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Chiral SU(4) × SU(4) Breaking, Axial Vector Current Divergences and Kaon PCAC

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ABSTRACT

The chiral $SU(4) \times SU(4)$ symmetry breaking of the hamiltonian is investigated assuming the symmetry breaking part of the hamiltonian belongs to a single $(4, \bar{4}) + (\bar{4}, 4)$ representation of $SU(4) \times SU(4)$. We classify the simplest possibilities, identify each with critical orbits in the space of the $(4, \bar{4}) + (\bar{4}, 4)$ representation and indicate their relations to quark masses. The divergences of the axial vector currents are discussed and their matrix elements are used to obtain relations among masses and coupling constants involving the recently discovered charmed pseudoscalar mesons. Finally, it is pointed out that soft kaon theorems can be obtained for certain processes involving charmed particles using kaon PCAC due to the relative smallness of the kaon mass. In this case the symmetry breaking is approximately on the critical orbit corresponding to the subgroup $SU(3) \times SU(3) \times U(1)^d$.

I. INTRODUCTION

With the discovery of new particles[1] carrying another quantum number, charm, it is quite natural to extend Gell-Mann's algebra of currents[2] from chiral $SU(3) \times SU(3)$ to chiral $SU(4) \times SU(4)$. Indeed several authors[3] have already considered various aspects of chiral $SU(4) \times SU(4)$ even prior to the discovery of charm.

Here we shall assume the correctness of the chiral $SU(4) \times SU(4)$ algebra for the equal-time commutars of the 15 vector charge operators F_i and the 15 axial vector charge operators F_i^5 and explore the chiral $SU(4) \times SU(4)$ symmetry breaking of the hamiltonian density. Some of our work is a direct generalization of the previous investigations[4,5] of the behavior of the hamiltonian density under chiral $SU(3) \times SU(3)$. In particular, we shall assume that the hamiltonian density is of the form $H = H_o + H'$ where H_o is $SU(4) \times SU(4)$ symmetric (but not $U(4) \times U(4)$ invariant) while H' belongs to a single $(4, \bar{4}) + (\bar{4}, 4)$ representation of chiral $SU(4) \times SU(4)$. There are several ways in which the $SU(4) \times SU(4)$ symmetry of H can be broken leaving only an $SU(2) \times U(1) \times U(1)$ invariance corresponding to isospin, strangeness and charm conservation. That is, there are hierarchies of subgroups of $SU(4) \times SU(4)$, each containing $SU(2) \times U(1) \times U(1)$. If any of these subgroups were exact symmetries of H' then H' would lie on a critical orbit[6] in the 32-dimensional space of the $(4, \bar{4}) + (\bar{4}, 4)$ representation as discussed in Section II.

In the real world these various subgroups, in particular applications, are quite useful approximate symmetries of H; c.f., the SU(3) and SU(2) $\times$ SU(2) subgroups of SU(3) $\times$ SU(3). In the context of the standard four quark model[7] H' being on a critical orbit corresponds to special values of the quark masses since these parameters determine the direction of H' in the sapce of the $(\bar{4}, 4) + (\bar{4}, 4)$ representation.[6]

Approximate symmetry under the subgroup $SU(4)^d$ [the superscript denotes the diagonal SU(4) subgroup of SU(4) $\times$ SU(4)] we shall assume is realized in nature by the approximate invariance of the vacuum as evidenced by the existence of SU(4) multiplets of particles. For example, the 15 pseudoscalar mesons[7] π, K, η, η', η_c, D and F at least approximately seem to belong to the adjoint representation of $SU(4)^d$. Of course, the smaller subgroup $SU(3)^d$ is no doubt a much better approximate symmetry and the breaking of the still smaller subgroup $SU(2)^d$ (isospin) is entirely negligible for our purposes.

However, in the case of the chiral subgroups of SU(4) $\times$ SU(4); e.g., chiral SU(3) $\times$ SU(3) and chiral SU(2) $\times$ SU(2), we shall assume the symmetry is realized in nature in the Nambu-Goldstone manner. That is, by the appearance of approximately massless pseudoscalar mesons rather than parity-doubled multiplets of particles. Clearly chiral SU(2) $\times$ SU(2) is a better approximate symmetry than chiral SU(3) $\times$ SU(3) as indicated by $M_\pi << M_K$. Nevertheless, as discussed below, there are cases where approximate chiral SU(3) $\times$ SU(3) symmetry is useful due to the fact that $M_K << M_D$ or M_F.

