. The decoupling is exact for the case where the Q_A and Q_B states are equally coupled to the $K^*\pi$ mode and is still a good approximation over a wide range of couplings. For example, as long as

$$\frac{1}{4} \le \frac{|<K^*\pi | Q_A>|^2}{|<K^*\pi | Q_B>|^2} \le 4 \quad , \tag{4b}$$

we still have

$$\frac{|\langle K\rho|Q_1\rangle|^2}{|\langle K^*\pi|Q_1\rangle|^2} = \frac{|\langle K^*\pi|Q_2\rangle|^2}{|\langle K\rho|Q_2\rangle|^2} \leq \frac{1}{9} \quad . \tag{4c}$$

A dynamical mechanism which naturally leads to this mixing is the SU(3) breaking in decay channels originally introduced to explain[5] $\omega\phi$ mixing before SU(6) and the quark model. The states Q_A and Q_B are coupled to one another via their decay channels $K^*\pi$ and $K\rho$.

$$|Q_A\rangle \leftrightarrow |K^*\pi\rangle \leftrightarrow |Q_B\rangle \tag{5a}$$

$$|Q_A\rangle \leftrightarrow |K\rho\rangle \leftrightarrow |Q_B\rangle \quad . \tag{5b}$$

In the SU(3) symmetry limit, the two transitions (5a) and (5b) exactly cancel one another and produce no mixing. This cancellation no longer occurs when SU(3) breaking introduces kinematic factors arising from the mass difference between the two intermediate states. These suppress the strength of the transition (5b) via the higher mass $K\rho$ intermediate state relative to the transition (5a) via $K^*\pi$.

The simple analysis of the transitions (5a) and (5b) gives 45° mixing for the eigenstates if $\langle K^*\pi|Q_A\rangle = \langle K^*\pi|Q_B\rangle$. This decouples the two states from $K^*\pi$ and $K\rho$ respectively. However, a more careful analysis shows that two partial waves are present in the decay, s-wave and d-wave, and the result is very sensitive to the relative amplitudes and phases of the s and d waves. In particular, for the ratio of s to d wave amplitudes predicted by the naive $SU(6)_W$ quark model, the transitions (5) vanish and cannot produce mixing, because the Q_A is coupled only to vector meson states with transverse polarization and the Q_B is coupled only to longitudinally polarized states.[3] For this reason the mechanism (5) for mixing was dropped.

Now that the $SU(6)_W$ predictions are known not to agree with experiment,[4] particularly in the closely related polarization predictions for B and A_1 decays, and the experimental data are consistent with pure s-wave for the Q decays, the mixing mechanism (5) should perhaps again be considered. However, a more realistic calculation would consider the coupled channels $K^*\pi$ and $K\rho$ through the resonance region, with phase space factors changing within the resonances because of the proximity to threshold.

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