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The Two-Component Pomeron and Hadron Total Cross Sections and Real Parts

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ABSTRACT

The predictions from a five parameter formula obtained from a two-component Pomeron model and fit to hadron-nucleon total cross sections from 2 to 200 GeV/c are in remarkable agreement with new ISR data on total proton-proton cross sections and real parts up to an equivalent P_{lab} of 2000 GeV/c and with total cross section data from cosmic rays up to 40,000 GeV/c. This oversimplified formula with a Regge term varying as $s^{-\frac{1}{2}}$, two Pomeron-like terms with slightly increasing and slightly decreasing power behavior and dependence upon quantum numbers given by simple quark-counting rules is adequate to fit all available data and can be useful for analysis of future data. Predictions for the ratios of real to imaginary parts of $\pi^{\pm}p$, $K^{\pm}p$, and $p^{\pm}p$ forward amplitudes are given.

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The total proton-proton cross section and the real part of the forward scattering amplitude has been recently measured[1] at ISR. Table I and Fig. 1 show that the new data in the energy range equivalent to P_{lab}=500 to 2000 GeV/c are in excellent agreement with predictions from a five parameter formula based on a two-component Pomeron model,[2] with no adjustment of the values of these parameters from already published values[2] fixed by fits to data below 200 GeV/c. Table I also lists predictions for higher energies and shows remarkable agreement with results from Cosmic Ray experiments[3] up to P_{lab} = 40,000 GeV/c. Whether these agreements confirm the validity of the oversimplified two-component model is unclear. However, the formula can certainly be used as a simple parametrization of the data and a guide to the physics of further experiments. The ISR group fit their data with a seven parameter formula.[1] Since data for the ratio ρ of the real to imaginary parts of the forward amplitudes for all hadron-proton scattering processes should soon be available, predictions are given in Table II. These are uniquely determined by the values of the five parameters already fixed.

The two-component Pomeron model describes hadron-nucleon total cross sections as the sum of a Regge term and two Pomeron-like components, one increasing slowly with energy and one decreasing slowly. The decreasing Pomeron component was introduced to describe the difference between pion-nucleon and kaon-nucleon cross sections, which shows this otherwise unexplained slowly decreasing behavior, and also the otherwise unrelated observation that exactly the same decreasing behavior is shown by the deviation of baryon-baryon cross sections from quark model predictions based on meson-baryon cross sections. The total cross section for a hadron H on a proton target in this model is given by

$$\sigma_{tot}(Hp) = C_1\sigma_1(Hp) + C_2\sigma_2(Hp) + C_R\sigma_R(Hp) \tag{1}$$

where $C_1 = 6.5$ mb., $C_2 = 2.2$ mb., $C_R = 1.75$ mb.,

$$\sigma_1(Hp) = N_q^H(P_{lab}/20)^{\epsilon} \tag{2a}$$

$$\sigma_2(Hp) = N_q^H N_{ns}^H(P_{lab}/20)^{-\delta} \tag{2b}$$

$$\sigma_R(Hp) = (N_{\bar{n}}^H + 2N_{\bar{p}}^H)(P_{lab}/20)^{-\frac{1}{2}} \quad , \tag{2c}$$

N_q^H is the total number of quarks and antiquarks in hadron H ($N_q^H = 2$ for mesons and 3 for baryons), N_{ns}^H is the total number of non-strange quarks and antiquarks in hadron H and $N_{\bar{n}}^H$ and $N_{\bar{p}}^H$ are the total number of $\bar{n}$ and $\bar{p}$ antiquarks in hadron H, $\epsilon = 0.13$ and $\delta = 0.2$.

The dependence of the individual terms in Eqs. (2a) and (2b) on the quantum numbers of H are determined by the model and discussed in ref. 2. The explicit form for the energy dependence is chosen to minimize the number of free parameters. Thus power behavior is chosen rather than logarithmic for the two components of the Pomeron, because two parameters are sufficient to describe a power and at least three are needed to describe logarithmic behavior. The Regge term was chosen to minimize the number of free parameters by assuming exact duality and exchange degeneracy for the leading trajectories with the conventional intercept of one-half.

