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MAXIMUM LIKELIHOOD ANALYSIS OF IONIZATION DATA FROM SAMM

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ABSTRACT

We present a maximum likelihood analysis of ionization information obtained from 3465 tracks with momentum less than 2.0 GeV/c in the 30" bubble chamber at Fermilab. The data were obtained from measurements on SAMM, the semi-automatic CRT measuring machine at Fermilab. We show that it is possible to separate pions from protons up to a laboratory momentum of ~1.4 GeV/c.

I. INTRODUCTION

The data being presented here were obtained from 100 GeV/c $\bar{p}p$ interactions in the 30" bubble chamber at Fermilab. The events were measured on SAMM, the CRT based semi-automatic measuring machine at Fermilab.[1] Tracks were measured by SAMM in the slice-scan mode, whereby each area of digitisation is raster-scanned by a 10µ spot of light perpendicular to the track. SAMM generates a hardware histogram of the track signal during a raster scan. The value of the histogram at the signal center is a direct measure of the number of hits on the track. Utilizing the ratio of the total number of hits/sweeps per track, it is possible to obtain information on the bubble density of the track on film. After three view reconstruction, the bubble density/ view can be transformed into the bubble density in space, leading directly to a velocity estimate of the particle.

II. ESTIMATION OF β FROM LACUNARITY MEASUREMENTS.

It can easily be shown[2] that the velocity β of a particle is related to its lacunarity (defined as the ratio of misses to total number of sweeps in the slice scan) by the following relation.

$$\frac{1}{\beta^2} = \frac{\ln(\ell)}{\ln(\ell_{ref})} \frac{ML_{film}}{L_{space}} \qquad (1)$$

where ℓ = lacunarity of track.

ℓ_{ref} = lacunarity of reference track

L_{film} = length of track on film

L_{space} = length of track in space

M = magnification from film to space.

The beam track being minimum ionizing was used as the reference track for each view. Only those events with beam track length greater than 10 cms were considered for this analysis. The three estimates of $1/\beta^2$ from the three stereo-views were averaged.

III. MAXIMUM LIKELIHOOD ANALYSIS.

In Figure 1, we introduce a plot of $1/\beta^2$, estimated as described above, versus $1/p^2$ for all tracks with laboratory momentum , p, less than 2.0 GeV/c. Since

$$\frac{1}{\beta^2} = 1+\frac{m^2}{p^2} \qquad (2)$$

particles of different mass, m, should lie on straight lines of slope m^2 and intercept 1. This is indeed the case in Figure 1, with two bands of points being visible clustered around the proton and pion lines. There is no kaon band visible, indicating that there are few slow kaons present. Each track, however, had three helix fits attempted on it (proton,pion and kaon) leading to three values of p. A helix fit was entered into Figure 1 only if the value of m^2

derived from it (using equation 2) was consistent with the mass squared associated with that helix fit within errors. In case of ambiguity, the one closest to the associated mass squared was used.

The problem now is to decide whether a track is a proton or pion from its closeness to the proton and pion lines. The problem is however non-trivial since the likelihood of a point being a given distance away from either line is not only a function of the distance but also of the momentum spectra of the protons and pions. In the following analysis, we neglect the error in $1/p^2$ and assume that the points in Figure 1 are displaced from their ideal lines by measurement error in $1/\beta^2$ only. This is justifiable, since the percentage error in $1/\beta^2$ is much greater than that in $1/p^2$, the ratio of the two being typically of the order ~15:1.

By initially separating each set of bands into protons and pions based on the distance from the ideal lines (the minimum distance method), it was established that the momentum spectra obey a power law, the power being dependent on the particle type.