Thus we adopt exactly the same point of view regarding the realization of approximate symmetries in the real world that was emphasized in reference 4.

In Section III we consider the divergences of the axial vector currents which are determined by H'. By taking the matrix elements of these current divergences between the vacuum and the pseudoscalar meson states we are able to obtain new relations among masses and coupling constants involving the charmed particles including a particularly interesting rather stringent inequality between the leptonic decay constants of the D and F mesons which can be tested experimentally. We also estimate the values of the parameters occuring in H' which are, in the context of the quark model, the quark masses.

Finally in Section IV we exploit the approximate invariance of H' under the chiral $SU(3) \times SU(3)$ subgroup of $SU(4) \times SU(4)$. It is pointed out that in certain processes involving the rather heavy charmed particles soft kaon theorems can be obtained using kaon PCAC since the kaon mass is relatively small. As one example of the use of kaon PCAC we consider the semileptonic decay of the D meson $D \to K + \ell + \nu_\ell$ in the soft kaon limit thus obtaining a relation analogous to the soft pion theorem obtained by Callan and Treiman[8] for the semileptonic decay of the K meson $K \to \pi + \ell + \nu_\ell$. One might expect this new soft kaon theorem to be valid to the order of $(M_K/M_D)^2$ just as the Callan-Treiman soft pion theorem should be good to the order of $(M_\pi/M_K)^2$.

II. SU(4) × SU(4) DIRECTIONS OF BREAKING

Starting from the simplest generalization of the $(3,\bar{3}) \oplus (\bar{3},3)$ model,[4,5] the strong hamiltonian density H is assumed to be approximately symmetric under the group chiral SU(4) × SU(4); i.e.,

$$H = H_o + H' \tag{2.1}$$

where H_o is symmetric under SU(4) × SU(4), but not U(4) × U(4). The symmetry breaking term H' is assumed to belong (at least approximately) to the $(4, \bar{4}) \oplus (\bar{4}, 4)$ representation of SU(4) × SU(4). Let us recall that the real 32-dimensional representation space G_{32} of $(4, \bar{4}) \oplus (\bar{4}, 4)$ can be realized as the complex 16-dimensional vector space of all 4 × 4 matrices M with complex coefficients. The action of the element (U, V) of SU(4) × SU(4) on M is defined by

$$M \xrightarrow{(U,V)} U M V^* = U M V^{-1} \quad . \tag{2.2}$$

More generally, we shall denote by u_i and v_i (i = 0, 1, ..., 15) the 16 scalar and 16 pseudoscalar components of the element (U, V). We then have the following commutation relations with the generators F_i and F_i^5 of SU(4) × SU(4):

$$[F_i, u_j] = i f_{ijk} u_k \tag{2.3}$$

$$[F_i, v_j] = i f_{ijk} v_k \tag{2.4}$$

$$[F_i^5, u_j] = -id_{ijk}v_k \tag{2.5}$$

$$[F_i^5, v_j] = id_{ijk}u_k \quad . \tag{2.6}$$

The f_{ijk} and d_{ijk} for SU(4) have been tabulated by Dicus and Mathur.[3] Therefore H' can be written in the form

$$H' = c_o u_o + c_8 u_8 + c_{15} u_{15} \tag{2.7}$$

requiring that isospin, hypercharge and charm be conserved. The symmetry of H', and therefore, of H will be determined by the values of the parameters c_i (i = 0, 8, 15).

In terms of the quark model

$$H' = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + m_c \bar{c}c \tag{2.8}$$

and the quark masses, by which we mean simply the parameters m_i in Eq. (2.8), are related to the constants c_i as follows:

$$m_u = \frac{1}{\sqrt{2}} c_o + \frac{1}{\sqrt{3}} c_8 + \frac{1}{\sqrt{6}} c_{15} = m_d \tag{2.9}$$

$$m_s = \frac{1}{\sqrt{2}} c_o - \frac{2}{\sqrt{3}} c_8 + \frac{1}{\sqrt{6}} c_{15} \tag{2.10}$$

$$m_c = \frac{1}{\sqrt{2}} c_o - \frac{3}{\sqrt{6}} c_{15} \quad . \tag{2.11}$$

A complete overview of the hierarchies of symmetry breaking is shown in Figure 1 which illustrates all the possible different intermediate unbroken subgroups between the largest symmetry case, SU(4) × SU(4), and the smallest symmetry, $SU(2)^d \times SU(1)^d \times U(1)^d$ (the superscript d denotes diagonal), allowed for H′ of the above form Eq. (2.7).