The extension of the formula (1) to the real part of the amplitude is a straightforward application of analyticity and crossing, which is particularly simple for terms with power behavior.[4]

The first two components have even signature and the ratios ρ of their real parts to their imaginary parts are simply given by the expressions

$$\rho_1 = \tan(\pi\epsilon/2) \tag{3a}$$

$$\rho_2 = -\tan(\pi\delta/2) \quad . \tag{3b}$$

The Regge term must be separated into its even and odd signature parts

$$\sigma_R(Hp) = \sigma_{Re}(Hp) + \sigma_{Ro}(Hp) \tag{4a}$$

where

$$\sigma_{Re}(Hp) = \left[\sigma_R(Hp) + \sigma_R(\overline{H}p)\right]/2 \tag{4b}$$

$$\sigma_{Ro}(Hp) = \left[\sigma_R(Hp) - \sigma_R(\overline{H}p)\right]/2 \quad . \tag{4c}$$

The corresponding ratios of the real to the imaginary parts of these components is given by

A convenient graphical test of the formula (1) for $\sigma_{tot}(pp)$ is shown in Fig. 1. Since $\sigma_R(pp) = 0$ by eq. (2c), a plot of $\sigma_{tot}(pp) \times (P_{lab}/20)^{\delta}$ vs. $(P_{lab})^{\epsilon+\delta}$ is predicted to give a straight line. Fig. 1 shows that the ISR and Cosmic Ray data fit very well on a straight line with a slope determined by the fit to the lower energy data. Similar plots of $\sigma_{tot}(K^+p)$ and linear combinations of cross sections for which there is no Regge contribution also show straight lines for the momentum range below 200 GeV/c where data are available. Similar plots with slightly different values of the parameters show straight lines over a range of values of δ, but that slight changes in ϵ destroy the straight line. It is difficult to determine the "best value" of δ because there is no clear criterion for what is a "best fit" without a model which defines the energy range and quantum numbers for which the model is expected to be valid. It is interesting that a good fit is obtained for $\delta = -0.185 = \frac{1}{2}(\epsilon - \frac{1}{2})$. This value makes the energy behavior of σ_2 like that of $\sqrt{\sigma_1 \sigma_R}$ and might suggest that σ_2 is due to an interference term between amplitudes responsible for σ_1 and σ_R.

$$\rho_{Re}(Hp) = -1 \tag{5a}$$

$$\rho_{R\delta}(Hp) = +1 \quad . \tag{5b}$$

Combining these equations gives the following expression for the real to the imaginary part of the $H\bar{p}$ amplitude

$$\rho(Hp) = \frac{C_1\sigma_1(Hp)\tan(\pi\epsilon/2) - C_2\sigma_2(Hp)\tan(\pi\delta/2) - C_R\sigma_R(\bar{H}p)}{\sigma_{tot}(Hp)} \quad . \tag{6}$$

This expression shows the expected qualitative behavior for the real part, a positive contribution from the increasing component and negative contributions from the two decreasing components. Thus ρ is negative at low energies and goes through zero and becomes positive at high energies, in agreement with experiment.

The good fits obtained to very high energy data indicate that these rather crude approximations are nevertheless adequate up to these energies. As long as this reasonable fit continues models containing more detailed assumptions will not be easily tested by the available data. For example, as long as a good fit is obtained with power behavior for the first component the necessity for logarithmic terms will be difficult to demonstrate since a considerably better fit is required to justify the use of additional parameters. The same is true for more detailed or realistic descriptions of the Regge component, since breaking exchange degeneracy or choosing a value different from one-half for the intercept necessarily requires more parameters.

However, as soon as data appear which fail to fit this formula, the underlying assumptions are so simple that the physics of the disagreement should be readily apparent. The nature of the disagreement might suggest, for example, that the rise of the cross sections is logarithmic rather than a power, that exchange degeneracy is breaking down, or that the Regge intercept is not one-half. There may also be a breakdown of the two-component pomeron picture if the dependence on the quantum numbers of hadron H no longer satisfies the simple relations of the model. Thus, regardless of the validity of the two component pomeron description, the formula (1) should be a valuable guide to the analysis of data on high energy total cross sections and real parts of scattering amplitudes.

Stimulating discussions with G. Cocconi, C. Quigg and G. Yodh are gratefully acknowledged.

