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An SU(3) $\otimes$ U(1) Theory of Weak and Electromagnetic Interactions

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ABSTRACT

We describe an SU(3) $\otimes$ U(1) gauge theory of weak and electromagnetic interactions and study its experimental implications. In this theory all nonsinglet fermions are assigned to triplet representations of SU(3). The theory satisfactorily accounts for the trimuon events recently observed in neutrino reactions. Furthermore, the model naturally insures quark-lepton an e-μ universality, absence of right-handed currents in β and μ decay, and absence of s-d neutral currents to order $G_F\alpha$. Various discrete symmetries play an important role in maintaining these properties. The model allows for μ- and e-type lepton number nonconservation at a naturally strongly suppressec level. The weak contributions to the anomalous magnetic moments of the electron and muon are calculated and shown to be in accord with experimental bounds. The predictions of the model for most neutral current reactions are in satisfactory agreement with the data on the processes. Other features of the theory include the prediction of an absolutely stable massive neutral lepton, the absence of a sizeable high-y anomaly in antineutrino charged current reactions, and the absence of large parity violation in heavy atoms.

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I. INTRODUCTION

Among unified renormalizable theories of weak and electromagnetic interactions,[1] those based on the original gauge group[2] $SU(2) \otimes U(1)$ have been quite successful in accounting for various properties of weak decays and of charged and neutral current neutrino reactions. Generalizations of the original model which add right-handed (and often also new left-handed) weak currents have been proposed in order to incorporate various experimental findings such as the anomalous antineutrino y-distribution and rise in the ratio $\sigma^{\bar{\nu}N}/\sigma^{\nu N}$ reported by the Harvard-Pennsylvania-Wisconsin-Fermilab (HPWF) experiment.[3] Recently, however, the observation of high energy trimuons by the same group[4] led S. Weinberg and one of us to propose a more extensive generalization of the original theory, namely a model based on an enlarged gauge group, $SU(3) \otimes U(1)$.[5] This model is noteworthy for the natural way in which it preserves the appealing features of the Weinberg-Salam model.

In this paper we shall elaborate upon the structure of the model and discuss certain of its phenomenological implications, especially those pertaining to neutral currents. Let us first consider the original motivation for this work. The FHPRW group concludes[4] that one source of its trimuon events is most probably the production and sequential decay of heavy leptons: $\nu_\mu + N \to M^- + \ldots$; $M^- \to \mu^- + M^0 + \ldots$; $M^0 \to \mu^- + \mu^+ + \ldots$ We shall tentatively accept this interpretation here. The measured rate of trimuon production, uncorrected for detection efficiency, is $R(\nu_\mu \to \mu^-\mu^-\mu^+)/R(\nu_\mu \to \mu^-) = 5 \times 10^{-4}$. Some fraction of this rate is probably due to "conventional" production mechanisms; the question of exactly how much is unresolved at present. From an analysis of the invariant mass distribution within the framework of this heavy lepton interpretation, the group infers that $m_{M^-} \simeq 7.0^{+3.0}_{-1.0}$ GeV and $m_{M^0} \simeq 3.5^{+1.5}_{-0.4}$. The kinematical suppression arising from the necessity of producing the heavy lepton M^-, and, in addition, the small branching ratio $BR(M^- \to \mu^-\mu^-\mu^+ \ldots)$, suggests that the initial $\nu_\mu \to M^-$ transition may occur at nearly full weak strength. This is not possible in the original $SU(2) \otimes U(1)$ model.[6] There are essentially two ways to render it possible. The first is to expand the lepton and quark content of the standard $SU(2) \otimes U(1)$ model. The crucial observation, as discussed elsewhere[7]

is that the experimental constraints on the mixing angles do not prevent there from being substantial mixing of leptons which can give rise to $\nu_\mu \to M^-$, as long as this mixing is approximately reflected in the quark multiplets and is approximately symmetric for e- and μ-type leptons. It was thus proposed in Ref. 7, with a particular V-A $SU(2) \otimes U(1)$ model as example, that such large leptonic mixing could serve as a source for trimuon production.[8]

The second way to make possible a full strength $\nu_\mu \to M^-$ transition, which seems to us perhaps more natural and richer in theoretical implications, is to introduce new gauge bosons to couple ν_μ to M^-, and thus to enlarge the weak gauge group. An obvious criterion is to choose a group with the minimal dimension and rank such that $SU(2) \otimes U(1)$ can be embedded in it. The group SU(3) satisfies this criterion and has the further advantages that it is simple and compact so that (a) electric charge is automatically quantized[9] and (b) the embedding of $SU(2) \otimes U(1)$ as a subgroup fixes the angle $\theta = \tan^{-1}(g'/g)$, where g and g' are the gauge coupling constants for the SU(2) and U(1) factors respectively. However, there is a serious problem with this group: one must assign the leptons to an octet representation (or more generally, a representation of zero triality) in order for them to have integral charges. With this assignment there is no natural way to have $\nu_\mu \to M^-$ rather than $\nu_\mu \to M^+$ and still satisfy experimental constraints such as quark-lepton weak universality, e-μ universality, absence of $\nu_\mu \to e^-$ and $\mu \to e\gamma$ (to their respective levels of precision), etc. The simplest way to remedy these problems is to adjoin a U(1) group to produce SU(3) $\otimes$ U(1).[10] This then allows one to assign both quarks and leptons to the fundamental representation of SU(3).

As was stressed in Ref. 5, although the original motivation for the SU(3) $\otimes$ U(1) gauge model was the FHPRW trimuon events, this model has so many attractive features that one might analyze it as a serious theory in its own right. The theory naturally insures quark-lepton and e-μ universality absence of right-handed currents in neutron, hyperon, and muon decay, suppression of strangeness-changing neutral currents to order G_F^2 in the processes $K^0 \longleftrightarrow \bar{K}^0$, and $K \to \mu\bar{\mu}$, and adequate suppression of μ- and e-type lepton number-nonconserving processes such as $\mu \to e\gamma$. It is in agreement with most available neutral current data and predicts a very small amount of parity violation in heavy atoms. The latter prediction is in accord with the results of recent experiments by the University of Washington and Oxford University groups[11] which seem to indicate that parity violation, if it is present at all, is smaller than the value that the best available estimates[12] imply for the original SU(2) $\otimes$ U(1) model. With regard to charged current (anti)neutrino reactions, the model does not predict a large high-y anomaly. Moreover, in the most straightforward version of the model in which there is a certain exact discrete symmetry, there is an absolutely stable massive neutral lepton with far-reaching cosmological consequences.[13] Finally, whatever the future of $SU(3)\otimes U(1)$ as a theory of weak and electromagnetic interactions, this work should be of methodological interest since it shows the problems which tend to arise in expanding the gauge group and how they might be resolved.

This paper is organized as follows. In Section II we discuss the Higgs content of the theory, the pattern of spontaneous symmetry breakdown, fermion multiplet assignments, and discrete symmetries. A useful generalizatic of the minimal $SU(3)\otimes U(1)$ model is constructed; this generalized model interpolates continuously between the minimal $SU(3)\otimes U(1)$ theory and the Weinberg-Salam theory. Next, in Section III we consider the predictions of the theory for deep inelastic, elastic, $\nu(\bar{\nu})p$ and leptonic neutrino and antineutrino neutral current reactions, and parity violation and hyperfine splitting in atoms. In Section IV various other experimental consequences of the model are examined. These include the anomalous magnetic moments of the muon and electron, the decay $\mu\rightarrow e\gamma$, the $K_L K_S$ mass difference, the decay rates and branching ratios of the heavy leptons, and an estimate of the rate for trimuon production. The summary and conclusions are contained in Section V. Finally, two appendices deal with technical aspects of the Higgs potential.

II. STRUCTURE OF THE SU(3) $\otimes$ U(1) MODEL

We shall first review the SU(3) $\otimes$ U(1) gauge model discussed in ref. 5. The generators of SU(3) take the form, in the fundamental representation, $\lambda_a/2$, $a = 1, 2, \ldots, 8$, where λ_a are the Gell-Mann matrices; we denote the generator of U(1) by y. The electric charge Q is

$$Q = \tfrac{1}{2}(\lambda_3 + \lambda_8/\sqrt{3}) + y \quad . \tag{2.1}$$

Quarks and leptons come in this theory in families. We assume as usual that each quark comes in three colors, and the strong color SU(3) gauge group commutes with the flavor SU(3) $\otimes$ U(1) gauge group, as is necessary in order to avoid order α violations of such strong interaction symmetries as parity and strangeness.[14] In that which follows, color indices are dropped everywhere, and SU(3) refers throughout to the flavor gauge group. A quark family consists of one $y = 0$ triplet and one $y = 2/3$ singlet of each chirality. Thus, a quark family may be written as

$$q_L'(2/3) \;,\; \begin{pmatrix} q\,(2/3) \\ q\,(-1/3) \\ q'(-1/3) \end{pmatrix}_L \;;\; q_R(2/3),\; \begin{pmatrix} q'(2/3) \\ q'(-1/3) \\ q\,(-1/3) \end{pmatrix}_R \quad , \tag{2.2}$$

where the numbers 2/3 and -1/3 in parentheses denote electric charges, R and L denote chiralities, and primes are used to distinguish between two independent quarks of the same electric charge and chirality. A lepton family consists of one $y = -2/3$ triplet of each chirality, and one $y = 0$

left-handed singlet. We may write a lepton family as

$$\ell_L'(0)\ ,\ \begin{pmatrix}\ell(\ 0)\\ \ell(-1)\\ \ell'(-1)\end{pmatrix}_L\ ;\ \begin{pmatrix}\ell'(\ 0)\\ \ell'(-1)\\ \ell\ (-1)\end{pmatrix}_R \tag{2.3}$$

where 0 and -1 denote electric charges. There may be more than one quark and lepton family. Furthermore, at this point we do not assume any specific identification of the fermion fields with particles of definite mass; this must be determined from the pattern of spontaneous symmetry breaking. We do not exclude the possibility of there being also a y = 0 right-handed singlet in a lepton family; in such a case all neutral leptons may be massive.

This model is quasi-vectorlike, by which we mean that the fermions which are nonsinglets under the gauge group of weak, electromagnetic, and strong interactions are distributed symmetrically into left- and right-handed multiplets. Thus in our model only $SU(3)\otimes U(1)$-invariant leptons can be (and are assigned to multiplets in a chirally asymmetric way. Quasi-vectorlike models are free of Adler-Bell-Jackiw triangle anomalies, as are vectorlike theories, since the asymmetrically arranged fermions which might be present do not couple to any of the gauge bosons.

Quasi-vectorlike theories such as the present $SU(3) \otimes U(1)$ model have another important property which we should like to stress here. Any spontaneous symmetry breakdown of the overall gauge group G to an unbroken subgroup G_0 in a quasi-vectorlike theory will obviously yield a new effective G_0-gauge theory which is itself quasi-vectorlike. This is of interest because we know that the gauge theory of strong and electromagnetic interactions based on the unbroken gauge group $SU(3)_{color} \otimes U(1)_{em}$ must be quasi-vectorlike; this is why parity is conserved in these interactions. Of course, a quasi-vectorlike theory of strong and electromagnetic interactions could always emerge by spontaneous symmetry breaking from a nonquasi-vectorlike theory, but in a quasi-vectorlike theory such as the present model this is automatic, and independent of the symmetry breaking mechanism.

The Higgs scalar fields which can have gauge-invariant interactions with these fermions include $y = -2/3$ SU(3) triplets Ω_i, $y = 0$ SU(3) octets Φ_i, and, in addition, SU(3) singlets. We assume that there are no Higgs singlets but do not constrain the number of triplets and octets of the above kind. The minimal theory consists of one complex octet and two triplets. It is also interesting to examine the effects of other Higgs fields which couple to gauge bosons but not to fermions. Specifically, we shall construct a generalization of the minimal theory which includes one or more $y = 1/3$ Higgs triplets η_i, with charges (1, 0, 0); this theory interpolates between the minimal SU(3) $\otimes$ U(1) model and the original SU(2) $\otimes$ U(1) model.[2]

In general, the fermion fields in Eqs. (2.2) and (2.3) would be arbitrary mixtures of all fermion mass eigenstates of the same charge and chirality. There are several important constraints on the amount and type of mixing allowed; these include (1) quark-lepton universality, (2) e-μ universality, (3) the successful Cabibbo theory of neutron and hyperon decays, (4) the absence to a high degree of accuracy, of right-handed weak currents involving only light fermions, which could appear in neutron or hyperon β decay, or in muon decay, (5) the nonobservation of the transition $\nu_\mu \to e^-$ in the two-neutrino experiment and the absence of μ- and e-type nonconserving decays such as $\mu \to e\gamma$ and $\mu \to ee\bar{e}$, to the levels probed by experiment, and finally (6) the absence of $\Delta S \neq 0$ neutral currents to lowest order and to order $G_F\alpha$, as required by the smallness of the $K_L K_S$ mass difference and $K_L \to \mu\bar{\mu}$ decay rate. In order to satisfy these constraints naturally,[15] we shall impose certain discrete symmetries which either completely prevent or adequately suppress unwanted mixings. We assume that the Lagrangian is invariant under a discrete symmetry R, which leaves the gauge bosons, right-handed fermions, and the scalar triplets η_i $(\bar{y} = 1/3)$ invariant, and changes the sign of left-handed fermions, the scalar triplets Ω_i $(y = -2/3)$ and the scalar octets Φ_i $(y = 0)$. The R symmetry forbids bare fermion mass terms, so that fermion masses must arise from (R-invariant)

Higgs-fermion Yukawa couplings, via certain vacuum expectation values of Ω_i and Φ_i. Yukawa interactions of the scalars Ω_i and Φ_i with fermions allowed by SU(3) $\times$ U(1) $\times$ R are of the form

$$c_1 \bar{\ell}\, \Omega^{\dagger} R + c_2 \bar{L} \Omega r + \bar{L}(c_3 \Phi + c_4 \Phi^{\dagger}) R + \text{h.c.} \tag{2.4}$$

where we have dropped inessential indices.

Another important consequence of the R symmetry is that, at least for a finite range of parameters of the Lagrangian, the symmetry of the vacuum is not just the U(1) gauge symmetry associated with electromagnetism, but U(1) ⊗RU, where U is a particular element of the SU(3)⊗U(1) group, which in the fundamental representation may be expressed as

$$U = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix} \tag{2.5}$$

in some basis. This residual symmetry of the vacuum is natural, in the sense that it is the maximal little group of the absolute minimum of the Higgs potential for a finite range of its parameters (as is proved in Appendix I); this symmetry is preserved in higher orders in perturbation theory. Further, in Appendix II we show that the SU(3) ⊗ U(1) ⊗ R-invariant Higgs potential does not admit a larger continuous symmetry group, and hence there are no pseudo-Goldstone bosons in this model. Conservation of electric charge and RU requires the vacuum expectation values of scalar fields to take the form

$$\langle\Omega_i\rangle_0 = \begin{pmatrix} v_i \\ 0 \\ 0 \end{pmatrix}, \quad \langle\Phi_i\rangle_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & a_i \\ 0 & b_i & 0 \end{pmatrix}, \quad \langle\eta_i\rangle_0 = \begin{pmatrix} 0 \\ 0 \\ v_i' \end{pmatrix} \tag{2.6}$$

in the basis in which U takes the form (2.5). We will adhere to this basis henceforth. The fermion fields transform under RU according to

$$\text{RU:} \quad \ell \equiv f_L \to -f_L \quad (f_L \text{ singlet}) ,$$

$$r \equiv f_R \to +f_R \quad (f_R \text{ singlet}) ,$$

$$L \equiv \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}_L \to \begin{pmatrix} +f_1 \\ +f_2 \\ -f_3 \end{pmatrix}_L , \tag{2.7}$$

$$R \equiv \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}_R \to \begin{pmatrix} -f_1 \\ -f_2 \\ +f_3 \end{pmatrix}_R .$$

Since RU is a symmetry of the vacuum, there is no mixing between fields of different RU parity.

At this stage, we can be more specific about the quark and lepton families needed on phenomenological grounds, and their compositions. By taking linear combinations of the triplets, we may always choose one member of all the left-handed triplets and, independently, another member of all the right-handed triplets to be ~~both~~ weak gauge group eigenstates and mass eigenstate We choose these to be the second (third) member of the left-handed (right-handed) triplets. We thus have d-family:

$$t_L\ ,\ \begin{pmatrix} u' \\ d \\ b' \end{pmatrix}_L \ ;\ u_R\ ,\ \begin{pmatrix} t' \\ b' \\ d \end{pmatrix}_R \ , \tag{2.8}$$

s-family:

$$g_L\ ,\ \begin{pmatrix} c' \\ s \\ h' \end{pmatrix}_L \ ;\ c_R\ ,\ \begin{pmatrix} g' \\ h' \\ s \end{pmatrix}_R \ , \tag{2.9}$$

e-family:

$$E_L^0\ ,\ \begin{pmatrix} \nu_e \\ e^- \\ E^{-\prime} \end{pmatrix}_L \ ;\ \begin{pmatrix} E^{0\prime} \\ E^{-\prime} \\ e^- \end{pmatrix}_R \ , \tag{2.10}$$

μ-family:

$$M_L^0\ ,\ \begin{pmatrix} \nu_\mu \\ \mu^- \\ M^{-\prime} \end{pmatrix}_L \ ;\ \begin{pmatrix} M^{0\prime} \\ M^{-\prime} \\ \mu^- \end{pmatrix}_R \ , \tag{2.11}$$

τ-family:

$$T_L^0\ ,\ \begin{pmatrix} \nu_\tau \\ \tau^- \\ T^{-\prime} \end{pmatrix}_L \ ;\ \begin{pmatrix} T^{0\prime} \\ T^{-\prime} \\ \tau^- \end{pmatrix}_R \ , \tag{2.12}$$

where primes indicate mixing among fermions of definite mass of the same color, charge, RU parity and chirality. The mixing need not be CP conserving. It is, however, always possible to define the phases of u_R and c_R such that $u'_L = u_L \cos\theta_C - c_L \sin\theta_C$, $c'_L = u_L \sin\theta_C + c_L \cos\theta_C$, where θ is the (real) Cabibbo angle.

It is worthwhile to ask what would have resulted if we had not imposed the R symmetry, and if RU had not been an invariance of the vacuum. There would then be bare fermion mass terms which would induce mixings between ν_μ and M_L^0, and between ν_e and E_L^0, for example, which would violate e-μ universality; there would also be mixings between u_R and t'_R, and between b'_R and d_R, which, together, would induce a right-handed current in β-decay, for example; universality would thus be lost in a plethora of mixing angles.

It is desirable to forbid mixing among b and h and among E^-, M^-, and T^- in order naturally to suppress higher order contributions to strangeness-changing neutral current processes such as $K^0 \leftrightarrow \bar{K}^0$ and $K_L \to \mu\bar{\mu}$, and the μ- and e-number violating decays $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ to an adequate degree (see Section IV). This can be accomplished if we assume further that the theory is invariant under another discrete symmetry S, which leaves gauge bosons, and the scalar fields Φ_i, Ω_i and η_i invariant, and transforms other fields according to

$$S : \begin{bmatrix} r_a \\ \ell_a \\ R_a \\ L_a \end{bmatrix} \to \eta_a \begin{bmatrix} r_a \\ \ell_a \\ R_a \\ L_a \end{bmatrix} , \tag{2.13}$$

$$\Omega_i \to \xi_i \, \Omega_i \quad (i \neq 1) \tag{2.14}$$

where r, ℓ, R, and L are as defined in Eq. (2.6), and the index a distinguishes between different families. η_a and ξ_i are phase factors, $|\eta_a| = |\xi_i| = 1$, such that $\eta_a^* \eta_b \neq 1$ for $a \neq b$ among quarks, and among leptons (lepton-number conservation is assured by a global gauge invariance). (There should be no confusion between the phase factors η_a and the Higgs fields η_i.) This prevents b and h, and E^-, M^- and T^- from mixing. In order that S be an element of a discrete group it is necessary that $S^n = 1$, for some integer n. The group in question is then the cyclic group of order n, Z_n. This requires that η_a and ξ_i be some powers of exp $(2\pi i/n)$.

To insure that there is Cabibbo mixing between u_L and c_L, it is necessary that there be at least one Ω_i, $i \neq 1$, such that $\eta_s^* \eta_d \xi_i^* = 1$ or $\eta_d^* \eta_s \xi_i^* = 1$, where η_s and η_d are phases of the d- and s-families under S. An economical choice of the phases under S, which does not contradict anything known phenomenologically, is: $\eta_d = \eta_e = 1$, $\eta_s = \eta_\mu = i$, $\eta_\tau = \exp(i\pi/4)$; we postulate one additional triplet with $y = -2/3$, Ω_2, with $\xi_2 = -i$ (or i). This choice will allow mixing between u and c, t and g, and E^0 and M^0, but leaves T^0 pure. When we consider cascade decays of M^- in a later section, we shall concentrate on this possibility exclusively for economy.

The family compositions of the fermions, with the above ansatz, then are as follows:

d-family:

$$t_L \ , \ \begin{bmatrix} u' \\ d \\ b \end{bmatrix}_L \ ; \quad u_R \ , \ \begin{bmatrix} t' \\ be^{i\phi_b} \\ d \end{bmatrix}_R \tag{2.15}$$

s-family:

$$g_L \ , \ \begin{bmatrix} c' \\ s \\ h \end{bmatrix}_L \ ; \quad c_R \ , \ \begin{bmatrix} g' \\ he^{i\phi_h} \\ s \end{bmatrix}_R \tag{2.16}$$

e-family:

$$E_L^0 \ , \ \begin{bmatrix} \nu_e \\ e^- \\ E^- \end{bmatrix}_L \ ; \quad \begin{bmatrix} E^{0\prime} \\ E^- e^{i\phi_E} \\ e^- \end{bmatrix}_R \tag{2.17}$$

μ-family:

$$M_L^0 \ , \ \begin{bmatrix} \nu_\mu \\ \mu^- \\ M^- \end{bmatrix}_L \ ; \quad \begin{bmatrix} M^{0\prime} \\ M^- e^{i\phi_M} \\ \mu^- \end{bmatrix}_R \tag{2.18}$$

τ-family:

$$T_L^0 \ , \quad \begin{bmatrix} \nu_\tau \\ \tau^- \\ T^- \end{bmatrix}_L \quad ; \quad \begin{bmatrix} T^0 \\ T^- e^{i\phi_T} \\ \tau^- \end{bmatrix}_R \tag{2.19}$$

In Eqs. (2.15 - 2.19) ϕ_b, ϕ_h, ϕ_E, ϕ_M, ϕ_T are CP-violating phases (we have defined the phases of b, h, E^-, M^- and T^- in such a way that the relative phase of d_L and b_L is zero, etc.), and

$$\begin{pmatrix} u' \\ c' \end{pmatrix}_L = \begin{pmatrix} \cos\theta_C & -\sin\theta_C \\ \sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}_L \ , \tag{2.20}$$

$$\begin{pmatrix} t' \\ g' \end{pmatrix}_R = \begin{pmatrix} e^{i\alpha}\cos\theta_C' \ , & -e^{i\alpha}\sin\theta_C' \\ e^{-i\alpha}\sin\theta_C' \ , & e^{-i\alpha}\cos\theta_C' \end{pmatrix} \begin{pmatrix} t \\ g \end{pmatrix}_L \ , \tag{2.21}$$

$$\begin{pmatrix} E^{0\prime} \\ M^{0\prime} \end{pmatrix}_R = \begin{pmatrix} e^{i\gamma}\cos\beta \ , & -e^{i\gamma}\sin\beta \\ e^{-i\gamma}\sin\beta \ , & e^{-i\gamma}\cos\beta \end{pmatrix} \begin{pmatrix} E^0 \\ M^0 \end{pmatrix}_R \ , \tag{2.22}$$

where θ_C is the Cabibbo angle, and α and γ are CP violating phases. In Eqs. (2.15 - 2.22) all unprimed fields are states of definite (real) mass.

An obvious generalization of the mixing of the neutral heavy leptons specified in Eq. (2.22) is to allow T_R^0 to mix with E_R^0 and M_R^0 to form $E_R^{0\prime}$, $M_R^{0\prime}$, and $T_R^{0\prime}$. This would require another Higgs triplet and different assignments of S-eigenvalues to the leptons. The mixing matrix would then be a 3×3 unitary matrix, which depends upon four parameters, one of which is CP-violating. We shall restrict ourselves here to the simpler form of the model in which the S symmetry constrains the mixing to occur only among ${E_R}^0$ and ${M_R}^0$.

The quark sector thus includes besides the familiar u, d, s, and c quarks, the heavy ones t, g, b, and h. These must have masses of at least 3.5 GeV in order not to have been observed at SPEAR. In order to maximize the trimuon production rate we shall assume that the t and g quarks are rather light; say ~4 GeV. The masses of the b and h quarks are not very severely constrained other than as already indicated; however if one wishes to have trimuon production by antineutrinos proceed with the same intrinsic rate (i.e. excluding flux considerations) as the production by neutrinos then m_b must not be too much greater than m_t and m_g. As is evident from Eq. (2.19), the leptonic sector includes an additional family of τ-type leptons including τ^-, identified as the heavy lepton discovered at SPEAR[16] and ν_τ, its associated neutrino. The M^- and the lighter two neutral leptons, defined as M^0 and E^0 are presumably responsible for the trimuon events recently observed at Fermilab.[4] With these identifications, we have $m_\tau \simeq 1.9$ GeV, $m_{M^-} \simeq 7 - 8$ GeV, and $m_{M^0} \simeq 4$ GeV.

As determined so far, the model satisfies all of the constraints discussed above on fermion mixing. This is obviously true for the constraints (1) - (4) on quark-lepton and e-μ universality, the Cabibbo theory of neutron and hyperon decays, and the absence of right-handed currents in neutron, hyperon, or muon decay. The fact that the model satisfies the other two constraints (5) and (6) on the suppression of μ- and e-number nonconservation and the absence of strangeness-changing neutral currents to order $G_F\alpha$ will be demonstrated explicitly in section IV.

These constraints significantly restrict the allowed assignments of fermions to multiplets. In particular, if one had tried to place the right-handed chiral component of the u quark in the position to which t_R is presently assigned, then there would be a serious problem: in order to give mass to the u quark one would need a nonzero Higgs vacuum expectation value $<(\Phi_i)_{11}>_0$, but this is forbidden by the exact RU symmetry.[17] Hence, in this model there is no natural way to incorporate a right-handed $\bar{b}_R\gamma_\mu u_R$ current which could be excited in antineutrino reactions and lead to a so-called high-y anomaly, including a large enhancement of $d\sigma^{\bar{\nu}N}/dy$ at high y, a consequent increase in $<y>^{\bar{\nu}N}$ and $\sigma^{\bar{\nu}N}/\sigma^{\nu N}$ and a decrease in $B^{\bar{\nu}N}$ with increasing energy E. The experimental situation concerning the high-y anomaly is currently in a state of flux. When our model was originally proposed the data from the HPWF experiment indicated a sizeable high-y anomaly.[3] The (statistically somewhat weaker) data from the Fermilab-IHEP-ITEP-Michigan (FIIM) antineutrino-neon bubble chamber experiment yielded values of $<y>^{\bar{\nu}N}$ which were consistent with being energy-independent or slightly increasing over the range $E \simeq 10$ to $E \simeq 100$ GeV.[18] More recently, data from the Caltech-Fermilab (CF) and CERN-Dortmund-Heidelberg-Saclay (CDHS), and CERN BEBC (anti)neutrino experiments has become available.[18,19]

These groups do not find any strong energy dependence in the quantities $<y>^{\bar{\nu}N}$, $B^{\bar{\nu}N}$, or $\sigma^{\bar{\nu}N}/\sigma^{\nu N}$ from incident $\nu(\bar{\nu})$ energy $E \simeq 30$ GeV to $E \simeq 200$ GeV; thus they do not observe any high-y anomaly of the magnitude originally reported by the HPWF group.

It should be noted parenthetically that at high energies above 100 GeV the model predicts the production of heavy leptons and heavy quarks (both must be produced together). A certain fraction of the decays of the heavy leptons will simulate regular charged current reactions; for example, $\bar{\nu}_\mu + N \rightarrow M^+ + X$, $M^+ \rightarrow \mu^+ + \ldots$ will simulate the process $\bar{\nu}_\mu + N \rightarrow \mu^+ + X$. Furthermore, in the cascade decay of the M^- a considerable amount of energy will be carried away in the form of neutrinos or $\overset{(-)}{E}{}^0$'s. Consequently, the apparent incident $\bar{\nu}_\mu$ energy will be substantially less than the actual incident energy, and effects of M^- production can appear at $E_{vis} \simeq 50 - 70$ GeV. Furthermore, because the μ^+ carries only a fraction (about 1/3 for the main decay chains; see Table 4) of the energy of the parent M^+ the simulated $\bar{\nu}_\mu + N \rightarrow \mu^+ + X$ events arising from M^+ production will preferentially populate the high y_{vis} part of the y_{vis} distribution, where $y_{vis} = \nu/E_{vis}$, with $E_{vis} = \nu + E_\mu$, and ν is the energy transfer in the lab frame. However for present neutrino experiments this will be a very small effect because M^+ production is kinematically suppressed by the necessity of producin concomitantly a heavy quark, and by a helicity factor which asymptotically is equal to 1/3. Detailed calculations using $m_{M^+} = 8$ GeV, $m_{M^0} = 4$ GeV, and $m_b = 4$ GeV and folding in the FHPRW quadrupole triplet neutrino flux spectrum yield the result that for $E > 100$ GeV,

$\sigma(\bar{\nu}_\mu + N \to M^+ + X)/\sigma(\bar{\nu}_\mu + N \to \mu^+ + X) \simeq 0.036$.[20] The branching ratio calculations discussed in section IV give $BR(M^+ \to \mu^+ + \ldots) \lesssim 30\%$. Hence, at most, M^+ production will contribute about a 1% increment to the cross section for the reaction $\bar{\nu}_\mu + N \to \mu^+ + X$. This has a negligible effect on $d\sigma^{\bar{\nu}N}/dy$, and $\sigma^{\bar{\nu}N}$. Thus our model predicts no sizeable high-y anomaly, in agreement with the present data from the FIIM, CF, and CDHS experiments.

Returning to the structure of the model, we note that so far the discrete symmetries, R, RU, and S, which have been imposed on the Lagrangian and/or vacuum have been for the purpose of preventing undesired mixings of fermions. The next discrete symmetry is an approximate one, imposed for the different purpose of accounting for the important property that the odd RU-parity fermions are much heavier than the corresponding even ones in the same family. Thus we shall assume that the Lagrangian is approximately invariant under the symmetry $\sqrt{R}$, which causes the various fields in the theory to transform as follows:

$$\begin{aligned}
\sqrt{R}: \quad & V_a{}^\mu \to V_a{}^\mu \qquad a = 0, \ldots, 8 \\
& r, R \to r, R \\
& \ell, L \to -i\ell, -iL \\
& \Omega_j \to -i\Omega_j \\
& \eta_j \to \eta_j \\
& \Phi_j \to i\Phi_j
\end{aligned} \tag{2.23}$$

If $\sqrt{R}$ were an exact symmetry of the Lagrangian, then among the possible Higgs-fermion couplings allowed by $SU(3) \times U(1) \times R$, as given in Eq. (2.4) the terms $L\Omega r$ and $L\Phi R$ would be absent, i.e. $c_2 = c_3 = 0$. The $\sqrt{R}$ symmetry is spontaneously broken by the Ω_i and Φ_i vacuum expectation values. However, if the $\sqrt{R}$ symmetry were an exact symmetry of the Lagrangian then it would be natural for the vacuum expectation values to be invariant under $\sqrt{R}\,\sqrt{U}$ where $\sqrt{U}$ is the SU(3) transformation

$$\sqrt{U} = \begin{pmatrix} i & & \\ & -i & \\ & & 1 \end{pmatrix} . \tag{2.24}$$

This would incidentally imply that $(\sqrt{R}\,\sqrt{U})^2$ is a symmetry of the vacuum; however, it is a stronger condition. (Of course there is an ambiguity in the choice of phases of the diagonal elements of the matrix $\sqrt{U}$. The choice made here gives the desired physical consequences.) If $\sqrt{R}\,\sqrt{U}$ were indeed a symmetry of the vacuum, then in the octets Φ_i, $b_i = 0$, although a_i, v_i, and v_i' can be arbitrary. Thus the approximate $\sqrt{R}$ symmetry of the Lagrangian implies that $|c_2|, |c_3| \ll |c_1|, |c_4|$ and the approximate $\sqrt{R}\,\sqrt{U}$ symmetry of the vacuum implies that $|b_i| \ll |a_i|, |v_i|, |v_i'|$. Taken together, these inequalities guarantee that fermions which are odd under RU are much heavier than the corresponding RU-even fermions in the same family.

We shall next discuss the vector boson mass generation. The gauge bosons of the theory consist of the SU(3) octet, U(1) singlet V_a^{μ}, a = 1, ..., 8, and the SU(3) singlet, U(1) gauge boson V_0^{μ}. These are coupled to the fermion currents by the terms

$$\mathscr{L}_{GJ} = g(J_a)_{\mu} V_a^{\mu} + g'(J_0)_{\mu} V_0^{\mu} \tag{2.25}$$

where

$$J_a^{\mu} = \sum_{\chi = L, R} \sum_i \bar{\psi}_{\chi, i} \gamma_{\mu} \left(\frac{\lambda_a}{2}\right) \psi_{\chi, i} \tag{2.26a}$$

and

$$J_0^{\mu} = \sum_{\chi = L, R} \sum_j \bar{\psi}_{\chi, j} \gamma_{\mu} \left(\frac{Y}{2}\right) \psi_{\chi, j} \quad . \tag{2.26b}$$

In Eqs. (2.26a, b) the first sum is over the chirality χ and the second is over all fermion triplets in the case of J_a^{μ} and all fermion triplets and singlets in the case of J_0^{μ}.

The vector bosons gain their masses through their couplings to Higgs bosons which are given by

$$\mathscr{L}_{GH} = \frac{1}{2} \operatorname{Tr}\left(D_{\mu}\Phi^{\dagger} D^{\mu}\Phi\right) + D_{\mu}\Omega^{\dagger} D^{\mu}\Omega + D_{\mu}\eta^{\dagger} D^{\mu}\eta \tag{2.27}$$

where

$$D_\mu \Phi = \partial_\mu \Phi - ig\left[V_\mu, \Phi\right] \tag{2.28a}$$

$$D_\mu \Omega = \partial_\mu \Omega - igV_\mu \Omega - ig'\frac{y_\Omega}{2}(V_0)_\mu \Omega \tag{2.28b}$$

and

$$D_\mu \eta = \partial_\mu \eta - igV_\mu \eta - ig'\frac{y_\eta}{2}(V_0)_\mu \eta \tag{2.28c}$$

with

$$V^\mu \equiv \tfrac{1}{2}\lambda_a V_a{}^\mu \tag{2.29}$$

and $y_\Omega = -2/3$, $y_\eta = 1/3$.

One can calculate the physical vector boson fields and their masses in the usual way by diagonalizing the vector boson mass matrix. For the sake of clarity we shall do this separately for the minimal version of the model with only the Higgs fields Φ and Ω, and the generalized version which includes also the η field. In the minimal version we find the physical fields

$$W^{\pm\mu} = \frac{\left(V_1{}^\mu \mp iV_2{}^\mu\right)}{\sqrt{2}} \tag{2.30}$$

$$U^{\pm\mu} = \frac{\left(V_4^\mu \mp iV_5{}^\mu\right)}{\sqrt{2}} \tag{2.31}$$

$$X_1{}^\mu = \cos(\epsilon/2)V_6^\mu - \sin(\epsilon/2)V_7{}^\mu \tag{2.32a}$$

$$X_2{}^\mu = \sin(\epsilon/2)V_6^\mu + \cos(\epsilon/2)V_7^\mu \tag{2.32b}$$

$$Z^\mu = \frac{1}{\sqrt{g^2 + g'^2/3}}\left[\frac{\sqrt{3}}{2}g\left(V_3{}^\mu + \frac{1}{\sqrt{3}}V_8\right) - \frac{g'}{\sqrt{3}}V_0{}^\mu\right] \tag{2.33}$$

$$Y^{\mu} = \tfrac{1}{2}\left(V_3{}^{\mu} - \sqrt{3}\, V_8{}^{\mu}\right) \tag{2.34}$$

and

$$A^{\mu} = \frac{1}{\sqrt{g^2 + g'^2/3}}\left[\frac{g'}{2}\left(V_3{}^{\mu} + \frac{1}{\sqrt{3}} V_8{}^{\mu}\right) + g\, V_0{}^{\mu}\right] \tag{2.35}$$

where

$$\epsilon = \arg\left\{\Sigma\, a_i{}^{*} b_i\right\} \quad . \tag{2.36}$$

The spectrum thus consists of the charged vector bosons $W^{\pm}$ and $U^{\pm}$ which mediate transitions in the $1 \pm i2$ and $4 \pm i5$ directions in SU(3) space, the electrically neutral X_1 and X_2 bosons, which mediate transitions in the $6 \pm i7$ directions in SU(3) space, the two neutral bosons Z and Y, and the massless photon A. Their RU parities are: $W^{\pm}(+)$, $U^{\pm}(-)$, $X_{1,2}(-)$, $Z(+)$, $Y(+)$, and $A(+)$.