Let x denote $1/p^2$ and y denote $1/\beta^2$. Let α be the percentage of protons and N be the total number of tracks. Then the number of protons between x and x+dx is given by

$$N_p dx = N\alpha \frac{x^{-s_1}}{I(s_1)} dx \qquad (3)$$

where s_1 is a constant to be determined.

and $$I(s_1) = \int_a^b x^{-s_1} dx \qquad (4)$$

a,b are the lower and upper limits of variation of x taken to be 0.25 and 10.0 $(\mathrm{GeV}/c)^{-2}$ respectively. Assuming that the points are normally distributed in y with standard deviation σ, the likelihood that any given point is a proton is given by

$$L_p = \frac{N_p}{\sqrt{2\pi}\,\sigma} e^{-(y-y_p)^2/2\sigma^2} \qquad (5)$$

where $y_p = 1+m_p^2 x$ (6)

m_p = mass of the proton.

then

$$\ln(L_p) = \ln(\alpha/\sigma)-s_1\ln(x)-(y-y_p)^2/2\sigma^2-\ln(I(s_1)) \qquad (7)$$

ignoring constant factors.

similarly,

$$\ell n(L_\pi) = \ell n((1-\alpha)/\sigma) - s_2 \ell n(x) - (y-y_\pi)^2/2\sigma^2 - \ell n(I(s_2)) \quad (8)$$

σ was determined to be 0.21 by separating the tracks using the minimum distance method.

If $\ell n(L_p) > \ell n(L_\pi)$ for a track for a given α, s_1, s_2 that track was declared a proton and $\ell n(L_p)$ was added to the total log-likelihood function and vice versa. The net log likelihood was maximised with α, s_1, s_2 as free parameters using the program MINUIT.[3] It was found best to keep σ fixed at 0.21 and not estimate it from the data, since the program tended to arrive at false maxima with too large values of σ that turned most of the protons into pions.

After the maximum was reached, each track was given a proton probability (defined as $L_p/(L_p+L_\pi)$) and a pion probability ($L_\pi/(L_p+L_\pi)$) Depending on which is greater. the track was declared a proton or a pion. The fraction of tracks thus declared protons was found to be 0.134. The fitted value of α was 0.134 ± 0.002. The two values are identical, indicating that the method is consistent.

Figure 2 gives a scatter plot of $1/\beta^2$ vs. $1/p^2$ for the tracks that have been declared pions by the maximum likelihood method. Figure 3(a) is a plot of the momentum spectrum of the protons with the fitted curve superimposed. Figure 3(b) is the corresponding figure for pions. The value of S_1 was found to be 0.69±0.05 and s_2 to be 0.84±0.02. Thus the protons tend to fall off slower with momentum than pions.

Figure 4 is a plot of the average proton and pion probabilities as a function of $1/p^2$. For $1/p^2 > 2.0$, corresponding to momenta <700 MeV/c both the probabilities are 1, indicating that the separation is complete below that momentum. For increasing momenta, the mean proton probability falls faster than the mean pion probability. This is a direct result of there being more pions than protons.

In Fig 5(a), we histogram the proton probability for both protons and pions below 700 MeV/c. The protons give the peak at 1.0 and the pions at 0.0. Figure 5(b) is the same plot in the momentum range 0.7-1.2 GeV/c. The proton and pion distributions now merge. Nevertheless, the two distributions are still distinct and separation is still possible. Figure 5(c) shows the proton probability in the momentum range 1.2 -1.4 GeV/c and Figure 5(d) between 1.4-1.8 GeV/c. The absence of a clear proton peak in Figure 5(c) does not imply that tracks with proton probability >0.5 are untrustworthy. Since for increasing momentum, the pion momentum spectrum rises much faster than the proton momentum spectrum, it becomes increasingly difficult for any track to have proton probabilities near 1. The efficiency for proton detection does however begin to drop rapidly at this point. In Figure 5(d) . the proton peak has vanished, being swamped by the pion peak. This does not imply that there are no protons in this momentum range, only that the errors in measurement make their identification impossible.

To conclude, we have carried out a maximum likelihood analysis on ionization information measured by SAMM and found that the separation of protons and pions is reliable up to a laboratory momentum of 1.4 GeV/c. To separate protons from pions above this momentum would demand more accuracy in the lacunarity measurements. The method outlined above, being automatic and computerised can be expected to yield more reliable information , especially in the momentum range 1.0-1.4 GeV/c than can be obtained by conventional techniques using the human eye.