TABLE I. Theoretical Predictions
and experimental data for $\sigma_{tot}(pp)$ and $\rho(pp)$

P_{lab} (GeV/c)	$\sqrt{s}$ (GeV)	$\sigma_{tot}(p\bar{p})$ Theory (mb)	$\sigma_{tot}(pp)$ Theory (mb)	$\sigma_{tot}(pp)$ Experiment (mb)	$\rho(pp)$ Theory	$\rho(pp)$ Experiment
498	30.6	41.8	40.0	40.1 ± 0.4	.025	.042 ± .011
1064	44.7	42.8	41.6	41.7 ± 0.4	.064	.062 ± .011
1491	52.9	43.5	42.5	42.4 ± 0.4	.079	.078 ± .010
2075	62.4	44.3	43.5	43.1 ± 0.4	.092	.095 ± .011
4600	92.9	46.8	46.2	47.0 ± 0.8	.118	
10000	137.	49.8	49.5	50.6 ± 1.2	.138	
25000	217.	54.3	54.0	53.8 ± 2.2	.156	
40000	274.	56.9	56.7	55.0 ± 3.0	.163	
100000	433.	62.7	62.6		.174	

TABLE II. Theoretical predictions for $\rho(Hp)$

P_{lab} (GeV/c)	$\rho(pp)$	$\rho(p\bar{p})$	$\rho(K^+p)$	$\rho(K^-p)$	$\rho(\pi^-p)$	$\rho(\pi^+p)$
2	-.76	-.098	-.68	-.0092	-.230	-.47
10	-.40	-.07	-.24	.037	-.120	-.24
15	-.33	-.059	-.17	.051	-.095	-.19
20	-.28	-.05	-.13	.060	-.076	-.16
25	-.25	-.043	-.098	.068	-.061	-.13
30	-.23	-.036	-.075	.074	-.050	-.11
35	-.21	-.031	-.057	.079	-.040	-.10
40	-.19	-.026	-.043	.083	-.032	-.087
45	-.18	-.022	-.031	.087	-.025	-.077
50	-.16	-.018	-.020	.09	-.019	-.068
70	-.13	-.006	.010	.10	.000	-.040
100	-.094	.007	.037	.11	.019	-.014
120	-.077	.014	.05	.11	.029	-.001
150	-.058	.023	.064	.12	.039	.013
170	-.048	.027	.071	.12	.045	.021
200	-.036	.033	.08	.13	.053	.030
240	-.022	.04	.089	.13	.061	.041
280	-.011	.046	.096	.14	.067	.049
500	.025	.066	.12	.15	.089	.076
1000	.061	.088	.14	.16	.11	.10
1400	.076	.098	.15	.16	.12	.11
2000	.090	.11	.16	.17	.13	.12

REFERENCES

[1]U. Amaldi et al., Physics Letters 66B, 390 (1977).

