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Why the Neutron Isn't All Neutral*

R.D. Carlitz† and S.D. Ellis

Department of Physics, University of Washington, Seattle, WA 98195 USA

and

R. Savit

Fermi National Accelerator Laboratory, Batavia, IL 60510 USA

Abstract

Experimental measurements of the electric form factor of the neutron indicate a spatially inhomogeneous distribution of charge. The quark model, with a spatially dependent spin-spin interaction, can accurately describe this inhomogeneity. We relate the neutron's charge radius to the nucleon-delta mass difference and discuss other experimental consequences of the inhomogeneity.

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†Alfred P. Sloan Foundation fellow on leave from the University of Chicago.

1. Introduction

The quark model[1] has provided a useful mnemonic for general features of hadronic structure. If this description is indeed a correct one, it must be capable of describing this structure more specifically as well. One aspect which has eluded description for many years concerns the charge distribution within the neutron[2]. The simplest quark models[1] indicate that the neutron's charge density should be everywhere zero; and, indeed, this density is small (relative to that of the proton). It is not exactly zero, however; and we must ask whether this fact is compatible with the quark description.

What we find is that the quark-spin-dependent interaction which breaks the mass degeneracy of the ground state baryons leads in general to a segregation of charge within the neutron. If this perturbing force is more repulsive for quarks with parallel than with antiparallel spins, then the nduced charge radius $\langle r^2 \rangle_n$ will be negative. Introducing a imple (harmonic oscillator) description of the bound state ave functions and a spin-spin force of the type suggested by uantum chromodynamics, we estimate the magnitude of the charge egregation and find impressive agreement with experiment.

Our model implies a specific description of the spatial istribution of quarks within the nucleon and leads to predicions for a variety of experiments which explore these disributions. Confirmation of these predictions would be a significant step toward the verification of the bound quark picture as a satisfactory dynamical model of hadrons.

2. The Argument

The lowest lying baryons of the quark model are characterized by a total symmetry under the interchange of the spin and isospin labels of each pair of quarks. The mass degeneracy of these levels is split by a spin-spin interaction: thus, for example, is the $\Delta(1232)$ (with spin 3/2) more massive than the nucleon (with spin 1/2). If the spin-spin force is repulsive for quark pairs in a spin 1 state, these quarks will on the average be further apart than those in a spin 0 state. Due to the symmetry of the wave function, those pairs with spin 1 must have isospin 1 as well; and those pairs with spin 0 must have isospin 0.

In the neutron there are two d quarks (charge -1/3 each) and one u quark (charge 2/3). The two d quarks always form an isospin 1 state, while the u-d pairs form mixtures of isospin 0 and isospin 1 states (with relative probabilities 3/4 and 1/4, respectively). Thus the two d quarks are more likely to be in a spin 1 state than are the u-d pairs. The repulsive force for quarks in this state will consequently drive the neutron's negative charge (the d quarks) further from the center of mass than its positive charge (the u quark). We conclude that $\langle r^2 \rangle_n$ must be negative[3].

3. A Model

To obtain a quantitative estimate of $\langle r^2 \rangle_n$ we must assume some specific form for the symmetric wave functions of the ground state baryons and for the spin-spin interaction which breaks their symmetry. We assume, for calculational simplicity, that the three quarks are bound by three coupled harmonic oscillators. The unperturbed Hamiltonian is thus[1)]

$$H_0 = \frac{1}{2m}\left(p_1^2 + p_2^2 + p_3^2\right) + \frac{k}{2}\left(r_{12}^2 + r_{23}^2 + r_{31}^2\right) , \qquad (1)$$

where $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ denotes the relative positions of the i^{th} and j^{th} quarks. The center of mass coordinates can be isolated if we define the conjugate variables $(\vec{R},\vec{x},\vec{y})$ and $(\vec{p},\vec{\xi},\vec{\eta})$:

