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Quark Analysis of Multibaryonic Systems

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ABSTRACT

Properties of the 3n-quark systems in the spherical cavity approximation of the MIT bag model are considered. Results concerning the kinematics--color restrictions, baryon composition--as well as dynamics--mass formulae, magnetic moments, gluon corrections--of such states are obtained via a group theoretical treatment. The appearance of "hidden color" baryonic mixtures in multibaryonic states is emphasized, and the role of the quark-gluon interaction illustrated.

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I. INTRODUCTION

The notion of exotic states in elementary particle physics is not a new one and has a long history. A natural and physical way of understanding such states lays on the consideration of multiquark systems--i.e. more than one quark-antiquark ($q\bar{q}$) or three quark (q^3) systems--which were extensively discussed in the last years.[1] The recent interest on this subject has been brought in part by the MIT bag model.[2] Indeed this model predicts the existence of unusual hadrons.[3] A developed study of $q^m\bar{q}^n$ ($m + n > 3$) system, as well as some results concerning q^6-system,[3] are already been proposed by Jaffe.[4,5]

In this paper, we shall present a detailed group-theoretical analysis of the quark content of six, nine and twelve quark systems, in the approximation of the spherical MIT bag. These states of baryonic number B = 2, 3, 4, etc. will be called multibaryonic states.

Our goals are the following. First, we would like to consider the properties which can be given by a group theoretical study of such states. These properties are naturally of two sorts: kinematical (color problem, baryon composition of the states) and dynamical (mass formulae, magnetic moments, gluon correction, etc.). Secondly, we will carry more particularly our attention on the multibaryonic states owning the same quantum numbers as well known states in nuclear physics, i.e. the deutron D and the light nuclei H^3, He^3 and He^4.

It is difficult at the present time to give an adequate physical interpretation of such states. However, we can expect that these multibaryonic states which emerge from the MIT bag model consideration could be observed as configuration admixture to the usual nuclei. As was argued in Ref. 6, in the deutron case, such a mixture of the 6q-system with deutron quantum numbers appearing in the real deutron can be estimated from the asymptotic behavior of the deutron electromagnetic form factor at the level of about 7%. One can also expect that multibaryonic systems with an exotic internal structure (hidden color, for instance) could appear as high level excitations in nuclear matter. It is worth to mention here the encouraging results in the experimental search for dibaryonic resonances in Λp and pp systems,[7,8] and also in $\Delta\Delta$ systems.[9]

The demand of the multibaryonic states to be singlets of color together with the Pauli principle lead to the possibility of assigning these states to completely anitsymmetric representations of the SU(12) group, which contain the direct product of the SU(4) spin-isospin group by the SU(3)-color group, as far as strange quarks are not involved; in this last case the appropriate group will be SU(18) (see section II). However this SU(12) group cannot be considered as an exact symmetry group, and the study of the possible masses of these states appears more covariant in the framework of the SU(6) color-spin group, which contains the product of SU(3) color group by the SU(2) spin group, and recently considered by Jaffe[5] (section III).

Our group theoretical analysis will be pursued in order to calculate the relative couplings of proton-neutron and $\Delta \cdot \Delta$ isobar contained in the 6 nonstrange quark system. The same type of analysis will be carried out for tri and tetrabaryonic states. It is interesting to point out that such a study also makes appear mixtures of "colored baryons" in these multibaryonic states (section IV).

The calculation of the magnetic moments of the multibaryonic states can be easily done in the framework of the MIT bag model. However the quark gluon interaction is expected to play a more important role for these states than in the free nucleons. Generalizing in Section V a formula giving the gluonic corrections to the magnetic moments, we notice that the gluonic corrections obtained by this way can be very large, which leads in particular to consider the use of perturbation theory difficult to justify for the multiquark systems.

Finally Appendix A is devoted to the study of certain representations of the SU(4) group relevant for our purposes, while in Appendix B Clebsch Gordan isoscalar factors of the group SU(12) are calculated.

II. SU(12) AND THE CLASSIFICATION OF MULTIBARYONIC STATES

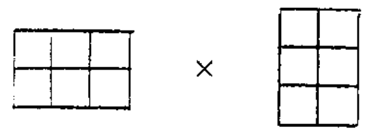
In all that follows, the quarks are supposed to stand on the same energy shell with spin $j = \frac{1}{2}$, which is the case of the MIT bag model in the spherical cavity approximation.

As far as we consider the usual baryons constituted by 3 quarks and classified by the group SU(6)--or SU(8) if we include charmed baryons--the principle of colorless baryons together with the Pauli principle lead to assign the low lying baryons to belong to the representation:

$$\underset{56 \text{ of } SU(6)}{\ydiagram{3}} \times \underset{1 \text{ of } SU(3)^C}{\ydiagram{1,1,1}} \tag{2.1}$$

if $SU(3)_C$ denotes the SU(3) color group. The representations $70 = \ydiagram{2,1}$ and $20 = \ydiagram{1,1,1}$ of SU(6) coming from the product $\ydiagram{1} \times \ydiagram{1} \times \ydiagram{1} = 6 \times 6 \times 6$ are also used to classified baryons of orbital momentum $L \neq 0$: the group SU(6) is then replaced by $SU(6) \times O(3)_L$ [10] and, for example, the completely symmetric combinations $(70, L = 1)$ or $(20, L = 1)$ are considered.

We can apply the same techniques to construct multibaryonic states. Let us take the example of the dibaryonic states supposed constitute of 6 quarks of the type p and n--no strange or charmed quark. Then the SU(6) classification group can be restricted to $SU(4) \supset SU(2)_{isospin} \times SU(2)_{spin}$. The fully antisymmetric functions could be obtained only from the combination of the two conjugate patterns



50 of SU(4) $\qquad$ 1 of $SU(3)^C$ $\qquad$ (2.2)

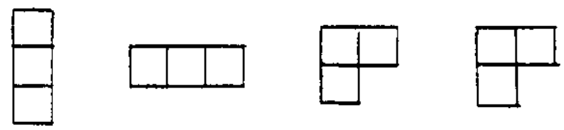
However, a simple check shows that in the product $4 \times \ldots \times 4 = 4^6$, in which 4 is the fundamental SU(4) representation, the 50 dimensional representati appears 5 times; and we can count the same number of singlet SU(3) representations in the product $3 \times \ldots \times 3 = 3^6$ of SU(3) representations. Let us note that we were not faced with such a problem for the usual one-baryon states. However, this problem is solved by applying the Pauli principle for multiquark wave functions. In the group theoretical language, this can be done by embedding the product group $SU(4) \times SU(3)$ into the group SU(12). Now, considering the product $12 \times \ldots \times 12 = 12^6$ where 12 is the fundamental representation of SU(12), we are interested in the completely antisymmetric SU(12) representation of dimension $924 = (1^6)$ using the Young diagram notation. The reduction of this representation with respect to $SU(4) \times SU(3)$ is[11]:

$$924 = (50,\ 1) + (64,\ 8) + (6,\ 27) + (10,\ 10) + (\overline{10},\ \overline{10}) \quad . \tag{2.3}$$

As expected, the 924 representation contains the representation (50, 1) once and only once! Thus the two-baryonic states are perfectly characterized as elements of the (50, 1) representation of $SU(4) \times SU(3)$ in the 924 representation of SU(12).