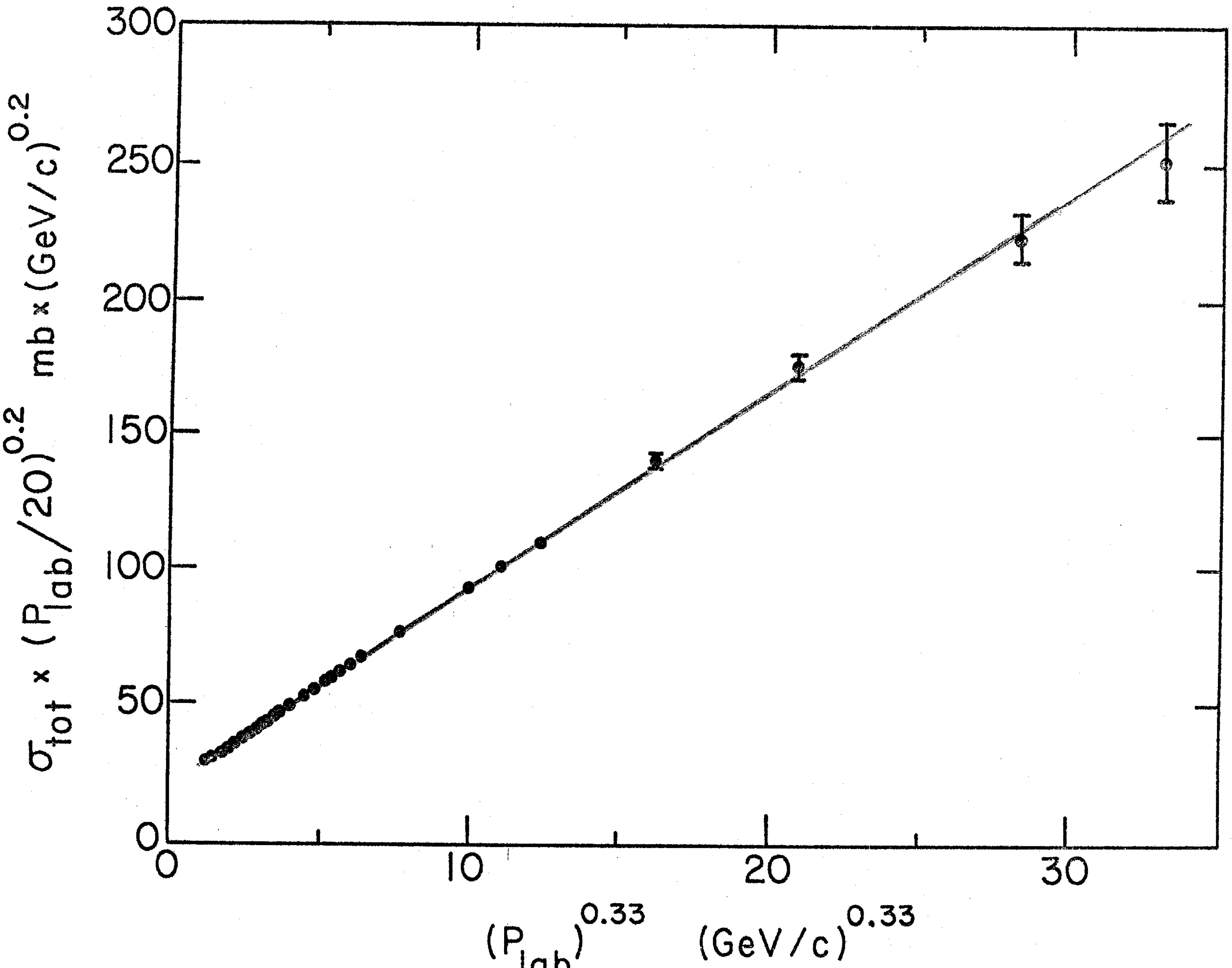
[2]H. J. Lipkin, Phys. Rev. D11, 1827 (1975). There are some errors in the values of the parameters quoted arising from a confusion between several notations. The correct values of the parameters actually used in this reference are given with eqs. (1) and (2) of the present paper. See also Nucl. Phys. B78, 381 (1974); Phys. Lett. 56B, 76 (1975); High Energy Physics, Proceedings of the 1975 EPS Conference, Palermo, Italy, edited by A. Zichichi, Editrice Compositori Bologna (1976) p. 1000.

[3]G. B. Yodh, to be published in the Proceedings of the Brookhaven Symposium on Prospects of Strong Interaction Physics at Isabelle, April 1977. See also F. Siohan, Ph.D. Thesis, University of Maryland, Publication No. PP77611 (1976) and T. K. Gaisser, G. B. Yodh, V. Barger and F. Halzen, Proceedings of the 14th International Cosmic Ray Conference, Munich (1975) Vol 7, p. 2161.

[4]R. J. Eden, High Energy Collisions of Elementary Particles, Cambridge University Press (1967), p. 194.

FIGURE CAPTION

Fig. 1 $\sigma_{tot}(pp) \times (P_{lab}/20)^{0.2}$ plotted against $(P_{lab})^{0.33}$. Formula (1) predicts that the data should lie on a straight line. The four cosmic ray points[3] above $P_{lab}{}^{0.33} = 15$ and the four ISR points[1] in the interval $7 < P_{lab}{}^{0.33} < 15$ are seen to lie on a straight line with a slope determined by fits to the lower energy data.


σtot x (Plab/20)0.2 mb x (GeV/c)0.2
300
250
200
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Dr. George Trigg, Editor
Physical Review Letters
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Dear Dr. Trigg,

Thank you for your letter of August 24, regarding my paper "Two-Component Pomeron and Hadron Total Cross Sections and Real Parts," LG1047. In my original letter submitting the paper for publication, I had indicated that it might be more suitable as a comment in Physical Review. I therefore accept the decision of the referee, and request its publication with the abstract originally submitted (before it was shortened to make it suitable for Phys. Rev. Letters, in response to your letter of July 12). You probably have the original abstract in your files. However, another copy is enclosed to be on the safe side.

In view of the more relaxed length requirements of Physical Review, I am also submitting a figure which should improve the clarity of the presentation, together with minor additions to the text to describe the figure. These include the words "and Fig. 1 show" replacing the word shows in the second line of page 2, and the additional text to be inserted at the end of the first paragraph on page 4 as indicated.

Sincerely yours,

Harry J. Lipkin

HJL/em
Encl.