The (nonvanishing) masses of these vector bosons are given by

$$m_W{}^2 = \tfrac{1}{4} g^2 \sum_i \left(|a_i|^2 + |b_i|^2\right) + \tfrac{1}{2} g^2 \sum_i |v_i|^2 \tag{2.37}$$

$$m_U{}^2 = m_W{}^2 \tag{2.38}$$

$$m_{X_{1,2}}{}^2 = \tfrac{1}{2} g^2 \left[\sum_i \left(|a_i|^2 + |b_i|^2\right) \mp 2 \left|\sum_i a_i{}^{*} b_i\right|\right] \tag{2.39}$$

$$m_Z{}^2 = \tfrac{2}{3}(g^2 + g'^2/3) \sum_i |v_i|^2 \tag{2.40}$$

$$m_Y{}^2 = g^2 \sum_i \left(|a_i|^2 + |b_i|^2\right) \quad . \tag{2.41}$$

It is convenient to write these masses in terms of m_W; to do so we introduce the following notation:

$$\ell = \frac{\frac{1}{2}\sum_i \left(|a_i|^2 + |b_i|^2\right)}{\sum_i |v_i|^2} \tag{2.42}$$

$$\delta = \frac{2\left|\sum_i a_i^* b_i\right|}{\sum_i \left(|a_i|^2 + |b_i|^2\right)} \tag{2.43}$$

and

$$w = \frac{g'^2}{g'^2 + 3g^2} \quad . \tag{2.44}$$

In terms of these quantities

$$m_{X_{1,2}}^2 = \frac{2\ell(1 \mp \delta)}{1+\ell} m_W^2 \tag{2.45}$$

$$m_Z^2 = \frac{4}{3(1-w)(1+\ell)} m_W^2 \tag{2.46}$$

and

$$m_Y^2 = \frac{4\ell}{1+\ell} m_W^2 \quad . \tag{2.47}$$

From the coupling of the photon field to the electromagnetic current we determine that

$$\begin{aligned} e &= \frac{1}{2} \frac{gg'}{\sqrt{g^2 + g'^2/3}} \\ &= \frac{\sqrt{3}}{2} g\sqrt{w} \quad . \end{aligned} \tag{2.48}$$

The relation between $g^2/m_W{}^2$ and G_F in this model is, as usual,

$$\frac{g^2}{8m_W{}^2} = \frac{G_F}{\sqrt{2}} \quad . \tag{2.49}$$

Combining Eqs. (2.37), (2.44), (2.48), and (2.49), we have

$$m_W = \left(\frac{4\pi\alpha}{3\sqrt{2}G_F w}\right)^{\frac{1}{2}} = \frac{43.06}{\sqrt{w}}\ \text{GeV} \quad . \tag{2.50}$$

A salient feature of this minimal model is that the $X_{1,2}$ and Y bosons derive their masses entirely from the vacuum expectation value of the Higgs octets Φ_i, whereas the Z picks up its mass completely from the vacuum expectation value of the Higgs triplets, Ω_i. It is also interesting that the $W^{\pm}$ and $U^{\pm}$ bosons are degenerate in this Higgs scheme.

Fig. 1 illustrates how in the minimal version of the model the vector boson masses depend on the parameter ℓ, which measures the relative size of the Φ and Ω vacuum expectation values. For this graph we take $w = 0.25$ and $\delta = 0.01$; these values are deduced, respectively from our analysis of neutral current phenomenology and fermion mass generation (see sections III and IV. 2). In Fig. 2 we show the dependence of the vector boson masses on the variable w for $\delta = 0.01$ and $(1+\ell)^2 = 1.4$, i.e. $\ell = 0.18$, a value which is derived from our fit to the neutral current data (see section III). If one chooses the three parameters which determine the gauge boson masses (in the minimal model) to be $\ell = 0.18$, $w = 0.25$, and $\delta = 0.01$, then the resulting nonzero masses of these vector bosons are, in units of GeV, $m_W = m_U \simeq 86$, $m_{X_1} \simeq 47$, $m_{X_2} \simeq 48$, $m_Y \simeq 67$, and $m_Z \simeq 106$.

Using the expressions for the physical vector boson fields we can rewrite the gauge boson-fermion coupling terms, Eq. (2.25) as

$$\mathscr{L}_{GJ} = \left\{\frac{g}{\sqrt{2}} W_\mu{}^\dagger J_W{}^\mu + \frac{g}{\sqrt{2}} U_\mu{}^\dagger J_U{}^\mu + \text{h.c.}\right\} + g\left(X_{1\mu} J_1{}^\mu + X_{2\mu} J_2{}^\mu\right)$$

$$+ \frac{1}{\sqrt{3}}\sqrt{g^2 + g'^2/3}\, Z_\mu J_Z{}^\mu + \frac{g}{2} Y_\mu J_Y{}^\mu + eA_\mu J_{em}{}^\mu \qquad (2.51$$

where
$$J_W{}^\mu = J_1{}^\mu + iJ_2{}^\mu \qquad (2.52$$

$$J_U{}^\mu = J_4{}^\mu + iJ_5{}^\mu \qquad (2.53$$

$$J_{X_1}{}^\mu = \cos(\epsilon/2) J_6^\mu - \sin(\epsilon/2) J_7{}^\mu \qquad (2.54$$

$$J_{X_2}{}^\mu = \sin(\epsilon/2) J_6^\mu + \cos(\epsilon/2) J_7^\mu \qquad (2.54$$

$$J_Z{}^\mu = \frac{3}{2}\left(J_3{}^\mu + \frac{1}{\sqrt{3}} J_8{}^\mu - wJ_{em}{}^\mu\right) \qquad (2.55$$

$$J_Y{}^\mu = -J_3{}^\mu + \sqrt{3}\, J_8{}^\mu \qquad (2.56$$

$$J_{em}{}^\mu = J_3{}^\mu + \frac{1}{\sqrt{3}} J_8{}^\mu + yJ_0{}^\mu \quad . \qquad (2.57$$

The neutral currents $J_Y{}^\mu$ and $J_Z{}^\mu$ are diagonal and remain so to order $G_F\alpha$. The currents $J_{X_1}{}^\mu$ and $J_{X_2}{}^\mu$ are neutral with respect to electric charge but not SU(3); i.e., they are flavor-changing. Because of our prevention of b, h mixing and E^-, M^-, or T^- mixing these latter two currents play no role in processes such as the $K^0 \longleftrightarrow \overline{K}^0$ transition and $K_L \rightarrow \mu\bar{\mu}$ decay or in μ- and e-number violating processes such as $\mu \rightarrow e\gamma$ and $\mu \rightarrow ee\bar{e}$. As will be seen, they do have an important effect on the magnetic moments of the muon and electron, and on the electric dipole moment of the neutron. However, in both cases an interesting gauge boson analogue of the GIM cancellation mechanism for fermions strongly suppresses their contributions.

We next consider the second version of the SU(3) $\otimes$ U(1) model which includes the $y = 1/3$ Higgs triplets η_i. A convenient measure of the size of the η_i vacuum expectation values v_i' is the ratio

$$r = \frac{\left(\sum_i |v_i'|^2\right)}{\left(\sum_i |v_i^2|\right)} \quad . \tag{2.58}$$

The usefulness of this generalized model is that it interpolates continuously between the SU(3) $\times$ U(1) theory and the Weinberg-Salam theory as r ranges from zero to infinity (with the identification $w = (4/3)\sin^2\theta_W$ in the latter case). The additional coupling term $|\partial_\mu \eta - igV_\mu \eta - ig' y_\eta V_0 \eta|^2$ modifies the vector boson fields and masses. The $W^\pm$, $U^\pm$, X_1, X_2, and A fields remain mass eigenstates but the Z and Y fields mix to form new eigenstates Z_1 and Z_2. Although the mass of the $W^\pm$ remains the same, all of the other nonzero masses change from their values at $v' = 0$. The new physical neutral vector bosons Z_1 and Z_2 are related to Z and Y by the orthogonal transformation V:

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}\begin{pmatrix} Z \\ Y \end{pmatrix} \equiv \begin{pmatrix} \cos\chi & \sin\chi \\ -\sin\chi & \cos\chi \end{pmatrix}\begin{pmatrix} Z \\ Y \end{pmatrix} \tag{2.59}$$

where

$$V_{11} = \sigma\left[\sigma^2 + \left(m_{Z_1}{}^2 - \rho\right)^2\right]^{-\frac{1}{2}} \tag{2.60}$$

$$V_{12} = \left(m_{Z_1}{}^2 - \rho\right)\left[\sigma^2 + \left(m_{Z_1}{}^2 - \rho\right)^2\right]^{-\frac{1}{2}} \tag{2.61}$$

$$V_{21} = \sigma \left[\sigma^2 + \left(m_{Z_2}{}^2 - \rho\right)^2\right]^{-\frac{1}{2}} \tag{2.62}$$

$$V_{22} = \left(m_{Z_2}{}^2 - \rho\right) \left[\sigma^2 + \left(m_{Z_2}{}^2 - \rho\right)^2\right]^{-\frac{1}{2}} \tag{2.63}$$

with

$$m_{Z_{1,2}}{}^2 = (\rho + \tau) \pm \tfrac{1}{2}\sqrt{(\rho - \tau)^2 + 4\sigma^2} \tag{2.64}$$

$$\rho = \frac{2}{3}\left(g^2 + g'^2/3\right)\left[\sum_i\left(v_i{}^2 + v_i'^2/4\right)\right] \tag{2.65}$$

$$\sigma = \frac{\sqrt{3}}{6} g\sqrt{g^2 + g'^2/3} \left(\sum_i v_i'^2\right) \tag{2.66}$$

$$\tau = g^2\left[\sum_i\left(|a_i|^2 + |b_i|^2\right)\right] + \frac{g^2}{2}\left(\sum_i v_i'^2\right) . \tag{2.67}$$

In Eq. (2.59) we have introduced a rotation angle χ defined in an obvious way as $\chi = \tan^{-1}(V_{12}/V_{11})$ to parametrize the orthogonal transformation V. In terms of the dimensionless quantities w, ℓ, and r

$$m_{Z_{1,2}}{}^2 = \frac{m_W{}^2}{2(1+\ell)}\left\{\left[\frac{r+4}{3(1-w)} + 4\ell + r\right] \pm \left[\left(\frac{r+4}{3(1-w)} - 4\ell - r\right)^2 + \frac{4r^2}{3(1-w)}\right]^{\frac{1}{2}}\right\} . \tag{2.68}$$

Thus $Z_1(Z_2)$ is defined as the mass eigenstate with the larger (smaller) eigenvalue $m_{Z_1}{}^2$ $(m_{Z_2}{}^2)$. As $r \to 0$ $Z_1 \to Z(Y)$ and $Z_2 \to Y(Z)$ if $m_Z > m_Y$ $(m_Z < m_Y)$. The condition that $m_Z > m_Y$ is the condition that $3\ell(1 - w) < 1$ and vice versa. With the values of w and ℓ which fit the neutral current data $m_Z > m_Y$ for $r \simeq 0$.

The masses of the physical massive vector bosons are given for $W^{\pm}$ by Eq. (2.50), for $m_{Z_{1,2}}{}^2$ by Eq. (2.68), and for the others as follows:

$$m_U{}^2 = \frac{(1 + \ell + r)}{(1 + \ell)} m_W{}^2 \tag{2.69}$$

and
$$m_{X_{1,2}}^{2} = \frac{[2\ell(1 \mp \delta) + r]}{(1+\ell)} m_W^{2} \quad . \tag{2.70}$$

As $r \to \infty$ only $m_{Z_2}^{2}$ (and trivially m_W^{2}) remain finite. We have

$$\lim_{r\to\infty} m_{Z_2}^{2} = \frac{m_W^{2}}{(1 - \frac{3}{4} w)} \quad . \tag{2.71}$$

In Fig. 3 we illustrate the dependence of the vector boson masses on r as this parameter ranges from 10^{-2} to 10^{3}. For this graph the values $(1+\ell)^2 = 1.4$, $w = 0.25$, and $\delta = 0.01$ are used. With the identification $w = (4/3)\sin^2\theta_W$ this becomes precisely the mass relation $(m_Z{}^2)_{WS} = (m_W{}^2)_{WS}\sec^2\theta_W$ of the Weinberg-Salam model (with only doublet Higgs multiplets). The corresponding eigenstate is

$$\lim_{r\to\infty} Z_2{}^{\mu} = \frac{-1}{\sqrt{4-3w}}\left[\sqrt{3(1-w)}\, Z^{\mu} + Y^{\mu}\right] \quad . \tag{2.72}$$

The new currents which couple to $Z_1{}^{\mu}$ and $Z_2{}^{\mu}$ are given by

$$\begin{pmatrix} J_{Z_1}{}^{\mu} \\ J_{Z_2}{}^{\mu} \end{pmatrix} = V \begin{pmatrix} \frac{1}{\sqrt{3}}\sqrt{g^2 + g'^2/3}\, J^{\mu}_{Z} \\ \frac{g}{2} J^{\mu}_{y} \end{pmatrix} \tag{2.73}$$

where for simplicity we have included the coupling constants in the definition of these new currents.

III. NEUTRAL CURRENT PROCESSES

In dealing with the neutral current phenomenology of the model, we shall concentrate mainly upon the minimal $r = 0$ version. Of the adjustable parameters in this version of the theory, two, namely ℓ and w, control the neutral current processes discussed here (δ plays no part). In order to demonstrate how the model can be changed continuously into the Weinberg-Salam theory we shall also study the effect of varying r from $r=0$ to $r=\infty$.

In the minimal version, there are two diagonal neutral currents, $J_Y{}^\mu$ and $J_Z{}^\mu$, which couple to the massive vector bosons Y and Z as specified in Eq. (2.51). The additional currents $J_{X_1}{}^\mu$ and $J_{X_2}{}^\mu$ which are neutral with respect to charge but not SU(3) play no role (in either the minimal or the generalized models) in the neutral current neutrino reactions, weak hyperfine effect, or atomic parity violation considered here. Thus, in particular, there are no nondiagonal neutral currents involving neutrinos in this theory. It is useful to write out the explicit form of the currents $J_Y{}^\mu$ and $J_Z{}^\mu$ for quarks and leptons. From Eq. (2.56) we have:

$$J^\mu_{Y,\text{ leptonic}} = \sum_{\ell,\, L^-} (-\bar{\ell}\gamma^\mu\gamma_5\ell + \overline{L^-}\gamma^\mu\gamma_5 L^-) \tag{3.1a}$$

where the sum is over $\ell = e, \mu, \tau$ and $L^- = E^-, M^-, T^-$, and

$$J^\mu_{Y,\text{ hadronic}} = \sum_{q_2, q_3} (-\bar{q}_2\gamma^\mu\gamma_5 q_2 + \bar{q}_3\gamma^\mu\gamma_5 q_3) \tag{3.1b}$$

where $q_2 = d, s$ and $q_3 = b, h$. There are two interesting features of the current $J_Y{}^\mu$ to note. First, $J^\mu_{Y,\text{leptonic}}$ contains no neutrino or neutral heavy lepton term and analogously, $J^\mu_{Y,\text{ hadronic}}$ contains no u, c, t, or g term. Secondly, J_Y^{μ} is purely axial-vector and hence neutral current processes which involve only Y-exchange are parity-conserving.

Neutral current neutrino reactions are due to the current J_Z^{μ};

$$J^{\mu}_{Z,\ \text{leptonic}} = \sum_{\ell,\ L^-,\ L^0} \left[\bar{\nu}_{\ell L}\gamma^{\mu}\nu_{\ell L} + \bar{L}_R^{\,0}\gamma^{\mu}L_R^{\,0} + \left(\frac{3}{2}w - \frac{1}{2}\right)(\bar{\ell}\gamma^{\mu}\ell + \bar{L}^-\gamma^{\mu}L^-)\right] \quad (3.2a)$$

where $(\ell-,\ L^-,\ L^0) = (e^-,\ E^-,\ E^0)$, etc. and

$$\begin{aligned} J^{\mu}_{Z,\,\text{hadronic}} &= (1-w)\bar{u}_L\gamma^{\mu}u_L - w\bar{u}_R\gamma^{\mu}u_R + \tfrac{1}{2}(w-1)\bar{d}\gamma^{\mu}d \\ &+ (1-w)\bar{t}_R\gamma^{\mu}t_R - w\bar{t}_R\gamma^{\mu}t_R + \tfrac{1}{2}(w-1)\bar{b}\gamma^{\mu}b \\ &+ (u,\ t,\ d,\ b) \longleftrightarrow (c,\ g,\ s,\ h) \quad . \end{aligned} \quad (3.2b$$

Thus the effective Lagrangian for $\nu(\bar{\nu})$-nucleon neutral current reactions is

$$\begin{aligned} \mathscr{L}_{\text{eff, n.c.}}(\nu(\bar{\nu}) + \text{nucleon}) &= \left(\frac{1}{\sqrt{3}}\sqrt{g^2 + g'^2/3}\right)^2 \left[\bar{\nu}_L\gamma_{\alpha}\nu_L\right]\left(\frac{1}{m_Z^{\ 2}}\right) J^{\alpha}_{Z,\,\text{hadronic}} \\ &= (1+\ell)\frac{G_F}{\sqrt{2}}\left[\bar{\nu}\gamma_{\alpha}(1-\gamma_5)\nu\right] J_{Z,\ \text{hadronic}}^{\alpha} \end{aligned} \quad (3.3)$$

For $r \neq 0$ the neutral current couplings are given by Eq. (2.69). It is actually simpler to work with the old currents and nondiagonal mass matrix:

$$\mathscr{L}_{\text{eff, n.c.}} = \sum_{i,j=Z,\,Y} c_i J_i^{\ \mu}(m^2)^{-1}_{ij}\, c_j J_{j\mu} \quad (3.4)$$

where
$$c_Z = \frac{1}{\sqrt{3}}\sqrt{g^2 + g'^2/3} \tag{3.5}$$

and
$$c_Y = \frac{g}{2} \tag{3.6}$$

and m^2 is the undiagonalized Z, Y sector of the vector boson mass matrix:

$$m^2 = \begin{pmatrix} \rho & -\sigma \\ -\sigma & \tau \end{pmatrix} \tag{3.7}$$

(i.e. $m^2_{ZZ} = \rho$, $m^2_{YY} = \tau$, etc.

In the limit $r \to \infty$ with the identification $w = (4/3)\sin^2\theta_W$ the neutral current interaction (3.4) becomes

$$\lim_{r\to\infty} \mathscr{L}_{\text{eff, n. c.}} = \frac{g^2}{m_W^2}\left(\left[3(1-w)\right]^{-\frac{1}{2}} J_Z{}^{\mu}, \tfrac{1}{2} J_Y{}^{\mu}\right)$$

$$\begin{pmatrix} \frac{3}{4}(1-w) & -\frac{1}{4}\sqrt{3(1-w)} \\ -\frac{1}{4}\sqrt{3(1-w)} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} \left[3(1-w)\right]^{-\frac{1}{2}}(J_Z)_{\mu} \\ \frac{1}{2}(J_Y)_{\mu} \end{pmatrix} \tag{3.8}$$

$$= \frac{G_F}{\sqrt{2}}\tilde{J}_Z{}^{\mu}\tilde{J}_{Z\mu}$$

where $\tilde{J}_Z{}^{\mu}$ is the neutral current in the Weinberg-Salam model;

$$\tilde{J}_Z{}^{\mu} = \tfrac{1}{2}(J_Z{}^{\mu} - \tfrac{1}{2}J_Y{}^{\mu})$$
$$= J_3{}^{\mu} - \sin^2\theta\ J_{em}{}^{\mu} \tag{3.9}$$

Thus in this limit the neutral current interaction assumes precisely the form of the Weinberg-Salam model. Observe that although the minimal $SU(3) \otimes U(1)$ theory is essentially a three-parameter theory (aside from the various fermion masses and CP-violating parameters), depending on ℓ, w, and δ, its four-parameter generalization reduces to the old one-parameter $SU(2) \otimes U(1)$ model in this limit; that is, as $r \to \infty$, all dependence on ℓ and δ disappears.

Returning to Eq. (3.4) with finite r, we calculate the effective Lagrangian for neutral current neutrino nucleon interactions to be

$$\mathscr{L}_{\text{eff, n.c.}}\left(\nu(\bar{\nu}) + \text{nucleon}\right) = (1+\ell)\frac{G_F}{\sqrt{2}}\left[\bar{\nu}\gamma_\alpha(1-\gamma_5)\nu\right] \times \\ \times \left[\bar{a}_{uL}\bar{u}_L\gamma^\alpha u_L + a_{uR}\bar{u}_R\gamma^\alpha u_R + \bar{a}_{dL}\bar{d}_L\gamma^\alpha d_L + \bar{a}_{dR}\bar{d}_R\gamma^\alpha d_R\right] \tag{3.10}$$

where

$$a_{uL} = \left(\frac{4\ell + r}{4\ell + \ell r + r}\right)(1 - w) \tag{3.11a}$$

$$a_{uR} = -\left(\frac{4\ell + r}{4\ell + \ell r + r}\right) w \tag{3.11b}$$

$$a_{dL} = -\tfrac{1}{2}\left[\frac{(4\ell + r)(1 - w) + r}{4\ell + \ell r + r}\right] \tag{3.11c}$$

and

$$a_{dR} = \tfrac{1}{2}\left[\frac{-(4\ell + r)(1 - w) + r}{4\ell + \ell r + r}\right] \quad . \tag{3.11d}$$

In Eq. (3.10) we have dropped non-valence quark currents since they make a very small contribution. (Note that the limits $\ell \to 0$ and $r \to 0$ do not commute; in order to specialize to the minimal version one must fix $\ell \neq 0$ and let $r \to 0$.)

For leptonic neutral current interactions with neutrinos we can write the effective Lagrangian for $\nu(\bar{\nu}) + \ell \to \nu(\bar{\nu}) + \ell$ as

$$\mathscr{L}_{\text{eff}}(\nu(\bar{\nu})\ell \to \nu(\bar{\nu})\ell) = \frac{G_F}{\sqrt{2}}\left[\bar{\nu}\gamma_\alpha(1-\gamma_5)\nu\right]\left[\bar{\ell}\gamma^\alpha(g_V - g_A\gamma_5)\ell\right] \tag{3.12}$$

where ℓ = e, μ, etc. (not to be confused with the parameter ℓ).

In the minimal version of the model only the Z-boson exchange contributes to this effective Lagrangian. Since the ℓ (=e, μ, etc.) part of $J_Z{}^\alpha$ is purely vecto

$$g_A = 0 \quad ; \tag{3.13a}$$

for g_V we find

$$g_V = \frac{3}{2}(1 + \ell)(w - \frac{1}{3}) \quad . \tag{3.13b}$$

In the extended version of the model the neutral current interaction arises from the exchange of the two massive vector bosons Z_1 and Z_2 coupling to (orthogonal) linear combinations of $(1/\sqrt{3})\sqrt{g^2 + g'^2/3}\, J_Z{}^\mu$ and $(g/2)J_Y{}^\mu$.

In this generalized version

$$g_A = -\frac{(1+\ell)r}{2(4\ell + \ell r + r)} \tag{3.14a}$$

and

$$g_V = \frac{3}{2}\frac{(1+\ell)(4\ell + r)(w - 1/3)}{(4\ell + \ell r + r)} \quad . \tag{3.14b}$$

We proceed to consider the predictions of both versions of the $SU(3) \otimes U(1)$ model for deep inelastic inclusive, elastic $\nu(\bar{\nu})p \rightarrow \nu(\bar{\nu})p$, and leptonic, neutral current reactions, as well as atomic parity violation.

1. Inclusive Neutral Current Reactions

We consider first the deep inelastic neutral current reactions $\nu(\bar{\nu})N \rightarrow \nu(\bar{\nu})X$ where N denotes an isoscalar nucleon target. For $r = 0$ we find in the valence quark approximation, for energies below threshold for heavy quark and heavy lepton production (which necessarily occur together in this model)

$$R^{\nu N} \equiv \frac{\sigma_{nc}^{\nu N}}{\sigma_{cc}^{\nu N}} = \frac{1}{3}(1 + \ell)^2 \left[\frac{5}{4} w^2 - 2w + 1\right] \qquad (3.15)$$

and

$$R^{\bar{\nu} N} \equiv \frac{\sigma_{nc}^{\bar{\nu} N}}{\sigma_{cc}^{\bar{\nu} N}} = (1 + \ell)^2 \left[\frac{5}{4} w^2 - w + \frac{1}{2}\right] \qquad (3.16)$$

where nc(cc) denotes neutral (charged) current. Sea quark contributions do not alter these ratios significantly. For example, with the values $\ell = 0.18$ and $w = 0.25$ which will be seen to fit the neutral current data, Eqs. (3.15) and (3.16) yield $R^{\nu N} = 0.27$ and $R^{\bar{\nu} N} = 0.45$. In comparison, a calculation which takes into account sea quark contributions and uses an SU(3) symmetric sea with $\int_0^1 x\left[\bar{u}(x) + \bar{d}(x)\right]dx \Big/ \int_0^1 x\left[u(x) + d(x)\right]dx = 0.1$ yields $R^{\nu N} = 0.29$ and $R^{\bar{\nu} N} = 0.45$. At energies beyond the respective thresholds, the charged current cross sections in the denominators of $R^{\nu N}$ and $R^{\bar{\nu} N}$ also receive contributions from the heavy lepton and heavy quark production processes $\nu_\mu + d \to M^- + t'$ and $\bar{\nu}_\mu + u \to M^+ + b$. (There is also a small contribution from the sea quark reaction $\nu_\mu + s \to M^- + g'$.) A certain fraction of these reactions will simulate neutral current and regular charged current $(\nu(\bar{\nu})N \to \mu^\mp + X)$ processes. As was discussed above, although the thresholds for these reactions are at about $E \simeq 100$ GeV, depending on the heavy lepton and heavy quark masses, they will appear to occur at lower visible incident energy. Hence one should determine their contributions to the $\sigma_{nc}^{(\nu,\bar{\nu})N}$ and $\sigma_{cc}^{(\nu,\bar{\nu})N}$. These are in fact negligible, primarily because the flux-averaged cross sections are so small. Specifically, for $E > 100$ GeV, with the quadrupole triplet beam, the event rates are in the ratios $R(\nu_\mu + N \to M^- + X)/R(\nu_\mu + N \to \mu^- + X) \simeq 2.3 \times 10^{-2}$ and $R(\bar{\nu}_\mu + N \to M^+ + X)/R(\bar{\nu}_\mu + N \to \mu^+ + X) \simeq 3.6 \times 10^{-2}$.[20] Of the events in which an $M^\pm$ is produced, only about two thirds will simulate neutral current reactions (see section IV and Fig. 22).

Finally, only a fraction of the events from the $M^{\pm}$ production reactions will appear to have E_{vis} in the range characterizing the data used by the HPWF, CF, and CERN experiments to determine the ratios $R^{\nu N}$ and $R^{\bar{\nu}N}$. At most, then, the corrections to the theoretical predictions for $R^{\nu N}$ and $R^{\bar{\nu}N}$ resulting from heavy lepton production are of the order of a few percent. For this reason we have not included them in Eqs. (3.15) and (3.16).

There are two free parameters, ℓ and w, at our disposal, so Eqs. (3.15) and (3.16) cannot by themselves be used to test the theory, but rather only to determine ℓ and w. In Fig. 4 we show the neutral to charged current cross section ratios $R^{\nu N}$ and $R^{\bar{\nu}N}$, Eqs. (3.15) and (3.16), as functions of ℓ and w. The three curves are for $(1+\ell)^2$ = (a) 1.0, (b) 1.2 (i.e. $\ell \simeq 0.095$) and (c) 1.4 (i.e. $\ell \simeq 0.18$); along each curve w increases from w = 0 to w = 1 in steps of 0.1. The data points are from the HPWF[21], CF[22], CERN Gargamelle (GGM)[23] and CERN CDHS[23] experiments. The specific experimental values are as follows: HPWF: $R^{\nu N} = 0.29 \pm 0.04$, $R^{\bar{\nu}N} = 0.39 \pm 0.10$; CF: $R^{\nu N} = 0.24 \pm 0.02$, $R^{\bar{\nu}N} = 0.34 \pm 0.09$, CERN GGM: $R^{\nu N} = 0.26 \pm 0.04$, $R^{\bar{\nu}N} = 0.39 \pm 0.06$; and CERN CDHS: $R^{\nu N} = 0.29 \pm 0.01$, $R^{\bar{\nu}N} = 0.35 \pm 0.03$. All of these ratios except the CERN GGM ones are raw, with no correction for the events excluded by the respective cuts in E_H ($E_H > 4$, 12, 1, and 12 GeV for the HPWF, CF, GGM, and CDHS experiments, respectively.) It should be stressed that these corrections require assumptions about the y distribution and hence depend on the particular (V, A) form assumed for the neutral current. One observes from Fig. 4 that all three values of ℓ yield reasonable fits to the data. The $\ell = 0$ curve is shown only for reference; values of ℓ too close to 0 are in fact excluded. The reason is that such values would yield intolerably low masses for the Y and $X_{1,2}$ vector bosons. As is evident from Fig. 1, if one takes the reasonable values $w = 0.25$, $\delta = 0.01$ then the conditions that the mass of X_1 (the lighter of the two $X_{1,2}$ vector bosons) be greater than 20 (40) GeV impose the mild requirement that $\ell > 0.046$ (0.135). The Y boson mass is roughly twice that of the $X_{1,2}$ bosons (which are roughly equal since $\delta \ll 1$). If one chooses $\ell = 0.18$ the optimal value of w is $w \simeq 0.25$. These are the rough values of ℓ and w which we shall use for the $r = 0$ version of the model.

In the $r \neq 0$ version of the model $R^{\nu N}$ and $R^{\bar{\nu} N}$ are given in the same approximation as was used above, by the obvious formulas

$$R^{\nu N} = \frac{1}{4}\left[(a_{uL}^2 + a_{dL}^2) + \frac{1}{3}(a_{uR}^2 + a_{dR}^2)\right] \tag{3.17a}$$

$$R^{\bar{\nu} N} = \frac{3}{4}\left[\frac{1}{3}(a_{uL}^2 + a_{dL}^2) + (a_{uR}^2 + a_{dR}^2)\right] \tag{3.17b}$$

where $a_{uL,uR}$ and $a_{dL,dR}$ are listed in Eq. (3.11). In Fig. 5 we show the variation in $R^{\nu N}$ and $R^{\bar{\nu} N}$ for $\ell = 0.18$ and w = (a) 0.2 and (b) 0.3, as a function of r as r ranges from zero to infinity. Qualitatively, for $0 \lesssim r \lesssim 1$ $R^{\bar{\nu} N}$ decreases while $R^{\nu N}$ increases very slightly. From later considerations of atomic parity violation and trimuon production we will restrict r to the range $0 \leq r \leq 0.1$.

For comparison, the Weinberg-Salam model (understood to include only Higgs doublets) predicts, in the same valence quark approximation, that

$$R^{\nu N}_{SU(2)\otimes U(1)} = \frac{1}{2} - \sin^2\theta + \frac{20}{27}\sin^4\theta \tag{3.18a}$$

and

$$R^{\bar{\nu} N}_{SU(2)\otimes U(1)} = \frac{1}{2} - \sin^2\theta + \frac{20}{9}\sin^4\theta \tag{3.18b}$$

where θ_W is the usual mixing angle. As was shown in general via Eqs. (3.8) and (3.9) these ratios are precisely the $r = \infty$ limit of the ratios in our generalized interpolating model, Eqs. (3.17a) and (3.17b), with the identification $w = (4/3)\sin^2\theta_W$. Numerically for $\sin^2\theta = 0.3$, one finds $R^{\nu N} = 0.26$ while $R^{\bar{\nu} N} = 0.40$. A calculation[24] of these ratios including sea quark contributions, with a 10% sea content as defined above, gives $R^{\nu N} = 0.31$ and $R^{\bar{\nu} N} = 0.42$ for $\sin^2\theta = 0.3$.

2. Elastic νp and $\bar{\nu}p$ Reactions

From Eq. (3.3) one can compute the cross sections for the elastic reactions $\nu(\bar{\nu})p \rightarrow \nu(\bar{\nu})p$.[24] The general form of the cross section is

$$\frac{d\sigma}{dQ^2}(\nu(\bar{\nu})p \rightarrow \nu(\bar{\nu})p) = \frac{G_F^{\;2}(1+\ell)^2}{32\pi m_N^{\;2}E^2}\Big[8m_N^{\;2}Q^2W_1(Q^2)$$

$$+\left\{(Q^2-4m_NE)^2 - Q^2(Q^2+4m_N^{\;2})\right\}W_2(Q^2) \pm 2Q^2(Q^2-4m_NE)W_3(Q^2)\Big] \tag{3.19}$$

where E is the incident neutrino energy, m_N is the nucleon mass, and $W_i(Q^2)$, i = 1, 2, 3 are the appropriate structure functions. These W_i can be determined by keeping only the isovector and isoscalar parts of the neutral current J_Z^μ and using SU(3) symmetry to relate these to the correspondi[ng] parts of the electromagnetic and charged weak currents. In doing this we mak[e] the reasonable assumption that the s, c, and heavier quark content in the nucleon is negligible. In terms of the isovector and isoscalar currents $J_3^{V,A}$ and $J_0^{V,A}$ (here (3) and (0) are U(2) indices) we thus write

$$J_Z^{\;\mu} = \alpha(J_3^{\;V})^\mu - \beta(J_3^{\;A})^\mu + \frac{1}{3}\gamma(J_0^{\;V})^\mu - \delta(J_0^{\;A})^\mu \quad . \tag{3.20}$$

The normalization is indicated by the form of the electromagnetic current:

$$J^\mu_{em} = (J_3^{\;V})^\mu + \frac{1}{3}(J_0^{\;V})^\mu \quad . \tag{3.21}$$

For our model

$$\alpha = 1 - \frac{3}{2} w \tag{3.22}$$

$$\beta = \frac{1}{2} \tag{3.22}$$

$$\gamma = - \frac{3}{2} w \tag{3.22}$$

and

$$\delta = \frac{1}{2} \tag{3.22}$$

For comparison, in the Weinberg-Salam model $\alpha = 1 - 2\sin^2\theta_W$, $\beta = 1$, $\gamma = -2\sin^2\theta_W$ and $\delta = 0$.

The structure functions W_i can be expressed in terms of the Sachs electric and magnetic form factors, G_E and G_M and the axial form factor G_A, by the relations

$$W_1 = (1 + \tau)(G_A{}^0)^2 + \tau(G_M{}^0)^2 \tag{3.23a}$$

$$W_2 = (G_A{}^0)^2 + \frac{\left[(G_E{}^0)^2 + \tau(G_M{}^0)^2\right]}{(1 + \tau)} \tag{3.23b}$$

and

$$W_3 = -2G_M{}^0 G_A{}^0 \tag{3.23c}$$

where

$$\tau = \frac{Q^2}{4m_N{}^2} \tag{3.24}$$

$$G_E{}^0(Q^2) = \frac{1}{2}(\alpha + \gamma)G_E(Q^2) \tag{3.25a}$$

$$G_M{}^0(Q^2) = \frac{1}{2}\left[\alpha + \frac{(1 + \mu_p + \mu_n)}{(1 + \mu_p - \mu_n)}\gamma\right]G_M(Q^2) \tag{3.25b}$$

and

$$G_A{}^0(Q^2) = \frac{1}{2}(\beta + \delta\epsilon)G_A(Q^2) \tag{3.25c}$$

where ϵ is the ratio of the isoscalar to the isovector axial vector form factors. In the Cabibbo theory, $\epsilon = (3F - D)/(D + F)$ where F and D are the antisymmetric and symmetric reduced matrix elements for octet currents. Inserting the measured values[25] of F and D, we find $\epsilon \simeq .368$.