In this framework, the nuclei and Δ isobars belong to the 220 dimensional representation of SU(12) and to the (20, 1) representation of SU(4) $\times$ SU(3), since in SU(12):

$$12 \times 12 \times 12 = 220 + 364 + 572 + 572 \tag{2.4}$$



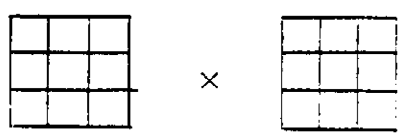
and reducing with respect to SU(4) $\times$ SU(3):

$$220 = (20, 1) + (20', 8) + (\bar{4}, 10) \quad . \tag{2.5}$$

Let us also note that the 924 representations can come only in the product $(12)^3 \times (12)^3$ from:

$$\begin{aligned} 220 \times 220 &= 924 + 8580 + 23166 + 15730 \\ &\ (1^6) + (2, 1^4) + (2^2, 1^2) + (2^3) \quad . \end{aligned} \tag{2.6}$$

For tri-baryonic states, we will consider the pattern:

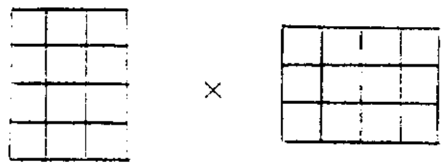


$$\times \tag{2.7}$$

20 of SU(4) 1 of SU(3)

in the representation: $220 = (1^9)$ of SU(12).

The tetrabaryonic state is unique: $(1^{12}) = 1$ and corresponds exactly to the case:

 (2.9)

1 of SU(4) 1 of SU(3)

III. GENERALIZATION OF JAFFE'S SPIN-COLOR ANALYSIS

The above introduced SU(12) group--or SU(18) if we consider strange quarks--is obviously not an exact symmetry even for these multibaryonic states constituted of quarks in the lowest mode. In the MIT bag model for instance, the energies of multibaryonic states are split due to the presence of the color gluon fields. Starting from the results of the MIT bag model for the gluon field contribution to the energy of multiquark system, R. Jaffe has introduced the notion of spin-color symmetry ($SU^{SC}(6)$-group), which provides an effective tool for the analysis of the mass spectrum of multibaryonic states in the cavity approximation. This result can be presented in the form[5, 12]

$$M_{bag} = m_p \left[\frac{2.04n - 1.84 + 0.0325\Delta}{3.50}\right]^{3/4} \tag{3.1}$$

where

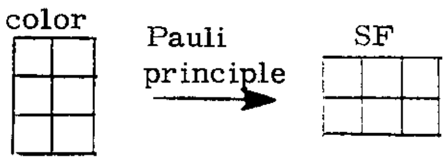
$$\Delta = 24n + 4J(J+1) - \frac{3}{2} C_6^{SC} \tag{3.2}$$

m_p being the proton mass, C_6^{SC} the quadratic Casimir operator of the $SU^{SC}(6)$ group, and n the number of the quarks in the lowest $j = 1/2$ mode.

Hereafter, we recall with some details the Jaffe's results for 6-quark systems, and present a generalization of these results for 9q and 12q systems.

1. 6q System (B = 2)

Starting from the Young diagram describing the color singlet wave function of 6 quarks one gets, via the Pauli principle, the type of the Young diagram for the spin-flavor wave function of the system, i. e.



$$= \begin{cases} 490 \text{ for } SU^{SF}(6) \\ 50 \text{ for } SU^{SI}(4) \end{cases} .$$

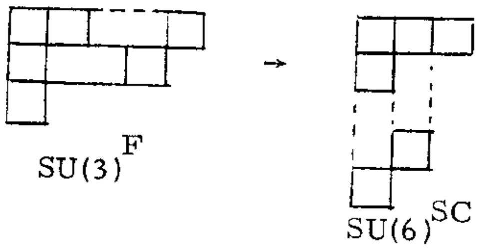
The spin-flavor decomposition $(SU^{SF}(6) \to SU^{S}(2) \times SU^{F}(3))$ of the 490 dimensional SU(6) representation is given by

$$490 = (7, \overline{10}) + (5, 27 + 8) + (3, 35 + 10 + \overline{10} + 8) + (1, 28 + 27 + 1) , \qquad (3.3)$$

while the spin-isospin decomposition of the corresponding representation 50 of $SU^{SI}(4)$, describing dibaryons without strangeness (or $Y = n/3$) is given by[13]:

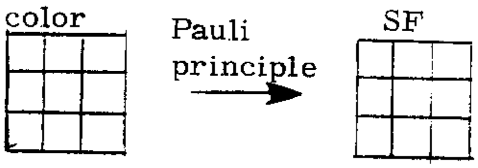
$$50 = (7, 1) + (5, 3) + (3, 5 + 1) + (1, 7 + 3) , \; Y = 2 \quad . \qquad (3.4)$$

Now, for each flavor (isospin) representation of $SU(3)^F$ $\left(SU(2)^I\right)$, there exists only one representation of the $SU(6)^{SC}$ group, which allows to construct the completely antisymmetric wave functions of the system. This representation of $SU(6)^{SC}$ can be determined as follows:



The results are presented in Table III. 1. Note that in Ref. 5 the state $S = 3$ is missing.

2. 9q system (B = 3)



$$= \begin{cases} 980 & SU(6)^{SF} \\ \overline{20} & SU(4)^{SI} \end{cases} .$$

The spin-flavor decomposition $\left(SU(6)^{SF} \rightarrow SU(2)^S \times SU(3)^F\right)$ is given by

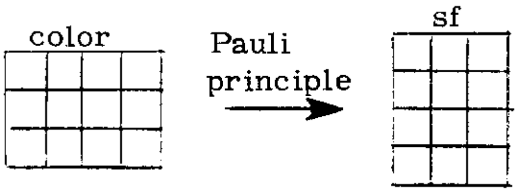
$$\begin{aligned} 980 &= (10,\ 1) + (8,\ 8) + (6,\ 27 + 8 + 1) \\ &+ (4,\ 64 + 27 + 10 + \overline{10} + 8 + 1) \\ &+ (2,\ 35 + \overline{35} + 27 + 8) \quad , \end{aligned} \tag{3.5}$$

while the spin-isospin decomposition $\left(SU(4)^{SI} \rightarrow SU(2)^S \times SU(2)^I\right)$ is :

$$\overline{20} = (4,\ 4) + (2,\ 2)\ , \quad Y = 3 \quad . \tag{3.6}$$

The result of analysis of spin-color content and the classic energies of the tribaryonic states are given in Table III.2.

3. 12q System (B = 4)



$$= \left\{ \begin{array}{ll} \overline{490} & SU(6)^{sf} \\ 1 & SU(4)^{SI} \end{array} \right. .$$

The spin-flavor decomposition $\left(SU(6)^{sf} \to SU(2)^{s} \times SU(3)^{f}\right)$ is given by

$$\overline{490} = (7,\ 10) + (5,\ 27 + 8) + (3,\ \overline{35} + 10 + \overline{10} + 8) + (1,\ \overline{28} + 27 + 1) \qquad (3.7)$$

while the spin-isospin decomposition $SU(4)^{SI} \to SU(2)^{S} \times SU(2)^{I}$ for the tetrabaryonic state without strangeness (Y = 4) is trivial: $1 = (1.1)$. The result of the analysis and the classical energies of tetrabaryonic states are given in Table III. 3.

Before concluding this section, let us add that, as is known, these multiquark systems are in general classically unstable against deformations of the surface of the bag and could undergo fission.[4, 15] The quantum picture of that fission process is not yet clear enough. However, due to the presence of the hidden color configurations (see section V) in multibaryonic systems, these fission processes should more likely be described in the framework of the multicharmed approach of Dashen et al.[16] which incorporates simultaneously confined and unconfined channels. We mention that this theory allows an appearance, in the Hamiltonian spectrum, of bound states above the threshold of continuum spectrum.