$$\begin{aligned}
\vec{p}_1 &= \tfrac{1}{3}\vec{P} - \sqrt{\tfrac{2}{3}}\,\vec{\xi} , & \vec{r}_1 &= \vec{R} - \sqrt{\tfrac{2}{3}}\,\vec{x} ,\\
\vec{p}_2 &= \tfrac{1}{3}\vec{P} + \tfrac{1}{\sqrt{6}}\vec{\xi} - \tfrac{1}{\sqrt{2}}\vec{\eta} , & \vec{r}_2 &= \vec{R} + \tfrac{1}{\sqrt{6}}\vec{x} - \tfrac{1}{\sqrt{2}}\vec{y} ,\\
\vec{p}_3 &= \tfrac{1}{3}\vec{P} + \tfrac{1}{\sqrt{6}}\vec{\xi} + \tfrac{1}{\sqrt{2}}\vec{\eta} , & \vec{r}_3 &= \vec{R} + \tfrac{1}{\sqrt{6}}\vec{x} + \tfrac{1}{\sqrt{2}}\vec{y} ,
\end{aligned} \qquad (2)$$

in terms of which

$$H_0 = \frac{P^2}{2(3m)} + \frac{\xi^2 + \eta^2}{2m} + \frac{3k}{2}\left(x^2 + y^2\right) . \qquad (3)$$

We shall choose the coordinate y to label the oscillator connecting the two like quarks (the d quarks in the neutron or the u quarks in the proton). Using Eqs. (2) we can express the proton and neutron charge radii $[\langle r^2\rangle = \sum_i Q_i \langle (r_i - R)^2\rangle]$ as

$$\langle r^2\rangle_p = \frac{2}{3}\langle y^2\rangle \,,$$
$$\langle r^2\rangle_n = \frac{1}{3}\left(\langle x^2\rangle - \langle y^2\rangle\right) \,. \tag{4}$$

For the unperturbed Hamiltonian, Eq. (3), the ground state wave function is symmetric in x and y, and hence we have $\langle r^2\rangle_n = 0$.

Suppose now we add to H_0 the perturbation

$$H_1 = \sum_{i<j} S_i \cdot S_j \; f(r_{ij}) \,, \tag{5}$$

where f is a function which we will specify below. This perturbation induces a shift in the ground state energy levels and, in particular, splits the nucleon and delta states by an amount

$$m_\Delta - m_N = \frac{3}{2}\langle f\rangle_0 \,. \tag{6}$$

Here $\langle f\rangle_0$ denotes the expectation value of $f(\sqrt{2}\, y)$ in the unperturbed ground state of the y oscillator.

The perturbation also modifies the wave functions. This effect is of second order for the mass shifts, but is of first order for a calculation of $\langle x^2 \rangle$ or $\langle y^2 \rangle$. We are interested primarily in the difference $\langle x^2 \rangle - \langle y^2 \rangle$ [see Eq. (4)] so we focus on that part of H_1 which distinguishes pairs of like quarks (dd or uu) from unlike pairs (ud). Thus it is sufficient to consider a perturbation of the form

$$\tilde{H}_1 = 3 S_2 \cdot S_3 \, f(\sqrt{2}\, y) = \frac{3}{4} f(\sqrt{2}\, y) , \tag{7}$$

which acts on the y-oscillator (i.e. the like quarks) alone.