Numerically, $\mu_p = 1.793$ and $\mu_n = -1.913$. The Sachs and axial-vector form factors are taken here as

$$G_E(Q^2) = \frac{G_M(Q^2)}{(1 + \mu_p - \mu_n)} = \frac{1}{(1 + Q^2/0.71)^2} \tag{3.26a}$$

$$G_A(Q^2) = \frac{1.24}{(1 + Q^2/m_A{}^2)^2} \quad . \tag{3.26b}$$

with $m_A = 0.9$ GeV. For the reaction $(\nu, \bar{\nu})n \rightarrow (\nu, \bar{\nu})n$, $\alpha \rightarrow -\alpha$ and $\beta \rightarrow -\beta$.

In order to compare the predictions of the $SU(3) \otimes U(1)$ model with data available from the Harvard-Pennsylvania-Wisconsin[26] (HPW) and Columbia-Illinois-Rockefeller[27] (CIR) experiments at Brookhaven we have computed the elastic $\nu(\bar{\nu})p$ cross sections and folded them with the Brookhaven neutrino flux spectrum.[24] We consider first the ratios

$$R_{el}^{\nu p} = \frac{\sigma(\nu p \rightarrow \nu p)}{\sigma(\nu n \rightarrow \mu^- p)} \tag{3.27}$$

and

$$R_{el}^{\bar{\nu} p} = \frac{\sigma(\bar{\nu} p \rightarrow \bar{\nu} p)}{\sigma(\bar{\nu} p \rightarrow \mu^+ n)} \quad . \tag{3.28}$$

In calculating these ratios we have incorporated the cuts appropriate for the HPW and CIR experiments; these are the requirement $0.3\ \mathrm{GeV}^2 < Q^2 < 0.9\ \mathrm{GeV}^2$ for the HPW data, and the requirements that $|\vec{p}|_{lab} > 0.5$ GeV and $\theta_{lab} > 25^{\circ}$ for the recoil proton in the CIR data. Corrections due to the fact that the target is a nucleus rather than a free proton are negligible.[28]

In Fig. 6 we present our results for the ratio $R_{el}^{\nu p}$ as a function of w, for r = 0, using two values of ℓ, viz. $\ell = 0$ and $\ell = 0.18$ (and the value $m_A = 0.9$ GeV). The former value is shown for reference only; it is actually excluded, for the reasons given above. The curves shown incorporate the HPW cut; the imposition of the CIR cuts yields very similar curves which are omitted for clarity. The data consists of a shaded band representing the HPW measurement[26] $R_{el}^{\nu p} = 0.17 \pm 0.05$ and a single data point representing the CIR result[27] $R_{el}^{\nu p} = 0.23 \pm 0.09$. Again, for clarity, the CIR point is positioned at a particular value of w; the reader may imagine it to be swept across horizontally to form a second shaded band. In order to show the sensitivity of the theoretical calculation to the assumed value of the axial vector mass parameter m_A, we plot in Fig. 7 $R_{el}^{\nu p}$ for $\ell = 0.18$ and three values of m_A, 0.8, 0.9, and 1.0 GeV. From Fig. 6 we observe that if one takes $\ell = 0.18$, good fits to the data can be obtained with $w = 0.1 - 0.3$. In this range of w, $\partial R_{el}^{\nu p}/\partial w < 0$ so that the same quality of fit can be achieved with either slightly lower or slightly higher values of both ℓ and w.

In Fig. 8 we present our predictions for $R_{el}^{\bar{\nu}p}$ as a function of w, again for $r = 0$, $\ell = 0$ and $\ell = 0.18$. The data for this graph comes only from the HPW measurement, $R_{el}^{\bar{\nu}p} = 0.2 \pm 0.1$, shown as a shaded band. The HPW cut has accordingly been imposed in the Q^2 integration. Again it is of interest to ascertain the dependence of $R_{el}^{\bar{\nu}p}$ upon m_A; Fig. 9 provides an illustration of this. It is evident from Fig. 8 that for $\ell = 0.18$ the range $w \simeq 0.1-0.25$ yields values of $R_{el}^{\bar{\nu}p}$ in satisfactory agreement with the measured value. Of course the fit is not unique; in particular since $\partial R_{el}^{\bar{\nu}p}/\partial w < 0$ in this range of w (as was the case for $R_{el}^{\nu p}$) an increase in ℓ can be compensated by an increase in w to produce the same value of $R_{el}^{\bar{\nu}p}$.

In contrast to the quantities $R_{el}^{\nu p}$ and $R_{el}^{\bar{\nu}p}$, the ratio of flux averaged neutral current cross sections $\sigma(\bar{\nu}p \to \bar{\nu}p)/\sigma(\nu p \to \nu p)$ is independent of ℓ (for $r = 0$). Aside from the ambiguity due to the imperfectly measured quantity m_A, which is present for any non-vector theory, there is thus only one adjustable parameter, w, in the minimal version of our model which enters into the calculation for this ratio. Fig. 10 shows our prediction for $\sigma(\bar{\nu}p \to \bar{\nu}p)/\sigma(\nu p \to \nu p)$ (with the HPW cuts) as a function of w, for $r = 0$ and $m_A = 0.9$ GeV, in comparison with the HPW measurement,[26] $\sigma^{\bar{\nu}p}/\sigma^{\nu p} = 0.4 \pm 0.2$.

We proceed to consider the differential cross sections for the reactions $\nu(\bar{\nu})p \to \nu(\bar{\nu})p$, using $\ell = 0.18$ and $m_A = 0.9$ GeV. In Fig. 11 we present plots of $d\sigma/dQ^2(\nu p \to \nu p)$, appropriately flux-averaged over the Brookhaven neutrino spectrum and compared with the HPW data. The two curves correspond to w = (a) 0.1 and (b) 0.2; both values yield satisfactory fits to the magnitude and shape (in Q^2) of the differential cross section. On the same graph is shown the HPW data on the quasielastic reaction $\nu n \to \mu^- p$ together with the prediction of the Cabibbo theory. Fig. 12 shows the analogous plots of $d\sigma/dQ^2(\bar{\nu}p \to \bar{\nu}p)$ for the same values of r, ℓ, and w, compared with the HPW data on this reaction. From this figure one can also s the relative magnitudes of the measured $d\sigma/dQ^2(\bar{\nu}p \to \nu p)$ and the Cabibbo theor prediction for $d\sigma/dQ^2(\bar{\nu}p \to \mu^+ n)$.

On the basis of this analysis we conclude that, roughly speaking, in the minimal version of our model the same values of ℓ and w yield good fits to both the deep inelastic reactions $\nu(\bar{\nu})N \to \nu(\bar{\nu})X$ and the elastic reactions $\nu(\bar{\nu})p \to \nu(\bar{\nu})p$. For example, the values (ℓ, w) = (0.18, 0.2) or (0.18, 0.25) are reasonably successful in accounting for both sets of data. Again, it is of interest to compare this fit with the one achieved by the Weinberg-Salam model. For $\sin^2\theta_W = 0.3$ and $m_A = 0.9$ GeV the latter model predicts that $R_{el}^{\nu p} = 0.11$ and $R_{el}^{\bar{\nu}p} = 0.13$. As is the case with the $SU(3) \otimes U(1)$ model both of these ratios are slightly smaller than, but in satisfactory agreement with, the measured values. For the same values of $\sin^2\theta_W$ and m_A the Weinberg-Salam model predicts for the ratio of antineutrino to neutrino cross sections (each flux-averaged and containing the HPWF cuts) $\sigma(\bar{\nu}p \to \bar{\nu}p)/\sigma(\nu p \to \nu p) = 0.57$, slightly higher than the HPWF measurement,[26] 0.4 ± 0.2. With regard to the differential cross sections, the prediction for $d\sigma^{\nu p}/dQ^2$ and $d\sigma^{\bar{\nu}p}/dQ^2$ are in satisfactory accord with the data, although the former is a little low, as reflected in the integrated quantity $R_{el}^{\nu p}$. The fit can be improved by taking $\sin^2\theta_W$ to be slightly smaller, say 0.3. We refer the reader to Ref. 24 for further details.

The generalized version of the $SU(3) \otimes U(1)$ model with nonzero r is able to achieve better agreement with the deep inelastic and elastic data. The first part of this statement was demonstrated before in Fig. 5. The second part of the statement is evident from Figs. 13 and 14 showing $R_{el}^{\nu p}$ and $R_{el}^{\bar{\nu}p}$ calculated with $r = 0.1$, $\ell = 0.18$, and $m_A = 0.9$ GeV, with the HPW cuts included. Qualitatively, the minima of the curves $R_{el}^{\nu p}$ and $R_{el}^{\bar{\nu}p}$ as functions of w move to somehwat smaller w and the magnitudes of both ratios increase. These changes in $R_{el}^{\nu p}$ and $R_{el}^{\bar{\nu}p}$ are also reflected in the differential cross sections, which for w in the region of 0.2 increase somewhat (but do not change shape appreciably). We only exhibit the results for $r = 0.1$ because larger values of r will later be shown to lead to sizeable atomic parity violation and undesirable suppression of trimuon production in neutrino reactions.

3. Leptonic Neutral Currents

There are three purely leptonic neutral current neutrino reactions on which there is presently experimental data, $\nu_\mu(\bar{\nu}_\mu)e \rightarrow \nu_\mu(\bar{\nu}_\mu)e$ and $\bar{\nu}_e e \rightarrow \bar{\nu}_e e$. These reactions are ideal in the sense that they are completely calculable and free of hadronic complications, but suffer from the fact that their rates are very small and the experimental cuts required to isolate candidate events from background are extremely severe.

For the reaction $\nu_\mu e \rightarrow \nu_\mu e$ the differential cross section is

$$\frac{d\sigma}{dE} = \frac{G_F{}^2 m_e}{2\pi}\left[(g_V + g_A)^2 + (g_V - g_A)^2\left(1 - \frac{E}{E_\nu}\right)^2 + \left(\frac{m_e E}{E_\nu{}^2}\right)(g_A{}^2 - g_V{}^2)\right] \tag{3.29}$$

where E_ν is the incident neutrino energy in the electron (laboratory) frame, E is the lab energy of the scattered electron, and the coupling constants g_V and g_A are given by Eq. (3.13). Eq. (3.29) also applies to the reaction $\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$ with the replacement of g_A by $-g_A$. Since $g_A = 0$ in the minimal form of the SU(3) $\otimes$ U(1) model there follows the prediction that $\sigma(\nu_\mu e \rightarrow \nu_\mu e) = \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e)$. In the third leptonic reaction of interest, $\bar{\nu}_e e \rightarrow \bar{\nu}_e e$, there are two graphs which contribute to the total amplitude, the neutral current t-channel graph and the charged current annihilation graph. The cross section for this reaction is given by Eq. (3.29) with the replacements $g_V \to g_V + 1$, $g_A \to g_A + 1$ and $g_A \to g_A + 1$.

In Fig. 15 we show a plot of g_V and g_A for $\ell = 0.18$ as functions of w and r. Since g_A is independent of w the ranges of (g_v, g_A) are horizontal line segments. As $r \to \infty$, g_V approaches $(3/2)(w - 1/3)$, which is just its value in the minimal model with $\ell = 0$, and g_A approaches $-\frac{1}{2}$; in both cases the ℓ-dependence drops out in the limit. If one takes $r \to \infty$ and, identifies, as was done previously, $w = (4/3)\sin^2\theta_W$, one reproduces the (g_V, g_A) values of the Weinberg-Salam model: $g_V = 2\sin^2\theta_W - \frac{1}{2}$, $g_A = -\frac{1}{2}$.

In Figs. 16 and 17 we show (as shaded areas) the regions in the (g_V, g_A) plane allowed by the Gargamelle experiment at CERN and the Reines experiment using a nuclear reactor. In the Gargamelle experiment, on the basis of three observed events a cross section

$$\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) = \left(1.0^{+2.1}_{-0.9}\right) \times 10^{-42}\left(\frac{E_{\bar{\nu}}}{\mathrm{GeV}}\right) \mathrm{cm}^2 \tag{3.30}$$

was deduced.[29] No candidates for the reaction $\nu_\mu e \to \nu_\mu e$ were bound, implying the 90% confidence level (CL) bound

$$\sigma(\nu_\mu e \to \nu_\mu e) \lesssim 2.6 \times 10^{-42}\left(\frac{E_\nu}{\mathrm{GeV}}\right) \mathrm{cm}^2 \quad . \tag{3.31}$$

The cuts made in this experiment are that the recoil electron energy E_e and angle θ_e (in the laboratory frame) satisfy $0.3\ \mathrm{GeV} < E_e < 2\ \mathrm{GeV}$ and $\theta_e < 5^\circ$. The Aachen-Padova spark chamber experiment at CERN[30] has also measured the reactions $\nu_\mu(\bar{\nu}_\mu)e \to \nu_\mu(\bar{\nu}_\mu)e$. However, since no final numbers have been published by this experiment at the present time, we have not included it in our comparison.

From Eq. (3.29) one can see that the cross section for the reactions $\nu_\mu(\bar{\nu}_\mu)e \rightarrow \nu_\mu(\bar{\nu}_\mu)e$ is invariant under the interchange $g_V \rightarrow -g_V$, $g_A \rightarrow -g_A$, which leads to a sign ambiguity in the determination of these coupling constants from the data. Furthermore, since the last term, proportional to $(m_e E/E^2_{\nu,\bar{\nu}})$ is negligible $(m_e/E_{\nu,\bar{\nu}} \lesssim 10^{-3})$ in the Gargamelle experiment the cross section is also invariant under the transformation $g_V \rightarrow \pm g_A$, $g_A \rightarrow \pm g_V$. The allowed regions are thus elliptical bands with major axes lying approximately along the directions 135° and 45° for the reactions $\nu_\mu e \rightarrow \nu_\mu e$ and $\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$, respectively.

The third reaction on which there is data, $\bar{\nu}_e e \rightarrow \bar{\nu}_e e$, does not share these symmetries in general, and thereby eliminates the $g_V \rightarrow -g_V$, $g_A \rightarrow -g_A$ ambiguity. However, again, to the extent that the last term in the cross section is small (it contributes only a few percent relative to the other terms in the experiment of Reines et al.[31]) there is still a symmetry under the interchange $g_V \rightarrow g_A$, $g_A \rightarrow g_V$. This remaining approximate symmetry is evident in Fig. 17. The two sections of elliptical bands correspond to the two bins in scattered electron lab energy, $1.5 < (E/\text{MeV}) < 3$, and $3 < (E/\text{MeV}) < 4.5$.

In Figs. 16 and 17 we have shown a point $(g_V, g_A) = (-0.24, 0)$ computed for a representative choice of parameters in the minimal

SU(3) $\otimes$ U(1) model, ℓ =0.18 , w = 0.2. This point is well within the allowed region of the Gargamelle data. The predicted point also agrees well with the lower energy band in the Reines data and is about one standard deviation from the higher energy band; at the 90% CL it is in good agreement with the region allowed by the intersection of these two bands.

Using Fig. 15 in conjunction with Figs. 16 and 17 we conclude that, if one chooses $\ell \simeq 0.18$, $w \simeq 0.2$ and $r < 0.1$, then the leptonic neutral current measurements are best fitted by the value $r = 0$.

4. Atomic Parity Violation

The parity-violating weak neutral current interaction between an electron and nucleon can be written as

$$\mathscr{H}_{\text{eff}}(e + N \to e + N) = \frac{G_F}{\sqrt{2}} \left\{ \left[\bar{e}\gamma_\alpha e\right]\left[\bar{N}\gamma^\alpha\gamma_5\left(\frac{f_0 + f_1\tau_3}{2}\right) N\right] + \left[\bar{e}\gamma_\alpha\gamma_5 e\right]\left[\bar{N}\gamma^\alpha\left(\frac{g_0 + g_1\tau_3}{2}\right) N\right]\right\} \tag{3.32}$$

where the constants $f_{0,1}$ and $g_{0,1}$ can be determined from Eqs. (3.1), (3.2) or (3.5). We shall consider here the case of a heavy atom which is relevant to the present experiments (on the atom Bi^{209}). In this case the axial-vector nucleon current gives a negligible contribution relative to the vector nucleon current because, in the nonrelativistic approximation, the former is proportional to the spin of the nucleus, whereas the latter is proportional to $\left[\frac{1}{2}(g_0 + g_1)Z + \frac{1}{2}(g_0 - g_1)N\right]\bar{\psi}\gamma_0\psi$, where Z and N denote respectively the number of protons and neutrons in the atom. Hence the nucleon axial-vector current term is of order unity, whereas the nucleon vector current term is enhanced by a factor of order Z or N. For a heavy atom Eq. (3.32) can thus be rewritten as

$$\mathscr{H}_{PV} = \frac{G_F}{\sqrt{2}} \frac{Q_{wk}(Z, N)}{2} \left[e^{\dagger} \gamma_5 e\right]\left[N^{\dagger} N\right] \tag{3.33}$$

where, following the conventional usage,[12] we define

$$Q_{wk}(Z, N) = (Z + N)g_0 + (Z - N)g_1 \quad . \tag{3.34}$$

This quantity serves to measure the dominant contribution to parity violation in heavy atoms.

In the minimal version of the $SU(3) \otimes U(1)$ model, $Q_{wk} = 0$; i.e. the dominant portion of the parity-violating amplitude vanishes. This is a consequence of the fact that $J_Y{}^{\mu}$ is purely axial-vector, and hence parity-conserving, while the electron part of $J_Z{}^{\mu}$ is purely vector. Specifically, the parity-violating neutral current amplitude in the minimal model arising from Z-boson exchange is (in terms of elementary fields) $-(G_F/\sqrt{2})(1 + \ell)[\bar{e}\gamma_{\mu} e]\,[\bar{u}\gamma^{\mu}\gamma_5 u]$. Although the experiments on the heavy atom Bi^{209} are not sensitive to this term, the forthcoming experiments[11] planning to search for parity violation in hydrogen will be sensitive to it and will hopefully serve as a test of this theory.

In the generalized version of the model we find

$$g_0 = \frac{(1+\ell)r}{(4\ell+\ell r+r)}\left(-\frac{3}{2}w\right) \qquad (3.35a)$$

and

$$g_1 = \frac{(1+\ell)r}{(4\ell+\ell r+r)}\left(1-\frac{3}{2}w\right) \quad . \qquad (3.35b)$$

From Eq. (3.34) we thus have

$$Q_{wk}(Z,\ N) = \frac{(1+\ell)r}{(4\ell+\ell r+r)}\left[Z(1-3w)-N\right] \quad . \qquad (3.36)$$

The quantity $-Q_{wk}$ is plotted in Fig. 18 for $\ell = 0.18$, w = (a) 0.2 and (b) 0.3, and $(Z, N) = (83,\ 126)$ as a function of r. The values of Z and N are those for Bi^{209}, the atom used for the atomic parity violation experiments at Oxford University and the University of Washington. For comparison the Weinberg-Salam theory predicts

$$Q_{wk}(Z,\ N) = \left[Z(1-4\sin^2\theta_W)-N\right] \quad . \qquad (3.37)$$

Thus for a typical value of $\sin^2\theta_W = 0.3$, $Q_{wk}(Z = 83,\ N = 126) = -143$.

At present, although the atomic physics which enters into the theoretical prediction of the magnitude and sign of the parity-violating optical rotation in bismuth is very complicated and somewhat uncertain,[12] the experimental measurements seem to be smaller than the Weinberg-Salam model prediction. The initial results were[11]

$$\text{Washington:}\quad R^{\exp}_{876\ \mathrm{nm}} = (-0.8 \pm 0.3) \times 10^{-7} \tag{3.38a}$$

$$\text{Oxford:}\quad R^{\exp}_{648\ \mathrm{nm}} = (\ 1.0 \pm 0.3) \times 10^{-7} \tag{3.38b}$$

while the Weinberg-Salam model, together with the relativistic central field approximation, gives[12]

$$R^{\text{theory}}_{876\ \mathrm{nm}} \simeq -3 \times 10^{-7} \tag{3.39a}$$

$$R^{\text{theory}}_{648\ \mathrm{nm}} \simeq -4 \times 10^{-7} \quad . \tag{3.39b}$$

Many-electron effects substantially reduce the expected values of R, but apparently not enough to yield agreement with the data.[11] It was stressed by the experimenters that there were systematic errors of order 10^{-7}, which were not fully understood. Recently, the two groups have quoted the results[11]

$$\text{Washington:}\quad R^{\exp}_{876\ \mathrm{nm}} = (-0.07 \pm 0.32) \times 10^{-7} \tag{3.40a}$$

$$\text{Oxford:}\quad R^{\exp}_{648\ \mathrm{nm}} = (\ 0.27 \pm 0.47) \times 10^{-7} \quad . \tag{3.40b}$$

In view of these experimental findings, which seem to indicate that parity violation, if present at all, is substantially smaller than predicted by the origin Weinberg-Salam model, we shall require that in our model $Q_{wk}/Q_{wk}(WS) \lesssim 0.1$ i.e. $Q_{wk} \lesssim 17$. The minimal version of the model automatically satisfies these constraints; in the generalized version, the constraint implies that $r \lesssim 0.1$.

5. Weak Hyperfine Splitting

In our model there is another interesting manifestation of weak neutral currents in atomic physics. This arises because we have a neutral current which is, in the r = 0 version of the model, purely axial vector, namely J_Y^{μ}. In the nonrelativistic limit the effective current-current interaction is thus a spin-spin interaction. There is available an exceedingly precisely known quantity, the hyperfine splitting in hydrogen, due to the electron-proton spin-spin interaction, and also a very accurate prediction from quantum electrodynamics. The spin-spin hamiltonian, to lowest order, is

$$
\begin{aligned}
H_{em}^{hfs} &= \frac{8\pi}{3} \langle \vec{\mu}_e \cdot \vec{\mu}_p \rangle \,|\psi(0)|^2 \\
&= \frac{8\pi}{3}\left(\frac{e}{2m_e}\right)\left(\frac{e}{2m_p}(1+\mu_p)\right) \langle \vec{\sigma}_e \cdot \vec{\sigma}_p \rangle \,|\psi(0)|^2
\end{aligned}
\tag{3.41}
$$

where $\vec{s}_e = \frac{1}{2}\vec{\sigma}_e$ and $\vec{s}_p = \frac{1}{2}\vec{\sigma}_p$ are the spin operators of the electron and proton, respectively, and $\psi(x)$ is the wave function for the hydrogen atom (assumed to be in the ground state.) Hence the hyperfine splitting between the two states F = 1 and F = 0 (where $\vec{F} = \vec{s}_e + \vec{s}_p$ is the total spin of the hydrogen atom) is determined by $\langle \vec{\sigma}_e \cdot \vec{\sigma}_p \rangle_{F=1} - \langle \vec{\sigma}_e \cdot \vec{\sigma}_p \rangle_{F=0} = 4$. The weak spin-spin interaction has the effective Hamiltonian

$$
\mathscr{H}_{wk} = \frac{G_F}{2\sqrt{2}}\left(\frac{1+\ell}{\ell}\right)\left[\bar{e}\gamma_\alpha\gamma_5 e\right]\left[\bar{d}\gamma^\alpha\gamma_5 d\right] \quad . \tag{3.42}
$$

Using

$$<p|\bar{d}\gamma^{\alpha}\gamma_5 d|p> \simeq g_A(1-\epsilon) \tag{3.43}$$

where $g_A = 1.24$ and ϵ is the ratio of isoscalar to isovector axial vector form factors, we find that

$$\mathscr{H}^{hfs}_{wk} = \frac{G_F}{2\sqrt{2}}\left(\frac{1+\ell}{\ell}\right) g_A(1-\epsilon)\langle\vec{\sigma}_e \cdot \vec{\sigma}_p\rangle|\psi(0)|^2 \tag{3.44}$$

from which the ratio of weak to electromagnetic hyperfine splittings is determined to be

$$\frac{\Delta E^{hfs}_{wk}}{\Delta E^{hfs}_{em}} = \frac{\frac{G_F}{2\sqrt{2}}\, g_A(1-\epsilon)\, \frac{1+\ell}{\ell}}{\frac{2\pi}{3}\left(\frac{\alpha}{m_e m_p}\right)(1+\mu_p)} \tag{3.45}$$

where $(1+\mu_p) = 2.79$ is the total magnetic moment (in units of $e/2m_p$) of the proton. For a typical value of ℓ, namely $\ell = 0.2$, this ratio is roughly equal to 2.2×10^{-7}. Unfortunately, although the hyperfine splitting in hydrogen is measured to $\sim$one part in 10^{12} the QED prediction, (with various corrections that extend Eq. (3.41)) is accurate only to about one part in 10^5.[32] Hence the J^{μ}_{Y} neutral weak current effect is roughly two orders of magnitude too small to be distinguished from the electromagnetic hyperfine effect. From Eq. (3.45) it is clear that $\Delta E^{hfs}_{wk}/\Delta E^{hfs}_{em}$ is larger by the factor (m_{μ}/m_e) in muonic hydrogen; however here the experimental measurement is not nearly accurate enough to test such a small effect of order 10^{-5}

IV. FURTHER PHENOMENOLOGICAL IMPLICATIONS OF THE MODEL

1. The Anomalous Magnetic Moments of the Muon and Electron

We shall consider here the weak contributions to the anomalous magnetic moments of the muon and electron. The anomalous magnetic moment a is defined as $a \equiv F_2(0) \equiv (\frac{g-2}{2})$, where $F_2(q^2)$ is the Pauli form factor.

The electromagnetic contribution to a_μ and a_e have been calculated[33] to order α^3. The hadronic vacuum polarization has also been determined with reasonable accuracy from the measured cross section $\sigma(e^+e^- \to \text{hadrons})$. Indeed a subset of the eighth order graphs has also been computed.[34] The results are[35]

$$a_\mu^{QED} = 1165851.8(2.4) \times 10^{-9} \tag{4.1a}$$

$$a_\mu^{hadronic} = 66.7(9.4) \times 10^{-9} \tag{4.1b}$$

and thus

$$a_\mu^{th} = 1165918.5(9.7) \times 10^{-9} \tag{4.1c}$$

where the numbers in parentheses are the one standard deviation errors. The most accurate measurement of a_μ (by the CERN group[36]) yields

$$a_\mu^{exp} = 1165922(9) \times 10^{-9} \quad . \tag{4.2}$$

Thus

$$a_\mu^{exp} - a_\mu^{theory} = +3.5(13) \times 10^{-9} \quad . \tag{4.3}$$

Therefore, the 1σ bound on the weak interaction contribution is

$$-9.5 \times 10^{-9} < a_\mu^{wk} < 17 \times 10^{-9} \tag{4.4}$$

Recently the measurement of the electron anomalous magnetic moment has increased considerably in accuracy. The older value obtained by the Michigan experiment[37] is

$$a_e^{exp} = 1159656.7\ (3.5) \times 10^{-9} \quad . \tag{4.5a}$$

The newer value obtained by the Washington group,[38] with an error smaller by more than a factor of 10, is

$$a_e^{exp} = 1159652.41\ (0.20) \times 10^{-9} \quad . \tag{4.5b}$$

The theoretical calculations by Levine and Wright and Levine and Roskies[33] yield

$$(a_e^{theory})_{LRW} = (1159652.71 \pm 0.60) \times 10^{-9} \tag{4.6a}$$

while those of Cvitanovic and Kinoshita[33] give

$$(a_e^{theory})_{CK} = (1159652.34 \pm 0.19) \times 10^{-9} \quad . \tag{4.6b}$$

Thus, using the Washington measurement, we have

$$a_e^{exp} - (a_e^{theory})_{LRW} = (-3.0 \pm 6.5) \times 10^{-9} \tag{4.7a}$$

and

$$a_e^{exp} - (a_e^{theory})_{CK} = (0.74 \pm 3.1) \times 10^{-3} \quad . \tag{4.7b}$$

Again, taking the most lenient limits, we find

$$-9.5 \times 10^{-9} < a_e^{wk} < 3.8 \times 10^{-9} \quad . \tag{4.8}$$

These, then, are the experimental bounds which our model must satisfy. We next proceed with the calculations.

The graphs which contribute in lowest order to a_μ^{wk}, the weak interaction correction to a_μ, are shown in Fig. 19. The graphs for a_e^{wk} are obtained from those for a_μ^{wk} by appropriate replacements of muon-type leptons with electron-type leptons. Only physical particles are shown; the unphysical Higgs scalars which appear in general R_ξ gauge are omitted. Graphs (e) and (f) each represent a sum of four graphs corresponding to LL, RR (for (e)) and LR, RL (for (f)) transitions mediated by the X_1 and X_2 gauge bosons. From our past calculations,[7,39] we find the following results, in the r = 0 version of the model, which are accurate to lowest order in m_L^2/m_V^2, where m_L represents a generic lepton mass and m_V a generic vector boson mass, and where the subscript ℓ = e or μ

$$a_\ell^{wk} = \frac{G_F m_\ell^2}{\sqrt{2}\pi^2} \bar{a}_\ell^{wk} \tag{4.9}$$

where

$$\bar{a}_{\ell}^{wk} = \sum_{i=a}^{f} \bar{a}_{\ell}^{(i)} \quad . \tag{4.10}$$

The $a_{\ell}^{(i)}$ arising from graphs $i = a$ through $i = f$ are

$$\bar{a}_{\ell}^{(a)} = \frac{5}{12} \tag{4.11a}$$

$$\bar{a}_{\ell}^{(b)} = \frac{5}{12} \tag{4.11b}$$

$$\bar{a}_{\ell}^{(c)} = \frac{3}{8}(1+\ell)(w-\frac{1}{3})^2 \tag{4.11c}$$

$$\bar{a}_{\ell}^{(d)} = -\frac{5}{24}\left(\frac{1+\ell}{\ell}\right) \tag{4.11d}$$

m

$$\bar{a}_{\ell}^{(e)} = -\frac{1}{3}\left(\frac{1+\ell}{\ell}\right)\left(\frac{1}{1-\delta^2}\right) \tag{4.11e}$$

$$\bar{a}_{\ell}^{(f)} = \frac{1}{2}\left(\frac{m_{L^-}}{m_{\ell}}\right)\left(\frac{1+\ell}{\ell}\right)\left(\frac{\delta}{1-\delta^2}\right) \cos\epsilon \tag{4.11f}$$

with ℓ, w, δ and ϵ as defined in Section 2. In Eq. (4.11f) $L^- = E^-, M^-$. In evaluating these contributions, we shall take the values $w = 0.25$, $\ell = 0.2$, and shall neglect CP violation, setting $\epsilon = 0$ or π. For δ it is natural, although not necessary, to use a value which roughly reflects the relevant fermion mass ratios; generically because of the approximate $\sqrt{R}$ symmetry of the Lagrangian and the approximat[e] $\sqrt{R}\,\sqrt{U}$ symmetry of the vacuum, $|b_i| \ll |a_i|$, and hence $\delta \ll 1$. From the fermion mass formula

$$\frac{m_e}{m_{E^-}} = \frac{\left|\sum_i (a_i c_{3i}^{(e)} + b_i^* c_{4i}^{(e)})\right|}{\left|\sum_i (b_i c_{3i}^{(e)} + a_i^* c_{4i}^{(e)})\right|} \tag{4.12}$$

where e labels the electron family terms of the form $\bar{L}(c_3\Phi + c_4\Phi^\dagger)R$ in Eq. (2.4), and from its counterparts for the μ, τ, u, and s families, together with the fact that $\delta << 1$, it is natural to take δ of the general order of these fermion mass ratios. Of course this is an approximate statement since they vary from m_e/m_E and m_d/m_b which are presumably of order 10^{-3} - 10^{-4}, to m_μ/m_{M^-} and m_s/m_h, of order 10^{-2}. We shall choose $\delta = 10^{-2}$ as a typical value and assume that the further suppression needed for mass ratios such as m_e/m_E or m_d/m_b is provided by the $c_{4i}^{(e)}/c_{3i}^{(e)}$ and $c_{4i}^{(u)}/c_{3i}^{(u)}$. Indeed a calculation of the electric dipole moment D_n of the neutron in (the $r = 0$ version of) this model gives

$$D_n \sim \frac{eG_F m_b}{\pi^2}\,\delta\,\sin\phi_b \quad . \tag{4.13}$$

Taking $m_b \simeq 5$ GeV and ϕ_b the typical size of a CP-violating phase, 10^{-3} we must require that $\delta \lesssim 10^{-2}$ in order that D_n be in agreement with the experimental bound[40] $D_n^{exp} = (0.4 \pm 1.1) \times 10^{-24}$ e - cm. It is interesting to determine independently how δ is bounded in magnitude by its contribution to the anomalous magnetic moments a_μ and a_e. From Eqs. (4.9) - (4.11) we compute

$$a_\mu^{wk} = 2.25 \times 10^{-8}\left(-1 + 94.105\,\frac{\delta\cos\epsilon}{1-\delta^2}\right) \tag{4.14}$$

and

$$a_e^{wk} = 5.265 \times 10^{-13}\left(-1 + 1.45935 \times 10^4\,\frac{\delta\cos\epsilon}{1-\delta^2}\right) \quad . \tag{4.15}$$

Taking $\delta = 10^{-2}$ and $\epsilon = 0$ or π we have

$$a_\mu^{wk} = \begin{cases} -1.325 \times 10^{-9} & (\epsilon = 0) \quad (4.16a) \\ -4.37 \times 10^{-8} & (\epsilon = \pi) \quad (4.16b) \end{cases}$$

and

$$a_e^{wk} = \begin{cases} 7.63 \times 10^{-11} & (\epsilon = 0) \quad (4.17a) \\ -7.74 \times 10^{-11} & (\epsilon = \pi) \quad (4.17b) \end{cases} .$$

For $\epsilon = 0$, a_μ^{wk} and a_e^{wk} both lie within their respective allowed ranges (4.4) and (4.8); for $\epsilon = \pi$, however, a_μ^{wk} is outside its permitted range. The value $\epsilon = 0$ is therefore favored. In the case of a_μ^{wk} all of the graphs except for (c) give comparable contributions. With our allowed range of w, graph (c) happens to be suppressed strongly by the $(w - 1/3)^2$ factor. The LR, RL graph, (f), is substantial but not dominant since the large factor (m_{M^-}/m_μ) is compensated by the small factor δ. For $\epsilon = 0$ there is considerable cancellation between the sum of graphs (d) and (e) on the one hand, and (f) on the other. When one chooses $\epsilon = \pi$ these graphs all add constructively, and thereby substantially increase the magnitude of a_μ^{wk}. In contrast, for the weak contribution to the electron moment, a_e^{wk}, because the factor $(m_{E^-}/m_e)\delta \simeq 10^2$, graph (f) is completely dominant; the sum of graphs (a) through (e) is 0.7% of graph (f). Hence in this case the second choice of ϵ merely causes a change in the sign of a_e^{wk}. Thus a_μ^{wk}, but not $|a_e^{wk}|$, is sensitively dependent upon the choices of parameters ℓ, w, and ϵ; both depend strongly on the choice of δ. For our values of ℓ, w, and ϵ, the upper bound on δ is set by a_e^{wk}. In order for a_e^{wk} to lie in the range (4.8) it is necessary that $\delta \lesssim 0.5$ if $\epsilon = 0$ and $\delta < 1.2$ if $\epsilon = \pi$. The muon weak anomalous magnetic moment a_μ^{wk} does not set a very useful bound on δ because of the strong cancellations involved and their dependence on ϵ. For $\epsilon = 0$, a_μ^{wk} would lie outside of the 1σ range (4.4) either if $\delta = 0$ or if $\delta \gtrsim 1.1 \times 10^{-2}$; the value $\delta = 10^{-2}$ happens to cause a near-cancellation as noted above. In contrast, for $\epsilon = \pi$, one favors $\delta < 10^{-2}$. Thus the electric dipole moment of the neutron yields a more parameter-independent constraint on δ.