Table III. 1

S	SU_3^F	SU_6^{SC}	Δ	M_{theor}	(2S + 1, 2I + 1) for Y = 2
0	28	1	144	2805	(1, 7)
	27	189	24	2241	S-like bag
	1	490	- 72	1753	
1	35	35	80	2510	(3, 5)
	$\overline{10}$	175	8	2163	D-like bag
	10	280	8	2163	
	8	896	- 28	1982	
2	27	189	48	2357	(5, 3)
	8	896	- 12	2063	
3	$\overline{10}$	175	48	2357	(7, 3)

Masses of dibaryonic S-wave states

Table III. 2

S	SU_3^F	SU_6^{SC}	Δ	M_{theor}	(2S + 1, 2I + 1) with Y = 3
1/2	35	70	120	3521	
	$\overline{35}$	$\overline{70}$	120	3521	(H^3, He^3)-like bag
	27	540	72	3317	
	8	1960	12	3057	
3/2	64	20	168	3721	(4, 4)
	27	540	84	3369	
	10	560	60	3266	
	$\overline{10}$	$\overline{560}$	60	3266	
	8	1960	24	3109	
	1	980	- 12	2950	
5/2	27	540	104	3454	
	8	1960	44	3197	
	1	980	8	3039	
7/2	8	1960	72	3317	
9/2	1	980	72	3317	

Masses of tribaryonic S-wave states

Table III. 3

S	SU_3^F	SU_6^{SC}	Δ	M_{theor}	States with $Y = 4$
0	$\overline{28}$	1	288	4932	He^4-like bag
	27	189	168	4474	
	1	$\overline{490}$	72	4096	
1	$\overline{35}$	35	224	4689	
	10	$\overline{175}$	152	4412	
	$\overline{10}$	$\overline{280}$	152	4412	
	8	$\overline{896}$	116	4270	
2	27	189	192	4567	
	8	$\overline{896}$	132	4333	
3	10	$\overline{175}$	192	4567	

Masses of tetrabaryonic S-wave states

IV. CONTENT OF MULTIBARYONIC STATES AND HIDDEN COLOR

A natural question which arises is the calculation of the relative mixtures of baryon- baryon contained in such dibaryonic states, and more generally the different combinations of baryons, dibaryonic and tribaryonic states appearing in the tri and tetrabaryonic states.

First consider the dibaryonic states. One can see in our framework that, if dibaryonic states contain mixtures of the nuclei and their Δ isobar belonging to the $(20, 1)$ representation of $SU(4) \times SU(3)$, they contain also mixtures of "colored" baryons standing in the $(20', 8)$ of $SU(4) \times SU(3)$. In order to prove this property, let us recall that the $SU(12)$ representation $924 = (1^6)$ appearing in the product $(12)^3 \times (12)^3$ is actually built only from the product $220' \times 220$, i.e. $(1^3) \times (1^3)$ in terms of Young diagrams. Then from Eq. (2.5) it is easy to check that the $(50, 1)$ states in the 924 come only from the two products:

$$(20, 1) \times (20, 1) = (50 + \ldots, 1)$$

$$(20', 8) \times (20', 8) = (50 + \ldots, 1 + \ldots) \qquad . \qquad (4.1)$$

The tribaryonic states contain mixtures of three baryons and one baryon-dibaryon. They belong to the representation $(\overline{20}, 1)$ of $SU(4) \times SU(3)$ in the $\overline{220}$ of $SU(12)$. The $\overline{220}$ representation is built from the products $220 \times 220 \times 220$ and 924×220 of representations of $SU(12)$. Using the decompositions given in Eqs. (2.3) and (2.5), we can compute that there exist the following mixtures of dibaryonic-baryon in 924×220:

$$
\begin{aligned}
(50,\ 1)\times(20,\ 1) &= (\overline{20}+\ldots,\ 1)\\
(64,\ 8)\times(20',\ 8) &= (\overline{20}+\ldots,\ 1+\ldots)\\
(\overline{10},\ \overline{10})\times(\overline{4},\ 10) &= (\overline{20}+\ldots,\ 1+\ldots)\quad .
\end{aligned}
\tag{4.2}
$$

These noncolored--(50, 1)--and colored--(64, 8) and $(\overline{10}, \overline{10})$--dibaryonic states containing themselves combinations of baryon - baryon states in the product 220×220:

$$
\begin{aligned}
(20,\ 1)\times(20,\ 1) &= (50+\ldots,\ 1)\\
(20',\ 8)\times(20',\ 8) &= (50+\ldots,\ 1+\ldots)\\
(20',\ 8)\times(20,\ 1) &= (64+\ldots,\ 8)\\
(20',\ 8)\times(20',\ 8) &= (64+\ldots,\ 8+\ldots)\\
(\overline{4},\ 10)\times(20',\ 8) &= (64+\ldots,\ 8+\ldots)\\
(20',\ 8)\times(20',\ 8) &= (\overline{10}+\ldots,\ \overline{10}+\ldots)\\
(\overline{4},\ 10)\times(\overline{4},\ 10) &= (\overline{10}+\ldots,\ \overline{10}+\ldots)\quad .
\end{aligned}
\tag{4.3}
$$

Ana analogous study can be done for the tetrabaryonic state. These different results are gathered in Table IV. 1.

Finally we have calculated the pn and $\Delta\Delta$ content for the "D-like" and "S-like" quark bag, i.e. the states belonging to the (3, 1) and (1, 3) respectively of the $SU(2)^S \times SU(2)^I$ group, in the 50 representations of SU(4) and in the 924 representation of SU(12). For such a calculation we used Clebsch-Gordan coefficients of the SU(12) group when reduced with respect to $SU(4) \times SU(3)$. Such a SU(12) C.G. coefficient can be written as the triproduct of an SU(12) isoscalar, a SU(4)- and a SU(3)-C. G. coefficient:

$$C_{SU(12)} = \left(\begin{matrix} x \\ (x_1, x_2) \end{matrix} \begin{matrix} y \\ (y_1, y_2) \end{matrix} \middle| \begin{matrix} z \\ (z_1, z_2) \end{matrix} \right) C_{SU(4)} \cdot C_{SU(3)} \tag{4.4}$$

if x, y, z are SU(12) representations containing respectively the couples (x_1, x_2), (y_1, y_2) and (z_1, z_2) of SU(4) $\times$ SU(3) representations.

The SU(12) isoscalar we were interested in is:

$$\left(\begin{matrix} 220 \\ (20, 1) \end{matrix} \begin{matrix} 220 \\ (20, 1) \end{matrix} \middle| \begin{matrix} 924 \\ (50, 1) \end{matrix} \right)$$

and has been calculated by the method given in Appendix B. Because the SU(3) C. G. coefficient part is trivial--we have only singlets of color--the second step is the SU(4) C. G. coefficients calculations. The details of these calculations are given in Appendix A, while the results are given in Tables IV. 2 and IV. 3. Thus the overall composition of the "D like" quark bag is the same as that of the "S like" quark bag, and such that

$$\begin{aligned} \text{nucleon-nucleon} &\simeq 11\% \\ \text{isobal components} &\simeq 9\% \\ \text{baryon pairs with hidden color} &\simeq 80\% \end{aligned}$$

We note the large percentage of hidden color baryon pairs. Then the "D like" quark bag appears quite different of the real deutron which is predominantly a loosely bound proton-neutron system. However, we have suggested in

Ref. 6, using experimental data on the deutron form factor $F_d(q^2)$ at large q^2, a possibility of tunneling transition of real deutron into the 6 quarks spherically symmetric state with probability about 7×10^{-2}. Following this line, the real deutron would contain nonnegligeable mixture of baryon-baryon with "hidden color."