Since the oscillator potential [Eq. (3)] is itself proportional to y^2, we can use the virial theorem to calculate the shift in $\langle y^2 \rangle$ induced by the perturbation $\tilde{H}_1$. This theorem gives the relation

$$\left\langle \frac{\eta^2}{2m} \right\rangle = \frac{3}{2} k \langle y^2 \rangle + \frac{1}{2} \langle \vec{y} \cdot \vec{\nabla}_y \, \tilde{H}_1(y) \rangle . \tag{8}$$

Here the expectation values are taken with the perturbed wave function. We can now write the total energy of the perturbed y oscillator as

$$\langle H_0(y)+\tilde{H}_1(y)\rangle = 3k\langle y^2\rangle + \langle \tilde{H}_1 + \frac{1}{2}\vec{y}\cdot\vec{\nabla}_y \tilde{H}_1\rangle , \qquad (9)$$

where $H_0(y) = \eta^2/2m + \frac{3}{2}k\,y^2$ [cf. Eq. (3)]. First order perturbation theory gives the alternate expression

$$\langle H_0(y)+\tilde{H}_1(y)\rangle \simeq \frac{3}{2}\omega_0 + \langle \tilde{H}_1\rangle_0 \quad , \qquad (10)$$

where ω_0 [$= 2k\langle y^2\rangle_0$] is the level spacing of the unperturbed system. Comparing these expressions, we conclude that to first order in f,

$$\frac{\langle y^2\rangle - \langle y^2\rangle_0}{\langle y^2\rangle_0} \simeq -\frac{\langle \vec{y}\cdot\vec{\nabla}_y f\rangle_0}{4\omega_0} . \qquad (11)$$

Since the perturbation (7) does not affect the x-oscillator, $\langle x^2\rangle = \langle y^2\rangle_0$, and Eqs. (11) and (4) provide the first order estimate

$$\frac{\langle r^2\rangle_n}{\langle r^2\rangle_p} \simeq \frac{\langle \vec{y}\cdot\vec{\nabla}_y f\rangle_0}{8\omega_0} . \qquad (12)$$

The experimental magnitude of the perturbation is specified by Eq. (6), which leads us to the relation

$$\frac{\langle r^2\rangle_n}{\langle r^2\rangle_p} \simeq \frac{m_\Delta - m_N}{12\,\omega_0}\left[\frac{\langle \vec{y}\cdot\vec{\nabla} f\rangle_0}{\langle f\rangle_0}\right] . \tag{13}$$

For any perturbation $f(y)$ which decreases with distance (and thus provides a repulsive force), $\langle r^2\rangle_n$ will be negative. This correspondence is quite general, independent of our approximations and of the detailed form of the potential H_0.

To estimate the f-dependent factors in Eq. (13) we turn for inspiration to quantum chromodynamics. This theory includes (by popular presumption[4]) a spin independent quark-confining force (analogous to our H_0) with smaller spin-dependent interactions which we estimate from elementary gluon exchange. The resulting spin-spin interaction resembles the hyperfine interaction of quantum electrodynamics but with a running coupling[5)] $\alpha_S(y)$ replacing the fine-structure constant. Modulo logarithms $f(y) \sim y^{-3}$, and the bracketed factor in Eq. (13) is simply -3. [Asymptotic freedom, i.e. the fact that $\alpha(y) \to 0$ as $y \to 0$, insures that there are no short distance divergences in $\langle f\rangle_0$ or $\langle \vec{y}\cdot\vec{\nabla} f\rangle_0$.]

The numerical value of ω_0 can be estimated from the slope of the nucleon's Regge trajectory to be $\omega_0 \approx (2m_N\alpha'_N)^{-1} \approx 530$ MeV. With $m_\Delta - m_N = 293$ MeV we thus obtain the result

$$\frac{\langle r^2\rangle_n}{\langle r^2\rangle_p} \simeq -0.14 . \tag{14}$$

$\frac{1}{2}\alpha'_N M_N (M_N - M_\Delta)$

$$-\frac{(M_\Delta - M_n)}{4} \cdot 2 M_N \alpha'_N = \frac{1}{2}(M_N - M_\Delta) M_N \alpha'_N$$

verse momentum distributions of the u and d quarks. One such experiment involves the production of pions in high energy collisions. One should select data[10] involving quarks with $0.1 < x_F < 0.5$, the region in which valence quarks are concentrated. Associating π^+ particles with u quarks in the initial proton and π^- particles with d quarks, we expect that $\langle p_\perp^2\rangle_{\pi^-} > \langle p_\perp^2\rangle_{\pi^+}$. Present data is somewhat inconclusive: hadroproduction[11] seems to support this inequality, while leptoproduction[12] data perhaps contradicts it.