An important thing to note from the calculation of a_μ^{wk} and a_e^{wk} is that the potentially large contributions of the LR, RL transitions in Fig. (f) are strongly suppressed by the fact that the X_1 and X_2 boson graphs contribute with opposite sign. This is analogous to the GIM mechanism for fermions; in both cases a transition occurs only because of the mixing of mass eigenstates to form weak gauge group eigenstates. The rate for these transitions is suppressed by the factor $(\Delta m_F^2)/m_V^2$ in the GIM fermion case and by $\Delta m_V^2/m_V^2$ in the vector boson case, where $\Delta m_F \equiv m_{F_1}^2 - m_{F_2}^2$ and $\Delta m_V^2 \equiv m_{V_1}^2 - m_{V_2}^2$ are the differences of fermion and vector boson masses squared, and m_V is a generic vector boson m

In the version of the model in which $r \neq 0$ the graphs which contribute in lowest order to the weak part of the anomalous magnetic moments $a_{\mu,e}^{wk}$ are the same, except that in graphs (c) and (d), the vector bosons Z and Y are replaced by Z_1 and Z_2. In the notation of Eqs. (4.9)-(4.10) (where again the subscript $\ell = e$ or μ) we find

$$\bar{a}_\ell^{(a)} = \frac{5}{12} \tag{4.18a}$$

$$\bar{a}_\ell^{(b)} = \frac{5}{12}\left(\frac{1+\ell}{1+\ell+r}\right) \tag{4.18b}$$

$$\bar{a}_\ell^{(c)} = \frac{1}{4}\left(\frac{m_W^{\;2}}{m_{Z_1}^{\;2}}\right)\left[2\cos^2\chi\,\frac{(w-1/3)^2}{(1-w)} - \frac{10}{3}\sin^2\chi\right] \tag{4.18c}$$

$$\bar{a}_\ell^{(d)} = \frac{1}{4}\left(\frac{m_W^{\;2}}{m_{Z_2}^{\;2}}\right)\left[2\sin^2\chi\,\frac{(w-1/3)^2}{(1-w)} - \frac{10}{3}\cos^2\chi\right] \tag{4.18d}$$

$$\bar{a}_\ell^{(e)} = -\frac{2}{3}\,\frac{(1+\ell)(2\ell+r)}{\left[(2\ell+r)^2 - 4\ell^2\delta^2\right]} \tag{4.18e}$$

$$\bar{a}_\ell^{(f)} = 2\left(\frac{m_{L^-}}{m_\ell}\right)\frac{\ell(1+\ell)\delta\cos\epsilon}{\left[(2\ell+r)^2 - 4\ell^2\delta^2\right]} \quad . \tag{4.18f}$$

The ratios $(m_W^{\;2}/m_{Z_{1,2}}^{\;2})$ are given by Eq. (2.68) and χ, by Eq. (2.59).

As $r \to \infty$ only the W and Z_2 contributions remain. The former is independent of r:

$$(a_\ell^{wk})^{(a)} = \frac{5}{12}\,\frac{G_F m_\ell^{\;2}}{\sqrt{2}\,\pi^2} \tag{4.19}$$

while the latter becomes

$$\lim_{r \to \infty} (a_\ell^{wk})^{(d)} = \frac{1}{24} \frac{G_F m_\ell^2}{\sqrt{2}\pi^2} \left[(3w - 1)^2 - 5 \right] \quad . \qquad (4.20)$$

With the usual identification $w = (4/3)\sin^2\theta_W$, Eqs. (4.19) and (4.20) are precisely the W and Z boson contributions to a_ℓ^{wk} in the original SU(2)⊗U(1) model.[39] For comparison, taking $\sin^2\theta_W = 1/3$, we have

$$a_\mu^{wk}(\mathrm{WS};\ \sin^2\theta_W = 1/3) = 1.99 \times 10^{-9} \qquad (4.21)$$

and

$$a_e^{wk}(\mathrm{WS};\ \sin^2\theta_W = 1/3) = 4.65 \times 10^{-14} \quad . \qquad (4.22)$$

2. The Decays $\mu \to e\gamma$ and $\mu \to ee\bar{e}$

In this model the mixing of E^0 and M^0 causes μ- and e-type lepton number nonconservation. We shall focus here on the most important μ- and e-lepton number violating decay, $\mu \to e\gamma$.[7,41,42,43] This decay proceeds via the transition $\mu_R \to E_R^0$, $M_R^0 \to e_R$ mediated by a virtual U^- vector boson, as shown in Fig. 20. Because this is the only transition which contributes, the decay depends only on the structure of the first and third members of the right handed e- and μ-type lepton triplets and their couplings to the $U^\pm$ vector bosons. Since these transitions are among members of SU(2) (V-spin) doublets of the same hypercharge y we can apply the general criteria for natural suppression of μ- and e-type lepton number nonconservation derived in Ref. 7 . These criteria are that (1) leptons of a given charge and chirality must have the same value of weak $\vec{V}^2$ and $(V)_3$, where[44] $V_\pm = V_1 = iV_4$; (2) leptons of a given charge and chirality must derive their masses from couplings to one and only one neutral Higgs field; and (3) leptons of charge Q = 0 do not belong to the same weak V-spin multiplet as μ or e for at least one chirality, to prevent LR, RL transitions (and similarly for Q = -2 leptons if there were any). All of these conditions are satisfied by our model and therefore the natural suppression mechanism discussed in Ref. 7 is operative.

Using our past calculations,[7,41,42] we find, that, in the minimal version of the model

$$BR(\mu \to e\gamma)_{r=0} = \frac{3\alpha}{32\pi}\left(\frac{\Delta m_{L^0}^2}{m_U^2(r=0)}\right)^2 \sin^2\beta\cos^2\beta \tag{4.23}$$

where $\Delta m_{L^0}^2 = m_{M^0}^2 - m_{E^0}^2$. For w = 0.25, m_{M^0} = 4 GeV, m_{E^0} = 1 GeV, and $\sin^2\beta\cos^2\beta = \frac{1}{4}$, we find $BR(\mu \to e\gamma)_{exp} = 4.66 \times 10^{-11}$, safely smaller than the present experimental limit, $BR(\mu \to e\gamma)_{exp} < 2.2 \times 10^{-8}$. In the $r \neq 0$ version of the model the branching ratio (4.23) becomes

$$BR(\mu \to e\gamma)_{r\neq 0} = \frac{3\alpha}{32\pi}\left(\frac{1+\ell}{1+\ell+r}\right)^4\left(\frac{\Delta m_{L^0}^2}{m_U^2(r=0)}\right)^2 \sin^2\beta\cos^2\beta \quad . \tag{4.24}$$

The related decay $\mu \to ee\bar{e}$ is similarly naturally suppressed by the leptonic GIM mechanism since the model satisfies, in addition to the three criteria listed above, the fourth criterion in Ref. 7, viz. that there be no doubly negatively charged heavy leptons which communicate with both

μ and e. The branching ratio for this decay is, in the r = 0 case

$$BR(\mu \to ee\bar{e})_{r=0} \sim \frac{\alpha^2}{\pi^2}\left(\frac{\Delta m_{L^0}^2}{m_U^2(r=0)}\right)^2 \ln^2\left(\frac{m_U^2(r=0)}{\Delta m_{L^0}^2}\right) \sin^2\beta \cos^2\beta \qquad (4.25)$$

For the values of w, m_{M^0}, m_{E^0}, and β taken above this branching ratio is $\sim 10^{-10}$, well below the experimental upper bound, $BR(\mu \to ee\bar{e})_{exp} < 6 \times 10^{-9}$. The branching ratio is further suppressed by an obvious factor in the $r \neq 0$ case.

If the model is generalized to include mixing of E^0 and M^0 with T^0 the expressions for the $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ become more slightly complicated but remain of the same general order of magnitude, since the four natural suppression criteria are still satisfied.

It is important to observe that if we had allowed E^- and M^- to mix then there would be a contribution to the $\mu \to e\gamma$ decay amplitude in which the μ makes a transition into e via a virtual E^- or M^- and virtual $X_{1,2}$ vector boson. There would be four such amplitudes, which could be characterized by the chirality of the initial and final weak vertices as LL, RR, LR, and RL. As was discussed in Ref. 7, LR and RL graphs would not be suppressed by a leptonic GIM mechanism and hence would give decay amplitudes larger than the one arising from the graphs of Fig. 20 by the factor

$$\left(\frac{m_U^2}{m_X^2}\right)^2\left(\frac{m_{L^-}}{m_\mu}\right)^2\left(\frac{m_U^2}{\Delta m_L^2}\right)^2$$

where $m_{L^-} \sim \max(m_{M^-}, m_{E^-})$. Unless the E^-, M^- mixing angle were extremely small the resultant branching ratio for $\mu \to e\gamma$ would be many orders of magnitude larger than the experimental upper bound $BR(\mu \to e\gamma)_{exp} < 2.2 \times 10^{-8}$. A similar remark applies to the decay $\mu \to ee\bar{e}$. Hence it is crucial to prevent, as we have done, any mixing between E^- and M^-.

3. The $K_L K_S$ Mass Difference

Among one-loop induced $|\Delta S| = 2$ neutral current processes such as the $K^0 - \bar{K}^0$ transition and the decays $K_L \to \mu\bar{\mu}$ and $K^\pm \to \pi^\pm e\bar{e}$ the first yields the most stringent constraint on the magnitude of such currents. This $K^0 - \bar{K}^0$ transition, which gives rise to the $K_L K_S$ mass difference, was computed previously[46] in the $SU(2) \otimes U(1)$ gauge model and implied a charmed quark mass $m_c \simeq 1.5$ GeV : in agreement with the value of m_c inferred from the mass of the subsequently discovered $J/\psi(3095)$ and the related charmonium spectrum. In the present model there are four diagrams which contribute in lowest order to the $K_L K_S$ mass difference, shown in Fig. 21.

From graphs (a) and (b) (which give the same contribution) we calculate an effective Lagrangian

$$\mathscr{L}_L \equiv \mathscr{L}(d_L\bar{s}_L \rightarrow s_L\bar{d}_L) = -\frac{g^4}{8m_W^2}\frac{1}{16\pi^2}\sin^2\theta_C\cos^2\theta_C I\left(\frac{m_c^2}{m_W^2}, \frac{m_u^2}{m_W^2}\right)$$

$$\times \left[\bar{s}\gamma_\alpha Ld\right]\left[\bar{s}\gamma^\alpha Ld\right] + \text{h. c.} \tag{4.26}$$

where

$$I(x, y) + I_2(x) + I_2(y) - 2J(x, y) \tag{4.27}$$

with

$$I_2(x) = \frac{2x \ln x}{(1-x)^3} + \frac{(1+x)}{(1-x)^2} \tag{4.28}$$

and

$$J(x, y) = \frac{1}{(x-y)}\left[\frac{x^2 \ln x}{(1-x)^2} - \frac{y^2 \ln y}{(1-y)^2}\right] + \frac{1}{(1-x)(1-y)} \tag{4.29}$$

The sum of graphs (c) and (d) gives, analogously,

$$\mathscr{L}_R \equiv \mathscr{L}(d_R\bar{s}_R \rightarrow s_R\bar{d}_R) = -\frac{g^4}{8m_U^2}\frac{1}{16\pi^2}\sin^2\theta'_C\cos^2\theta'_C I\left(\frac{m_g^2}{m_U^2}, \frac{m_t^2}{m_U^2}\right)$$

$$\times \left[\bar{s}\gamma_\alpha Rd\right]\left[\bar{s}\gamma^\alpha Rd\right] + \text{h. c.} \tag{4.30}$$

In $\mathscr{L}_L$, because $m_u^2/m_W^2 \ll m_c^2/m_W^2 \ll 1$, the function I can be approximated as

$$I\left(\frac{m_c^2}{m_W^2}, \frac{m_u^2}{m_W^2}\right) \simeq \frac{m_c^2}{m_W^2} \quad . \tag{4.31}$$

In $\mathscr{L}_R$ the GIM mechanism is operative, although this is not manifest in Eq. (4.27). In order to render it manifest we re-express I(x, y) as a function of $\xi = \frac{1}{2}(x + y)$ and $\eta = x - y$, and approximate this function for $\xi, \eta \ll 1$:

$$I(x, y) \simeq \frac{1}{x - y}\left[2xy \ln (y/x) + x^2 - y^2\right]$$

$$\simeq \frac{1}{3}\frac{\eta^2}{\xi} \quad . \tag{4.32}$$

The factor η^2 which arises from the difference of the two fermion propagators, all squared, is thus made explicit in Eq. (4.32). Note however that, as was discussed in Refs. 7 and 46 there is really only a single GIM suppression factor for small but nondegenerate fermion masses, since the additional factor of η is accompanied by a factor ξ^{-1}. The expression (4.32) reflects the fact that GIM suppression consists of two parts: (a) for fixed $m_{F_i}^2/m_V^2$, the property that the amplitude vanishes as $m_{F_1}^2 \to m_{F_2}^2$ where F_1 and F_2 are the two fermions participating in the suppressed transition, and (b) for fixed $|m_{F_1}^2 - m_{F_2}^2|$ the suppression due to the fact that $|m_{F_1}^2 - m_{F_2}^2|/m_V^2 \ll 1$.

It is convenient to rewrite these effective Lagrangians in the equivalent form

$$\mathscr{L}_L = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi}\left(\frac{m_c^2}{wm_W^2}\right) \sin^2\theta_C \cos^2\theta_C \left[\bar{s}\gamma_\alpha Ld\right]\left[\bar{s}\gamma^\alpha Ld\right] + \text{h.c.} \tag{4.33}$$

$$\mathscr{L}_R = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi}\left(\frac{1+\ell}{(1+\ell+r)w}\right)\sin^2\theta'_C\cos^2\theta'_C\, I\left(\frac{m_g^2}{m_U^2}, \frac{m_t^2}{m_U^2}\right)$$

$$\times \left[\bar{s}\gamma_\alpha Rd\right]\left[\bar{s}\gamma^\alpha Rd\right] + \text{h.c.} \qquad (4.34)$$

In order to estimate the matrix elements of the total effective Lagrangian between K^0 and K^0 states, we shall use the same approximation as was employed previously,[46] namely to insert a complete sum over states and saturate it with the vacuum state, thereby obtaining the result

$$\frac{m_L - m_S}{m_K} = \frac{G_F f_K^2}{\sqrt{2}}\frac{\alpha}{4\pi}\left[\left(\frac{m_c}{42.1\ \text{GeV}}\right)^2\sin^2\theta_C\cos^2\theta_C + \left(\frac{1+\ell}{(1+\ell+r)w}\right)\sin^2\theta'_C\cos^2\theta'_C\, I\left(\frac{m_g^2}{m_U^2}, \frac{m_t^2}{m_U^2}\right)\right]. \qquad (4.35)$$

Experimentally, $(m_L - m_S)/m_K \simeq 0.71 \times 10^{-14}$. Inserting the value $m_c = 1.5$ GeV, we compute the first term on the right hand side of Eq. (4.35) to be 0.6×10^{-14}. It thus follows that, to the accuracy of this free quark approximation and the accuracy of our approximation of the Feynman parameter integrals, the size of the second term arising from the heavy t and g quark contributions is restricted to lie in the range from zero to $\sim 1 \times 10^{-5}$. To see what this implies for the t and g quark masses, let us take $w = 0.25$ and assume for simplicity that $r = 0$. Then it follows that

$$\sin^2\theta_C' \cos^2\theta_C' \, I\left(\frac{m_g^2}{m_U^2}, \frac{m_t^2}{m_U^2}\right) \lesssim 3 \times 10^{-6} \tag{4.36}$$

with $m_U = 86.1$ GeV. From considerations of the trimuon production rate we shall choose $m_t \simeq 4$ GeV; if $m_g = 8$ GeV, say, then Eq. (4.36) implies that $\sin^2\theta_C' \cos^2\theta_C' \lesssim 4 \times 10^{-3}$. This bound should not be taken too literally, however, since it is based on the free quark approximation. The contributions of the right handed heavy t and g quarks to the $K_L K_S$ mass difference can of course be further suppressed by increasing r.

4. Heavy Lepton Decay Rates and Branching Ratios

In this section we shall present the results of a calculation of the total decay rates and branching ratios of the heavy leptons. We shall concentrate on M^- and M^0, since these play an important role in heavy lepton production and sequential decay processes yielding multilepton events in high energy neutrino and antineutrino reactions. The trimuon branching ratio will be used to derive rough estimates of the trimuon rate predicted for the FHPRW experiment. In order to avoid complicating the situation with additional unknown parameters we will work in the minimal (r = 0) version of the model and assume either that $m_{E^-} > m_{M^-}$ or, if $m_{M^-} > m_{E^-}$, that the mass difference is sufficiently small that decays of M^- into E^- are rendered negligible by phase space suppression.

Furthermore, we shall assume that θ'_C, the t, g mixing angle, is small (or equivalently, here, $m_t \simeq m_g$), and that m_t, $m_g < m_b$, m_h. One can, of course, consider the most general case, but only at the price of having the resulting branching ratios depend on the additional unknown constants m_{E^-}, θ'_C, m_b, and m_h. For the numerical work we shall take $m_t = 4$ GeV.

Given (1) the requirement that $m_{t,g} \gtrsim 3.5 - 4$ GeV from SPEAR; (2) the approximate HPWF determination of the M^0 mass; (3) the fact that $m_{E^0} < m_{M^0}$ (which effectively defines the angle β); and (4) the exact RU symmetry, it follows that E^0 has no available decay channels and is absolutely stable. In passing, we note that if one did not invoke (2) above, then it would be reasonable to consider the possibility that $m_{E^0} > m_t$ where E^0 and t are the lightest leptons and quarks. In this case the t quark would be absolutely stable and one would have the striking prediction of absolutely stable hadrons in addition to the proton, viz. the lightest t-flavored meson and baryon, presumably $t\bar{u}$ and tuu, respectively. This possibility will not be considered further here.

From the cosmological argument given in Ref. 13 $m_{E^0} \lesssim 40$ eV or $m_{E^0} \gtrsim 1 - 4$ GeV. We shall consider both of these cases. In the former case the mixing angle β is constrained to be quite small. The reason for this is that for nonzero β the muon will have, in addition to its regular decay channel, another: $\mu^- \to E^0 e^- \overline{E^0}$, via the U^- boson. Assuming that $m_{E^0} \ll m_\mu$ the total muon decay rate is, to lowest order (for general r)

$$\begin{aligned}\Gamma_\mu &= \Gamma(\mu^- \to \nu_\mu e^- \overline{\nu}_e) + \Gamma(\mu^- \to E^0 e^- \overline{E^0}) \\ &= \frac{G_F{}^2 m_\mu{}^5}{192\pi^2}\left[1 + \left(\frac{1+r}{1+\ell+r}\right)^2 \sin^2\beta\cos^2\beta\right] .\end{aligned} \tag{4.37}$$

In order to avoid circularity, take the Fermi coupling constant G_F to be determined by neutron and hyperon decay measurements. Then Γ_μ is observed to be equal to $\Gamma(\mu^- \to \nu_\mu e^- \nu_e)$ to within $\sim 1\%$, where the level of precision comes from the combined determination of G_F and θ_C.[47] Hence it is sufficient to require, from considerations of the total μ

decay rate, that $\sin^2\beta\cos^2\beta < 0.01$. Another constraint is provided by the experimental measurement of the longitudinal electron polarization P_e in μ decay. In the original $SU(2)\otimes U(1)$ model, $P_e=1$; in the present model, for $m_{E^0} << m_e$

$$P_e = \frac{(1-\sin^2\beta\cos^2\beta)}{(1+\sin^2\beta\cos^2\beta)} \quad . \tag{4.38}$$

The electron polarization has been measured to be[47] $(P_e)_{exp} = 1.00 \pm 0.13$. In the absence of scalar, pseudoscalar, or tensor terms in the weak interaction the asymmetry parameter ξ is equal to P_e; measurements yield $(\xi)_{exp} = 0.972 \pm 0.013$. From the direct polarization measurement we infer $0 < \sin^2\beta < 0.070$, while from the measurement of ξ we obtain $0.76\times 10^{-2} < \sin^2\beta < 2\times 10^{-2}$. As will be evident, because of the constraint that $\sin^2\beta \lesssim 10^{-2}$ this light E^0 option is not favored by the trimuon data.

In order to calculate the contributions of various decay modes we need the general formula for the leptonic or semileptonic $F \to f_1\bar{f}_2 f_3$ where F, f_i $i = 1, 2, 3$ are fermions. For all cases of interest here, the amplitude is of the form

$$A_{\chi\chi'}\Big(F(p) \to f_1(p_1) + \bar{f}_2(p_2) + f_3(p_3)\Big) =$$

$$4\kappa\frac{G_F}{\sqrt{2}}\left[\bar{u}_{f_1}(p_1)\gamma_\mu P_\chi u_F(p)\right]\left[\bar{u}_{f_3}(p_3)\gamma^\mu P_{\chi'} v_{f_2}(p_2)\right] \tag{4.39}$$

where P_χ and $P_{\chi'}$ are chiral projection operators $P_L = \frac{1}{2}(1-\gamma_5)$, or $P_R = \frac{1}{2}(1+\gamma_5)$. The factor κ is defined as $\kappa = (m_W{}^2/m_V{}^2)^2$, where m_V is the mass of the vector boson mediating the decay. Since we have assumed that M^- does not decay into E^- and moreover have specialized to the $r = 0$ version of the model, $\kappa = 1$ in all cases. A simplifying fact is that the amplitudes A_{LL}, A_{RR}, A_{LR}, and A_{RL} all yield the same total decay rate. There are several equivalent ways to write any one of these decay rates; the simplest form is, in units where $m_F = 1$,

$$\frac{\Gamma}{\Gamma_0} = f(m_1, m_2, m_3)$$

$$= \int_{Q^2_{min}}^{Q^2_{max}} dQ^2 \lambda^{\frac{1}{2}}(1, m_1{}^2, Q^2)\left[2AQ^2(1 + m_1{}^2 - Q^2)\right. \tag{4.40}$$

$$\left. + B(\left\{1 - m_1{}^2\right\}^2 - Q^4)\right]$$

where

$$\Gamma_0 = \frac{G_F{}^2 m_F{}^5}{192\,\pi^3} \tag{4.41}$$

$$A = \left[\frac{\lambda^{\frac{1}{2}}(Q^2, m_2{}^2, m_3{}^2)}{Q^2}\right]^3 \tag{4.42}$$

$$B = \frac{2\lambda^{\frac{1}{2}}(Q^2, m_2{}^2, m_3{}^2)}{Q^6}\left[Q^4 - 2(m_2{}^2 - m_3{}^2)^2 + Q^2(m_2{}^2 + m_3{}^2)\right] \tag{4.43}$$

$$Q^2_{min} = (m_2 + m_3)^2 \tag{4.44a}$$

and

$$Q^2_{max} = (1 - m_1)^2 \tag{4.44b}$$

The function λ is defined as

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx) \quad . \tag{4.45}$$

As is clear from Eqs. (4.40)-(4.45), the decay rate is symmetric under interchange of m_2 and m_3.

The heavy lepton M^- decays in the following modes:

(1)

$$\begin{array}{l} M_R^- \to M_R^0 + W^- \\ \qquad\qquad\qquad \left\{ \begin{array}{l} e_L^- \bar{\nu}_{eL} \\ \mu_L^- \bar{\nu}_{\mu L} \\ \tau_L^- \bar{\nu}_{\tau L} \\ d_{\theta L} \bar{u}_L \\ s_{\theta L} \bar{c}_L \end{array} \right. \end{array} \tag{4.46}$$

(2)

$$\begin{array}{l} M_R^- \to E_R^0 + W^- \\ \qquad\qquad\qquad \left\{ \begin{array}{l} e_L^- \bar{\nu}_{eL} \\ \mu_L^- \bar{\nu}_{\mu L} \\ \tau_L^- \bar{\nu}_{\tau L} \\ d_{\theta L} \bar{u}_L \\ s_{\theta L} \bar{c}_L \end{array} \right. \end{array} \tag{4.47}$$

and

(3)
$$M_L^- \to \nu_{\mu L} + U^- \qquad U^- \to \begin{cases} e_R^- \overline{E}_R^0 \\ \mu_R^- \overline{E}_R^0 \\ e_R^- \overline{M}_R^0 \\ \mu_R^- \overline{M}_R^0 \\ d_{\theta' R} \overline{t}_R \end{cases} \tag{4.48}$$

where $d_\theta = d\cos\theta_C + \sin\theta_C$, $s_\theta = -d\sin\theta_C + s\cos\theta_C$ and $d_{\theta'} = d\cos\theta_C' + s\sin\theta_C'$. In order to estimate the semileptonic decays of M^- we use the usual free quark counting method. The resulting rate is, neglecting e, μ, u, d, and s masses,

$$\Gamma_{M^-} = \frac{G_F^2 m_{M^-}^5}{192\pi^3}\left[\Gamma_1 + \Gamma_2 + \Gamma_3\right] \tag{4.49}$$

where

$$\Gamma_1 = \cos^2\beta\left[5f\left(\frac{m_{M^0}}{m_{M^-}}, 0, 0\right) + f\left(\frac{m_{M^0}}{m_{M^-}}, 0, \frac{m_\tau}{m_{M^-}}\right) + 3f\left(\frac{m_{M^0}}{m_{M^-}}, 0, \frac{m_c}{m_{M^-}}\right)\right] \tag{4.50a}$$

$$\Gamma_2 = \sin^2\beta\left[5f\left(\frac{m_{E^0}}{m_{M^-}}, 0, 0\right) + f\left(\frac{m_{E^0}}{m_{M^-}}, 0, \frac{m_\tau}{m_{M^-}}\right) + 3f\left(\frac{m_{E^0}}{m_{M^-}}, 0, \frac{m_c}{m_{M^-}}\right)\right] \tag{4.50b}$$

and

$$\Gamma_3 = \left[f\left(0, \frac{m_{E^0}}{m_{M^-}}, 0\right) + f\left(0, \frac{m_{M^0}}{m_{M^-}}, 0\right) + \cos^2\theta' f\left(0, \frac{m_t}{m_{M^-}}, 0\right) \right. \tag{4.50c}$$

$$\left. + \sin^2\theta' f\left(0, \frac{m_g}{m_{M^-}}, 0\right)\right].$$

Numerically, for $m_{E^0} = 1$ GeV,

$$\tau_{M^-} = \frac{1}{\Gamma_{M^-}} = \frac{(8.8 \times 10^{-16}\ \mathrm{sec})}{(6.2 \sin^2\beta + 2.7)} \tag{4.51}$$

while for $m_{E^0} < 40$ eV,

$$\tau_{M^-} = \frac{(8.8 \times 10^{-16}\ \mathrm{sec})}{(7.3 \sin^2\beta + 2.8)} \quad . \tag{4.52}$$

The M^0 decay channels are more restricted because of its lower mass, taken here to be 4 GeV:

(1)
$$M_R{}^0 \to \mu_R{}^- + U^+ \quad \hookrightarrow \begin{cases} e_R{}^+ E_R{}^0 \\ \mu_R{}^+ E_R{}^0 \end{cases} \tag{4.53}$$

(2)
$$M_R{}^0 \to e_R{}^- + U^+ \quad \hookrightarrow \begin{cases} e_R{}^+ E_R{}^0 \\ \mu_R{}^+ E_R{}^0 \end{cases} \tag{4.54}$$

These yield

$$\Gamma_{M^0} = \frac{G_F{}^2 m_{M^0}{}^5}{192\pi^3}\, f(0,\ 0,\ m_{E^0}/m_{M^0}) \tag{4.55}$$

or $$\tau_{M^0} = 1.8 \times 10^{-14}\ \mathrm{sec}\ (\tau_{M^0} = 2.8 \times 10^{-14}\ \mathrm{sec}) \tag{4.56}$$

for $$m_{E^0} = 1\ \mathrm{GeV}\ (m_{E^0} < 40\ \mathrm{eV}) \quad . \tag{4.57}$$

The third heavy lepton whose decays are important for determining the final states in M^- decay is τ. It decays according to

$$\tau_L^- \to \nu_{\tau L} + W^- \hookrightarrow \begin{cases} e_L^- \bar{\nu}_{eL} \\ \mu_L^- \bar{\nu}_{\mu L} \\ d_{\theta L} \bar{u}_L \end{cases} \tag{4.58}$$

with a rate $\Gamma_\tau \simeq 5\, G_F^2 m_\tau^5/(192\,\pi^3)$. Note that we have neglected the decay into $s_\theta \bar{c}$ because there is little or no phase space available for the actual hadronic states (D^-, F^-) which would be produced.

To estimate the semileptonic decays of c- and t-flavor hadrons we use the free quark model. Because of the already approximate nature of our computation of M^- leptonic branching ratios it seems out of place to try to estimate the enhancement of nonleptonic weak decays of these hadrons. In our model the c quark decays via the usual channels with a free quark rate $\Gamma_c = (m_c/m_\tau)^5 \Gamma_\tau$. It is an interesting feature of the model that, given our assumption that m_t, $m_g < m_b$, m_h, the t and g quarks must decay semileptonically:

$$t_R \to d_{\theta' R} + U^+ \hookrightarrow \begin{cases} e_R^+ E_R^0 \\ \mu_R^+ E_R^0 \end{cases} \tag{4.59a}$$

and

$$g_R \to s_{\theta' R} + U^+ \hookrightarrow \begin{cases} e_R^+ E_R^0 \\ \mu_R^+ E_R^0 \end{cases} \tag{4.59b}$$

where $d_{\theta' R}$ was defined above and $s_{\theta' R}$ is the orthogonal rotation of d and s. The rates are $\Gamma_j = (G_F{}^2 m_j{}^5/(192\pi^3))f(0,\ 0,\ m_{E^0}/m_j)$ for j = t, g, respectively. This concludes the specification of inputs for the branching ratio calculation.

In order to compute the branching ratios for M^- to decay into various one-, two- and three-lepton inclusive final states, one traces through all the decay chains (4.46) - (4.48) to their final states. In Tables 1 - 5 we list the different decay modes of M^- yielding $\mu^-\mu^-\mu^+$, $\mu^-\mu^-$, $\mu^-\mu^+$, μ^-, μ^+, and no $\mu^\pm$, where, for example, in the first case we mean $\mu^-\mu^-\mu^+$ plus electrons, neutral stable leptons, and hadrons, and similarly for the other four cases. This general classification is appropriate for electronic counter experiments such as the FHPRW, Caltech-Fermilab, and CERN-Dortmund-Heidelberg-Saclay experiments, in which electrons are indistinguishable from hadrons. However, bubble chamber experiments such as the Brookhaven-Columbia-Fermilab $\nu(Ne + H_2)$ experiment are able to detect electrons, and accordingly we have further classified each of the five types of muon final states according to the electrons present. Note that none of the $\mu^-\mu^-\mu^+$ modes has any electrons, and that all the $\mu^-\mu^-$ modes have an e^+. The stable heavy leptons E^0 or $\overline{E}^0$ appear as final decay products in all the modes.

In Figs. 22 and 23 we plot the branching ratios for the six types of muonic final states: $\mu^-\mu^-\mu^+$, $\mu^-\mu^-$, $\mu^-\mu^+$, μ^-, μ^+, and no $\mu^\pm$, as a function of $\sin^2 \beta$. The mass of E^0 is taken to be 1 GeV in Fig. 22 and less than 40 eV in Fig. 23. It is clear from these graphs that the sizes of these branching ratios are strongly dependent on the mixing angle β. In particular, the trimuon branching ratio vanishes if $\beta = 0$, $\pi/2$, or π.

These results can be used to obtain rough estimates of the sizes of heavy lepton production and decay contributions to multimuon events observed in high energy neutrino experiments. In order to do this it is necessary to compute the cross section for the M^- production process $\nu_\mu + N \to M^- + X$ arising from the elementary reaction(s) $\nu_\mu + d \to M^- + t'$, (where t' is given by Eq. (2.21)), i.e. $\nu_\mu + d \to M^- + t$ and $\nu_\mu + d \to M^- + g$, with respective weightings $\cos^2 \theta_C'$ and $\sin^2 \theta_C'$. This cross section must then be properly flux-averaged over the neutrino spectrum of a given experiment.

We shall specialize here to the minimal model with $r = 0$. Even if r is allowed to be nonzero, the requirement that the theory must account for the observed trimuon rate places a stringent upper bound on this parameter. The reason for this is that the production reaction $\nu_\mu + d \to M^- + t'$ involves the exchange of a U vector boson; as r increases from zero the U boson mass also increases, as prescribed by Eq. (2.69). The cross section for this reaction and hence, approximately, the rate for trimuon production are thus scaled down by the factor $((1 + \ell)/(1 + \ell + r))^2$ arising from the U-boson propagator. As will be seen, our prediction for the trimuon rate is slightly smaller than the (uncorrected) rate measured by the FHPRW experiment, so that optimal agreement is obtained with $r = 0$. In order to avoid conflict with the FHPRW rate, say by a factor of 2, it is necessary to limit r to be less than ~0.1. We recall that the constraint that the enhanced parity violation in heavy atoms be less than 10% of the value predicted[12] by the original SU(2)⊗U(1) model yielded a similar upper bound on r.

In our model M^- production by neutrinos is suppressed firstly by phase space because of the necessity of producing the heavy quarks t and g along with the heavy lepton M^-, and secondly by the fact that the chiralities of the ν_μ and M^- are opposite to those of the d and t' quarks. Since θ' is, for simplicity assumed to be small, only the reaction $\nu_\mu + d \to M^- + t$ is important. Again, one could always generalize this and obtain cross section estimates which depend on the three parameters m_t, m_g, and θ' rather than just on the one parameter m_t. In addition to the t quark mass, one must choose the corresponding threshold in hadronic invariant mass, W_t; this is taken as $W_t = (m_t + m_N)$ in units of GeV. A calculation[20] of the production cross section for $m_{M^-} = 8$ GeV, $m_t = 4$ GeV, and $E > 100$ GeV, with the quadrupole triplet beam used by the FHPRW collaboration, gives $\sigma(\nu_\mu + N \to M^- + X)/\sigma(\nu_\mu + N \to \mu^- + X) \simeq 2.3 \times 10^{-2}$. Thus from Fig. 22, for $0.1 < \sin^2\beta < 0.4$ one can estimate that

$$\frac{\sigma(\nu_\mu + N \to \mu^-\mu^-\mu^+ + X)}{\sigma(\nu_\mu + N \to \mu^- + X)} \simeq 3 \times 10^{-4} \tag{4.60}$$

which agrees approximately with the experimental number from the FHPRW group,[4] viz., 5×10^{-4}. In view of the fact that this latter number has large statistical uncertainties and may be due in part to production mechanisms other than the heavy lepton cascade, we consider the agreement between the theoretical prediction and the data to be reasonably good. In the case of a light E^0, with $m_{E^0} < 40$ eV, since $\sin^2\beta \lesssim 0.02$, the trimuon branching ratio is too small by an order of magnitude to give a trimuon rate in accord with experiment. We therefore choose the heavy E^0 alternative, in which $m_{E^0} \gtrsim 1$ GeV.

For antineutrino beams the corresponding heavy lepton-heavy quark production reaction is, in terms of elementary fields, $\bar{\nu}_\mu + u \to M^+ + b$; this yields trimuons of the form $\mu^+\mu^+\mu^-$. For $E \gg m_{M^-}$, m_t, m_b

$$\frac{\sigma(\bar{\nu}_\mu + N \to M^+ + X)}{\sigma(\bar{\nu}_\mu + N \to \mu^+ + X)} \to 3\,\frac{\sigma(\nu_\mu + N \to M^- + X)}{\sigma(\nu_\mu + N \to \mu^- + X)} \tag{4.6}$$

Insofar as the trimuons arise dominantly from the decay of the $M^\pm$, which is a good approximation (see below), the trimuon production rates satisfy the same asymptotic relation:

$$\frac{\sigma(\bar{\nu}_\mu + N \to \mu^+\mu^+\mu^- + X)}{\sigma(\bar{\nu}_\mu + N \to \mu^+ + X)} = 3\,\frac{\sigma(\nu_\mu + N \to \mu^-\mu^-\mu^+ + X)}{\sigma(\nu_\mu + N \to \mu^- + X)} \quad . \tag{4.6}$$

At subasymptotic energies these rates depend on the masses of the t, g, and b quarks, the corresponding thresholds in invariant hadronic mass, and the mixing angle θ'. Moreover, because the antineutrino flux at high energies $E \gtrsim 100$ GeV is considerably smaller than the neutrino flux the actual event rate for the reaction $\bar{\nu}_\mu + N \rightarrow \mu^+\mu^+\mu^-$ is commensurately smaller than that for $\nu_\mu + N \rightarrow \mu^-\mu^-\mu^+$. This prediction is in agreement with the observation of no $\mu^+\mu^+\mu^-$ trimuon events by the FHPRW group using their quadrupole triplet beam (tuned to focus neutrinos and defocus antineutrinos).