Table IV. 2

Color analysis of multibaryonic states

Dibaryonic states

$$(50,\ 1) \in 220_{SU(12)}$$
$$\uparrow$$

1-baryonic state	×	1-baryonic state
(20, 1)	×	(20, 1)
(20', 8)	×	(20', 8)

Tribaryonic states

$$(\overline{20},\ 1) \in \overline{220}_{SU(12)}$$
$$\uparrow$$

1-baryonic state	×	2-baryonic state	←	1-bar. state	×	1-bar. state
(20 , 1)	×	(50, 1)	←	(20 , 1)	×	(20 , 1)
				(20', 8)	×	(20', 8)

(cont'd)

$(20', 8) \times (64, 8)$	$\leftarrow$	$(20', 8) \times (20, 1)$
		$(20', 8) \times (20', 8)$
		$(\bar{4}, 10) \times (20', 8)$
$(\bar{4}, 10) \times (\overline{10}, \overline{10})$	$\leftarrow$	$(20', 8) \times (20', 8)$
		$(\bar{4}, 10) \times (\bar{4}, 10)$

Tetrabaryonic state

$$(1, 1) \in 1_{SU(12)}$$
$$\uparrow$$

1-bar. state × 3-bar. state		1-bar. state × 2-bar. state		1-bar. state × 1-bar. state
$(20, 1) \times (\overline{20}, 1)$	$\leftarrow$	$(\overline{20}, 1) \times (50, 1)$	$\leftarrow$	$(20, 1) \times (20, 1)$
				$(20', 8) \times (20', 8)$
		$(20', 8) \times (64, 8)$	$\leftarrow$	$(20', 8) \times (20, 1)$
				$(20', 8) \times (20', 8)$
				$(\bar{4}, 10) \times (20', 8)$
		$(\bar{4}, 10) \times (\overline{10}, \overline{10})$	$\leftarrow$	$(20', 8) \times (20', 8)$
				$(\bar{4}, 10) \times (\bar{4}, 10)$
$(20', 8) \times (20', 8)$	$\leftarrow$	$(20, 1) \times (64, 8)$		
		$(20', 8) \times (64, 8)$		
		$(\bar{4}, 10) \times (64, 8)$		
		$(20', 8) \times (6, 27)$	$\leftarrow$	$(20', 8) \times (20', 8)$
		$(\bar{4}, 10) \times (6, 27)$		$(\bar{4}, 10) \times (20', 8)$
		$(20', 8) \times (\overline{10}, \overline{10})$		$(\bar{4}, 10) \times (\bar{4}, 10)$

$$
\begin{array}{lllll}
 & & (\bar{4},\ 10) \times (\overline{10},\ \overline{10}) & & \left\{\begin{array}{l} (\bar{4},\ 10) \times (20,\ 1) \\ (20',\ 8) \times (20',\ 8) \\ (\bar{4},\ 10) \times (20',\ 8) \end{array}\right. \\
 & & (20',\ 8) \times (10,\ 10) & \leftarrow & \\
(\bar{4},\ 10) \times (\bar{4},\ 10) & \leftarrow & (20',\ 8) \times (64,\ 8) & & \\
 & & (20',\ 8) \times (6,\ 27) & & \\
 & & (\bar{4},\ 10) \times (6,\ 27) & & \\
 & & (20,\ 1) \times (\overline{10},\ \overline{10}) & & \\
 & & (20',\ 8) \times (\overline{10},\ \overline{10}) & & \\
 & & (\bar{4},\ 10) \times (10,\ 10) & &
\end{array}
$$

$$
\begin{array}{c}
(1,\ 1) \in 1_{SU(12)} \\
\uparrow
\end{array}
$$

$$
\begin{array}{lcl}
\underline{\text{di-bar. state}} \times \underline{\text{di-bar. state}} & \leftarrow & \underline{\text{1-bar. state}} \times \underline{\text{1-bar. state}} \\
(64,\ 8) \times (64,\ 8) & \leftarrow & \left\{\begin{array}{l} (20',\ 8) \times (20,\ 8) \\ (20',\ 8) \times (20',\ 8) \\ (\bar{4},\ 10) \times (20',\ 8) \end{array}\right. \\
(6,\ 27) \times (6,\ 27) & \leftarrow & \left\{\begin{array}{l} (20',\ 8) \times (20',\ 8) \\ (\bar{4},\ 10) \times (20',\ 8) \\ (\bar{4},\ 10) \times (\bar{4},\ 10) \end{array}\right. \\
(\overline{10},\ \overline{10}) \times (10,\ 10) & \leftarrow & \left\{\begin{array}{l} (\bar{4},\ 10) \times (20,\ 1) \\ (20',\ 8) \times (20',\ 8) \\ (\bar{4},\ 10) \times (20',\ 8) \end{array}\right.
\end{array}
$$

$$(10,\ 10) \times (\overline{10},\ \overline{10}) \leftarrow \begin{cases} (20',\ 8) \times (20',\ 8) \\ (\overline{4},\ 10) \times (\overline{4},\ 10) \end{cases}$$

$$(50,\ 1) \times (50,\ 1) \leftarrow \begin{cases} (20,\ 1) \times (20,\ 1) \\ (20',\ 8) \times (20',\ 8) \end{cases}$$

Table IV. 2

Couplings of "D-like" quark bag with $S_Z = 1$ to the $p \cdot n$ and $\Delta \cdot \Delta$ systems (SU(4) part)

spin / charge	1/2, 1/2	3/2, -1/2	-1/2, 3/2
$\Delta^{++}\Delta^{-}$	$-\frac{2}{3\sqrt{10}}$	$\frac{1}{\sqrt{30}}$	$\frac{1}{\sqrt{30}}$
$\Delta^{+}\Delta^{0}$	$\frac{2}{3\sqrt{10}}$	$-\frac{1}{\sqrt{30}}$	$-\frac{1}{\sqrt{30}}$
pn	$\frac{5}{3\sqrt{10}}$	---	---

Couplings of the "S-like" quark bag with $I_Z = 1$ to the $p \cdot n$ and $\Delta \cdot \Delta$ systems (SU(4) part)

spin / charge	1/2 -1/2	3/2 -3/2
$\Delta^{++}\Delta^{0}$	$\frac{1}{\sqrt{30}}$	$-\frac{1}{\sqrt{30}}$
$\Delta^{0}\Delta^{++}$	$\frac{1}{\sqrt{30}}$	$-\frac{1}{\sqrt{30}}$
$\Delta^{+}\Delta^{+}$	$-\frac{2}{3\sqrt{10}}$	$\frac{2}{3\sqrt{10}}$
pp	$\frac{5}{3\sqrt{10}}$	

V. ROLE OF THE GLUON EXCHANGE CORRECTIONS

As it is known, the gluon-quark interaction gives quite large excess of energy in the multibaryonic states described by a spherical bag, as compared with energies of the corresponding systems of free nucleons. Thus we can expect that the dynamics of multibaryonic systems is much more sensitive to the gluon exchange effects than in the case of usual baryons and mesons. Here we illustrate this point by calculating the gluon exchange corrections to the magnetic moments of multibaryonic ssytems. A formula giving the gluonic corrections $\delta\mu^{(2)}$ to the second order in the strong coupling constant α_s, and valid for any number of nonstrange quarks is obtained, generalizing the results of Ref. 17.

First, let us recall that the magnetic moment of a multiquark system in the MIT bag model is given by[2,10]:

$$2M\,\mu = 1.08\, g_M \cdot h = 0.2\, R \cdot g_M \tag{5.1}$$

with
$$M = 0.62\ \mathrm{GeV} \cdot h^{3/4}$$

and
$$h = n - 0.9 + 0.06\left[n(n-6) + S(S+1) + 3T(T+1)\right]$$

where R is the radius of the quark bag, n the number of (nonstrange) quarks, and $g_M = \langle \sum_i (Q\sigma_Z)_i \rangle$ the "nonrelativistic" group theoretical part of the magnetic moment.