It is instructive to follow Michel and Radicati's approach[6] and recognize in this simple classification in which cases H′ is on a critical orbit in the space of the representation $(4, \bar{4}) \oplus (\bar{4}, 4)$. Indeed these authors have noticed that the directions of breaking of the hadronic internal symmetry have in general special mathematical properties. More precisely, these directions can be related to idempotents or nilpotents of an algebra, and they are critical; namely, every function invariant under the action of the symmetry group has an extremum in these directions. We have summarized in Appendix A the essential mathematical definitions.

As an example, let us recall that in the framework of the chiral group $SU(3) \times SU(3)$ there are in the quotient space $E_{17} = E_{18}/R - \{0\}$ critical orbits admitting as a little group, or invariance group, $SU(3)^d$ and $SU(2) \times SU(2) \times U(1)^d$. This latter group corresponds to the massless pion case in the $(3,\bar{3}) \oplus (\bar{3},3)$ model.[4,5]. Using the notations of Michel and Radicati H′ in this model can be written in the form[6]

$$H' \propto (\sqrt{2} - 0.058)y + 0.058\sqrt{3}\, n \qquad (2.12)$$

where y and n are respectively the directions belonging to the $SU(2) \times SU(2) \times U(1)^d$ and $SU(3)^d$ critical orbits. That is,

$$y = \frac{1}{\sqrt{3}} I - \lambda_8 \qquad (2.13)$$

and

$$n = \frac{\sqrt{2}}{3} I \qquad . \qquad (2.14)$$

Notice that in this model H' is approximately in the direction of y.

In the case of chiral SU(4) × SU(4) two critical orbits have previously been fou by Mott[9] in the quotient space $E_{31} = E_{32}/R - \{0\}$, the little groups of which are $SU(3) \times SU(3) \times U(1)^d$ and $SU(2) \times SU(2) \times U(2)^d$. Representative elements of each of these orbits are

$$r = \sqrt{2} \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 1 \end{pmatrix} \qquad (2.15)$$

and

$$s = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \qquad . \qquad (2.16)$$

It is interesting to notice that the kaon PCAC problem which is considered below corresponds exactly to the case Eq. 2.15 for which the symmetry group of H' is then $SU(3) \times SU(3) \times U(1)^d$.

III. MESON COUPLING CONSTANTS AND QUARK MASSES

Next we proceed to obtain relations among masses and meson coupling constants. From the symmetry breaking hamiltonian H' one can readily compute the divergences of the axial vector currents in terms of the operators v_i and the quark masses from the relation

$$\partial A_j = -i\left[F_j^5,\ H'\right] \quad . \tag{3.1}$$

One then finds that these axial current divergences have the following matrix elements between the vacuum and the various pseudoscalar meson states:

$$<0|\,\partial A_\pi\,|\pi> \;=\; M_\pi^{\;2}F_\pi \;=\; m<0|\,v_\pi\,|\,\pi> \tag{3.2}$$

$$<0|\,\partial A_K\,|K> \;=\; M_K^{\;2}F_K \;=\; \tfrac{1}{2}(m+m_s)<0|\,v_K\,|\,K> \tag{3.3}$$

$$<0|\,\partial A_D\,|D> \;=\; M_D^{\;2}F_D \;=\; \tfrac{1}{2}(m+m_c)<0|\,v_D\,|\,D> \tag{3.4}$$

$$<0|\,\partial A_F\,|F> \;=\; M_F^{\;2}F_F \;=\; \tfrac{1}{2}(m_s+m_c)<0|\,v_F\,|F> \tag{3.5}$$

where $m = m_u = m_d$ and the coupling constants F_i are determined by the leptonic widths of the pseudoscalar mesons; e.g., $F_\pi \simeq 0.94\ M_\pi$. Solving these equations for the ratios of quark masses yields

$$\frac{m_s}{m} = 2\,\frac{M_K^{\;2}F_K}{M_\pi^{\;2}F_\pi}\;\frac{<0\,|v_K|\,K>}{<0\,|v_\pi|\,\pi>}\;-1 \tag{3.6}$$

$$\frac{m_c}{m} = 2\,\frac{M_D{}^2 F_D}{M_\pi{}^2 F_\pi}\,\frac{\langle 0\,|v_D|\,D\rangle}{\langle 0\,|v_\pi|\,\pi\rangle} - 1 \tag{3.7}$$

and

$$\frac{m_s + m_c}{m} = \frac{2M_F{}^2 F_F}{M_\pi{}^2 F_\pi}\,\frac{\langle 0\,|v_F|\,F\rangle}{\langle 0\,|v_\pi|\,\pi\rangle} \quad . \tag{3.8}$$