The mechanism of M^- production and cascade decay is able to explain not only the general magnitude of the observed FHPRW trimuon rate, but also the kinematic characteristics of these high energy events.[48] These include th following properties: (1) all of the trimuons are of the form $\mu^-\mu^-\mu^+$; there are no "wrong-sign" trimuons, $\mu^+\mu^+\mu^-$; (2) in all events at least two of the muons have large energies; (3) in four of the six events the opening angles (in the laboratory) between the muons are rather small; (4) the distribution in azimuthal angle (defined relative to the beam direction) between pairs of muons in the trimuon events is flat; (5) the distribution in the invariant mass of the $\mu^-\mu^-\mu^+$ system shows no peaking, but (6) the invariant mass of $\mu^-\mu^+$ pairs does show some peaking. The model accounts for the first property by construction. It is true that some μ^+'s from the decay of the t quark will combine with opposite-sign dimuons $\mu^-\mu^+$ from the decay of M^- to form wrong-sign trimuons. However, when one imposes the experimental cut of $E_\mu > 4$ GeV (see below) most such $\mu^+\mu^+\mu^-$ events are eliminated. Properties (2)-(4) are general characteristics of leptons arising from heavy lepton production and decay in neutrino reactions. The reason for points (5) and (6) is that one of the μ^-'s comes from the primary decay of the M^-, whereas the μ^- and remaining μ^- arises from the decay of the M^0 or $\overline{M}^0$. Hence the total $\mu^-\mu^-\mu^+$ combination will not exhibit any peaking but a fraction of the $\mu^+\mu^-$ pairs which originate from the M^0 or $\overline{M}^0$ will. Even in the case in which the μ^- and μ^+ both come from the decaying M^0 or $\overline{M}^0$ there will not be any sharp peak in the invariant mass distribution because of the undetected E^0 or $\overline{E}^0$, respectively. As has been stressed before, in this model M^- production by neutrinos is necessarily accompanied by t production. Hence the hadronic invariant mass W must satisfy $W \gtrsim (m_t + m_N)$, where we have approximated the mass of the lowest t-flavored baryon or nucleon + t-flavored meson final state as $m_t + m_N$. Experimentally, however, this will not be a very stringent test since a sizeable fraction of the hadronic energy will be carried off by the unobservable E^0 resulting from t quark decay.

In a more complete analysis of multimuon events arising from M^- production in neutrino reactions one would fold in detection efficiency and, moreover, would take account of the μ^+ which can arise from the semileptonic decay of the t quark.[49] The μ^+ which results half of the time from this decay will combine with $\mu^-\mu^-$ from M^- decay to yield additional $\mu^-\mu^-\mu^+$ events, and with $\mu^-\mu^+$ to yield wrong-sign trimuon events, $\mu^+\mu^+\mu^-$. It will also give rise to 4-muon events of the form $\mu^-\mu^-\mu^+\mu^+$, etc. However, leptons arising from the semileptonic decay of hadrons tend to have considerably lower energies than those arising from the decay of a heavy lepton produced at the leptonic vertex. Indeed, in most such events the μ^+ from a decaying hadron would have an energy lower than the cutoff value of 4 GeV used in the FHPRW experiment and consequently would not be observed. To compute the rate for trimuon production taking into account the cuts and acceptance in the FHPRW experiment would require a complicated Monte Carlo simulation; however, for the reason given above, the trimuon branching ratio of the M^- combined with its relative production cross section probably gives a reasonable estimate. The same comments apply, with appropriate changes, to M^+ production and the resulting trimuons of the form $\mu^+\mu^+\mu^-$ in antineutrino reactions.

Finally, in Figs. 24 and 25 we show the branching ratios for specific electron final states for various muonic final states. The mass of E^0 is taken to be 1 GeV. The $\mu^-\mu^-\mu^+$ and $\mu^-\mu^-$ modes are completely characterized by the curves in Fig. 22 since no electrons occur in the former case and all $\mu^-\mu^-$ states are of the form $\mu^-\mu^-e^+$; hence they are not drawn in Figs. 24 or 25.

V. SUMMARY AND CONCLUSIONS

Until the recent observation of trimuons by the FHPRW neutrino experiment there was no serious phenomenological motivation for enlarging the gauge group of weak and electromagnetic interactions beyond the group $SU(2) \otimes U(1)$ of Weinberg and Salam. A reasonable explanation

for most of these events seems to be the production and sequential decay of a heavy lepton. In order that this mechanism account for a large trimuon rate in the most natural way, it would be necessary to invoke a new gauge boson coupling ν_μ to M^- and hence to enlarge the gauge group.

The $SU(3) \otimes U(1)$ theory analyzed in this paper is a particularly appealing generalization of the original $SU(2) \otimes U(1)$ model with rich experimental implications. We summarize here its main features. All nonsinglet fermions are assigned to $\underset{\sim}{3}$ representations of SU(3). The new fermion content of the theory includes, in addition to the charmed quark, four heavy quarks, (t, g, b, and h), and in addition to the SPEAR heavy lepton τ and its neutrino ν_τ, six heavy leptons (E^0, M^0, T^0; E^-, M^-, T^-). It is reasonable to propose that the M^0 and M^- may have already manifested their presence in the FHPRW trimuon events. If indeed m_t or $m_g \sim 4$ GeV and $m_{M^-} \simeq 7$-8 GeV these fermions could be discovered in the next generation of colliding beam machines. The gauge boson spectrum of the theory consists of the photon, four massive charged bosons $W^\pm$ and $U^\pm$, and four neutral bosons X_1, X_2, Y and Z. The $W^\pm$, Y, and Z bosons can be searched for via the usual means under present consideration, e.g.

pp, $\bar{p}p$, and $e\bar{e}$ colliding rings. For the favored values of the parameters ℓ, w, and δ characterizing the (minimal) model, the masses of these vector particles are predicted to be generally comparable to the $W^{\pm}$ and Z of the original SU(2) $\otimes$ U(1) theory. Unfortunately, since J_Y^{μ} is purely axial-vector, while the electron and muon parts of J_Z^{μ} are purely vector, there will not be any parity violation in the reaction $e\bar{e} \to \mu\bar{\mu}$ resulting from the exchange of Y or Z gauge bosons. However, at sufficiently high energies there will be a sizeable front-back asymmetry in the cross section resulting from the interference of the amplitude arising from Y exchange with the amplitude arising from γ and Z exchange. Furthermore, at the appropriate energies the Y and Z bosons would be observed as resonances in the cross secti Because the $X_{1,2}$ and $U^{\pm}$ vector bosons couple to nondiagonal fermion currents they cannot of course be produced as resonances in the s-channel in the reaction $e\bar{e} \to \mu\bar{\mu}$. Furthermore, because they couple light quarks to heavy quarks it will be very difficult to produce them via pp or $\bar{p}p$ collisions.

The theory naturally incorporates quark-lepton and e-μ universality, and insures that J_Y^{μ} and J_Z^{μ} are diagonal to order $G_F\alpha$, thereby precluding neutral strangeness-changing currents, to the same order, in such processes as $K^0 \longleftrightarrow \bar{K}^0$ and $K_L \to \mu\bar{\mu}$. Furthermore there are no right-handed charged currents which involve only light fermions and could appear in neutron, hyperon, or muon decay. The model allows μ- and e-number nonconservation but at a naturally strongly suppressed level. The electrically neutral flavor-changing currents $J_{X_1}^{\mu}$ and $J_{X_2}^{\mu}$ contribute to the anomalous magnetic dipole moments of the muon and electron, and to the electric dipole moment of the neutron; however, a natural cancellation mechanism analogous to the GIM mechanism for fermions operates to severely reduce their contributions. Discrete symmetries such as the R and S invariance of the Lagrangian and the RU invariance of the vacuum are crucial in preventing undesirable fermion mixing and maintaining the properties mentioned above. A particularly interesting feature which follows from the exact RU symmetry is the absolute stability of a massive neutral lepton, E^0, with cosmological implications concerning the closure of the universe. Together with these exact discrete symmetries of the Lagrangian or vacuum the imposition of the approximate discrete symmetries $\sqrt{R}$ and $\sqrt{R}\sqrt{U}$ naturally establishes a hierarchy of fermion masses, ordered such that RU even fermions are light compared to the corresponding RU-odd fermions. The theory is quasi-vectorlike, which guarantees that it is free of triangle anomalies. Furthermore, in contrast to theories which are not quasi-vectorlike the present $SU(3) \otimes U(1)$ model makes it automatic that the interactions corresponding to the residual exact gauge group, $SU(3)_{color} \otimes U(1)_{charge}$, will be parity conserving, independent of the mechanism of spontaneous symmetry breaking.

The minimal model contains two Higgs triplets with U(1) hypercharge $y = -2/3$ and a complex Higgs octet. By adding another Higgs triplet with $y = 1/3$ we have constructed a useful generalization which interpolates continuously between the minimal $SU(3) \otimes U(1)$ model and the original $SU(2) \otimes U(1)$ model.

As regards the confrontation with experiment, the minimal $SU(3) \otimes U(1)$ theory agrees reasonably well with all available data with which its predictions can be compared. The weak contribution to the hyperfine splitting in hydrogen is too small to be measured. The weak contributions to the anomalous magnetic moments of the muon and electron are calculated and found to be in accord with the high precision experimental bounds. CP violation is adjusted to be of milliweak strength; the electric dipole moment of the neutron is then estimated to be of order 10^{-24} e-cm, close to the present experimental bound. Concerning neutrino reactions, in this model there is no right-handed valence strength current of the type which could be excited by antineutrinos and lead to a high y anomaly. This agrees with the preponderance of experimental data on this controversial question. A simplifying feature of the theory is that of the two neutral vector bosons Y and Z only the latter couples to neutrinos. With one choice of parameters, $\ell \simeq 0.18$ and $w \simeq 0.25$, the model agrees with charged and most neutral current data. The fit to charged and neutral current reactions is comparable in quality to that provided by the original $SU(2)\otimes U(1)$ model; however, it is true that in the present model this fit is attained by adjusting two free parameters in contrast to the one free parameter in the original $SU(2)\otimes U(1)$ theory. On the other hand, the present theory is also successful in accounting for both the general magnitude and the kinematic characteristics of the FHPRW trimuon events which motivated its initial construction. A specific feature of the model is that heavy lepton and hence trimuon production must be accompanied by heavy quark production; this can be tested experimentally. Finally, again unlike the original $SU(2)\otimes U(1)$ model, the (minimal version of the) present theory predicts no enhanced parity violation in heavy atoms; present experiments appear to favor this prediction. The minimal $SU(3)\times U(1)$ theory thus seems to be an attractive and relatively economical extension of the original $SU(2)\times U(1)$ unified gauge model of weak and electromagnetic interactions.

(Notes added by R. E. S.)

1. Recently a set of resonances in the reaction $p + (\text{Be or Cu}) \rightarrow \mu^+\mu^- + \text{anything}$ has been observed in the Columbia-Fermilab-Stony Brook experiment with masses in the region of 9.5 GeV.[50] The resonances can be resolved into at least two peaks, $\Upsilon(9.4)$, and $\Upsilon'(10.0)$, with some evidence for a third peak. In analogy with the J/ψ, ψ', and ψ'', it is plausible to assume that these peaks are due to the production and decay into dimuons of a $J^P = 1^-$ bound state Υ of heavy quarks Q and $\bar{Q}$, together with its radial excitations Υ' etc. In our model the heavy quark Q, which thus has an effective mass $m_Q \simeq 4.7$ GeV, could be identified as t or g or alternatively as b or h, depending on its charge. Unfortunately a decisive determination of the charge will probably require production of the Υ by e^+e^- colliding beams. The implications of various quark assignments for Q are analyzed further in Ref. 49.

2. Regarding the neutral current sector of the model, it is of interest to compare the predictions for the ratios $R^{\nu p} = \sigma_{nc}^{\nu p}/\sigma_{cc}^{\nu p}$ and $R^{\bar{\nu}p} = \sigma_{nc}^{\bar{\nu}p}/\sigma_{cc}^{\bar{\nu}p}$ with data which has recently become available. The Fermilab-IHEP-ITEP--Michigan νH_2 bubble chamber experiment[51] has measured a raw ratio $R_{raw}^{\nu p} = 0.40 \pm 0.14$. When this ratio is corrected for events excluded by

the hadron energy cut, assuming the Weinberg-Salam model with $\sin^2\theta_W = 1/3$, it becomes $R^{\nu p}_{corr} = 0.48 \pm 0.17$. For the corresponding antineutrino ratio the Argonne-Carnegie Mellon $\bar{\nu}H_2$ bubble chamber experiment[52] finds $R^{\bar{\nu}p}_{corr} = 0.49 \pm 0.14$, where again the correction has been made using the Weinberg-Salam model with $\sin^2\theta_W = 1/3$. For $r = 0$, our model yields, in the valence quark approximation, the predicted ratios

$$R^{\nu p} = (1+\ell)^2(\frac{7}{12} - \frac{7}{6}w + \frac{3}{4}w^2)$$

and

$$R^{\bar{\nu}p} = (1+\ell)^2(\frac{3}{8} - \frac{3}{4}w + \frac{9}{8}w^2) \quad .$$

Taking $\ell = 0.18$ and $w = 0.25$, which fit the other neutral current data, we find $R^{\nu p} = 0.47$ and $R^{\bar{\nu}p} = 0.36$. These values are both in agreement with the (corrected) measured values to within the experimental errors. Of course, in order to make this comparison, one would ideally use experimental ratios with a correction based on the structure of the neutral currents in the SU(3)×U(1) model. The inclusion of a 10% SU(3)-symmetric sea, as defined in Sec. 2, increases these ratios by a few per cent. In comparison, an evaluation of the predictions of the Weinberg-Salam model, assuming a similar sea content and taking $\sin^2\theta_W = 1/3$, gives $R^{\nu p} = 0.43$ and $R^{\bar{\nu}p} = 0.33$.

3. As Sehgal has recently pointed out,[53] an analysis of semi-inclusive neutral current pion production in the reaction $\nu(\bar{\nu}) + N \rightarrow \nu(\bar{\nu}) + \pi^{\pm} + X$ enables one to determine the isospin structure of the weak neutral hadronic

current (which couples to neutrinos). In contrast, the use of data on the fully inclusive reaction on an isoscalar target $\nu(\bar{\nu}) + N \rightarrow \nu(\bar{\nu}) + X$ only yields information on isoscalar combinations of (squares of) neutral current couplings. From his study of the Gargamelle data on semi-inclusive neutral current pion production, Sehgal finds that $u_L^2 = 0.082 \pm 0.035$, $u_R^2 = 0.055 \pm 0.025$, $d_L^2 = 0.158 \pm 0.035$, and $d_R^2 = 0.001 \pm 0.025$ in a notation where the neutral current amplitude is given by $A(\nu + q \rightarrow \nu + q) = (G_F/\sqrt{2})\,[\bar{\nu}\gamma_\alpha(1-\gamma_5)\nu]\,[u_L\bar{u}\gamma^\alpha(1-\gamma_5)u + u_R\bar{u}\gamma^\alpha(1+\gamma_5)u + d_L\bar{d}\gamma^\alpha(1-\gamma_5)d + d_R\bar{d}\gamma^\alpha(1+\gamma_5)d]$. In the model (with $r = 0$), one can see from Eqs. (3.2b) and (3.3) or (3.11) that $u_{L,R} = \frac{1}{2}(1+\ell)a_{uL,uR}$ and $d_{L,R} = \frac{1}{2}(1+\ell)a_{dL,dR}$. Thus for $\ell = 0.18$ and $w = 0.25$, the model would predict that $u_L^2 = 0.20$, $u_R^2 = 0.022$, and $d_L^2 = d_R^2 = 0.05$. These values are not in good agreement with the data. For comparison, the Weinberg-Salam model gives $u_L = (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)$, $u_R = -\frac{2}{3}\sin^2\theta_W$, $d_L = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W$, and $d_R = \frac{1}{3}\sin^2\theta_W$. For $\sin^2\theta = 0.3$, we thus find the values $u_L^2 = 0.09$, $u_R^2 = 0.04$, $d_L^2 = 0.16$, and $d_R^2 = 0.01$, all of which are in agreement with the data as analyzed by Sehgal.

4. Finally, we note that six additional trimuon events have been observed by the FHPRW group.[54] The heavy lepton production and cascade decay mechanism contained in our $SU(3)\otimes U(1)$ model is again reasonably successful in accounting for the characteristics of most of these events.

On June 16, 1977, our dear friend Benjamin W. Lee was killed in an automobile accident. This is an inestimably great loss to the world of physics, to the Fermi National Accelerator Laboratory, and to his family. Those physicists who were fortunate enough to talk to or work with him will forever remember his tremendous creativity, analytical powers and depth of understanding. Ben Lee was one of the most outstanding physicists of our day, and no words can adequately express our deep sorrow over his tragic passing.

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APPENDIX I.

Let us consider a potential V of the fields Ω, Φ (the last taken here to be complex: $\Phi = (\Phi_1 + i\,\Phi_2)/\sqrt{2}$, $\Phi_{1,2}^{\dagger} = \Phi_{1,2}$) which is invariant under the group $G = SU(3) \otimes U(1) \otimes R$. The action of G on the manifold M formed by Ω and Φ is described in the text. We define the G-invariant norms λ^2 and μ^2 of a point $m = (\Omega, \Phi) \in M$ by

$$\lambda^2 = \Omega^{\dagger}\Omega, \quad \mu^2 = \mathrm{Tr}\ \Phi^{\dagger}\Phi \tag{A. 1}$$

which are positive semidefinite. The potential V is a C^{∞} function of m, bounded from below, which has the property

$$\lim_{\lambda^2 + \mu^2 \to \infty} \frac{V}{(\lambda^2 + \mu^2)} = \infty \quad . \tag{A. 2}$$

A point $m \in M$ has a stabilizer (little group) $G_m \subset G$ which leaves this point invariant. The action of G on m generates an orbit O(m) consisting of all points $m' = gm$, $g \in G$. The stabilizers of m and m' are conjugated: $G_{m'} = g\,G_m g^{-1}$. Orbits whose stabilizers are equivalent to G_m form a stratum S(m). We refer the reader to Michel's paper[55] and references cited therein for a more rigorous description of the spaces of orbits and strata, and differential geometry therein. What concerns us here is the second theorem of Michel, which states that

Theorem (Michel): If S(m) is compact, and has an infinite number of orbits, every C^{∞} G-invariant function has at least one (local) minimum on an orbit in S(m).

For our purpose it is sufficient to note that the orbits O(m), where m take the form

$$\Omega = \begin{pmatrix} \lambda \\ 0 \\ 0 \end{pmatrix} \quad ; \quad \Phi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \mu\cos\theta \\ 0 & \mu e^{i\phi}\sin\theta & 0 \end{pmatrix} \tag{A. 3}$$

$\lambda, \ \mu > 0$

form a compact stratum $S_{\lambda\mu}(m)$ for given $\lambda, \mu > 0$, corresponding to the stabilizer $G_m = U(1) \otimes RU$, where RU is defined in the text. Orbits in this stratum may be labeled by two parameters θ, ϕ $(0 \leq \theta \leq \pi, -\pi/2 \leq \phi \leq \pi/2)$. Michel's theorem then states that any V takes a minimum on an orbit in $S_{\lambda\mu}(m)$, say at $\theta = \theta_0(\lambda, \mu)$, $\phi = \phi_0(\lambda, \mu)$, for given $\lambda, \mu > 0$.

The gradient of the potential V evaluated at this orbit lies necessarily in the $\lambda\mu$ plane. It is therefore possible to adjust, for example, the coefficients of λ^2 and μ^2 in V, so that a minimum of V occurs at $\lambda = \lambda_0$, $\mu = \mu_0$, $\theta = \theta_0(\lambda_0, \mu_0) \equiv \theta_0$, $\phi = \phi_0(\lambda_0, \mu_0) \equiv \phi_0$.

It still remains to show that for some range of parameters this local minimum is in fact an absolute minimum. For this purpose consider a new potential V' defined as

$$V' = V + a(\Omega^\dagger\Omega - \lambda_0^2)^2 + b(\mathrm{Tr}\ \Phi^\dagger\Phi - \mu_0^2)^2 \tag{A. 4}$$

$$+ c(\mathrm{Tr}\,\Phi^2 - \mu^2 e^{i\phi_0}\sin 2\theta_0)^2 + c(\mathrm{Tr}\,\Phi^{\dagger 2} - \mu^2 e^{-i\phi_0}\sin 2\theta_0)^2 + d(\Omega^\dagger\Phi^\dagger\Phi\Omega) + e(\Omega^\dagger\Phi\Phi^\dagger\Omega)$$

where the coefficients a, ..., e are all positive definite. We note that $V' - V$ is positive semidefinite, taking its minimum on the orbit $O(m_0)$. Thus, for sufficiently large a, ..., e > 0, V' takes the absolute minimum on $O(m_0)$ whose stabilizer is $U(1) \otimes RU$.

Since V is arbitrary, we have shown that for at least a finite range of parameters of V, the stabilizer of the orbit on which V becomes absolute minimum is $U(1) \times RU$. This proof can be readily extended to the case in which there are several Ω's, Φ's and η's.

APPENDIX II.

In the Lagrangian, local interaction terms of the Higgs fields Ω and Φ are at most quartic. Furthermore the discrete R symmetry forbids all cubic invariants. At first glance it seems possible that the resulting quartic potential might be invariant under a bigger group than $SU(3) \otimes U(1) \otimes R$. If the connected continuous group were bigger than $SU(3) \otimes U(1)$, we would have pseudo-Goldstone bosons[56] in the theory. We shall prove that this is not the case.

Consider first the terms $\mathrm{Tr}\,\Phi^\dagger\Phi$, $\mathrm{Tr}\,\Phi^{\dagger 2}$, $\mathrm{Tr}\,\Phi^2$, $\mathrm{Tr}(\Phi^\dagger\Phi)^2$, $\mathrm{Tr}\,\Phi^4$, $\mathrm{Tr}\,\Phi^{\dagger 4}$, which are even under R. The maximal invariance group of these terms is $SU(3) \times G$ where G acts trivially on Φ. Now consider terms involving Ω: $\Omega^\dagger\Omega$, $\Omega^\dagger\Phi^\dagger\Phi\Omega$, $\Omega^\dagger\Phi^2\Omega$, $\Omega^\dagger\Phi^{\dagger 2}\Omega$, etc. which are also even under R. Since G acts trivially on Φ, G must act not on the SU(3) index of Ω, but on all components of Ω uniformly. Therefore G can only be U(1):

$$\mathrm{U}(1):\quad \Omega \to e^{i\phi}\,\Omega$$
$$\Omega^\dagger \to \Omega^\dagger e^{-i\phi} \quad .$$

We have thus shown that the maximal connected invariance group of a generic quartic polynomial of Ω and ϕ which is invariant under $SU(3) \otimes U(1) \otimes R$ is $SU(3) \otimes U(1)$, and not a larger group.

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[4]A. Benvenuti, et al., Phys. Rev. Lett. 38, 1110, 1183 (1977). The initials stand for Fermilab-Harvard-Pennsylvania-Rutgers-Wisconsin. The data reported upon in this paper consists of six trimuon events. From a selected subset of $\mu^+\mu^-$ dimuons, inferred to arise from M^0 production and decay the FHPRW group derives another estimate of the M^0 mass, viz. $m_{M^0} = 4.9 \pm 1.5$ GeV. The Caltech-Fermilab collaboration had also observed and partially reconstructed two trimuon events; c.f. B. C. Barish, et al., Phys. Rev. Lett. 38, 577 (1977). The charges of the muons were not all measured, although the highest energy one

was a μ^- in both events. In the one event where the energies of both of the other muons were measured, they were quite small, ~5.2 and 7.2 GeV; their invariant mass was also quite small, 0.5 ± 0.05 GeV. Partly because of these characteristics the CF group attributed their trimuon events to conventional production mechanisms. These included (1) the production of low invariant mass muon pairs from virtual photons and/or the decay of vector mesons and (2) the associated production and semileptonic decay of new-flavored hadrons. The CERN-Dortmund-Heidelberg-Saclay experiment has also observed trimuons similar to the CF events; see M. Holder, et al., CERN preprint.

[5] Benjamin W. Lee and Steven Weinberg, Phys. Rev. Lett. 38, 1237 (1977).

[6] It might be noted parenthetically, that if the trimuons had been of the form $\mu^+\mu^+\mu^-$ rather than $\mu^-\mu^-\mu^+$ then one could have modified the original $SU(2) \times U(1)$ theory by assigning the leptons and quarks to triplets, $(M^+, \nu_\mu, \mu^-)_L$, $(E^+, \nu_e, e^-)_L$, $(M^+, M^0, \mu^-)_R$, $(E^+, E^0, e^-)_R$ and similarly with the τ-type leptons and quarks. The leptons E^+, M^+, T^+ would be mixtures of mass eigenstates and the production-decay chain would be $\nu_\mu + N \to M^+ + \ldots$, $M^+ \to \mu^+ + M^0 + \ldots$, $M^0 \to \mu^- + \mu^+ + \ldots$

[7] B. W. Lee and R. E. Shrock, Fermilab-Pub-77/21-THY (February, 1977) to be published in Physical Review D.

[8] For an analysis of trimuon production based on a mechanism similar to that proposed in Ref. 7, see V. Barger, et al., Phys. Rev. Lett. 38, 1190 (1977); V. Barger, et al., Wisconsin preprints COO-596, 597, 598, 602, and 603.

[9] C. N. Yang, Phys. Rev. D1, 2360 (1970).

[10] For an exhaustive bibliography on previous SU(3) or SU(3) × U(1) models up to 1974, see C. H. Albright, C. Jarlskog, and M. Tjia, Nucl. Phys. B86, 535 (1975). See also P. Fayet, Nucl. Phys. B78, 14 (1974); H. Fritzsc and P. Minkowski, Phys. Lett. 63B, 99 (1976); F. Gürsey and P. Sikivie, Phys. Rev. Lett. 36, 775 (1976); Phys. Rev. Lett. 36, 775 (1976); P. Ramonc Nucl. Phys. B110, 214 (1976); M. Yoshimura, Prog. Theor. Phys. 57, 327 (1977). The main difference between all these models and ours is that, in our theory, universality, suppression of right-handed currents, flavor conservation in neutral currents, etc., are consequences of the pattern of spontaneous symmetry breaking that emerges naturally in the theory. SU(3) ⊗ U(1) models have also been considered by Segrè and collaborators:

G. Segrè and J. Weyers, Phys. Lett. 65B, 243 (1976); G. Segrè and M. Golshani, University of Pennsylvania preprint UPR-0075T; P. Langack and G. Segrè, Univ. of Penn. preprints UPR-0072T, 0073T. There are several similarities between the original work of Segrè and Weyers and our model, e.g. the use of a discrete symmetry to make the symmetry-breaking pattern natural. However, their model is different from ours in a number of important respects. Their model contains only fermion triplets and is strictly vectorlike; their discrete symmetry corresponds to our $\sqrt{R}$; and they assume the limit $|a| << |v'|$ with $b = v = 0$. In consequence, the phenomenological implications of the two models are quite different.

[11] For the original results quoted in Eqs. (3.38a, b), see P. Baird, et al., Nature, 264, 549 (1976) (see also D. Soreide, et al., Phys. Rev. Lett. 36, 352 (1976)). For the more recent results cited in Eqs. (3.40a, b); see P. G. H. Sanders, invited talk given at the 1977 International Symposium on Lepton and Photon Interactions at High Energies (Hamburg) and M. Bouchiat, talk given at the Seventh International Conference on High Energy Physics and Nuclear Structure, Zurich (1977). It is important to note that the error estimates given for the earlier results did not fully include systematic errors, whereas those given for the recent results do include both statistical and systematic errors.

[12] M. Brimicombe, C. Loving, and P. Sanders, J. Phys. B9, L1 (1976); E. N. Henley and L. Wilets, Phys. Rev. A, 1411 (1977), I. Khriplovich, talk given at the XVIII International Conference on High Energy Physics, Tbilisi (1976); S. Meshkov and S. P. Rosen, ERDA preprint (1976); C. E. Loving and P. G. H. Sandars, calculation reported at the Hamburg Conference, 1977. For earlier discussions of atomic parity violations (in hydrogen, normal heavy atoms, and muonic atoms) see Ya. B. Zeldovich, JETP 36, 964 (1959); F. Curtis Michel, Phys. Rev. 138B, 408 (1965); G. Feinberg and M. Y. Chen, Phys. Rev. D10, 190 (1974); M. A. Bouchiat and C. Bouchiat, Phys. Lett. 48B, 111 (1974); J. Bernabeu, et al., Phys. Lett. 50B, 467 (1974); I. B. Khriplovich, JETP Lett. 20, 315 (1974); R. R. Lewis and W. L. Williams, Phys. Lett. 59B, 70 (1975).

[13] Benjamin W. Lee and Steven Weinberg, Phys. Rev. Lett, 39, 165 (1977). The question of whether such absolutely stable neutral heavy leptons exist is of course more general than the particular $SU(3) \otimes U(1)$ gauge theory

proposed in Ref. 5. The astrophysical implications of such leptons are examined further in J. E. Gunn, B. W. Lee, I. Lerche, D. N. Schramm, G. Steigman, and S. Weinberg, University of Chicago preprint, in preparation.

[14] Steven Weinberg, Phys. Rev. D8, 605 (1973); D8, 4482 (1973).

[15] By "natural" we mean in a way which depends only on the group structure and representation content of the theory and not on the values taken by the parameters of the theory.

[16] M. Perl, et al., Phys. Rev. Lett. 35, 1489 (1975) and Phys. Lett. 63B, 466 (1976).

[17] Recall that if $<(\Phi_i)_{11}>_0$ were nonzero, the neutral members of the left-handed lepton triplets (currently the neutrinos) would not in general also be mass eigenstates. There would still necessarily be one massless lepton in each of the three lepton families since the neutral leptonic sector consists of six left-handed chiral components and only three right-handed chiral components. However in a given family the left-handed chiral component of the massless lepton would have both a triplet part and a singlet part. Since the triplet component would be different for the e- and μ-families e-μ universality would be violated. Another problem with placing u_R' in the right-handed triplet where t_R' is at present, is that this would give

rise to a right-handed light quark current $\bar{u}_R{}'\gamma_\mu d_R$ coupled to the gauge boson U. If the E^0 were light (i.e. if $m_{E^0} < 40$ eV in accordance with the results of Ref. 13) then such a current would contribute to neutron and and hyperon decay via the elementary decays $d_R \to u_R e^- \bar{E}^0$ and $s_R \to u_R e^- \bar{E}^0$ (weighted by appropriate mixing angles). Unless suppressed by small mixing angles, etc. this would contradict experiment. Moreover, even if E^0 is heavy, which is the preferred option, the $\bar{u}_R{}'\gamma_\mu d_R$ current would again enter in neutron and hyperon decays if there were $W^\pm$-$U^\pm$ mixing. Such mixing would be allowed if RU symmetry were broken.

[18]The FIIM data is reported in J. P. Berge, et al., Phys. Rev. Lett. 39, 382 (197 The $B^{\bar{\nu}N}$ values measured in this experiment do exhibit a steady decrease, which leads the authors of the above paper to describe their data as consistent with either no high-y anomaly or one which is somewhat smaller, as manifested in $\langle y\rangle^{\bar{\nu}N}$ and $B^{\bar{\nu}N}$, than the effect reported by the HPWF group, Ref. 3. For the Caltech-Fermilab data, see B. C. Barish, et al., Caltech preprints CALT 68-605, 68-606, and 68-607 (1977).

[19]The CDHS data is reported in M. Holder, et al., Phys. Rev. Lett. 39, 433 (1977); the BEBC data is discussed by K. Schultze, invited talk at the 1977 International Symposium on Lepton and Photon Interactions at High Energies (Hamburg).

[20]C. H. Albright, J. Smith, and J. Vermaseren, SUNY at Stony Brook preprint ITP-77-32. See also C. H. Albright, J. Smith, and J. Vermaseren, Phys. Rev. Lett. 38, 1187 (1977) and SUNY at Stony Brook preprint ITP-77-43; and C. H. Albright, R. E. Shrock, and J. Smith, Fermilab-Pub-77/70-THY.

... in the theoretical calculation of the ratios

$\sigma(\bar{\nu}_\mu + N \to M^+ + X)/\sigma(\bar{\nu}_\mu + N \to \mu^+ + X)$ and $\sigma(\nu_\mu + N \to M^- + X)/\sigma(\nu_\mu + N \to \mu^- + X$ in order that one may justifiably compare them directly with the corresponding experimentally measured ratios, in which the denominators (the regular charged current cross sections) have this cut imposed. The requirement $E > 100$ GeV has very little effect on the numerators of the theoretically computed ratios, i.e. the $M^\pm$ production cross sections, since with the masses $m_{M^-} = 8$ GeV, m_t or $m_b = 4$ GeV the thresholds for the reactions $\nu_\mu(\bar{\nu}_\mu) + N \to M^\mp + X$ are at $E_{Th} \simeq 90$ GeV and the cross sections do not become substantial until $E - E_{Th} \gtrsim 50$ GeV.

[21] A. Benvenuti, et al., Phys. Rev. Lett. 37, 1039 (1976). The values of $R^{\nu N}$ and $R^{\bar{\nu}N}$ given in this paper supersede the earlier results $R^{\nu N} = 0.11 \pm 0.05$ and $R^{\bar{\nu}N} = 0.32 \pm 0.09$ given by B. Aubert, et al., Phys. Rev. Lett. 32, 1454 (1974); 32, 1457 (1974).

[22] D. Buchholz, in the Proceedings of the International Neutrino Conference, Aachen, 1976, eds. H. Faissner, et al., (Vieweg, Braunschweig, 1977), p. 289. Subsequent to the writing of this paper, slightly different values of the (uncorrected) neutral to charged current ratios have been reported by the CF group in F. S. Merrit, et al., Caltech preprint CALT 68-601: $R^{\nu N} = 0.28 \pm 0.03$ and $R^{\bar{\nu}N} = 0.35 \pm 0.11$. See also F. S. Merrit, et al., Caltech preprint CALT 68-600.

[23] J. Blietschau, et al., Nucl. Phys. B118, 218 (1977) (for CERN Gargamelle data); M. Holder, et al., CERN preprint (for CDHS data).

[24] Our calculations of $\nu(\bar{\nu})p \rightarrow \nu(\bar{\nu})p$ cross sections use the methods of C. H. Albright, C. Quigg, R. E. Shrock, and J. Smith, Phys. Rev. D14, 1780 (1976). In particular the Brookhaven neutrino flux distribution is taken to be $dN^{\nu}/dE(E) = 0.12 \exp\left[-0.8(E - 1.6)^2\right]$ for $0.5 < E < 2.4$ and $dN^{\nu}/dE(E) = \exp(-E) + 0.0133 \exp(-0.3E)$ for $E > 2.4$, where E is in

units of GeV. The antineutrino spectrum is taken to have the same shape. This flux distribution is based on the Sanford-Wang π, k spectra [see BNL Report No. 11299, 1967 (unpublished)].

[25]K. Kleinknecht, in Proceedings of the XVII International Conference on High Energy Physics, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England), p. III-23. Our use of the symbol ϵ in Eq. (3.22d) follows the notation of Ref. 24; no confusion should arise between this symbol and the CP-violating parameter ϵ defined in Eq. (2.36).

[26]D. Cline, et al., Phys. Rev. Lett. 37, 252 (1976); 37, 648 (1976).

[27]W. Lee, et al., Phys. Rev. Lett. 37, 186 (1976).

[28]This is determined from the analysis in Y. P. Yao, Phys. Rev. 176, 1680 (1968).

[29]J. Blietschau, et al., Nucl. Phys. B114, 189 (1976) and references therein.

[30]F. Bobisut, in Proceedings of the International Neutrino Conference, Aachen, 197 eds. H. Faissner, et al. (Vieweg, 1977), p. 223; H. Reithler, invited talk given at the 1977 International Symposium on Lepton and Photon Interactions at High Energies (Hamburg). The allowed region in g_V and g_A implied by the values of $\sigma(\nu_\mu e \to \nu_\mu e)$ and $\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$ given in the latter reference is ~1 standard deviation away from the point $(g_V, g_A) = (-0.24, 0)$ predicted in the minimal $SU(3) \otimes U(1)$ model with $\ell = 0.18$ and $w = 0.2$.

[31]F. Reines, H. S. Gurr, and H. W. Sobel, Phys. Rev. Lett. 37, 315 (1976).

[32]See e.g. B. E. Lautrup, A. Peterman, and E. de Rafael, Phys. Rept. 3C, 193 (1972); V. Hughes and C. S. Wu, eds. Muon Physics (Academic Press, New York, 1975).

[33] For reviews of these calculations and references to the extensive earlier literature, see B. E. Lautrup, A. Peterman, and E. de Rafael, Phys. Rept. 3C, 193 (1972) op cit. (Ref. 29); S. Brodsky, SLAC-PUB-1699 ('975); J. Calmet, S. Narison, M. Perrottet, and E. de Rafael, Rev. Mod. Phys., 49, 21 (1977). Among the most recent and most accurate calculations of the α^3 contribution to a_μ and/or a_e are P. Cvitanovic and T. Kinoshita, Phys. Rev. D10, 4007 (1974); R. Barbieri and E. Remiddi, Nucl. Phys. B90, 223 (1975); J. Calmet and A. Peterman Phys. Lett. 58B, 449 (1976); M. Levine and J. Wright, Phys. Rev. D8, 3171 (1973); M. J. Levine, R. C. Perisho, and R. Roskies, Phys. Rev. D13, 997 (1976); M. Levine and R. Roskies, Phys. Rev. D14, 2191 (1976) and R. Carol, Phys. Rev. D12, 2344 (1975).