In order to calculate the quantity g_M corresponding to a specific multiquark system, it is necessary to know the spin and isospin quark content of the considered state.

For such a purpose we will be concerned only with the $SU(4)^{SI}$ part of the SU(12) group introduced in Sec. II, as far as strange quarks are not involved. Let us consider the different dibaryonic states. Using Eq. (3.4) we have to look only at the states of the 50 dimensional representation of $SU(4)^{SI}$ which belongs to one of the $SU(2)^{S} \times SU(2)^{I}$ representations: (3, 1), (3, 5), (5, 3), and (7, 1), since the states of the (1, 3) and (1, 7) have no magnetic moments, being singlets of spin.

The quark content of these states has been calculated using the Young tableau techniques, and we have, for example, the following results:

$$
\begin{aligned}
&|\,50;\ (7,\ 1);\ S_Z = 3,\ I = 0\rangle &&= \begin{pmatrix} \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \\ \mathrm{pppnnn} \end{pmatrix} \\
&|\,50;\ (3,\ 5);\ S_Z = 1,\ I_Z = 2\rangle &&= \begin{pmatrix} \uparrow\uparrow\uparrow\downarrow\downarrow\uparrow \\ \mathrm{pppppn} \end{pmatrix} \\
&|\,50;\ (5,\ 3);\ S_Z = 2,\ I_Z = 1\rangle &&= \begin{pmatrix} \uparrow\uparrow\uparrow\downarrow\uparrow\uparrow \\ \mathrm{ppppnn} \end{pmatrix}
\end{aligned}
\tag{5.2}
$$

the notations on the r.h.o. of Eq. (5.2) being defined in Appendix A. It is also very easy to construct the states of the (3, 5) and (5, 3) multiplets with $I_Z = -2$ and $I_Z = -1$ respectively. Then using the relation:

$$
g_M = \frac{S}{3} + g_A \cdot \frac{I_Z}{2I} \tag{5.3}
$$

where $g_A = \langle \sum_i (\sigma_Z \tau_Z)_i \rangle$ with $\tau_{x,y,z}$ the generators of $SU(2)^I$, we deduce that the different g_M corresponding to the above states satisfy: $g_M = S/3$.

The same property is of course valid for the states of the multiplets (7, 1) and (3, 1). We know already that the tribaryonic states are classified in the $\overline{20}$ of SU(4), the decomposition of which with respect to SU(2) $\times$ SU(2) is given by Eq. (3.6). Using the conjugation property between the representations $\overline{20}$ and 20, this later containing the usual baryons (see Appendix A), it is almost straightforward to obtain the values of g_M corresponding to tribaryonic states. As an example, we will consider the physically interesting case of the tribaryonic states in the (2, 2) with $S_Z = \frac{1}{2}$ and $I_Z = +\frac{1}{2}$ and $I_Z = -\frac{1}{2}$, which own the quantum numbers of the He^3 and H^3 states respectively. These states are directly related to the neutron and proton states with $S_Z = -\frac{1}{2}$, respectively, and we can write[18]:

$$
\begin{aligned}
|\overline{20}; (2, 2); S_Z = \tfrac{1}{2}, I_Z = \tfrac{1}{2}\rangle &= -\frac{1}{\sqrt{3}} \begin{pmatrix} \uparrow\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow\downarrow\downarrow \\ \mathrm{pppppnnnn} \end{pmatrix} \begin{array}{|c|c|c|}\hline \; & \; & \; \\ \hline \; & \; & \; \\ \hline \; & \; & \; \\ \hline \end{array} \\
&\quad + \sqrt{\frac{2}{3}} \begin{pmatrix} \uparrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow \\ \mathrm{pppppnnnn} \end{pmatrix} \begin{array}{|c|c|c|}\hline \; & \; & \; \\ \hline \; & \; & \; \\ \hline \; & \; & \; \\ \hline \end{array} \\
|\overline{20}; (2, 2); S_Z = \tfrac{1}{2}; I_Z = -\tfrac{1}{2}\rangle &= \frac{1}{\sqrt{3}} \begin{pmatrix} \uparrow\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow \\ \mathrm{ppppnnnnn} \end{pmatrix} \begin{array}{|c|c|c|}\hline \; & \; & \; \\ \hline \; & \; & \; \\ \hline \; & \; & \; \\ \hline \end{array} \\
&\quad - \sqrt{\frac{2}{3}} \begin{pmatrix} \uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow\downarrow\downarrow \\ \mathrm{ppppnnnnn} \end{pmatrix} \begin{array}{|c|c|c|}\hline \; & \; & \; \\ \hline \; & \; & \; \\ \hline \; & \; & \; \\ \hline \end{array}
\end{aligned}
\qquad (5.3)
$$

It follows that:

$$g_M(\mathrm{He}^3\text{-like}) = -2/3; \quad g_M(\mathrm{H}^3\text{-like}) = 1$$

which are exactly the values of g_M corresponding to the neutron and proton respectively.

Now, let us determine the contribution of the second order gluon exchange diagrams, given in Fig. V.1, to the magnetic moments of the multibaryonic states.

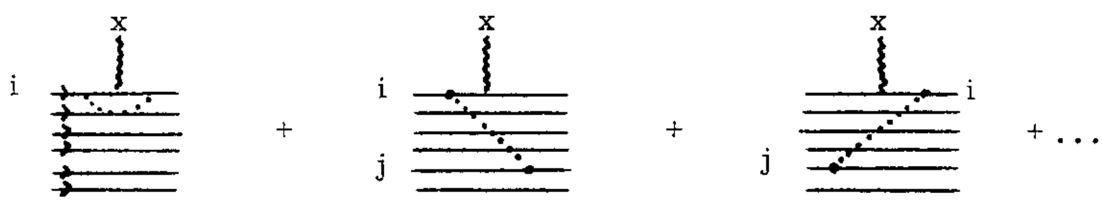


Fig. V.1 Gluon Exchange Diagrams

Actually one can generalize the results of calculation of gluon exchange corrections to the nucleon magnetic moment[17] to obtain the equation:

$$\delta\mu^{(2)}_{\text{gluon exch.}} = cR\sum_a\left[\sum_i(\vec{\Gamma}^a\sigma_Z Q\vec{\Gamma}^a)_i + \sum_{i\neq j}\left\{\vec{\Gamma}^a, \sigma_Z Q\right\}_i(\vec{\Gamma}^a)_j\right] \tag{5.4}$$

where: $\vec{\Gamma}^a = \lambda^a\cdot\vec{\sigma}$, λ^a being the SU(3) color matrices and: $4/3\ c = 0.005$.

Using the relation:

$$-\sum_a(\lambda^a\vec{\sigma})_i(\lambda^a\vec{\sigma})_j = \frac{8}{3} + \frac{2}{3}\vec{\sigma}_i\vec{\sigma}_j + 2\vec{\tau}_i\vec{\tau}_j + \sum_a\lambda^a_i\lambda^a_j \tag{5.5}$$

which follows from the requirement of complete antisymmetry of the multiquark wave function, one can bring eq. (5.4) into the form:

$$\delta\mu^{(2)}_{\text{gluon exch.}} = -0.005\, R\, \frac{S_Z}{S}\left[g_A(I+1) + S(8Q-n-2) + 4g_M(n-3)\right] . \qquad (5.6)$$

Contrary to the case of the nucleon where the gluon exchange corrections which has been found constitute only small corrections to the magnetic moment,[17] the contribution of gluon exchange diagrams to the multibaryonic magnetic moments is generally very large. Considering as an example the six quark bag, one gets from Eq. (5.6)

$$\left(\frac{\delta\mu^{(2)}}{\mu}\right)_{6q} = -0.3\,(2Q-1) \qquad (5.7)$$

which corresponds to gluon exchange contributions up to 150% (!) for $Q = Q_{max} = 3$. Thus the perturbation theory cannot be applied in finding magnetic moments of the multibaryonic systems.