Assuming only that the pseudoscalar meson states are SU(3) symmetric one then finds the well known result

$$\frac{m_s}{m} = 2\,\frac{M_K{}^2 F_K}{M_\pi{}^2 F_\pi} - 1 \simeq 30 \tag{3.9}$$

where we have taken $F_K/F_\pi = 1.25$. Furthermore, since the F and D belong to the same SU(3) triplet we have the relation

$$\frac{F_D}{F_F} = \frac{M_F{}^2}{M_D{}^2}\left(\frac{1 + m/m_c}{1 + m_s/m_c}\right) < \frac{M_F{}^2}{M_D{}^2} \simeq 1.19 \tag{3.10}$$

where we have used the values[10] $M_{D^+} = 1868$ MeV and $M_F = 2040$ MeV in obtaining the numerical result. We emphasize that the inequality Eq. (3.10) follows only from the SU(3) symmetry of the states and is independent of their SU(4) purity. (Note also that the derivation is free of any assumptions about soft-meson limits.) Clearly it is very important to test Eq. (3.10) experimentally.

However, if we do further assume that the pseudoscalar mesons belong purely to the 15 of SU(4) then we obtain two additional relations:

$$\frac{m_c}{m} = 2\,\frac{M_D^{\,2} F_D}{M_\pi^{\,2} F_\pi} - 1 \tag{3.11}$$

$$M_F^{\,2} F_F - M_D^{\,2} F_D = M_K^{\,2} F_K - M_\pi^{\,2} F_\pi \quad . \tag{3.12}$$

To obtain a very crude numerical estimate of the ratio m_c/m one might take[10] $F_\pi \simeq F_K \simeq F_D \simeq F_F$. Then, from Eq. (3.11) one finds

$$\frac{m_c}{m} \sim 2\left(\frac{M_D}{M_\pi}\right)^2 - 1 \sim 360 \tag{3.13}$$

However, Eq. (3.12) is in very poor agreement with the data since[11]

$$M_F^{\,2} - M_D^{\,2} \simeq (825\ \mathrm{MeV}/c^2) = 0.683\ (\mathrm{GeV}/c^2)^2 \tag{3.14a}$$

while

$$M_K^{\,2} - M_\pi^{\,2} \simeq (475\ \mathrm{MeV}/c^2)^2 = 0.225\ (\mathrm{GeV}/c^2)^2 \quad . \tag{3.14b}$$

Consequently, the validity of (3.13) is doubtful.

However, we emphasize that this estimate of the ratio m_c/m is only very crude. It is quite possible that while $F_\pi \simeq F_K$ and $F_D \simeq F_F$, which follow from only approximate SU(3), that F_D/F_π is not at all close to unity. For example, if one writes Eq. (3.12) in the form

$$\frac{F_D}{F_\pi} = \frac{M_K{}^2\left(\frac{F_K}{F_\pi}\right) - M_\pi{}^2}{M_F{}^2\left(\frac{F_F}{F_D}\right) - M_D{}^2} \tag{3.15}$$

and takes $F_K/F_\pi = 1.25$, $F_F/F_D = 1$ and $M_F = 2040$ MeV then one obtains $F_D/F_\pi \sim 0.4$ and $m_c/m \sim 150$. It is important to note that m_c/m is <u>very</u> sensitive to F_D/F_π which in turn is <u>very</u> sensitive to $M_F{}^2(F_F/F_D)$. Clearly firm data are required to check Eq. (3.12) and to obtain m_c/m from Eq. (3.11) since phenomenological estimates differ. For example, Preparata[12] finds $F_D/F_\pi = 1.1$ in his geometrodynamical model of hadronic matter while Quigg and Rosner[13] estimate $F_D/F_\pi \simeq 0.3$ in their phase space model.

So far we have only discussed ratios of quark masses. Leutwyler[14] has found that $m \simeq 5.4$ MeV and $m_s \simeq 125$ to 150 MeV. Using our estimate $m_s/m \simeq 30$ [Eq. (3.9)] and taking $m = 5.4$ MeV we find $m_s \simeq 160$ MeV in reasonable agreement. Assuming $m_c/m \backsim 360$ (Eq. 3.13), which is of doubtful validity, one finds $m_c \backsim 1940$ MeV/c^2. However, the estimate $m_c/m \backsim 150$, which follows from Eq. (3.15) and is probably more reliable, gives $m_c \backsim 810$ MeV/c^2.