[34] M. A. Samuel and C. Chlouber, Phys. Rev. Lett. 36, 442 (1976); M. J. Levine and R. Roskies, op cit., Ref. 30.

[35] J. Calmet, et al., op cit., Ref. 30; S. Brodsky, op cit., Ref. 30. The theoretical calculations reviewed in these papers must be modified for application to our model because of the new contributions to the fourth order vacuum polarization graph arising from the heavy charged fermions in this model. However, as will be shown, these contributions are negligible. The contribution to a_ℓ from a heavy lepton loop in the fourth order vacuum polarization graph has been calculated in B. E. Lautrup and E. de Rafael, Phys. Rev. 174, 1835 (1968); the result is

$$a_\ell(\text{heavy lepton}) = (\alpha/\pi)^2\left[(m_\ell/M)^2/45 + \mathcal{O}\left\{(m_\ell/M)^4\ln(M/m_\ell)\right\}\right],$$

where ℓ = e or μ and M denotes the mass of the heavy lepton. Evaluating this formula for the lowest-lying charged heavy leptons, we find $a_\mu(\tau^-; m_\tau = 1.95\text{ GeV}) = 3.52 \times 10^{-10}$, and $a_\mu(M^-; m_{M^-} = 8\text{ GeV}) = 2.09 \times 10^{-11}$. These contributions are all negligibly small compared to the present error of $\pm 9.7 \times 10^{-9}$ on the theoretical estimate of a_μ. It should be noted in passing that the main part of this error, $\pm 9.4 \times 10^{-9}$, comes from the uncertainty in the hadronic vacuum polarization contribution, as determined from the cross section $\sigma(e\bar{e} \to \text{hadrons})/\sigma(e\bar{e} \to \mu\bar{\mu})$. In turn, the main uncertainty in the experimental measurement comes not from the high energy region where there might be new narrow resonances, but from the region in $\sqrt{s}$ below the $\rho(770)$. For the electron anomaly one can also easily check that the contributions of τ^-, M^-, etc. to the fourth order vacuum polarization graph are negligible. For example, $a_e(\tau; m_\tau = 1.95\text{ GeV}) = 8.23 \times 10^{-15}$, whereas the error quoted by Cvitanovic and Kinoshitafor their calculation of a_e is $\pm 1.9 \times 10^{-10}$. Similar comments apply for the contributions from the heavy quarks t,g, b, and h, (or, more precisely,the hadrons comprised of them).

[36] J. Bailey, et al., Phys. Lett. 67B, 225 (1977). For a review of earlier work, see F. Combley and E. Picasso, Phys. Rept. 14C, 1 (1974).

[37] A. Rich and J. Wesley, Rev. Mod. Phys. 44, 250 (1972).

[38] R. S. Van Dyck, Jr., P. B. Schwinberg, and H. G. Dehmelt, Phys. Rev. Lett. 38, 310 (1977).

[39] B. W. Lee, K. Fujikawa, and A. I. Sanda, Phys. Rev. D6, 2923 (1972). The anomalous magnetic moments of the muon and electron in the original $SU(2)\otimes U(1)$ theory (c.f. Eqs. (4.19) and (4.20)) were calculated by Roman Jackiw and Steven Weinberg, Phys. Rev. D5, 2396 (1972).

[40] W. B. Dress, P. D. Miller, J. M. Pendleton, Paul Perrin, and Norman F. Ramsey, Phys. Rev. D15, 9 (1977).

[41] J. D. Bjorken, Kenneth Lane, and Steven Weinberg, SLAC-PUB-1925 (1977).

[42] B. W. Lee, S. Pakvasa, R. E. Shrock, and H. Sugawara, Phys. Rev. Lett. 38, 937 (1977); B. W. Lee and R. E. Shrock, Fermilab-Pub-77/21-THY, op cit., Ref. 7.

[43] The present mechanism for $\mu \rightarrow e\gamma$ is similar to that discussed by S. Petcov, Dubna preprint JINR-E2-10176; S. Bilenky, S. Petcov, and B. Pontecorvo, Dubna preprint JINR-E2-10374; and T. P. Cheng and L. F. Li, Phys. Rev. Lett. 38, 381 (1977), and University of Missouri preprints UMSL-77-2 and UMSL-77-3 (revised version). Other recent papers on μ- and e-number nonconservation include James D. Bjorken and Steven Weinberg, SLAC-PUB-1925; F. Wilczek

and A. Zee, Phys. Rev. Lett. 38, 531 (1977); S. B. Treiman, F. Wilczek, and A. Zee, Princeton preprint; W. Marciano and A. I. Sanda, Phys. Lett. 67B, 303 (1977) and Phys. Rev. Lett. 38, 1512 (1977); W. K. Tung, Phys. Lett. 67B, 52 (1977); P. Minkowski, Phys. Lett. 67B, 421 (1977); H. Fritzsch, Phys. Lett. 67B, 451 (1977); M. A. B. Bég and A. Sirlin, Phys. Rev. Lett. 38, 1113 (1977); V. Barger and D. V. Nanopoulos, Wisconsin preprint COO-881-583 (also Errata and Addenda), among other references. There is also an earlier literature on μ- and e-number nonconservation extending back to B. Pontecorvo, Sov. Phys. JETP 33, 549 (1957); 34, 247 (1958) and G. Feinberg, Phys. Rev. 110, 1482 (1958).

[44] There should be no confusion between $(V)_3 \equiv \frac{1}{2}\left[V_+, V_-\right] = \sqrt{3}V_8 + V_3$ and $V_3 = \frac{1}{2}\lambda_3$. We are using the same letter V for the generators of SU(3) and for V-spin.

[45] In Ref. 10 it was shown that in order for the mean mass density due to an absolutely stable neutral heavy lepton (E^0, in our model) to be less than the presently established upper limit for the mean mass density of the universe, it is necessary that $m_{E^0} \lesssim 40$ eV or $m_{E^0} \gtrsim 1$-4 GeV. For our estimate of BR($\mu \to e\gamma$) we have chosen to take $m_{E^0} = 1$ GeV and for illustrative purposes have chosen the maximum value of $\sin^2\beta\cos^2\beta$. If we had taken the E^0 to be very light, then the angle β would be constrained such that $\sin^2\beta \lesssim 0.02$ (see Section IV). Thus for $w = 0.25$, $m_{M^0} = 4$ GeV, and $m_{E^0} < 40$ GeV, $BR(\mu \to e\gamma)_{r=0} \simeq 2.2 \times 10^{-11}$.

[46] M. K. Gaillard and B. W. Lee, Phys. Rev. D10, 897 (1974). The $K_L \to \mu\bar{\mu}$ decay rate provides a less stringent bound on the mass of the charged quark; see M. K. Gaillard, B. W. Lee, and R. E. Shrock,

Phys. Rev. D13, 2674 (1976). In Eq. (4.35), $f_K = f_\pi$ where f_π is the pion decay constant.

[47]M. M. Nagels, et al., Nucl. Phys. B109, 1 (1976); K. Kleinknecht, in Proceedings of the SVII International Conference on High Energy Physics, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. III-23; Particle Data Group, Review of Particle Properties (1977). The model-dependent (but nuclear wavefunction-independent) radiative correction to the beta decay coupling constant $G_F{}^\beta$ is expected to be of order 1% and involves terms such as $\ln(m_Z{}^2/m_p{}^2)$ and $\ln(m_Z{}^2/m_A{}^2)$, where m_A is the mass parameter entering into the axial vector form factor. This has been calculated by A. Sirlin, Nucl. Phys. B71, 29 (1974). See also D. H. Wilkinson, Nature 257, 189 (1975); D. H. Wilkinson and D. E. Alburger, Phys. Rev. C13, 2517 (1976); D. H. Wilkinson, University of Sussex preprint.

[48]For calculations of trimuon production via heavy lepton production and cascade decay trimuons see C. H. Albright, J. Smith, and J. Vermaseren, op cit., Ref. 20; V. Barger, et al., Phys. Rev. Lett. 38, 1190 (1977), Wisconsin preprints COO-596, 597, 598, 600 (and the revised version), 602 and 603; and R. M. Barnett and L. N. Chang, SLAC-PUB-1932. For discussions of higher gauge groups motivated by the FHPRW observation of trimuons, see P. Langacker and G. Segrè, op cit., Ref. 10, and A. Zee, F. Wilczek, and S. B. Treiman, Phys. Lett. 68B, 369 (1977). An analysis of associated production of a pair of hadrons carrying new quantum numbers as the source of the trimuon events, see F. Bletzacker, H. T. Nieh, and A. Soni, Phys. Rev. Lett. 38, 1241 (1977) and SUNY at Stony Brook preprint ITP-SB-77-43.

[49]C. H. Albright, R. E. Shrock, and J. Smith, Fermilab-Pub-77/70-THY.

[50]S. W. Herb, et al., Phys. Rev. Lett. 39, 252 (1977); S. W. Herb, et al., Fermilab preprint.

[51]F. A. Harris, et al., Phys. Rev. Lett. 39, 437 (1977).

[52]This data is reported by A. F. Garfinkel, invited talk given at the 1977 International Symposium on Lepton and Photon Interactions at High Energies (Hamburg).

[53]L. Sehgal, Aachen preprint.

[54]A. Benvenuti, et al., HPWF preprint.

[55]L. Michel, C. R. Acad. Sc. Paris, 272A, 433 (1971); L. Michel, in Statistical Mechanics and Field Theory, R. N. Sens and C. Weil, eds., (Gordon and Breach, London, 1972), p. 133.

[56]Steven Weinberg, Phys. Rev. Lett. 29, 1698 (1972).

TABLE CAPTIONS

Table 1: Specific final states in M^- decay containing $\mu^-\mu^-\mu^+$.

Table 2: Specific final states in M^- decay containing $\mu^-\mu^-$. All of these states are of the form $\mu^-\mu^-e^+$ + neutral stable leptons + possible hadrons.

Table 3: Specific final states in M^- decay containing $\mu^-\mu^+$. A further classification according to the electrons present is: (a) $\mu^-\mu^+$ (no $e^\pm$) (b) $\mu^-\mu^+e^-$.

Table 4: Same as Table 3, for states containing μ^-. The classification according to electrons present is (a) μ^- (no $e^\pm$) (b) μ^-e^+ (c) $\mu^-e^-e^+$.

Table 5: Same as Table 3, for states containing μ^+; (a) μ^+e^- (b) $\mu^+e^-e^-$.

Table 6: Same as Table 3, for states containing no $\mu^\pm$; (a) no $\mu^\pm$ or $e^\pm$ (b) e^- (c) e^-e^+ (d) $e^-e^-e^+$.

TABLE 1

$M^- \to \mu^-\mu^-\mu^+ + \ldots$

1. $\mu^-\mu^-\mu^+E^0\bar{\nu}_\mu$
2. $\mu^-\mu^-\mu^+\bar{E}^0\nu_\mu$
3. $\mu^-\mu^-\mu^+E^0\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau$
4. $\mu^-\mu^-\mu^+E^0\bar{\nu}_\mu s_\theta\bar{s}_\theta$

TABLE 2

$M^- \to \mu^-\mu^- + \ldots$

$(\mu^-\mu^-e^+)$

1. $\mu^-\mu^-e^+E^0\bar{\nu}_\mu$
2. $\mu^-\mu^-e^+\bar{E}^0\nu_\mu$
3. $\mu^-\mu^-e^+E^0\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau$
4. $\mu^-\mu^-e^+E^0\bar{\nu}_\mu s_\theta\bar{s}_\theta$

TABLE 3

$$M^- \to \mu^- \mu^+ + \ldots$$

(A) ($\mu^-\mu^+$; no $e^\pm$)

1. $\mu^-\mu^+E^0 d_\theta \bar{u}$
2. $\mu^-\mu^+E^0 \nu_\tau \bar{\nu}_\tau d_\theta \bar{u}$
3. $\mu^-\mu^+E^0 d_\theta \bar{u} s_\theta \bar{s}_\theta$

(B)($\mu^-\mu^+e^-$)

1. $\mu^-\mu^+e^-E^0 \bar{\nu}_e$
2. $\mu^-\mu^+e^-E^0 \bar{\nu}_\mu$
3. $\mu^-\mu^+e^-\bar{E}^0 \nu_\mu$
4. $\mu^-\mu^+e^-E^0 \bar{\nu}_e \nu_\tau \bar{\nu}_\tau$
5. $\mu^-\mu^+e^-E^0 \bar{\nu}_\mu \nu_\tau \bar{\nu}_\tau$
6. $\mu^-\mu^+e^-E^0 \bar{\nu}_e s_\theta \bar{s}_\theta$
7. $\mu^-\mu^+e^-E^0 \bar{\nu}_\mu s_\theta \bar{s}_\theta$

TABLE 4

$M^- \to \mu^- + \ldots$

(A) (μ^- ; no $e^\pm$)

1. $\mu^- E^0 \bar{\nu}_\mu$
2. $\mu^- \bar{E}^0 \nu_\mu$
3. $\mu^- E^0 \bar{\nu}_\mu \nu_\tau \bar{\nu}_\tau$
4. $\mu^- E^0 \bar{\nu}_\mu s_\theta \bar{s}_\theta$
5. $\mu^- \bar{E}^0 \nu_\mu d_\theta \bar{d}_{\theta'}$

(B) ($\mu^- e^+$)

1. $\mu^- e^+ E^0 d_\theta \bar{u}$
2. $\mu^- e^+ E^0 \nu_\tau \bar{\nu}_\tau d_\theta \bar{u}$
3. $\mu^- e^+ E^0 d_\theta \bar{u} s_\theta \bar{s}_\theta$

(C) $\mu^- e^- e^+$

1. $\mu^- e^- e^+ E^0 \bar{\nu}_e$
2. $\mu^- e^- e^+ E^0 \bar{\nu}_\mu$
3. $\mu^- e^- e^+ \bar{E}^0 \nu_\mu$
4. $\mu^- e^- e^+ E^0 \bar{\nu}_e \nu_\tau \bar{\nu}_\tau$
5. $\mu^- e^- e^+ E^0 \bar{\nu}_e s_\theta \bar{s}_\theta$
6. $\mu^- e^- e^+ E^0 \bar{\nu}_\mu s_\theta \bar{s}_\theta$

TABLE 5

$M^- \to \mu^+ + \ldots$

(A) $\mu^+ e^-$

1. $\mu^+ e^- E^0 d_\theta \bar{u}$
2. $\mu^+ e^- E^0 \nu_\tau \bar{\nu}_\tau d_\theta \bar{u}$
3. $\mu^+ e^- E^0 s_\theta \bar{s}_\theta d_\theta \bar{u}$

(B) $\mu^+ e^- e^-$

1. $\mu^+ e^- e^- E^0 \bar{\nu}_e$
2. $\mu^+ e^- e^- \bar{E}^0 \nu_\mu$
3. $\mu^+ e^- e^- E^0 \bar{\nu}_e \nu_\tau \nu_\tau$
4. $\mu^+ e^- e^- E^0 \bar{\nu}_e s_\theta \bar{s}_\theta$

TABLE 6

$M^- \to (\text{no } \mu^{\pm}\text{'s}) + \ldots$

(A) (no $e^{\pm}$'s)

1. $E^0 d_\theta \bar{u}$
2. $E^0 \nu_\tau \bar{\nu}_\tau d_\theta \bar{u}$
3. $E^0 s_\theta \bar{s}_\theta d_\theta \bar{u}$

(B) (e^-)

1. $e^- E^0 \bar{\nu}_e$
2. $e^- \bar{E}^0 \nu_\mu$
3. $e^- E^0 \bar{\nu}_e \nu_\tau \bar{\nu}_\tau$
4. $e^- E^0 \bar{\nu}_e s_\theta \bar{s}_\theta$

(C) ($e^- e^+$)

1. $e^- e^+ E^0 d_\theta \bar{u}$
2. $e^- e^+ E^0 \nu_\tau \bar{\nu}_\tau d_\theta \bar{u}$

(D) ($e^- e^- e^+$)

1. $e^- e^- e^+ E^0 \bar{\nu}_e$
2. $e^- e^- e^+ \bar{E}^0 \nu_\mu$
3. $e^- e^- e^+ \bar{\nu}_e \nu_\tau \bar{\nu}_\tau$
4. $e^- e^- e^+ E^0 \bar{\nu}_e s_\theta \bar{s}_\theta$

FIGURE CAPTIONS

Fig. 1: Vector boson masses as functions of ℓ, for $r = 0$, $w = 0.25$, and $\delta = 0.01$.

Fig. 2: Vector boson masses as functions of w for $r = 0$, $\ell = 0.18$, and $\delta = 0.01$.

Fig. 3: Vector boson masses as functions of r for $\ell = 0.18$, $w = 0.25$, and $\delta = 0.01$.

Fig. 4: Deep inelastic charged current to neutral current isoscalar cross section ratios $R^{\nu N}$ and $R^{\bar{\nu} N}$ for $r = 0$ and (a) $\ell = 0$, (b) $(1+\ell)^2 = 1.2$, i.e. $\ell = 0.095$, (c) $(1+\ell)^2 = 1.4$, i.e. $\ell = 0.18$. Along each curve w increases from $w = 0$ to $w = 1$ in steps of 0.1 as indicated by tick marks. See text for details of the calculation and references for the data points.

Fig. 5: Deep inelastic charged current to neutral current isoscalar cross section ratios $R^{\nu N}$ and $R^{\bar{\nu} N}$ as functions of r for $\ell = 0.18$ an (a) $w = 0.2$, (b) $w = 0.3$. Along each curve r increases from $r = 0$ to $r = 0.5$ in steps of 0.1; thereafter we show only the values $w = 1, 2, \infty$.

Fig. 6: The ratio $R_{el}^{\nu p} = \sigma(\nu p \rightarrow \nu p)/\sigma(\nu n \rightarrow \mu^- p)$ as a function of w for $r = 0$, $\ell = 0$ (dashed curve) and $\ell = 0.18$ (solid curve). The HPW data is shown as a shaded band and the CIR data is represented by a point at $w = 0.2$. The cross sections are flux-averaged for the Brookhaven neutrino spectrum,

and the cuts applied are those for the HPW experiment. See text for details.

Fig. 7: The ratio $R_{el}^{\nu p}$ as a function of w for r = 0, ℓ = 0.18, and m_A = 0.8 GeV (solid curve), 0.9 GeV (dashed curve), and 1.0 GeV (dotted curve). Data are as in Fig. 6.

Fig. 8: The ratio $R_{el}^{\bar{\nu}p} = \sigma(\bar{\nu}p \to \bar{\nu}p)/\sigma(\bar{\nu}p \to \mu^+ n)$ as a function of w for r = 0, ℓ = 0 (dashed curve) and ℓ = 0.18 (solid curve). The shaded band represents the HPW data. The cross sections are flux-averaged for the Brookhaven neutrino spectrum, and the cuts applied are those for the HPW experiment. See text for details.

Fig. 9: The ratio $R_{el}^{\bar{\nu}p}$ as a function of w for r = 0, ℓ = 0.18, and m_A = 0.8 GeV (solid curve), 0.9 GeV (dashed curve), and 1.0 GeV (dotted curve). The shaded band represents the HPW data.

Fig. 10: The ratio of flux-averaged neutral current cross sections, $\sigma(\bar{\nu}p \to \bar{\nu}p)/\sigma(\nu p \to \nu p)$, for r = 0, with the HPW cuts, as a function of w. The shaded band represents the HPW measurement.

Fig. 11: The neutrino neutral current and, for comparison, the charged current differential cross sections $d\sigma/dQ^2(\nu p \to \nu p)$ and $d\sigma/dQ^2(\nu n \to \mu^- p)$, flux-averaged for the Brookhaven neutrino spectrum. For the neutral current reaction,

$r = 0$, $\ell = 0.18$, and (a) $w = 0.1$, (b) $w = 0.2$. The solid and dashed data points are the charged and neutral current data from the HPW experiment.

Fig. 12: The antineutrino neutral current and, for comparison, the charged current differential cross sections $d\sigma/dQ^2(\bar{\nu}p \to \bar{\nu}p)$ and $d\sigma/dQ^2(\bar{\nu}p \to \mu^+ n)$, flux-averaged for the Brookhaven neutrino spectrum. For the neutral current reaction curves, $r = 0$, $\ell = 0.18$, and $w =$ (a) 0.1, (b) 0.2. The neutral current data is from the HPW experiments.

Fig. 13: The ratio $R_{el}^{\nu p}$ as a function of w for $r = 0.1$ and $\ell = 0.18$.

Fig. 14: The ratio $R_{el}^{\bar{\nu} p}$ as a function of w for $r = 0.1$ and $\ell = 0.18$.

Fig. 15: Values of g_V (horizontal axis) and g_A (vertical axis) as functions of w and r, for $\ell = 0.18$. The line segments correspond to (a) $r = 0$, (b) $r = 0.1$, (c) $r = 1.0$, and (d) $r = \infty$. Along each line segment w increases from 0 to 1 in units of 0.2, indicated by tick marks and explicitly labelled for segment (a).

Fig. 16: Regions in the (g_V, g_A) plane allowed by the Gargamelle data on the reactions $\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$, Eqs. (3.30) and (3.31). The outer contours are 90% confidence level (CL) limits. The dot at $(g_V, g_A) = (-0.24, 0)$ represents the prediction for $r = 0$, $\ell = 0.18$, and $w = 0.2$.

Fig. 17: Regions in the (g_V, g_A) plane allowed by the measurement of the reaction $\bar{\nu}_e e \to \bar{\nu}_e e$ by Reines et al. (Ref.31). The two different elliptical regions correspond to the two bins 1.5 - 3.0 MeV and 3.0 - 4.5 MeV in scattered electron lab energy. The dot at $(g_V, g_A) = (-0.24, 0)$ represents the prediction for $r = 0$, $\ell = 0.18$, and $w = 0.2$.

Fig. 18: The enhanced atomic parity violation parameter Q_{wk} for Bi^{209} as a function of r. For this plot $\ell = 0.18$ and (a) $w = 0.2$, (b) $w = 0.3$.

Fig. 19: Graphs contributing in lowest order to the weak interaction part of the muon anomalous magnetic moment. Fig. (19e) represents a sum of four graphs: LL and RR transitions mediated by the vector bosons X_1 and separately by X_2. Fig. (19f) also represents a sum of four graphs: LR and RL transitions mediated by X_1 and, separately, by X_2.

Fig. 20: Graph for the decay $\mu \to e\gamma$.

Fig. 21: Graphs contributing in lowest order (g^4) and in the free quark approximation, to the $K_L K_S$ mass difference.

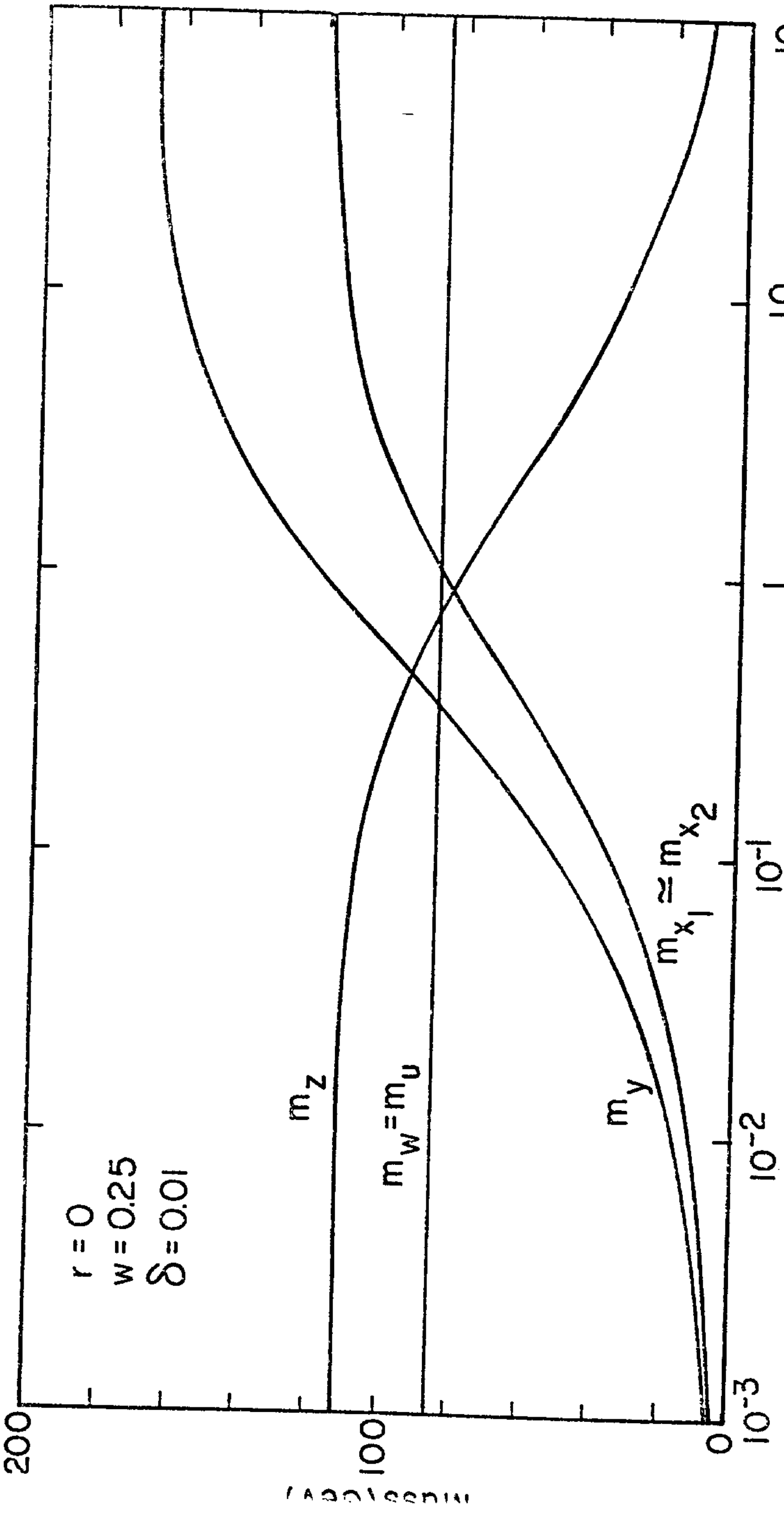
Fig. 22: Branching ratios for muonic final states in M^- decay, as a function of $\sin^2\beta$. The mass of E^0 is taken to be 1 GeV. For clarity the $BR(\mu^-\mu^-)$ curve is dashed.

Fig. 23: Branching ratios for muonic fixed states in M^- decay, as a function of $\sin^2\beta$. Here $m_{E^0} < 40$ eV. For clarity the $BR(\mu^-\mu^-)$ curve is dashed.

Fig. 24: Branching ratios for specific electron and muon final states in M^- decay, as a function of $\sin^2 \beta$. The curves shown are for the sub-classes of the muonic modes $\mu^- \mu^+$ and μ^-. The mass of E^0 is taken to be 1 GeV. The $\mu^- e^- e^+$ curve is dashed.

Fig. 25: Continuation of Fig. 24. The curves shown are for the sub-classes of the muonic modes μ^+ and "no $\mu^\pm$." For clarity the $e^- e^+$ curve is dashed and the $\mu^+ e^- e^-$ curve is dotted.

Fig. 1



r = 0
w = 0.25
δ = 0.01
m_z
$m_W = m_u$
m_y
$m_{x_1} \simeq m_{x_2}$
ℓ
0
100
200
10^{-3}
10^{-2}
10^{-1}
1
10
10

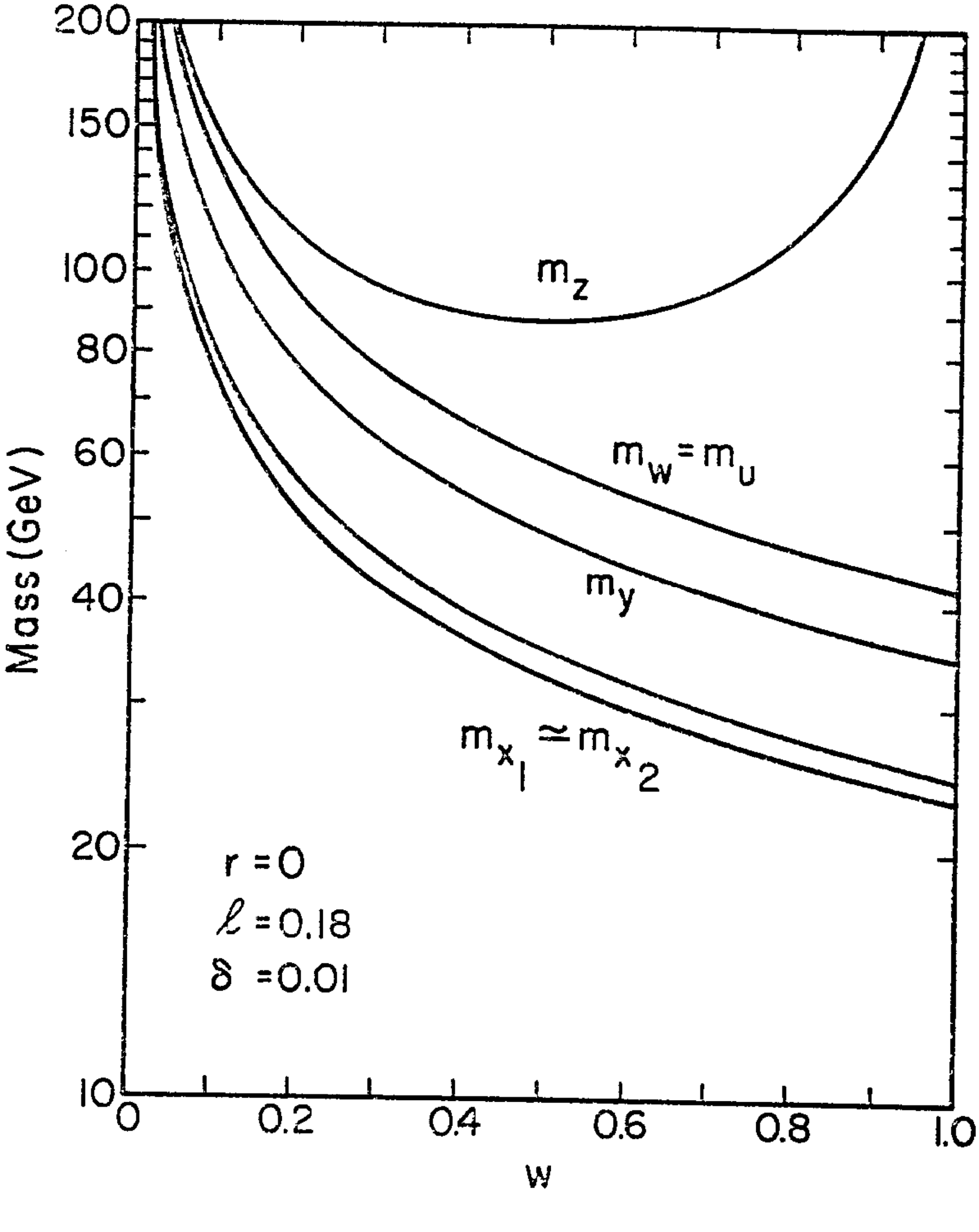

200
150
100
80
60
40
20
10
Mass (GeV)
m_z
$m_W = m_u$
m_y
$m_{x_1} \simeq m_{x_2}$
r = 0
ℓ = 0.18
δ = 0.01
0
0.2
0.4
0.6
0.8
1.0
w

Fig. 2

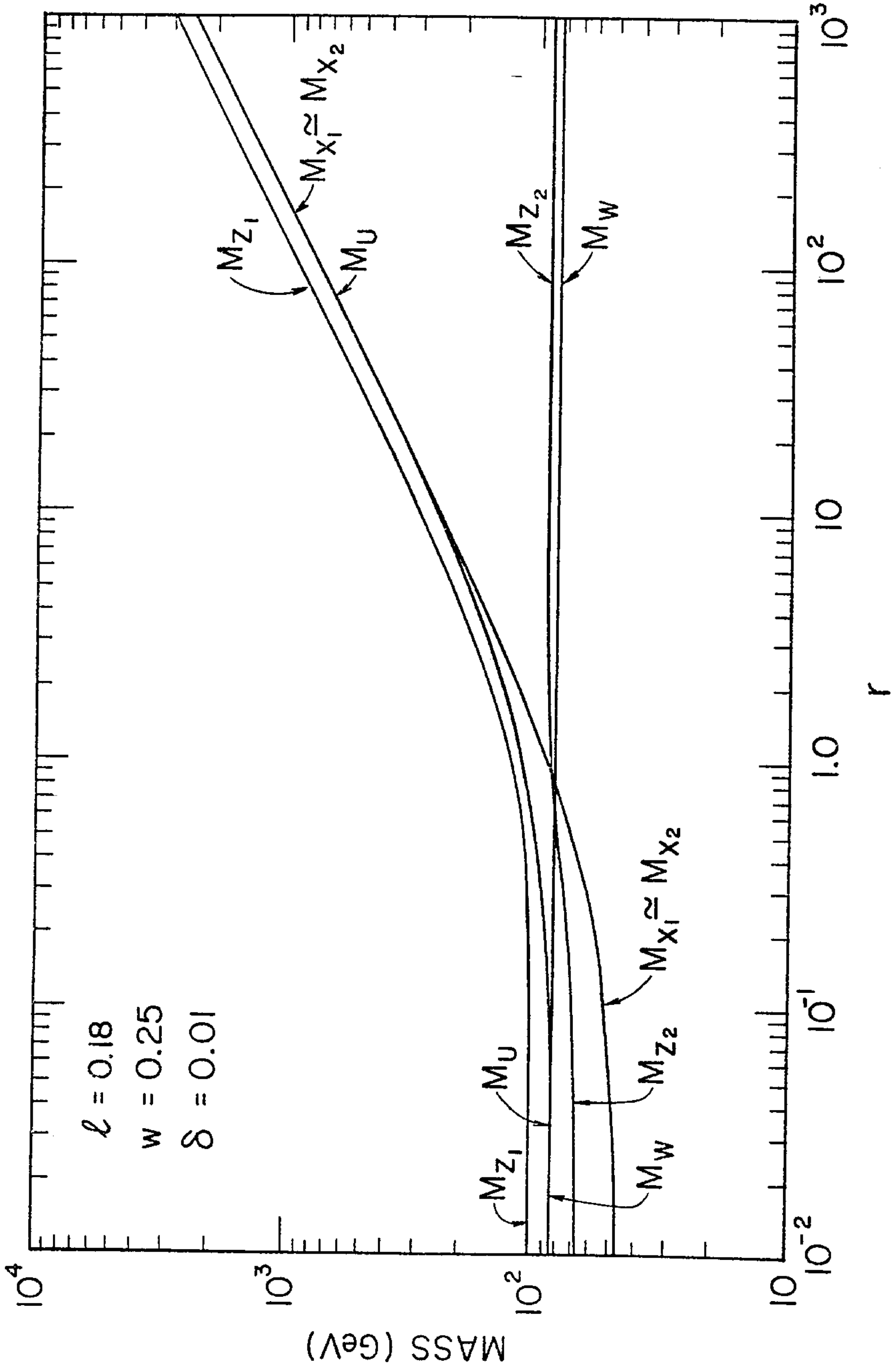
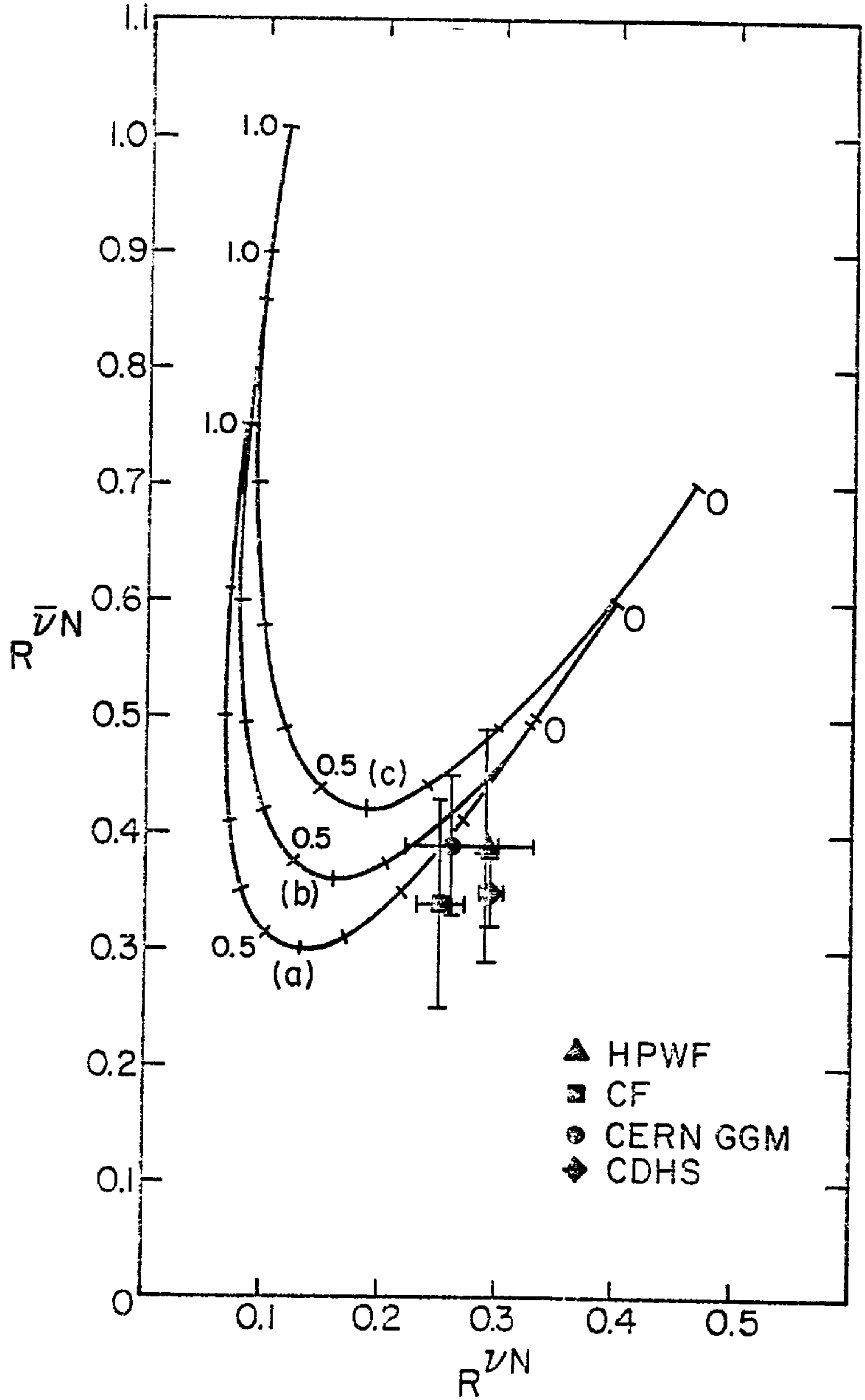