Hereafter we give, as an information, a tableau of the calculated values of μ and $\delta\mu^{(2)}$.

<u>6 quark system:</u>

(S, I)	Q	g_M	$(2M_M)_{bag}$	$(2M\mu)^{19}_{nuclei}$	$\delta\mu^{(2)}/\mu$
(3, 1)	1	1/3	1.88	1.71	- 30%
(7, 1)	3	1	6.29		-150%
(3, 5)	1	1/3	2.27		- 30%
(5, 3)	2	2/3	4.19		- 90%

<u>9 quark system:</u>

(S, I)	Q	g_M	$(2M_M)_{bag}$	$(2M\mu)^{19}_{nuclei}$	$\delta\mu^{(2)}/\mu$
$(\frac{1}{2}, \frac{1}{2})$ $I_2 = \frac{1}{2}$	2	-2/3	-77.13	-6.4	- 60%
$(\frac{1}{2}, \frac{1}{2})$ $I_2 = -\frac{1}{2}$	1	1	10077	8.96	- 50%

Table V.1 Calculation of multibaryonic magnetic moments in the spherical bag approximation

CONCLUSION

This study can be considered as a first step in the multibaryonic states problem, in the sense that we have restricted ourselves to quarks in the lowest mode $j = \frac{1}{2}$. However we feel this approach necessary and already instructive. First, from a technical point of view, one can expect from these results a straightforward group theoretical development to the study of systems containing quarks in excited modes. Moreover, the calculation of the magnetic moments gluonic corrections on one hand, and of the multibaryonic masses, on the other hand, confirm the important role of the gluon-quark interaction in multibaryonic systems. Finally the existence of mixtures of "colored baryons" we pointed out in these 3n-quark systems could be an interesting field of investigation in the study of nuclear matter.

ACKNOWLEDGMENT

This work was just finished the day of the tragic accident which happened to B. W. Lee. The authors would like to humbly dedicate this paper to his memory.

APPENDIX A: THE REPRESENTATIONS 20 AND 50 OF SU(4)

The calculations of the relative couplings of baryons in dibaryonic states is performed with the help of the SU(4) Clebsch-Gordan coefficients. Tables of SU(4) Clebsch-Gordan coefficients are given in the literature[20] considering the reduction of SU(4) with respect to SU(3) × U(1). In order to use these tables we have to express the multibaryonic states known in the spin-isospin basis--$SU(2)^S \times SU(2)^I$--in terms of the SU(3) × U(1) basis states. Note that we could also use the method described in Appendix B, for the computation of isoscalar factors to calculate the Clebsch-Gordan coefficients of SU(4) reduced with respect to SU(2) × SU(2). Indeed such a C. G. coefficient will be the triproduct of an isoscalar factor and two SU(2) C. G. coefficients corresponding to the spin and isospin respectively. Anyway, it is obvious that tables of C. G. coefficients of SU(4) reduced with respect to SU(2) × SU(2) would be very useful.[21]

The reduction with respect to SU(3) of the representations 20 and 50 of SU(4) are:

$$\begin{aligned} 20 &= 10 + 6 + 3 + 1 \\ 50 &= \overline{10} + \overline{15} + 15 + 10 \end{aligned} \qquad (A.1)$$

These representations are represented in the 3-dimensional space (SU(4) being of rank 3) in Tables AI and AII. The four states of the fundamental representation of SU(4) are chosen to be: $p^{\uparrow}$, $p^{\downarrow}$, $n^{\uparrow}$, $n^{\downarrow}$, and by the notation $\overset{\uparrow\downarrow\uparrow}{(p\,p\,n)}_{\begin{array}{|c|c|c|}\hline \; & \; & \; \\ \hline\end{array}}$ for example in Table AI, we mean the state:

$$\overset{\uparrow\downarrow\uparrow}{(p\,p\,n)}_{\begin{array}{|c|c|c|}\hline \; & \; & \; \\ \hline\end{array}} = \mathscr{Y}\begin{bmatrix}\uparrow\downarrow\uparrow \\ p\,p\,n\end{bmatrix} \tag{A.2}$$

$\mathscr{Y}$ being the Young symmetrizer corresponding to the Young tableau: $\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline\end{array}$ ·

Let us add that in Table A. II the Young tableau will be: $\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline\end{array}$.

Considering again Table AII, we note that there are two independent states on each top of the inside triangle appearing in the SU(3) representations 15 and $\overline{15}$. In other words, in the reduction of the 15 (resp. $\overline{15}$) representation of SU(3) with respect to $SU(2)_J \times U(1)_Y$, we find for the value $Y = 1/3$ (resp. $-1/3$) a $J = 3/2$ and a $J = 1/2$ SU(2) representation, while for $Y = -2/3$ (resp. $2/3$) there exist a $J = 1$ and a $J = 0$ SU(2) representation. There the states of the 50 representation of SU(4) corresponding to $I = 0$, $S = 1$ and $I = 0$, $S = 0$, to which are respectively associated the deutron-like six quark system and its so-called deutron S-companion can be written:

$$\begin{aligned}
|d_{S_Z=1}\text{-like}\rangle &= \sqrt{\frac{2}{15}}\,\left|15; \frac{3}{2}, \frac{3}{2}\right\rangle - \frac{\sqrt{5}}{3}\left|\overline{15}; \frac{1}{2}, \frac{1}{2}; Y = -\frac{1}{3}\right\rangle \\
&\quad - \frac{2\sqrt{2}}{3\sqrt{5}}\left|\overline{15}; \frac{3}{2}, \frac{1}{2}\right\rangle + \sqrt{\frac{2}{15}}\left|\overline{10}; \frac{3}{2}, -\frac{1}{2}\right\rangle \\
|S_{I_Z=1}\text{-like}\rangle &= \sqrt{\frac{2}{15}}\,|15; 1, 1\rangle - \sqrt{\frac{3}{5}}\,|\overline{15}; 0\rangle \\
&\quad - \sqrt{\frac{2}{15}}\,|\overline{15}; 1, 0\rangle + \sqrt{\frac{2}{15}}\,|\overline{10}; 1, -1\rangle \quad .
\end{aligned} \tag{A.3}$$

while the nuclei and their isobar states appear as:

$$|p_{1/2}\rangle = \sqrt{\frac{2}{3}}\,|6;1,1\rangle - \frac{1}{\sqrt{3}}\,|10;1,0\rangle$$

$$|n_{1/2}\rangle = \sqrt{\frac{2}{3}}\,|10;\tfrac{1}{2},-\tfrac{1}{2}\rangle + \frac{1}{\sqrt{3}}\,|6;\tfrac{1}{2},\tfrac{1}{2}\rangle$$

$Q = 2$ $$|\Delta^{++}{}_{S_Z}\rangle = |10;\tfrac{3}{2},S_Z\rangle \quad S_Z = \tfrac{3}{2},\ \pm\tfrac{1}{2}$$

$Q = 1$ $$|\Delta^{+}{}_{3/2}\rangle = |10;1,1\rangle$$

$$|\Delta^{+}{}_{1/2}\rangle = \sqrt{\frac{2}{3}}\,|10;1,0\rangle + \frac{1}{\sqrt{3}}\,|6;1,1\rangle$$

$$|\Delta^{+}{}_{-1/2}\rangle = \sqrt{\frac{2}{3}}\,|6;1,0\rangle + \frac{1}{\sqrt{3}}\,|10;1,-1\rangle \qquad \text{(A.4)}$$