Another, more direct, test of Eq. (16) is possible with the production of massive lepton pairs in πp collisions. This process[13] can proceed by the annihilation of an incident antiquark from the pion with a valence quark of the target proton. One thus measures in a more or less direct fashion the momentum distribution functions of Eq. (16). Once again we have a prediction valid for quarks in the range[14] $0.1 < x_F < 0.5$. Assuming that the transverse momentum distribution of antiquarks in the incident pion is approximately the same as the quark distribution of the target proton, we predict that

$$\frac{\langle p_\perp^2\rangle_{\pi^+p} - \langle p_\perp^2\rangle_{\pi^-p}}{\frac{1}{2}\left(\langle p_\perp^2\rangle_{\pi^+p} + \langle p_\perp^2\rangle_{\pi^-p}\right)} \simeq 0.1 \quad . \tag{18}$$

Here $p_\perp$ denotes the transverse momentum of the produced lepton pair.

The experimental test of these predictions is an important and challenging task. The results will help define the rôle played by quarks in the pageant of hadronic physics.

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This is in remarkably good agreement with the experimental value[6] of this ratio, -0.146 ± 0.005. The preceding analysis can also be applied to the corresponding magnetic properties of nucleons with the result[7]

$$\left.\frac{dG_n^M(q^2)/dq^2}{dG_p^M(q^2)/dq^2}\right|_{q^2=0} = -\frac{2}{3}\left[1-\frac{m_\Delta - m_N}{12\omega_0}\right]. \quad (15)$$

The factor $-2/3$ is the familiar quark model prediction for the ratio μ_n/μ_p. The term $(m_\Delta - m_N)/12\omega_0$ arises from the wave function shift induced by the interaction (5); its numerical value is only 0.05. Present data[8] are consistent with Eq. (15) but cannot really distinguish our prediction from the naive ratio $-2/3$.

4. Discussion

We have argued that in any dynamical quark model, the spin-spin interaction which breaks the mass degeneracy of the nucleon and delta will necessarily alter the relative spatial distributions of u and d quarks within the nucleon. Quantum chromodynamics suggests a possible form for this interaction. For the neutron this results in a broader spatial distribution for the d quarks and a negative value for the neutron's charge radius. We believe this result to be quite general. Previous treatments of the problem have missed this point by (i) assuming the spin-spin interaction to be independent of inter-quark separation or (ii) neglecting to calculate the wave function shift induced by a spatially dependent interaction.

The preceding argument has dealt with the properties of a static nucleon. Differences in the spatial distributions of the u and d quarks should, however, affect high energy interactions as well. A natural assumption is that the valence quark contribution to the momentum distribution[9] $u(p_\perp, x_F)$ (the u quarks in the proton) will be narrower (in $p_\perp$) than the contribution to $d(p_\perp, x_F)$ (the d quark distribution). Thus, if for moderate x_F we parametrize these functions as

$$
\begin{aligned}
u(p_\perp, x_F) &= u(0, x_F)\, \exp(-r_u^2 p_\perp^2) , \\
d(p_\perp, x_F) &= d(0, x_F)\, \exp(-r_d^2 p_\perp^2) ,
\end{aligned}
\tag{16}
$$

then the calculations of the previous section suggest that

$$
\frac{r_u^2 - r_d^2}{r_d^2} \simeq -\frac{3\langle r^2\rangle_n}{2\langle r^2\rangle_p} \simeq -0.2 \quad . \tag{17}
$$