Fig. 3



1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$R^{\bar{\nu}N}$
0.1
0.2
0.3
0.4
0.5
$R^{\nu N}$
1.0
1.0
1.0
0.5
0.5
0.5
(a)
(b)
(c)
O
O
O
HPWF
CF
CERN GGM
CDHS

Fig. 4

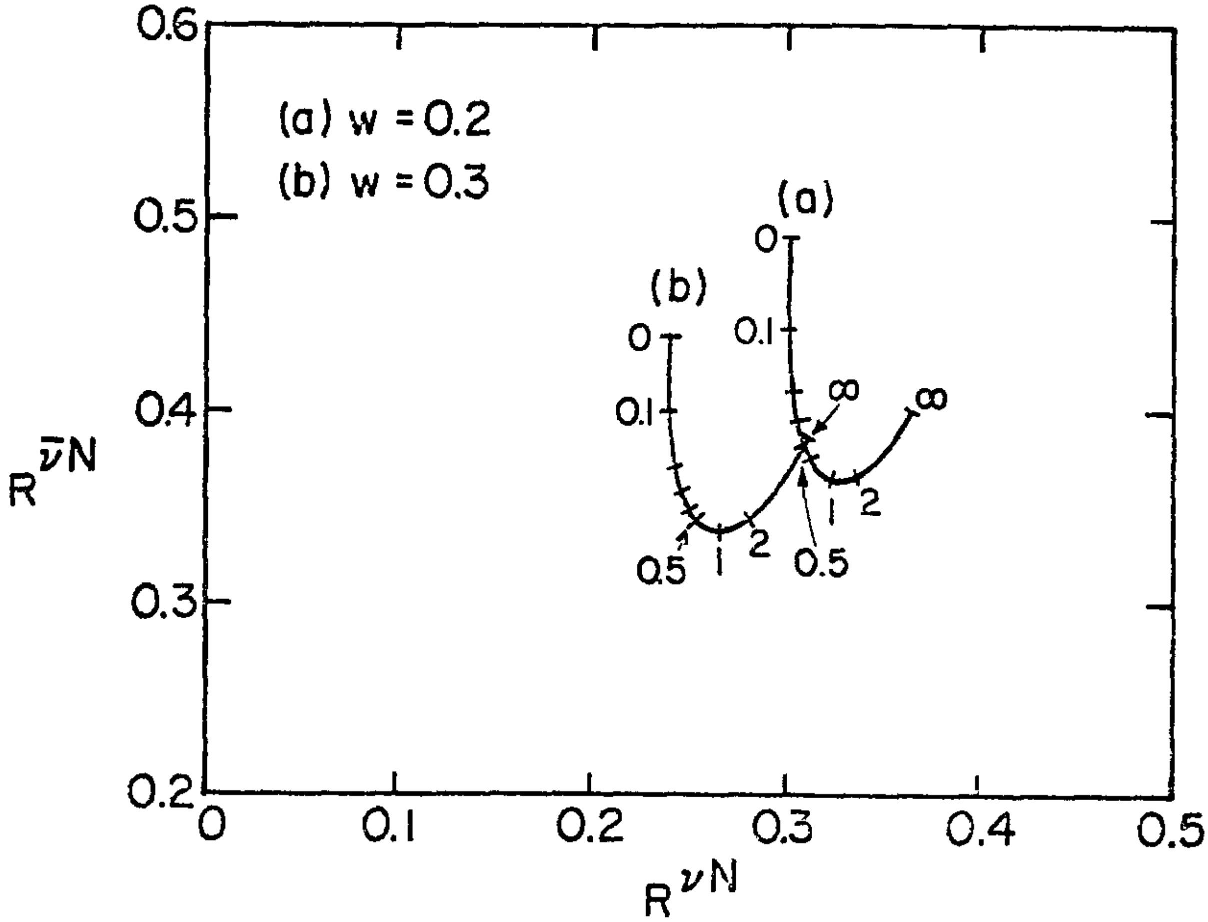


Fig. 5

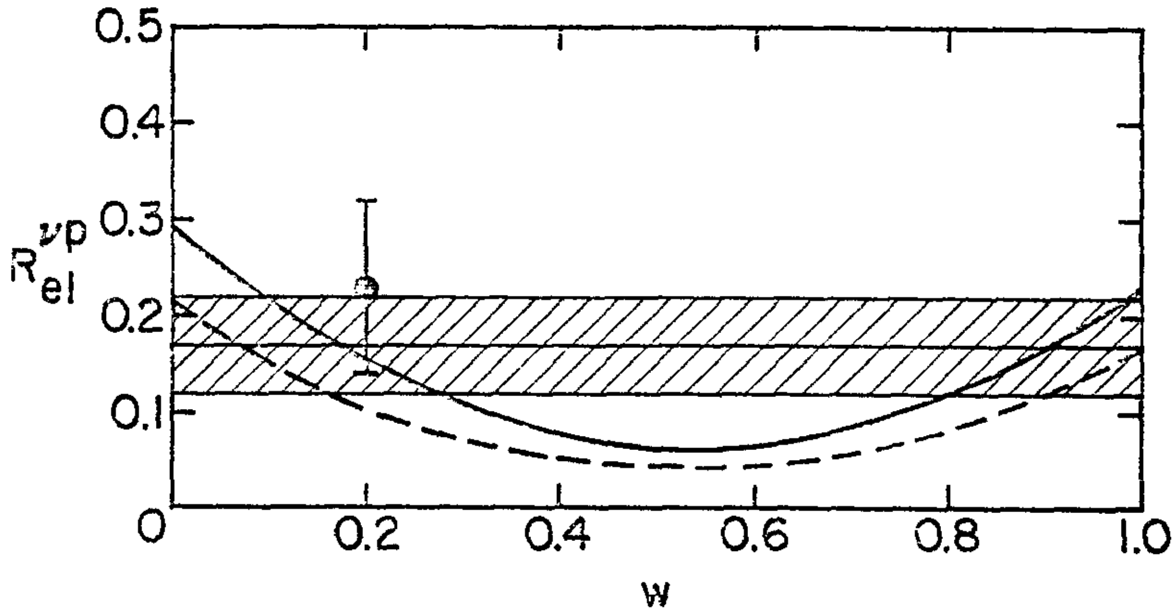


Fig. 6

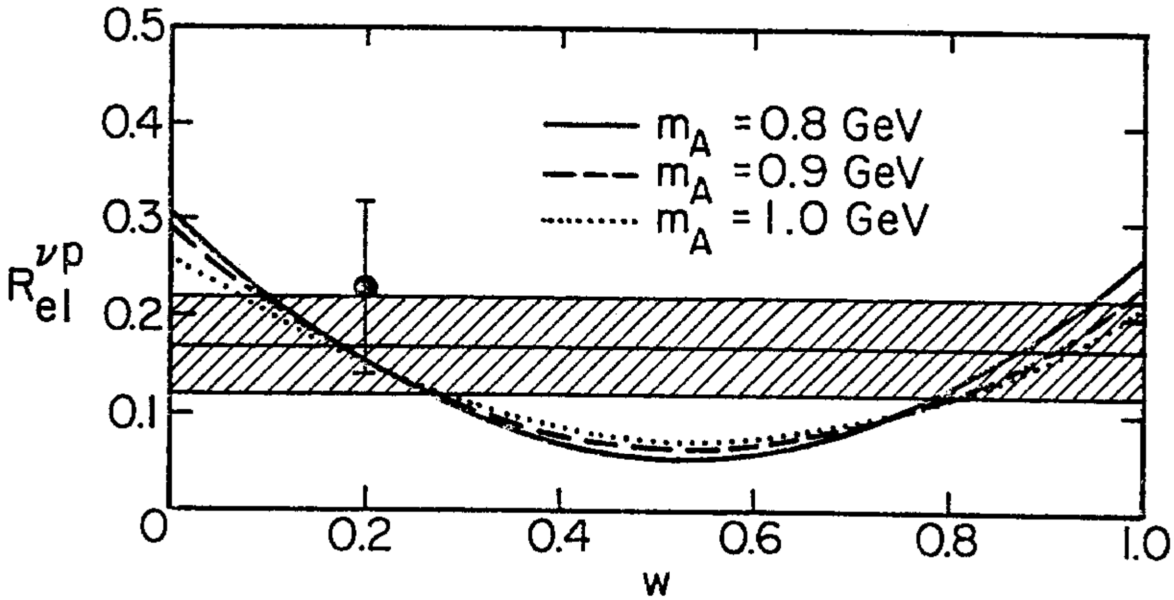


Fig. 7

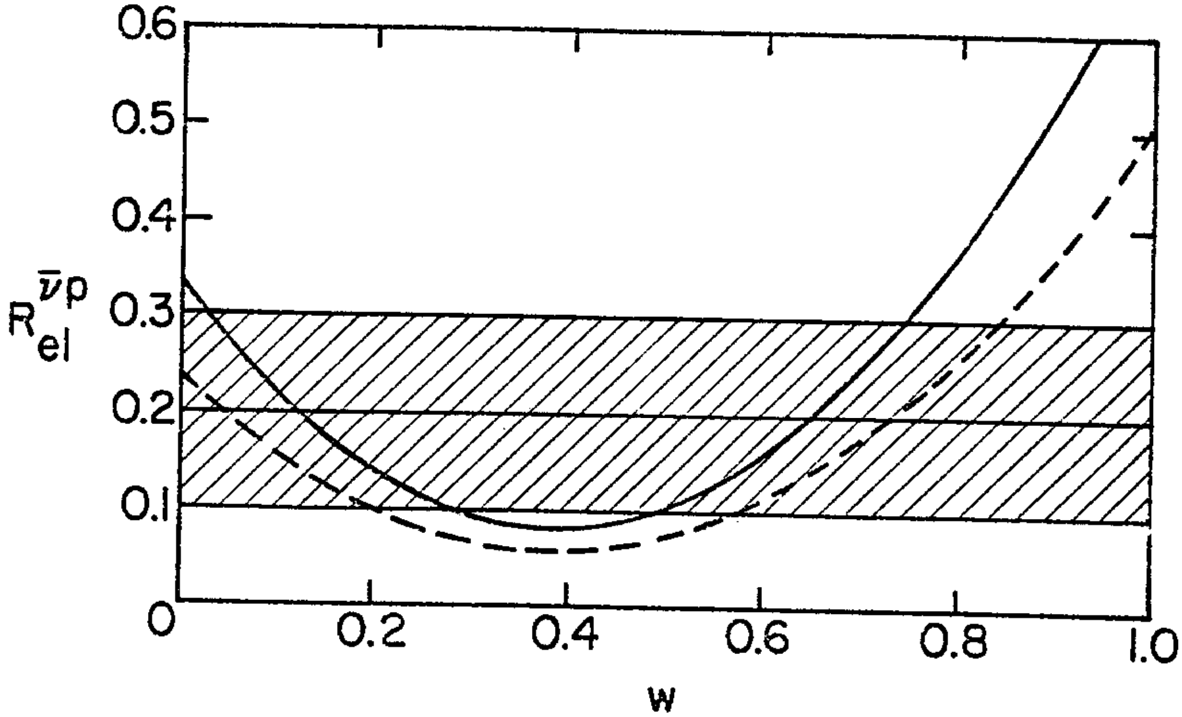
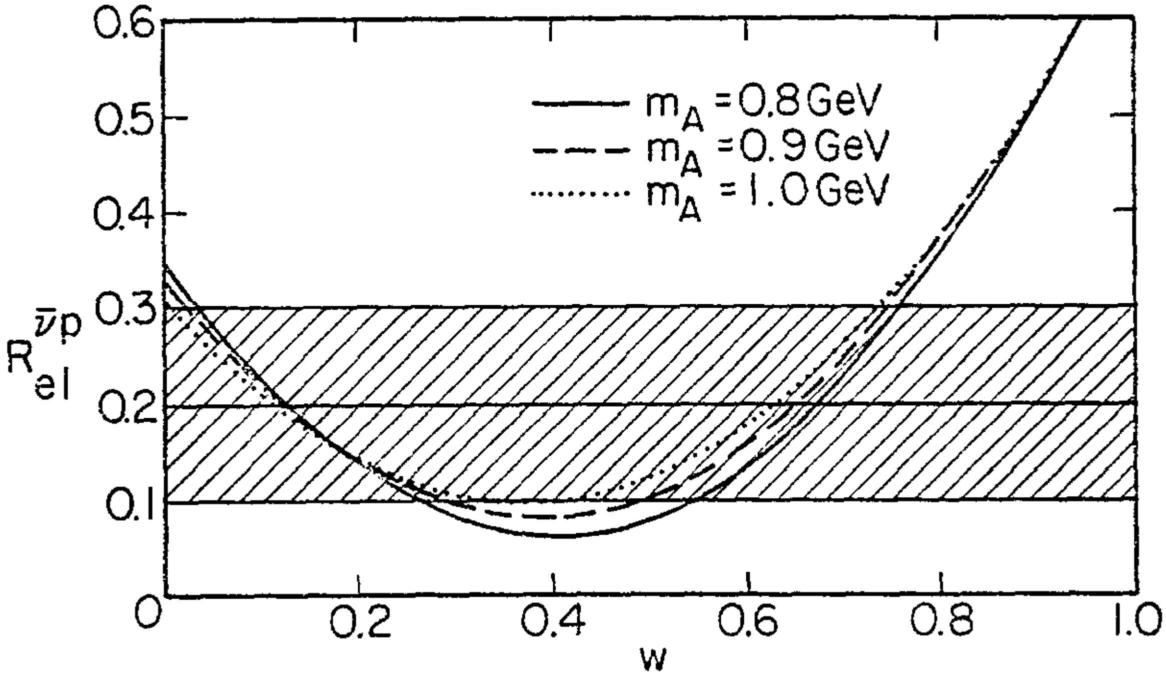


Fig. 8



0.6
0.5
0.4
0.3
0.2
0.1
0
m_A = 0.8 GeV
m_A = 0.9 GeV
m_A = 1.0 GeV
$R_{el}^{\bar{\nu}p}$
0.2
0.4
0.6
0.8
1.0
w

Fig. 9

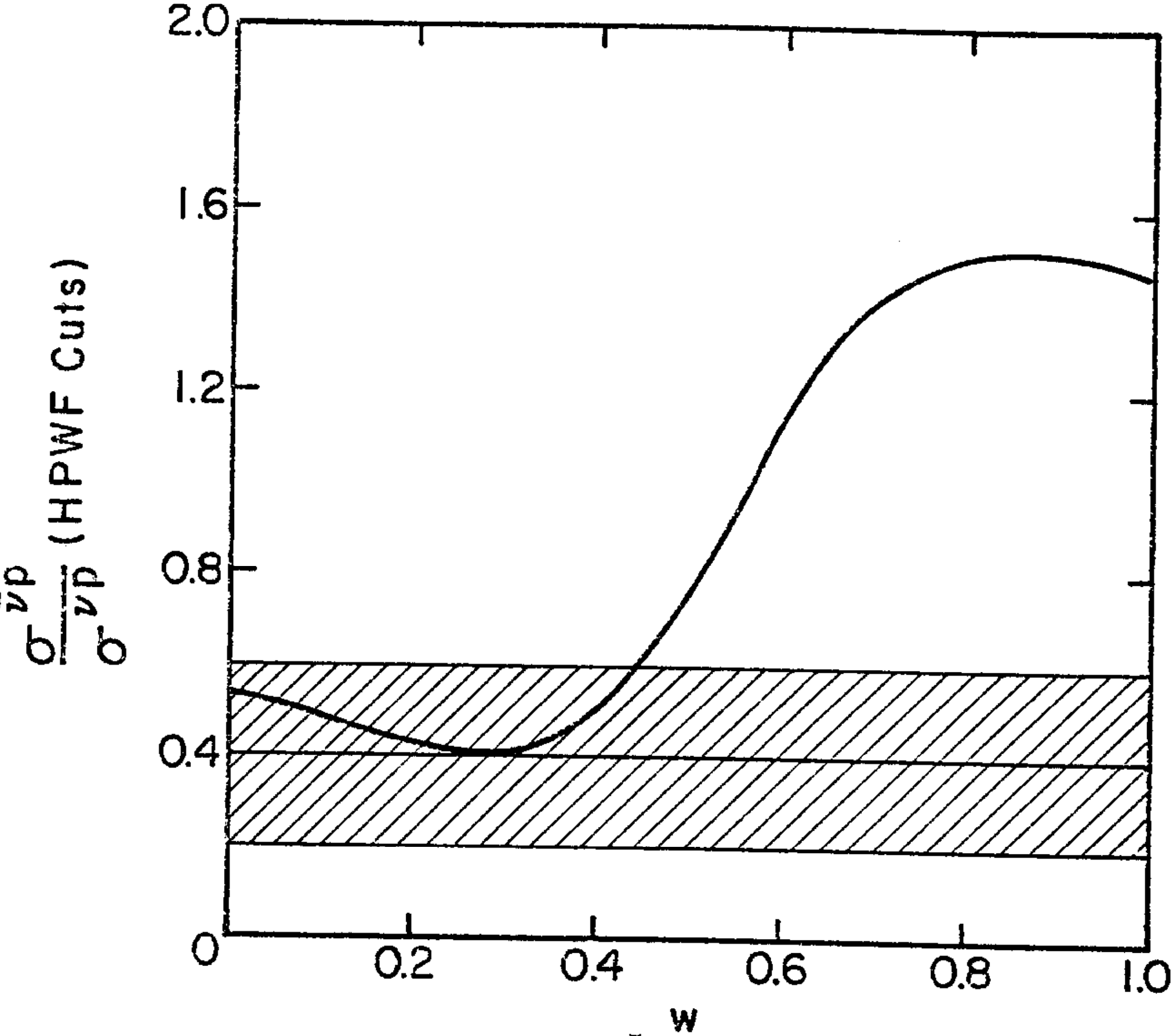
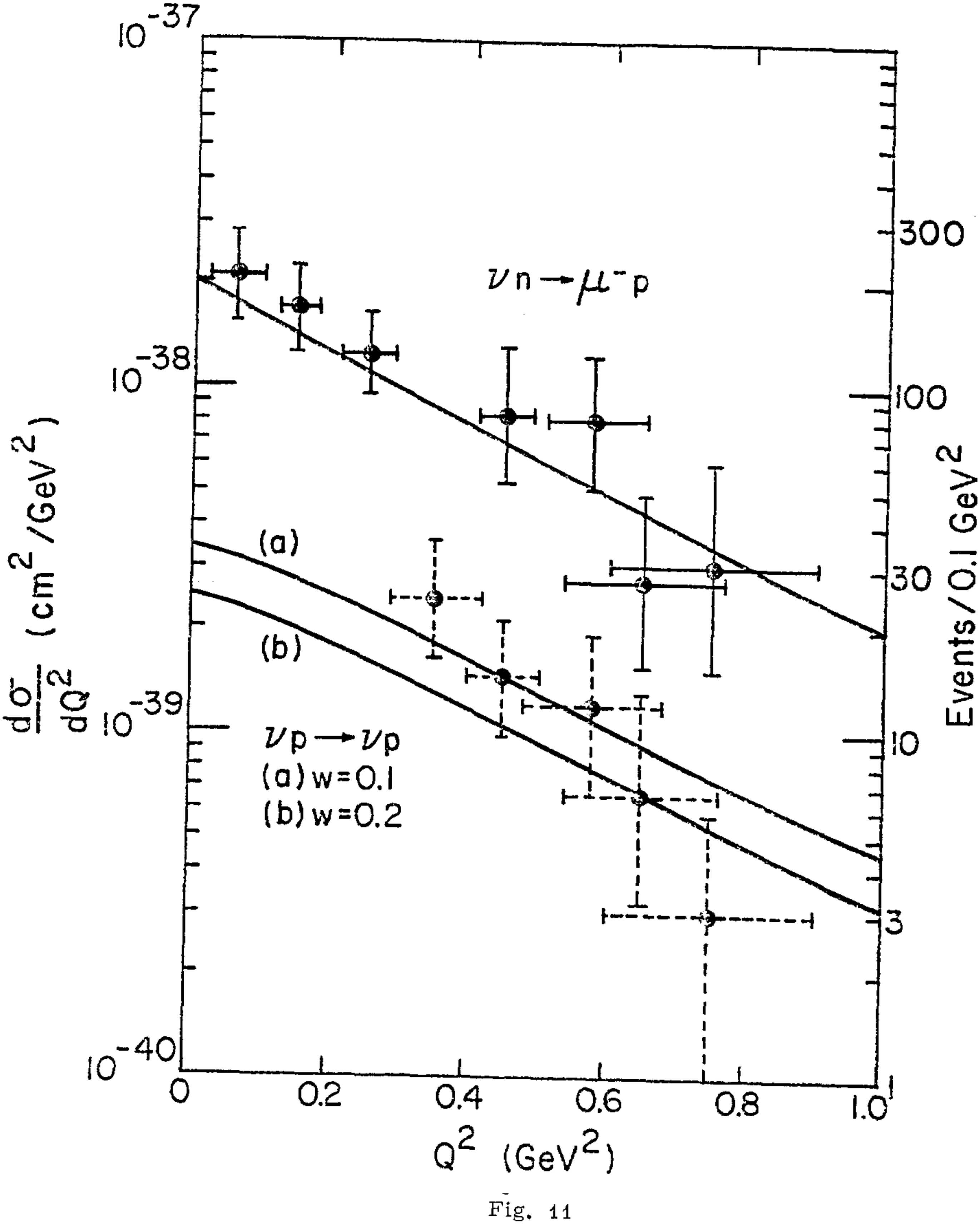
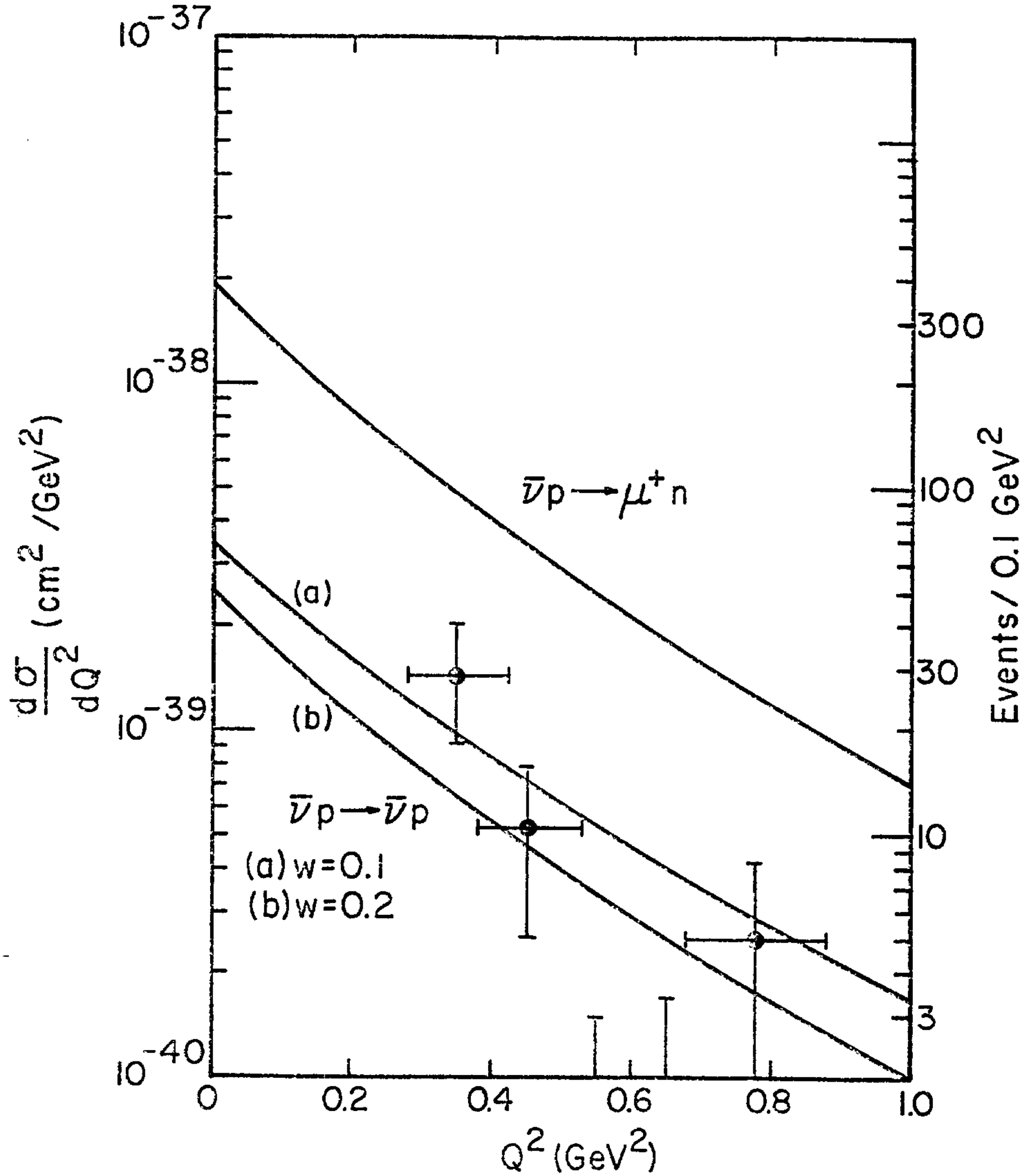

2.0
1.6
1.2
0.8
0.4
0
0.2
0.4
0.6
0.8
1.0
W
$\frac{\sigma^{\bar{\nu}p}}{\sigma^{\nu p}}$ (HPWF Cuts)

Fig. 10



10^{-37}
10^{-38}
10^{-39}
10^{-40}
$\frac{d\sigma}{dQ^2}$ (cm^2/GeV^2)
$\nu n \rightarrow \mu^- p$
(a)
(b)
$\nu p \rightarrow \nu p$
(a) w=0.1
(b) w=0.2
300
100
30
10
3
1
Events/0.1 GeV^2
0
0.2
0.4
0.6
0.8
1.0
Q^2 (GeV^2)

Fig. 11



10^{-37}
10^{-38}
10^{-39}
10^{-40}
$\frac{d\sigma}{dQ^2}$ (cm^2 /GeV^2)
$\bar{\nu}p \to \mu^+ n$
(a)
(b)
$\bar{\nu}p \to \bar{\nu}p$
(a) w=0.1
(b) w=0.2
0
0.2
0.4
0.6
0.8
1.0
Q^2 (GeV^2)
300
100
30
10
3
Events / 0.1 GeV^2