$Q = -1$ $$|\Delta^{-}{}_{3/2}\rangle = |10;0,0\rangle$$

$$|\Delta^{-}{}_{1/2}\rangle = |6;0,0\rangle$$

$$|\Delta^{-}{}_{-1/2}\rangle = |3;0,0\rangle$$

$Q = 0$ $$|\Delta^{0}{}_{3/2}\rangle = |10;\tfrac{1}{2},\tfrac{1}{2}\rangle$$

$$|\Delta^{0}{}_{1/2}\rangle = \sqrt{\frac{2}{3}}\,|6;\tfrac{1}{2},\tfrac{1}{2}\rangle + \frac{1}{\sqrt{3}}\,|10;\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$|\Delta^{0}{}_{-1/2}\rangle = \sqrt{\frac{2}{3}}\,|6;\tfrac{1}{2};-\tfrac{1}{2}\rangle + \frac{1}{\sqrt{3}}\,|3;\tfrac{1}{2},\tfrac{1}{2}\rangle \ .$$

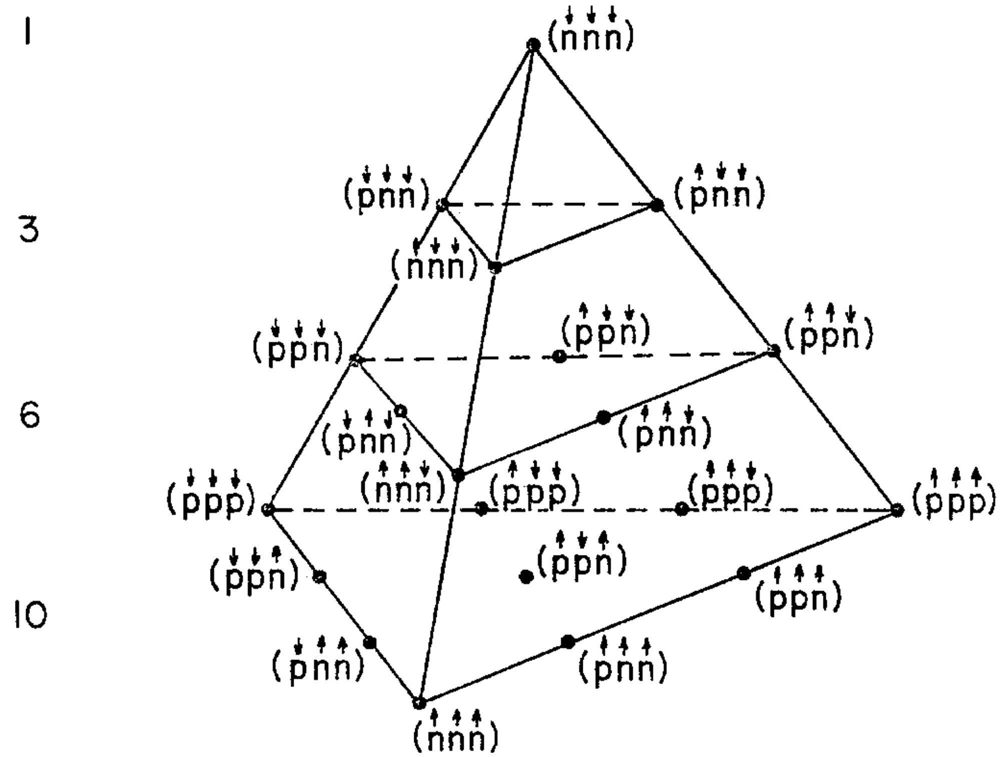


Table AI. The representation $20 = (1^3)$ of SU(4).

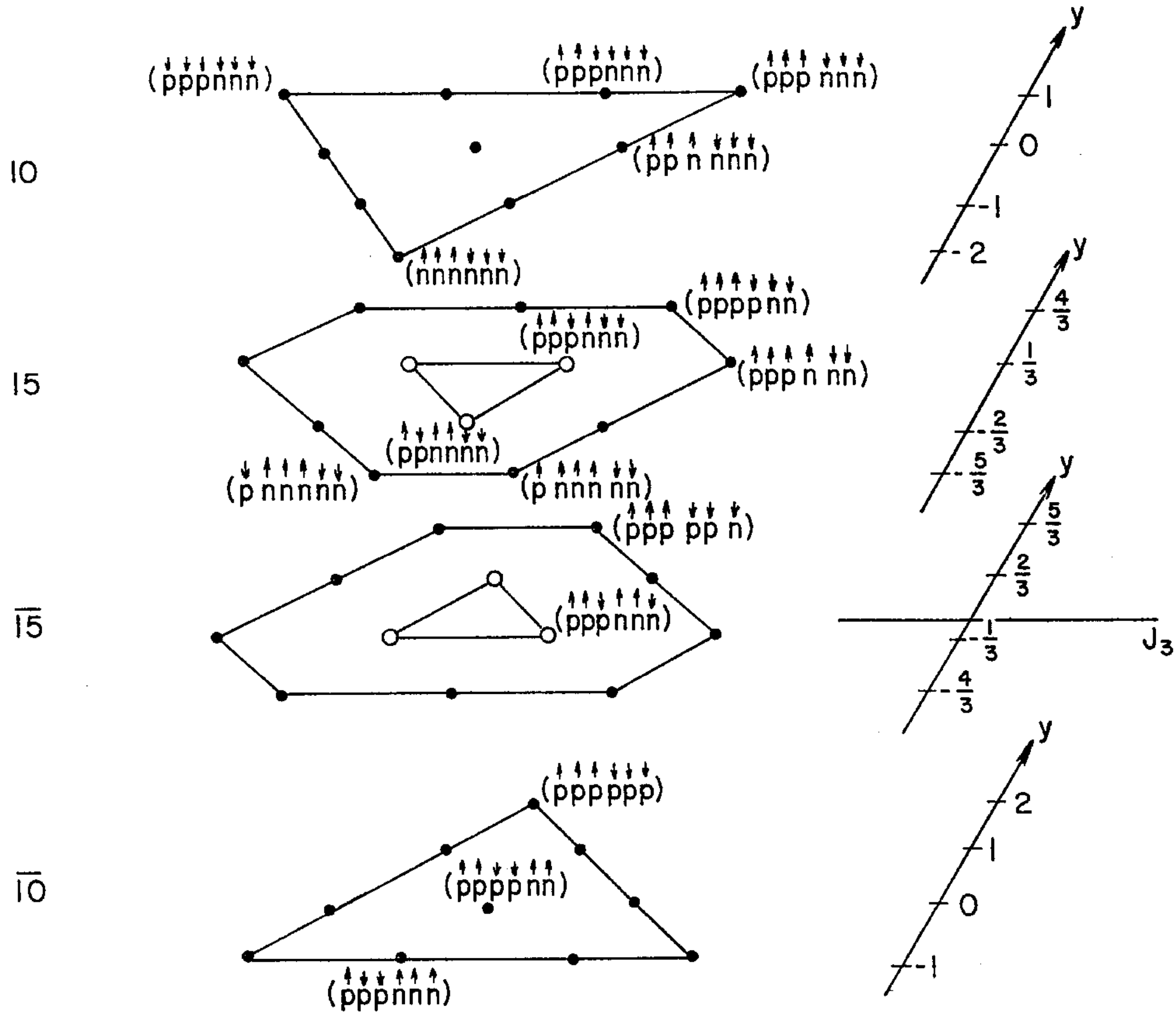


Table AII. The representation $50 = (3^2)$ of SU(4).