In the parametrization (16) this asymmetry is the same as the transverse momentum asymmetry $(\langle p_\perp^2\rangle_d - \langle p_\perp^2\rangle_u)/\langle p_\perp^2\rangle_u$. Equation (17) thus suggests asymmetries in the momentum distributions of experiments which probe differences in the trans-

Footnotes and References

1. See, for example, R.P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D3 (1971) 2706; R. Horgan and R.H. Dalitz, Nucl. Phys. B66 (1973) 135; R. Horgan, Nucl. Phys. B71 (1974) 514. These papers use a harmonic oscillator potential to confine quarks; another approach, using a self-consistent "bag" model is illustrated by T. DeGrand et al., Phys. Rev. D12, (1975) 2060.

2. Recent discussions of this question include L.M. Sehgal, Phys. Lett. 53B, (1974) 106; D. Parashar and R.S. Kaushal, Phys. Rev. D13 (1976) 2684; A. Niégawa and D. Kiang, Phys. Rev. D14 (1976) 3235; A. Le Yaouanc, L. Oliver, O. Pène, and J.C. Raynal, Phys. Rev. D15 (1977) 844; and F.E. Close, F. Halzen, and D.M. Scott, Rutherford Laboratory preprint (1977).

3. This result has seemed difficult to achieve in the models in Ref. 2. Recent work by R.R. Horgan (private communication) has shown that an analysis of SU(6) configuration mixing [see Le Yaouanc et al., Ref. 2] can yield the proper sign for $\langle r^2 \rangle_n$.

4. See, for example, K. Wilson, Phys. Rev. D10 (1974) 2445.

5. D.J. Gross and F. Wilczek, Phys. Rev. D8 (1973) 3633 and D9 (1974) 980; H. Georgi and H.D. Politzer, ibid. D9 (1974) 416.

6. V.E. Krohn and G.R. Ringo, Phys. Rev. D8 (1973) 1305; R.W. Berard et al., Phys. Lett. B47 (1973) 355; F. Borkowski et al., Nucl. Phys. A222 (1974) 269.

7. The proton's magnetic radius, $-6\, G_p^{M'}(0)/G_p^M(0)$, need not be the same as its charge radius, $\langle r^2\rangle_p = -6\, G_p^{E'}(0)/G_p^E(0)$. Our model predicts that $G_p^{M'} G_p^E / G_p^{E'} G_p^M = 1-(m_\Delta - m_N)/6\,\omega_0 = 0.9$. The data of Borkowski et al. [Ref. 6] support this result.

8. See, for example, W. Bartel et al., Nucl. Phys. B58 (1973) 429.

9. The variables $p_\perp$ and x_F refer to the transverse momentum and longitudinal momentum fraction of quarks within a rapidly moving nucleon. See R.P. Feynman, "Photon-Hadron Interactions," W.A. Benjamin (1972).

10. In hadroproduction one should select produced pions with longitudinal momentum fractions in the range 0.1 to 0.5. These pions presumably arise from the fragmentation of quarks with the same or larger momentum fractions. In leptoproduction one can directly select the struck quark's momentum fraction by an appropriate choice of momentum and energy transfer from the incident lepton. The pions relevant to our discussion are those which result from the fragmentation of this quark and carry a finite fraction z of the quark's momentum.

11. P. Capiluppi et al., Nucl. Phys. B70 (1974) 1.

12. J.T. Dakin et al., Phys. Rev. D10 (1974) 1401.

13. S.D. Drell and T-M Yan, Phys. Rev. Lett. 25 (1970) 316.

14. The corresponding range of lepton pair masses M is approximately $0.1 < M/\sqrt{s} < 0.5$, where $\sqrt{s}$ is the available energy. Note that the dominance of valence quarks implies a cross-section ratio $\sigma_{\pi^+p}/\sigma_{\pi^-p} \approx 1/8$. See P. Mockett et al., Fermilab proposal P-332 (1974). Measurement of a larger a ratio would imply the presence of competing processes and a subsequent reduction of the predicted asymmetry (18).