Fig. 12

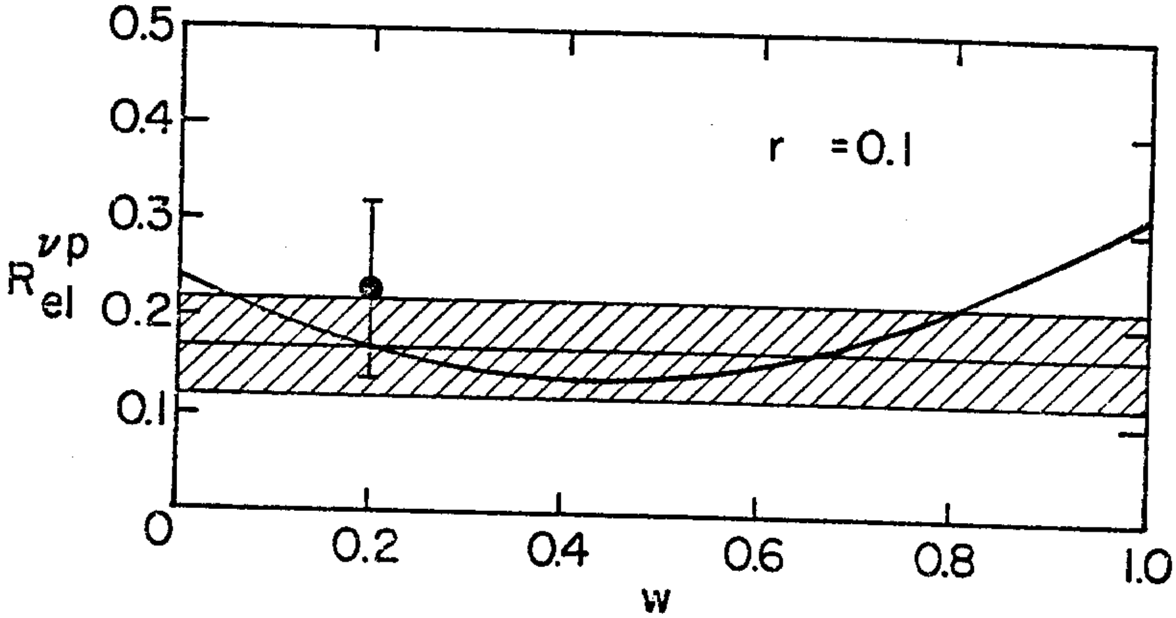


Fig. 13

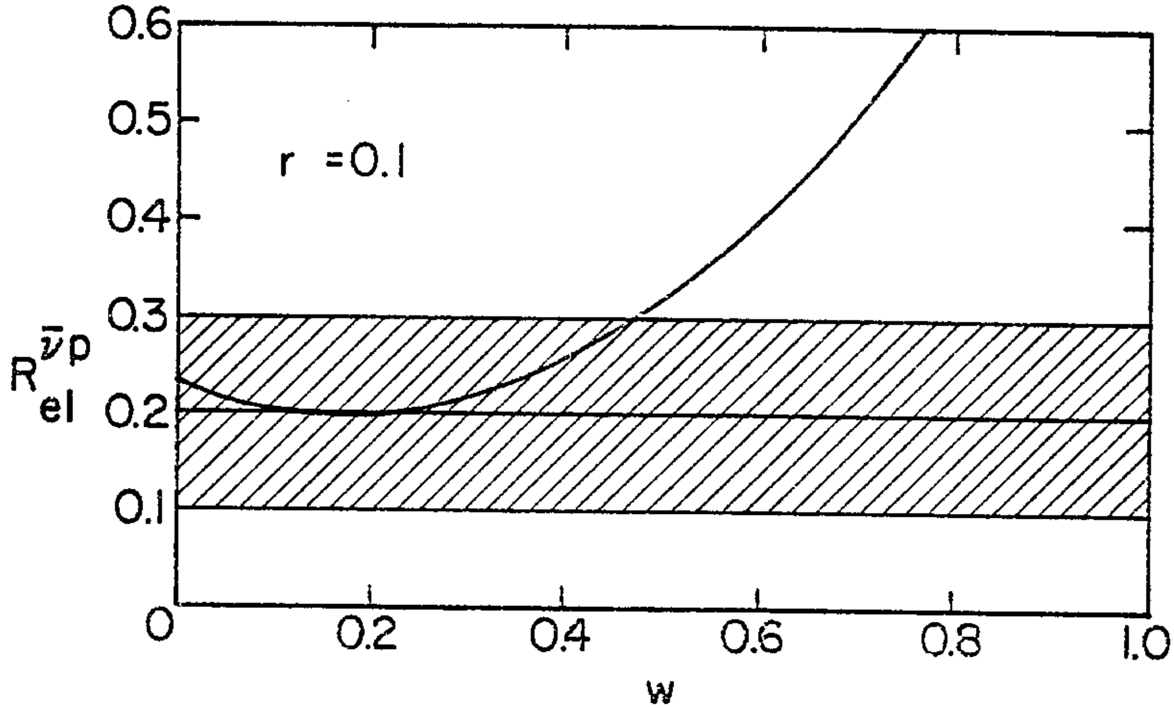


Fig. 14

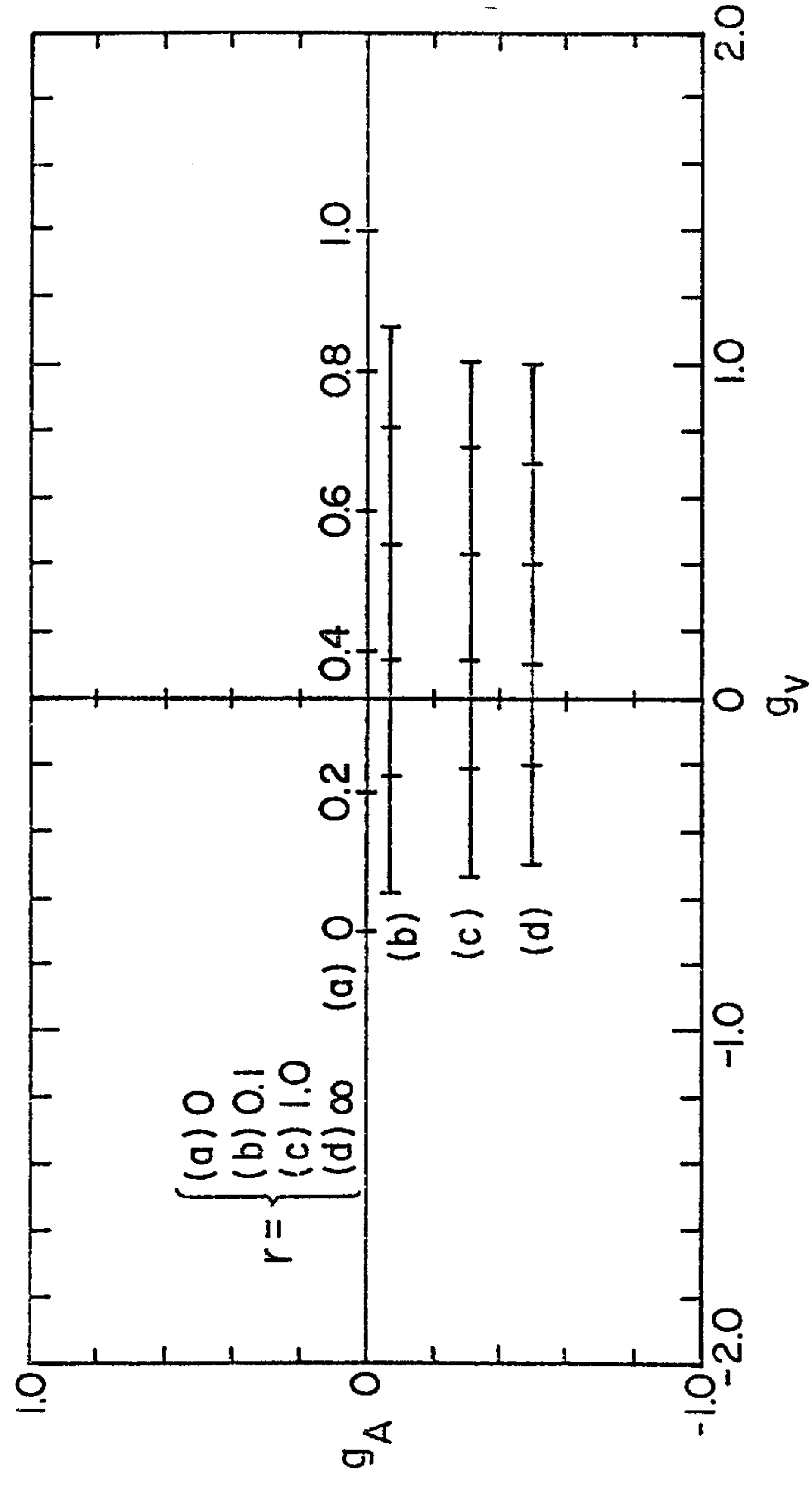
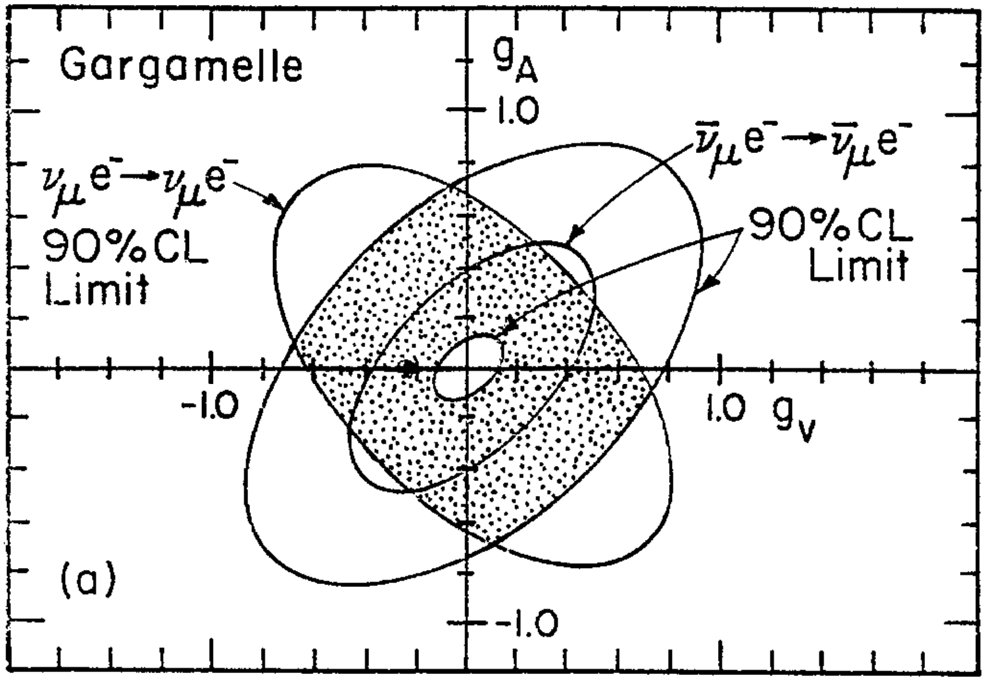
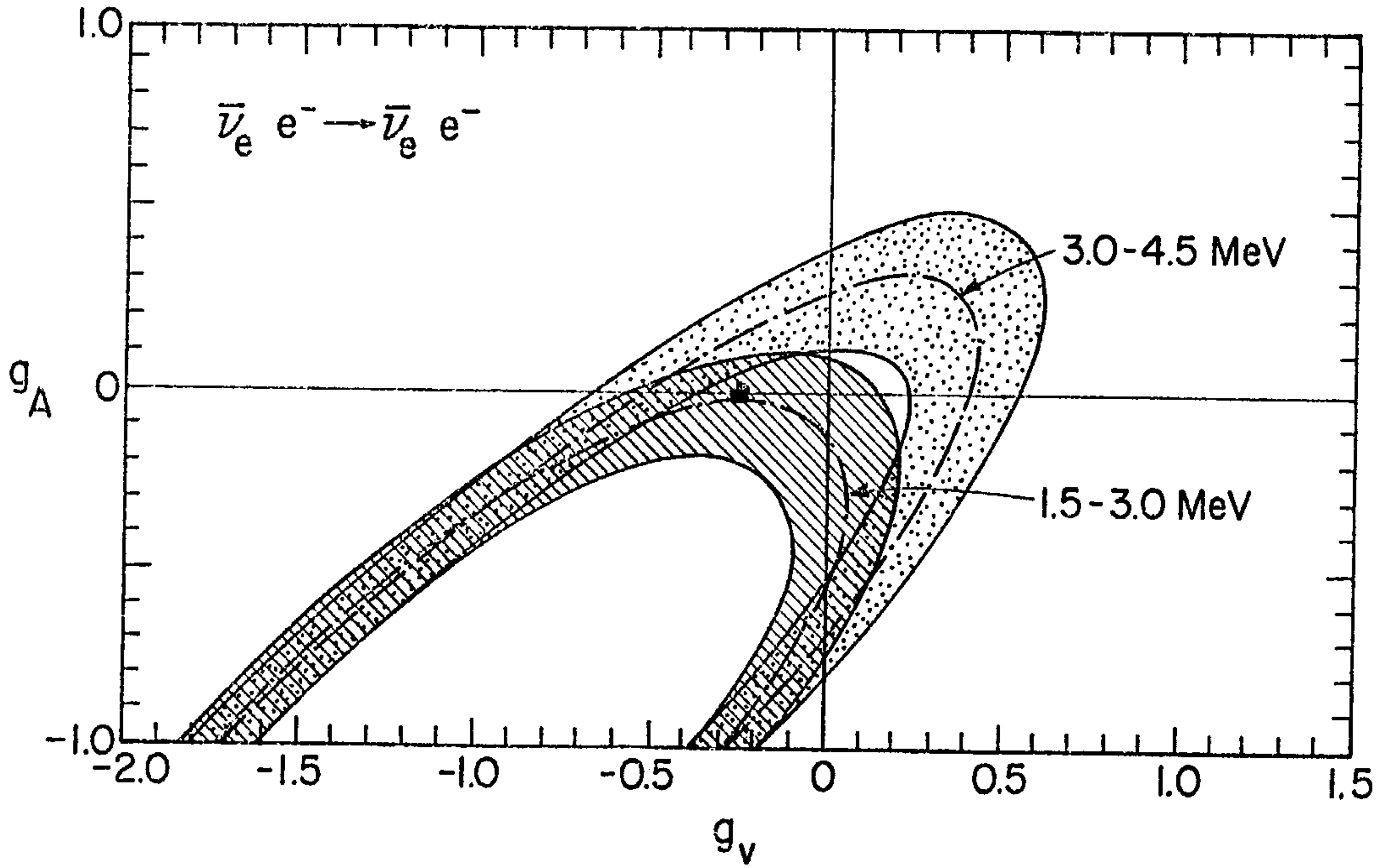


Fig. 15



Gargamelle
g_A
1.0
$\nu_\mu e^- \to \nu_\mu e^-$
90% CL
Limit
$\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$
90% CL
Limit
-1.0
1.0 g_V
(a)
-1.0

Fig. 16



$\bar{\nu}_e e^- \to \bar{\nu}_e e^-$
3.0-4.5 MeV
1.5-3.0 MeV
g_A
g_V
1.0
0
-1.0
-2.0
-1.5
-1.0
-0.5
0
0.5
1.0
1.5

Fig. 17

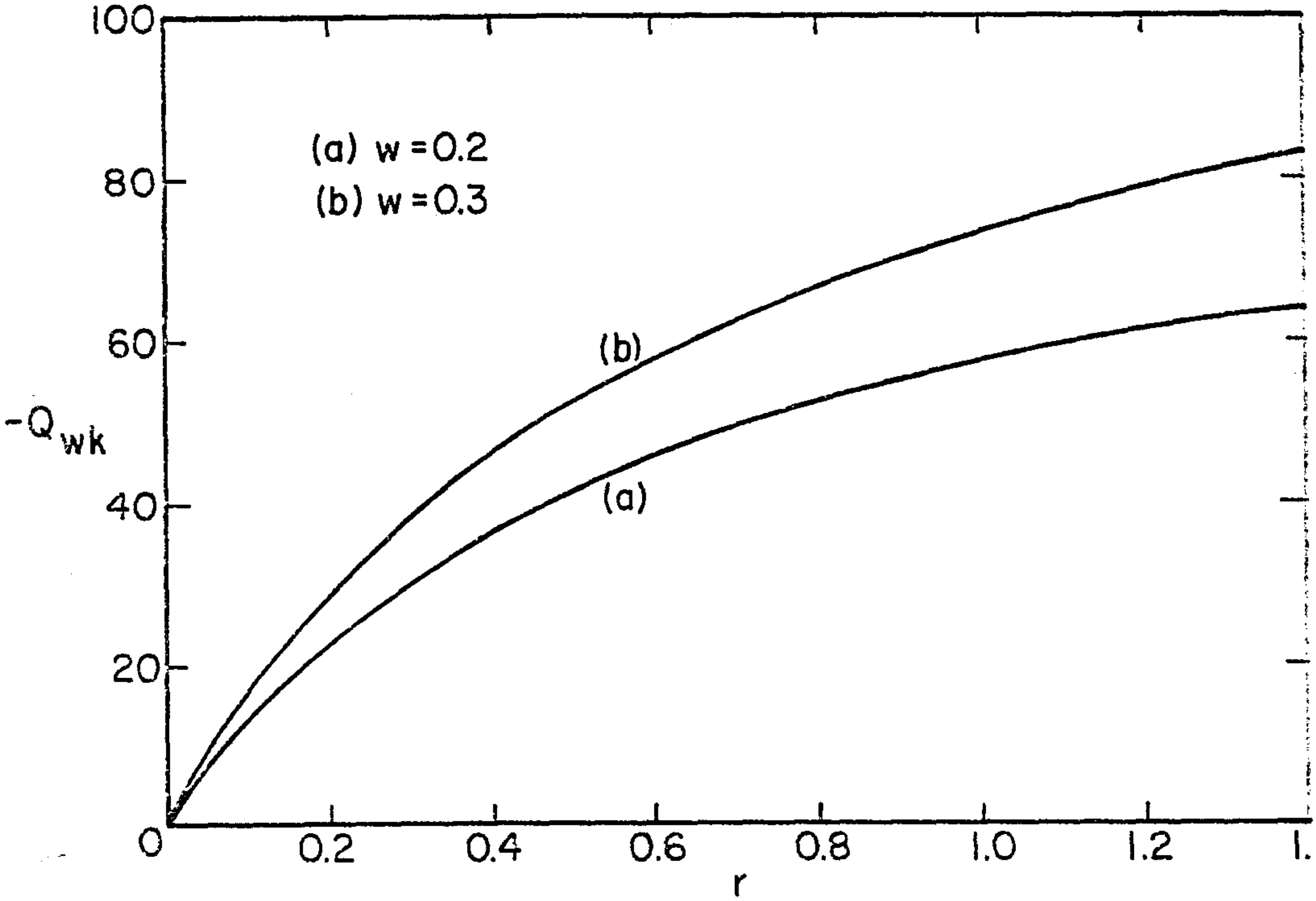
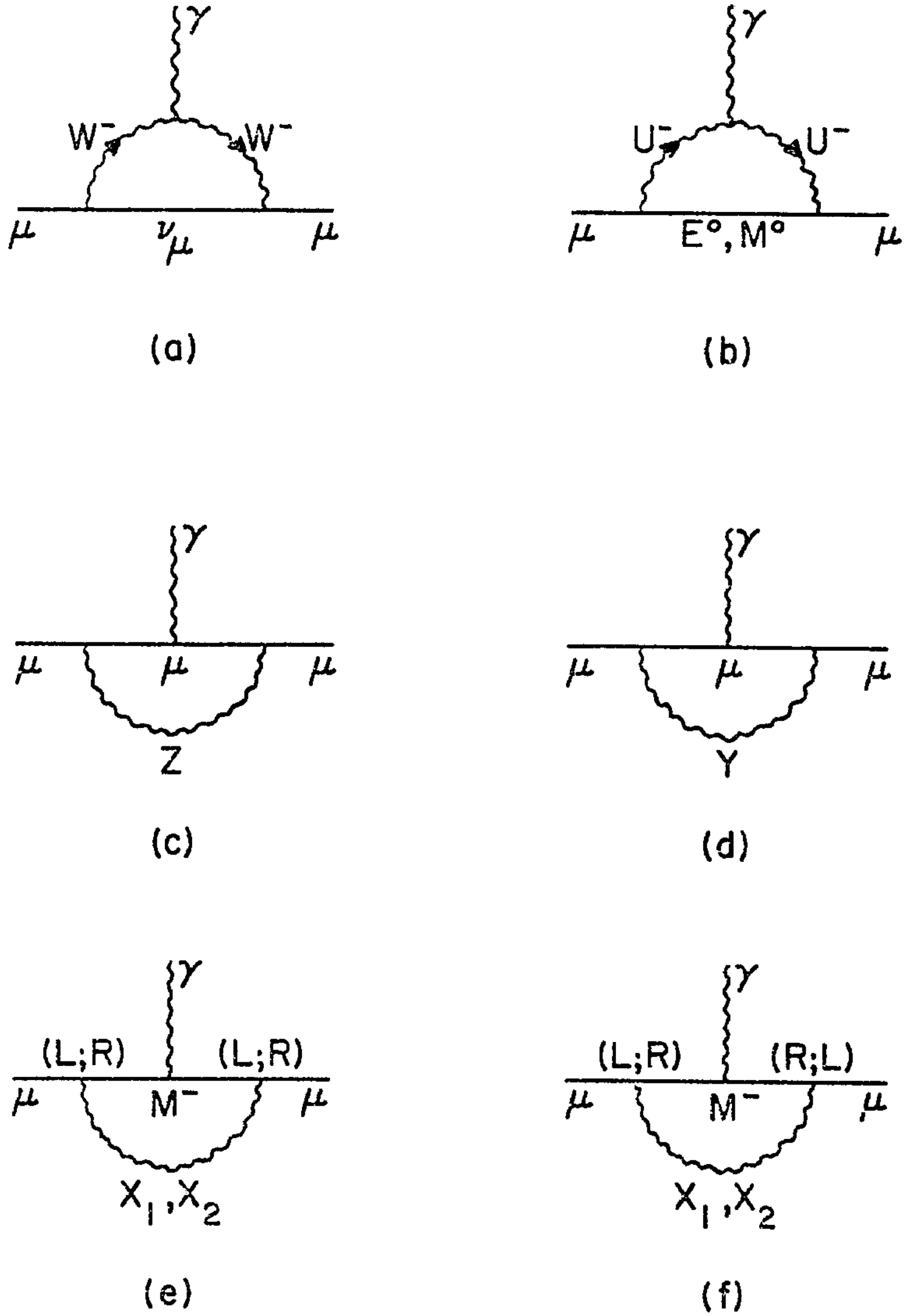


Fig. 18



γ
W⁻
W⁻
μ
ν_μ
μ
(a)
γ
U⁻
U⁻
μ
E°, M°
μ
(b)
γ
μ
μ
μ
Z
(c)
γ
μ
μ
μ
Y
(d)
γ
(L;R)
(L;R)
μ
M⁻
μ
X1, X2
(e)
γ
(L;R)
(R;L)
μ
M⁻
μ
X1, X2
(f)

Fig. 19

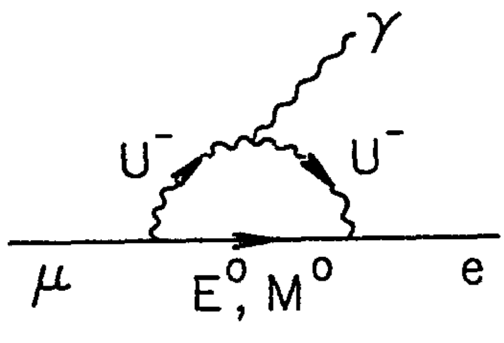
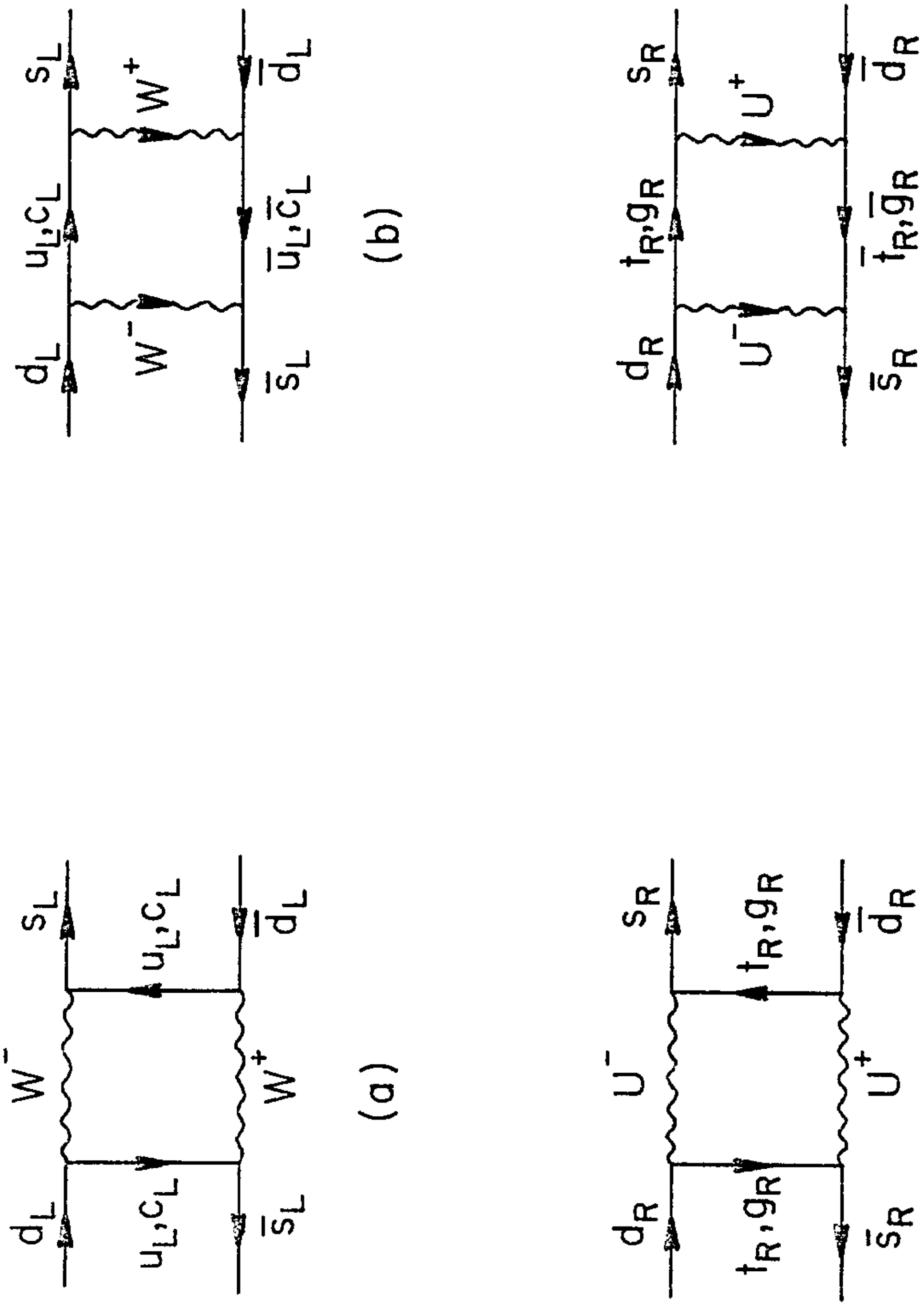
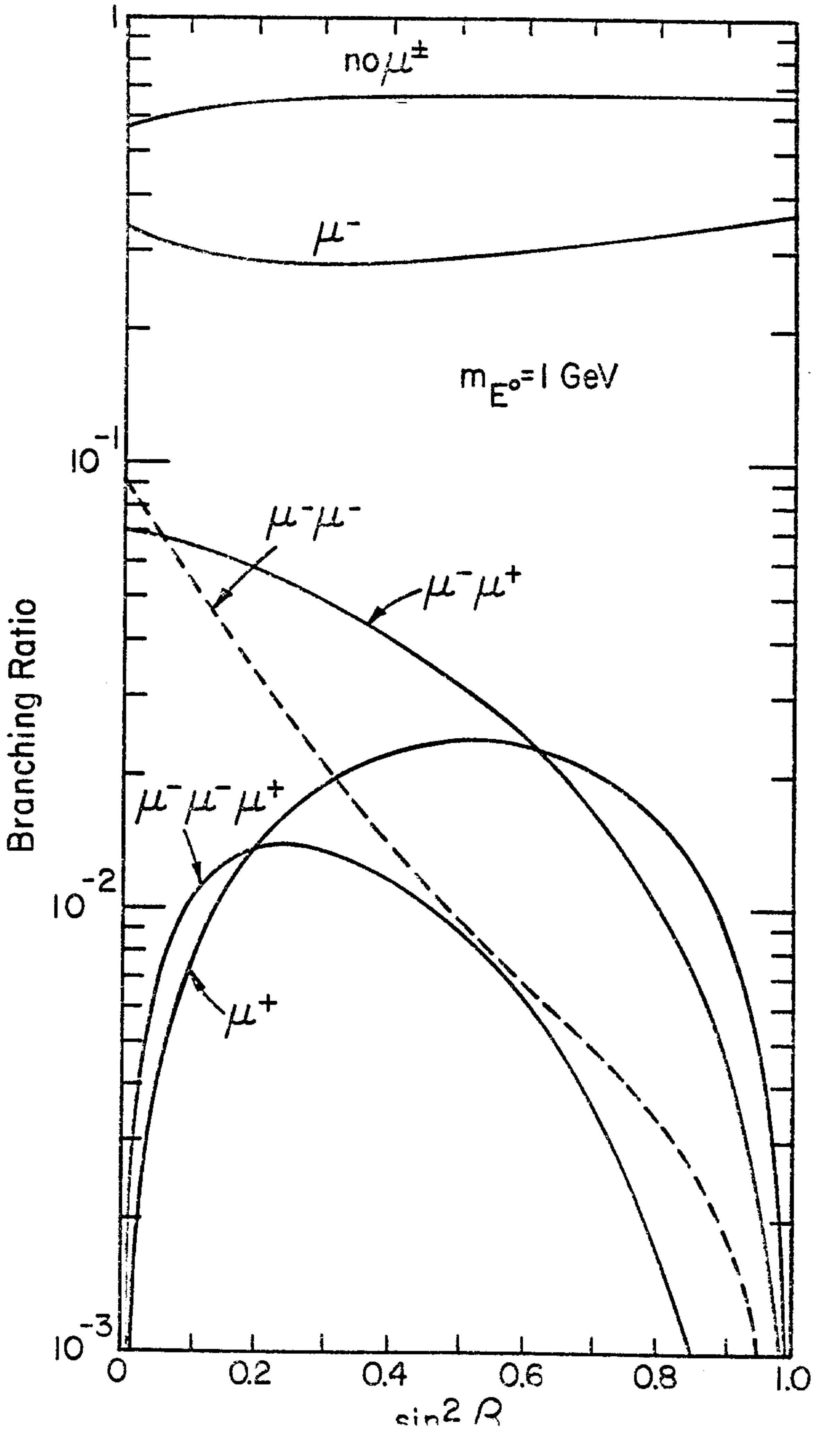


Fig. 20



d_L
s_L
W^-
W^+
u_L, c_L
$\bar{u}_L, \bar{c}_L$
$\bar{s}_L$
$\bar{d}_L$
(a)
(b)
d_R
s_R
U^-
U^+
t_R, g_R
$\bar{t}_R, \bar{g}_R$
$\bar{s}_R$
$\bar{d}_R$
(c)
(d)

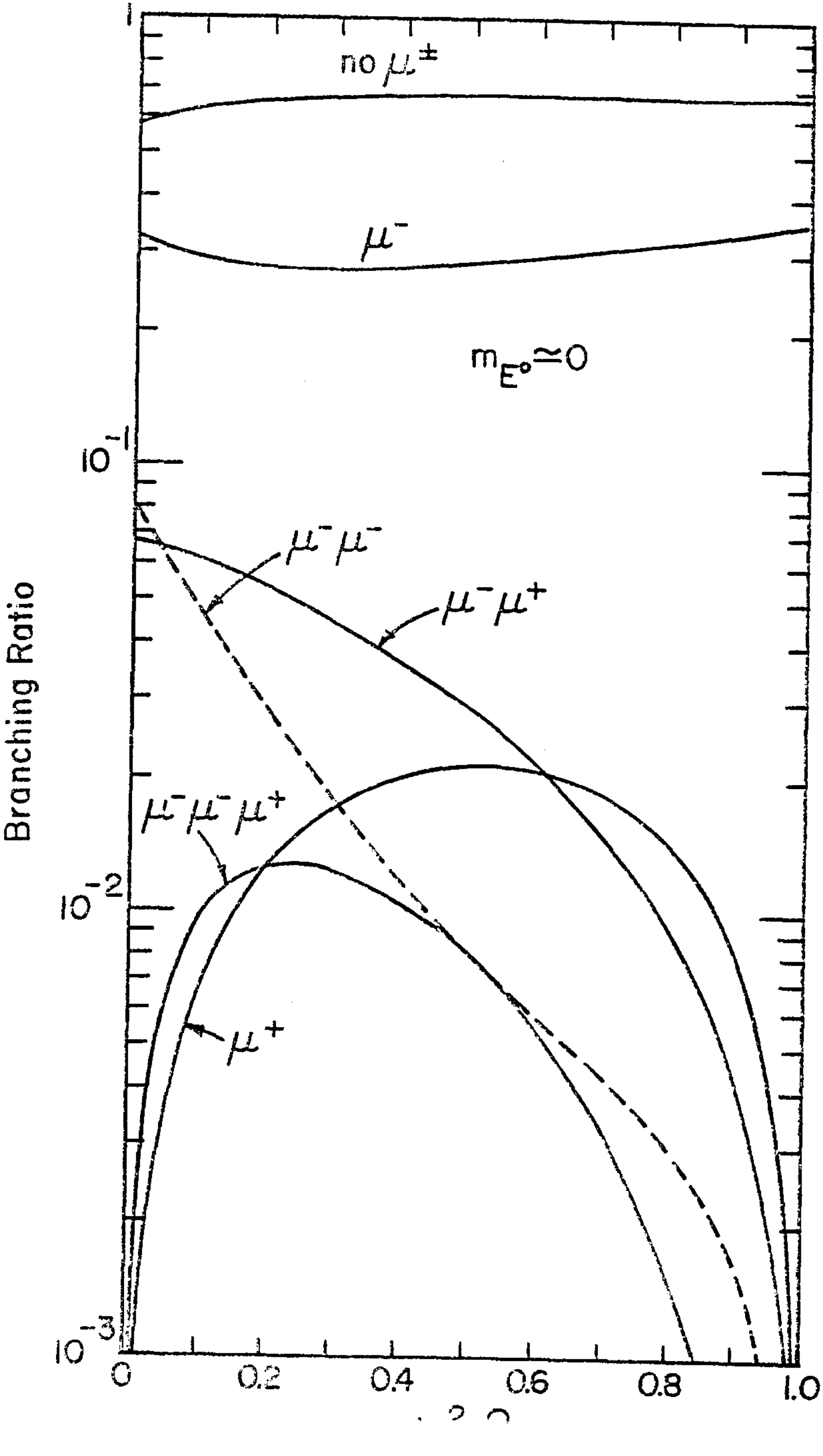
Fig. 21

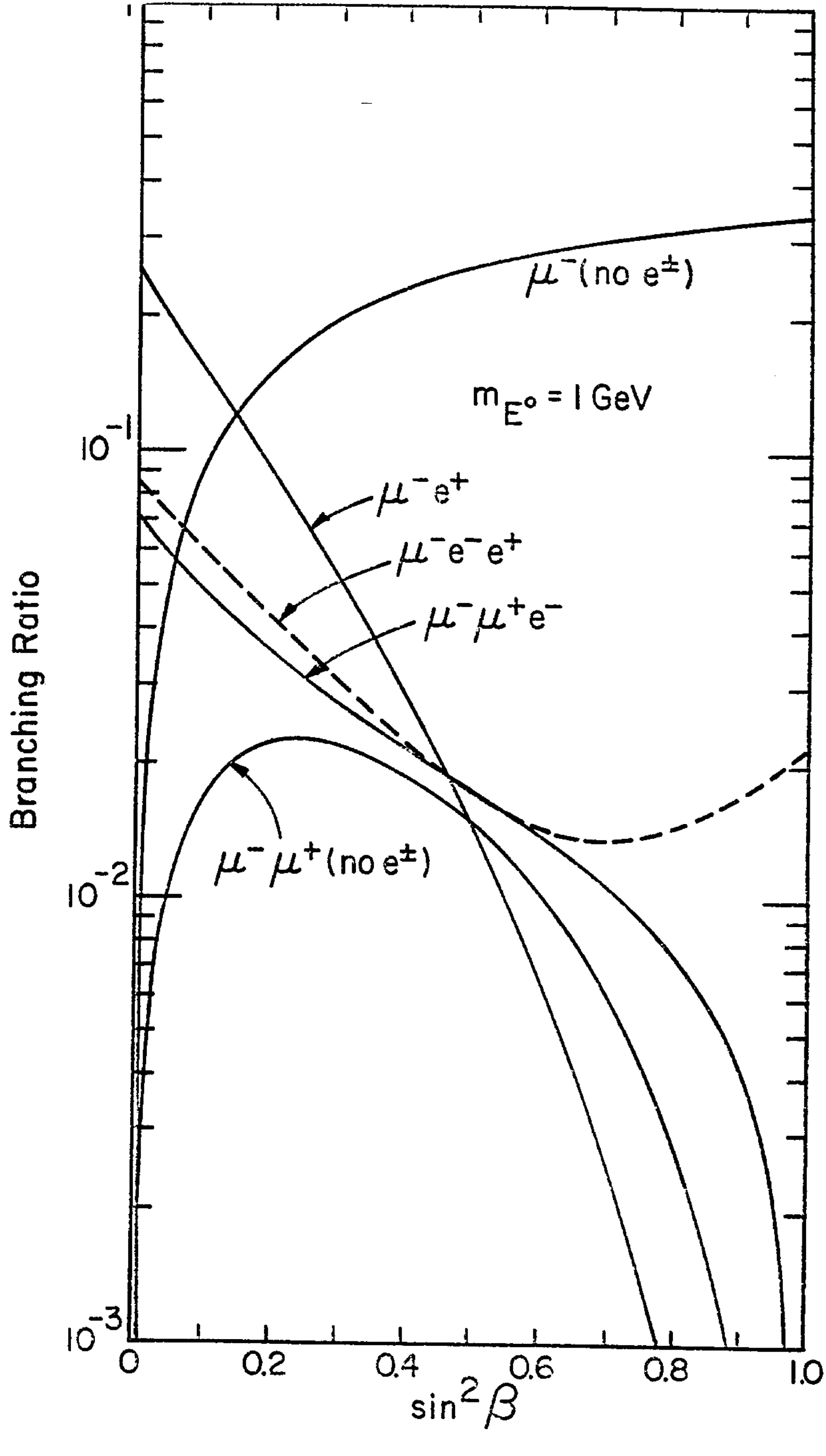


no $\mu^\pm$
μ^-
m_{E°=1 GeV
$\mu^-\mu^-$
$\mu^-\mu^+$
$\mu^-\mu^-\mu^+$
μ^+
Branching Ratio
1
10^{-1}
10^{-2}
10^{-3}
0
0.2
0.4
0.6
0.8
1.0
$\sin^2\beta$

Fig. 22

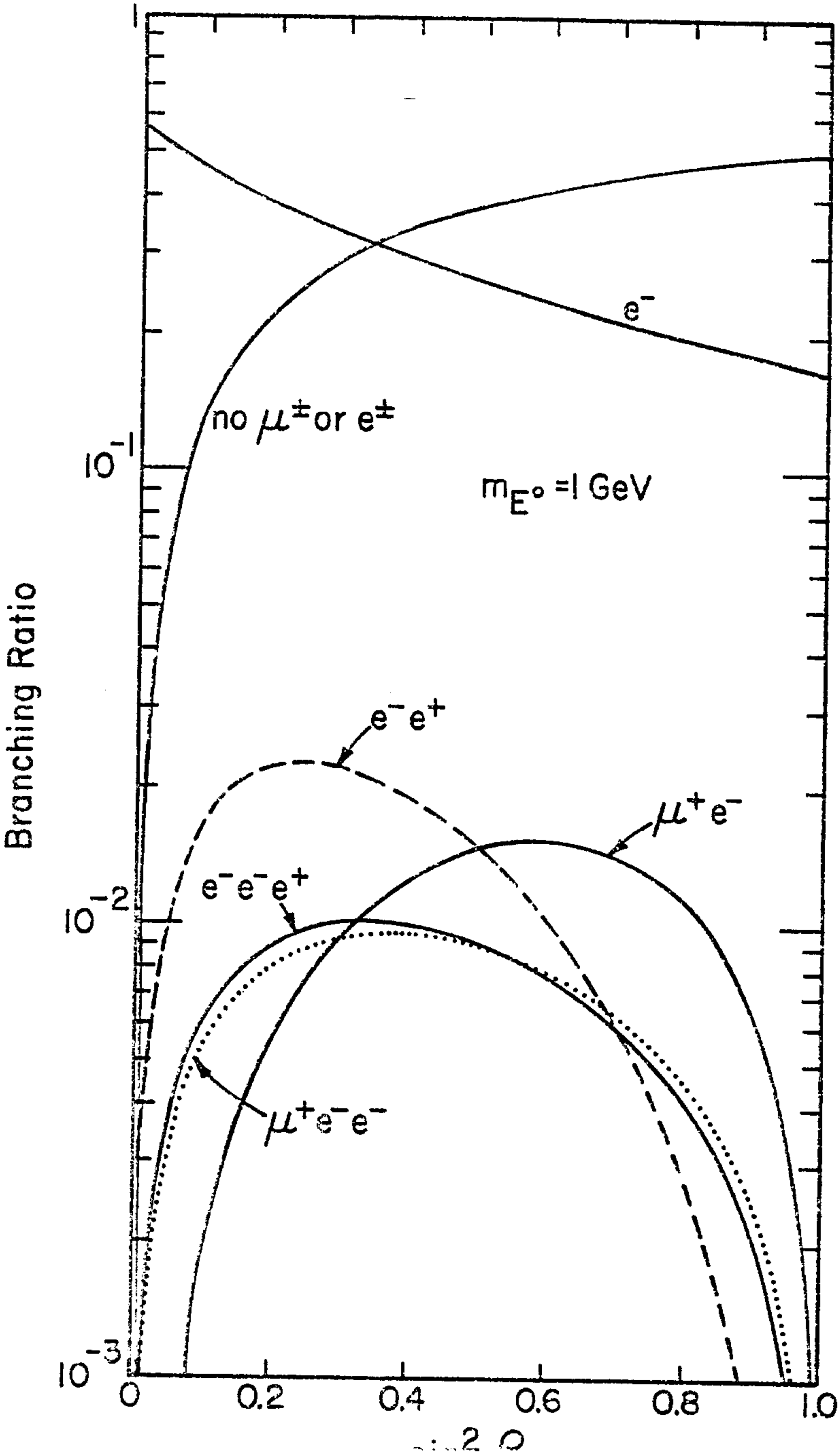
Fig. 23





μ^-(no $e^\pm$)
m_{E° = 1 GeV
$\mu^- e^+$
$\mu^- e^- e^+$
$\mu^- \mu^+ e^-$
$\mu^- \mu^+$(no $e^\pm$)
Branching Ratio
10^{-1}
10^{-2}
10^{-3}
0
0.2
0.4
0.6
0.8
1.0
$\sin^2\beta$

Fig. 24



Branching Ratio
1
10^-1
10^-2
10^-3
0
0.2
0.4
0.6
0.8
1.0
e^-
no μ^± or e^±
m_E° =1 GeV
e^-e^+
μ^+e^-
e^-e^-e^+
μ^+e^-e^-

Fig. 25