APPENDIX B: ON THE CLEBSCH-GORDAN ISOSCALAR FACTORS OF THE SU(12) GROUP

Hereafter we will be concerned only with baryonic and dibaryonic states belonging to the 220 and 924 dimensional representations of SU(12) respectively. First, let us determine such states by the formulae:

$$
\begin{aligned}
|b\rangle &= \sum_{i_1, i_2, i_3} \begin{array}{|c|}\hline i_1 \\ \hline i_2 \\ \hline i_3 \\ \hline \end{array} \; \frac{a^+_{i_1} a^+_{i_2} a^+_{i_3}}{\sqrt{3!}} |0\rangle \\
|d\rangle &= \sum_{i_1, \dots, i_6} \begin{array}{|c|}\hline i_1 \\ \hline i_2 \\ \hline i_3 \\ \hline i_4 \\ \hline i_5 \\ \hline i_6 \\ \hline \end{array} \; \frac{a^+_{i_1} a^+_{i_2} \dots a^+_{i_6}}{\sqrt{6!}} |0\rangle
\end{aligned}
\qquad (B.1)
$$

For the sake of simplicity we shall suppress below the sign of summation in the quark indices for which we shall preserve only their order number. So the matrix elements we are interested in take the form:

$$
\langle d | b_1, b_2 \rangle = \sqrt{20} \; \overline{\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 3 \\ \hline 4 \\ \hline 5 \\ \hline 6 \\ \hline \end{array}} \; \cdot \; \begin{array}{c} \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 3 \\ \hline \end{array} \\ \\ \begin{array}{|c|}\hline 4 \\ \hline 5 \\ \hline 6 \\ \hline \end{array} \end{array}
\qquad (B.2)
$$

where the factor $6!/3!\,3! = 20$ appears as a statistical weight of a system of two baryon states, each made of three quarks.

The completeness or normalization equation takes the form:

$$\frac{1}{20} \sum_{b_1 b_2} \left| \langle d \,|\, b_1 b_2 \rangle \right|^2 = 1 \tag{B.3}$$

for each state d.

The structure of the complete antisymmetric wave functions of baryonic states in terms of the color and spin-isospin degrees of freedom can be presented in the following form:

$$(20\,,\,1) = \begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array} = \begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array}_{\,C} \times \begin{array}{|c|c|c|}\hline 1&2&3\\\hline\end{array}_{\,SI}$$

$$(20',\,8) = \begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array} = \frac{\sqrt{2}}{3}\left(1 - \sum_{\substack{i=1,2\\ j=4,5,6}} P_{i3}\right) \begin{array}{|c|c|}\hline 2&3\\\hline \multicolumn{1}{|c|}{1}\\\cline{1-1}\end{array}_{\,C} \times \begin{array}{|c|c|}\hline 2&1\\\hline \multicolumn{1}{|c|}{3}\\\cline{1-1}\end{array}_{\,SI} \tag{B.4}$$

$$(50\,,\,1) = \begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline 4\\\hline 5\\\hline 6\\\hline\end{array} = \frac{1}{\sqrt{20}}\left(1 - \sum_{\substack{i=1,2,3\\ j=4,5,6}} P_{ij}\right) \begin{array}{|c|c|}\hline 1&4\\\hline 2&5\\\hline 3&6\\\hline\end{array}_{\,C} \times \begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}_{\,SI}$$

Using the symmetry properties of the corresponding color and spin-isospin wave functions, e.g.:

$$\left(1+\sum_{i=1,2,3} P_{ij}^{\ SI}\right) \left[\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}\right]_{SI} = 0 \quad \text{(for each j)}$$

$$\left(1+\sum_{i=1,2,3} P_{ij}^{\ C}\right) \left[\begin{array}{|c|c|}\hline 1&4\\\hline 2&5\\\hline 3&6\\\hline\end{array}\right]_{C} = 0 \quad \text{(for each j)} \tag{B.5}$$

$$\left(1+\sum_{i=1,2} P_{i3}^{\ SI}\right) \left[\begin{array}{|c|c|}\hline 2&1\\\hline 3\\\cline{1-1}\end{array}\right]_{SI} = \left(1-\sum_{i=1,2} P_{i3}^{\ C}\right) \left[\begin{array}{|c|c|}\hline 2&3\\\hline 1\\\cline{1-1}\end{array}\right]_{C} = 0.$$

One can bring the matrix elements of the problem to the form:

$$<(50,\ 1)\,|\,(20\ ,\ 1)^2> \; = \; 2\left(\overline{\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}} \cdot \begin{array}{c}\begin{array}{|c|c|c|}\hline 1&2&3\\\hline\end{array}\\ \\ \begin{array}{|c|c|c|}\hline 4&5&6\\\hline\end{array}\end{array}\right)_{SI} \tag{B.6}$$

$$<(50,\ 1)\,|\,(20',\ 8)^2> \; = \; -\frac{3}{2}\left(\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array} \quad \begin{array}{|c|c|}\hline 2&1\\\hline 3\\\cline{1-1}\end{array} \quad \begin{array}{|c|c|}\hline 5&4\\\hline 6\\\cline{1-1}\end{array}\right)_{SI} \times \mathrm{Tr}(B_1 B_2)$$

where the B^a_b are octet 3×3 matrices representing the color wave functions of the baryons which belong to the (20', 8) family.

These formulas allow to find the total weights of the "normal" $(20,\ 1)\times(20,\ 1)$--and the "hidden color" $(20',\ 8)\times(20',\ 8)$--components of the dibaryonic states. These weights can be related to the isoscalar factors which appear in finding the Clebsch-Gordan coefficients of the SU(12) group via the decomposition with respect to $SU(4)^{SI}\times(SU(3))^{color}$, namely:

$$\alpha = \left(\begin{array}{cc|c} 220 & 220 & 924 \\ \\ (20,\ 1) & (20,\ 1) & (50,\ 1)\end{array}\right)^2 = \frac{1}{20}\sum_{b_1 b_2}\left|<(50,\ 1)\,|\,(20,\ 1)^2>\right|^2$$

$$\beta = \left(\begin{array}{cc|c} 220 & 220 & 924 \\ (20', 8) & (20', 8) & (50, 1) \end{array}\right)^2 = \frac{1}{20} \sum_{b_1 b_2} \left| \langle 40, 1) \middle| (20', 8)^2 \rangle \right|^2 \quad . \qquad \text{(B. 7)}$$

Starting from Eqs. (B. 6) and using the completeness equations for the $SU(4)^{SI}$ wave functions:

$$\sum_{b \in 20} \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline \end{array} \times \overline{\begin{array}{|c|c|c|}\hline 1' & 2' & 3' \\ \hline \end{array}} = \hat{S}_3 \text{ (symmetrizer operator)}$$

$$\sum_{b \in 20'} \begin{array}{|c|c|}\hline 2 & 1 \\ \hline 3 \\ \cline{1-1} \end{array} \times \overline{\begin{array}{|c|c|}\hline 2' & 1' \\ \hline 3' \\ \cline{1-1} \end{array}} = \frac{1}{12} (1 + P_{12})(2 - P_{13} - P_{23})(1 + P_{12}) \qquad \text{(B. 8)}$$

and for the octet $SU(3)^{\text{color}}$ wave function

$$\sum_{b_1, b_2} \mathrm{Tr}(B_1 B_2) \mathrm{Tr}(\overline{B}_1 \overline{B}_2) = 8 \qquad \text{(B. 9)}$$

one can get the result:

$$\alpha = \frac{1}{5} \; ; \; \beta = \frac{4}{5} \quad . \qquad \text{(B. 10)}$$

This says that the 6-quark dibaryonic state consists of 20% of "normal" and as much as 80% of the "hidden color" components.

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