While our estimates of m_c are rather crude the important conclusion is that $m_c \gg m_s \gg m$ in which case there is a regime of phenomena involving charmed particles for which chiral SU(3) $\times$ SU(3) is a useful approximate symmetry of the hamiltonian.

IV. KAON PCAC

We consider next the possibility that for certain processes one can obtain soft kaon theorems. In the breaking of chiral $SU(4) \times SU(4)$ if the subgroup chiral $SU(3) \times SU(3)$ remains an approximate symmetry by virtue of the kaon mass being small compared to any other masses involved then one expects kaon PCAC to be valid. Specifically we have in mind processes where the errors in taking the soft kaon limit are expected to be of the order of, for example, $(M_K/M_D)^2$ or $(M_K/M_F)^2$; or in terms of quarks m_s/m_c which is small.

The situation is entirely analogous to the familiar case in which chiral $SU(3) \times SU(3)$ is broken leaving the subgroup chiral $SU(2) \times SU(2)$ as an approximate symmetry of the hamiltonian due to the relative smallness of the pion mass. As is well known for processes in which the limit $M_\pi \to 0$ is smooth one can use pion PCAC to derive soft pion theorems valid to the order of, for example, $(M_\pi/M_K)^2$.

To illustrate the case of $SU(4) \times SU(4)$ breaking in which an approximate $SU(3) \times SU(3)$ invariance of the hamiltonian remains useful by virtue of the relative smallness of the kaon mass we consider as an example the processes

$$D^+ \to \overline{K}^0 + \ell^+ + \nu_\ell$$

and

$$D^0 \to K^- + \ell^+ + \nu_\ell \quad .$$

The hadronic part of the $D_{\ell 3}$ matrix element is of the form

$$<K(q)|V_\mu|D(p)> = f_+(p+q)_\mu + f_-(p-q)_\mu \quad . \tag{4.1}$$

Using kaon PCAC a standard current algebra calculation immediately gives the analogue of the Callan-Treiman relation:

$$f_+ + f_- = F_D/F_K \quad . \tag{4.2}$$

Corrections to this result should be small, of order $(M_K/M_D)^2$, and therefore testing it experimentally is particularly important.

Clearly there are also a number of other processes involving charmed particles for which soft kaon theorems can be obtained using kaon PCAC and standard current algebra techniques.

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APPENDIX: CRITICAL ORBITS ON A MANIFOLD

Here we recall some definitions and properties helpful in understanding Section II. More details will be found in the original work of Michel and Radicati.[6]

Let us consider the 32 dimensional space E_{32} of the $(4, \bar{4}) + (\bar{4}, 4)$ representation of the compact group $G = SU(4) \times SU(4)$. Each H' defined by Eq. (2.7) is a point of this manifold. Such a point H', or M, is on the orbit G(M); i.e., the set of all points transformed by G from M. Moreover the set of transformations $g \in G$ which leave M invariant is called the little group g_M of M. It is easy to prove that all points on the same orbit have conjugated little groups; i.e., $G_{gM} = g\, G_M g^{-1}$. Finally, the set of all points of E_{32} with conjugated little groups is called a stratum; in other words, the stratum S(M) is the union of all orbits such that the little groups of their points are all conjugated.

The following theorem has been proved by Michel:[15]

<u>Theorem</u>: Let G be a compact lie group acting smoothly (i.e., in an infinitely differentiable way) on the real manifold $\mathscr{M}$, and let $M \in \mathscr{M}$. The two properties (a) and (b) are equivalent:

(a) The orbit G(M) is critical; i.e., the differential $df_{M'}$ of every smooth real G-invariant function f on $\mathscr{M}$ vanishes for $M' \in G(M)$.

(b) The orbit G(M) is isolated in its stratum; i.e., there exists a neighborhood V_M of M such that if $p \notin G(M)$, $p \in V_M$, then G_p is not conjugated to G_M.

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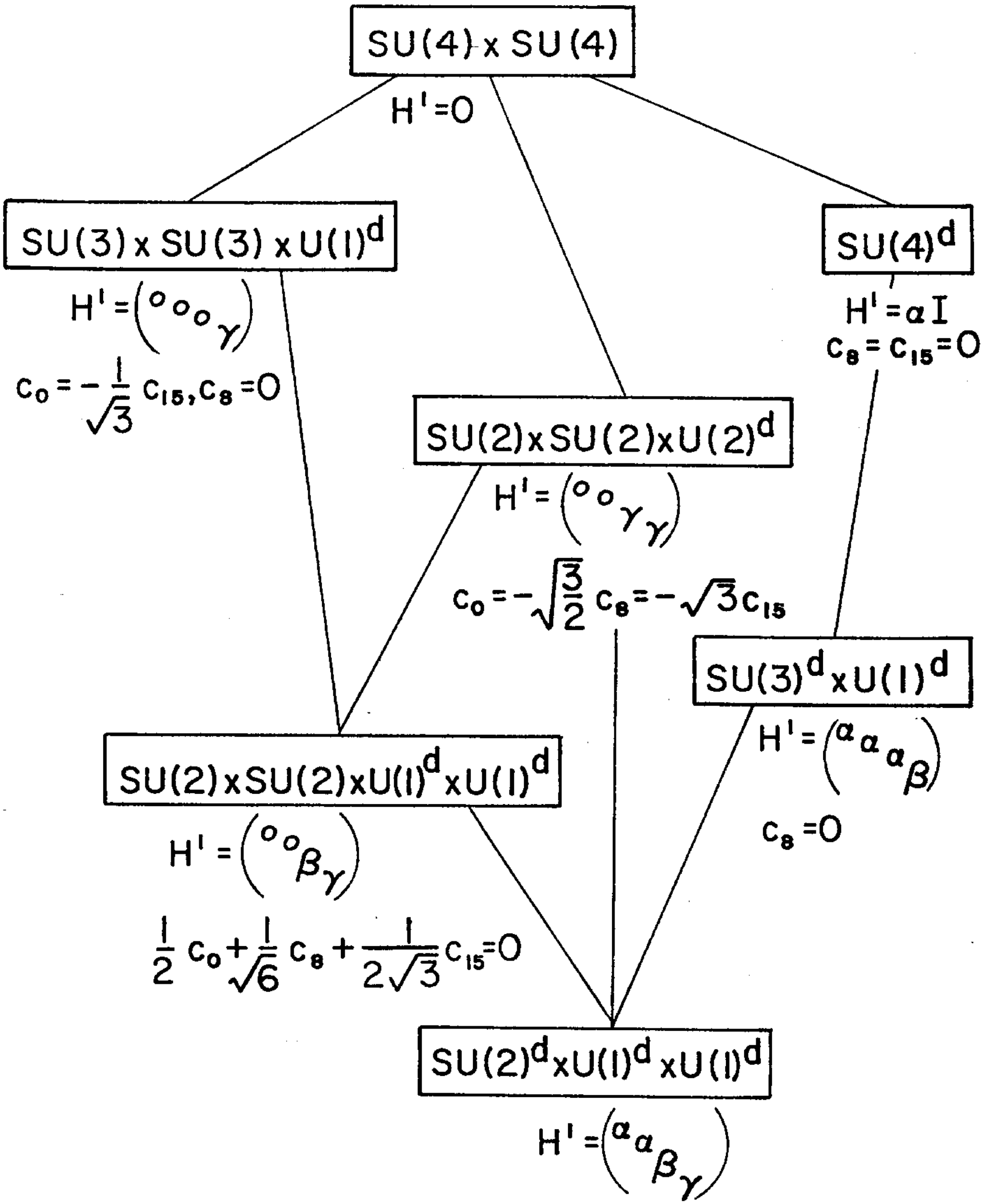
FIGURE CAPTION

The possible ways in which SU(4) $\times$ SU(4) can be broken leaving an exact subgroup are shown. Each unbroken subgroup corresponds to H' being on a critical orbit and the relevant values of the parameters c_i are indicated. For each case the form of H' is given in the context of the quark model to illustrate the structure of the corresponding quark mass matrix.



SU(4) x SU(4)
H' = 0
SU(3) x SU(3) x U(1)^d
H' = (0 0 0 γ)
c_0 = −(1/√3) c_15, c_8 = 0
SU(4)^d
H' = α I
c_8 = c_15 = 0
SU(2) x SU(2) x U(2)^d
H' = (0 0 γ γ)
c_0 = −√(3/2) c_8 = −√3 c_15
SU(3)^d x U(1)^d
H' = (α α α β)
c_8 = 0
SU(2) x SU(2) x U(1)^d x U(1)^d
H' = (0 0 β γ)
(1/2) c_0 + (1/√6) c_8 + (1/(2√3)) c_15 = 0
SU(2)^d x U(1)^d x U(1)^d
H' = (α α β γ)