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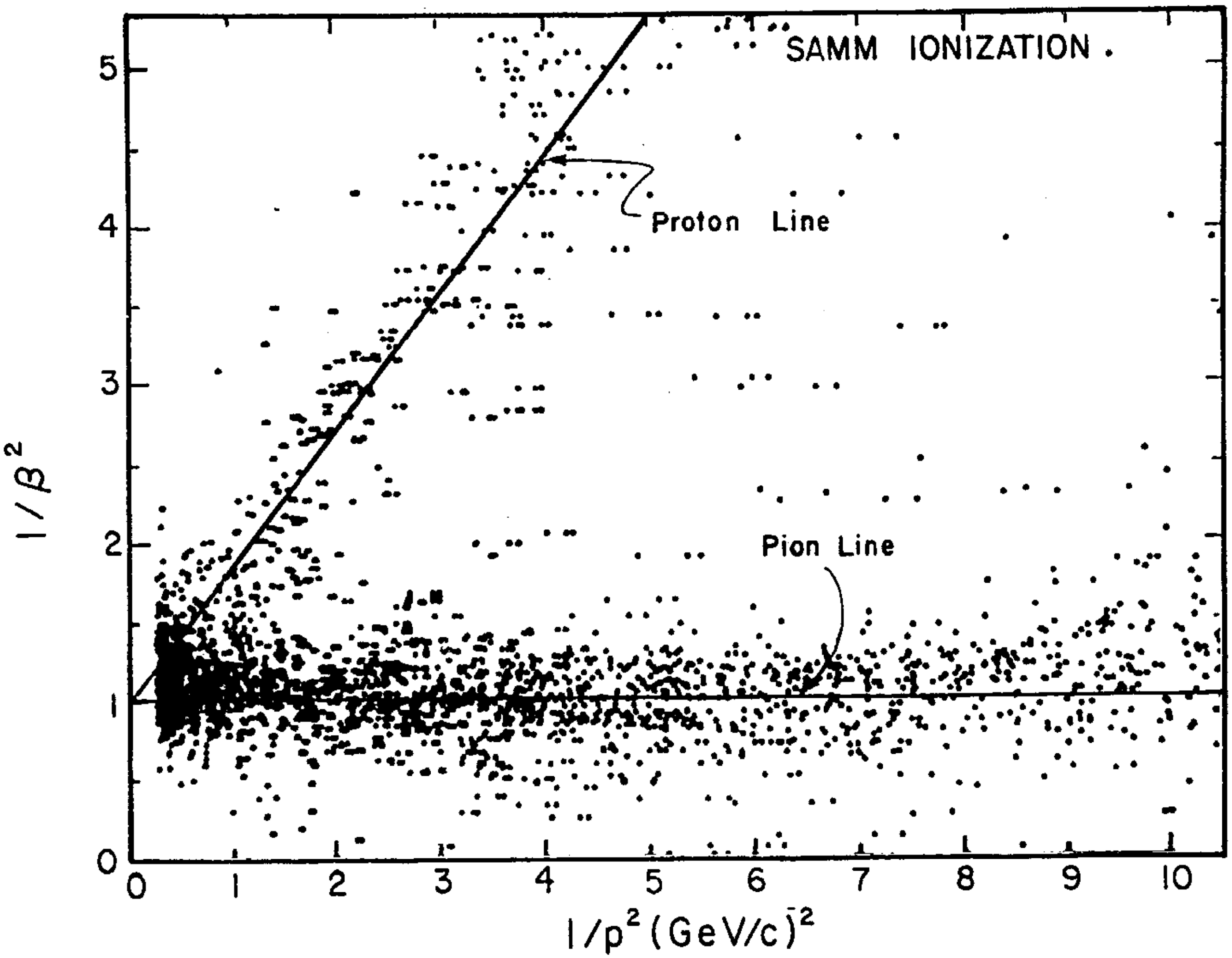

Fig. 1

Scatter plot of $1/\beta^2$ vs $1/p^2$ for all tracks,

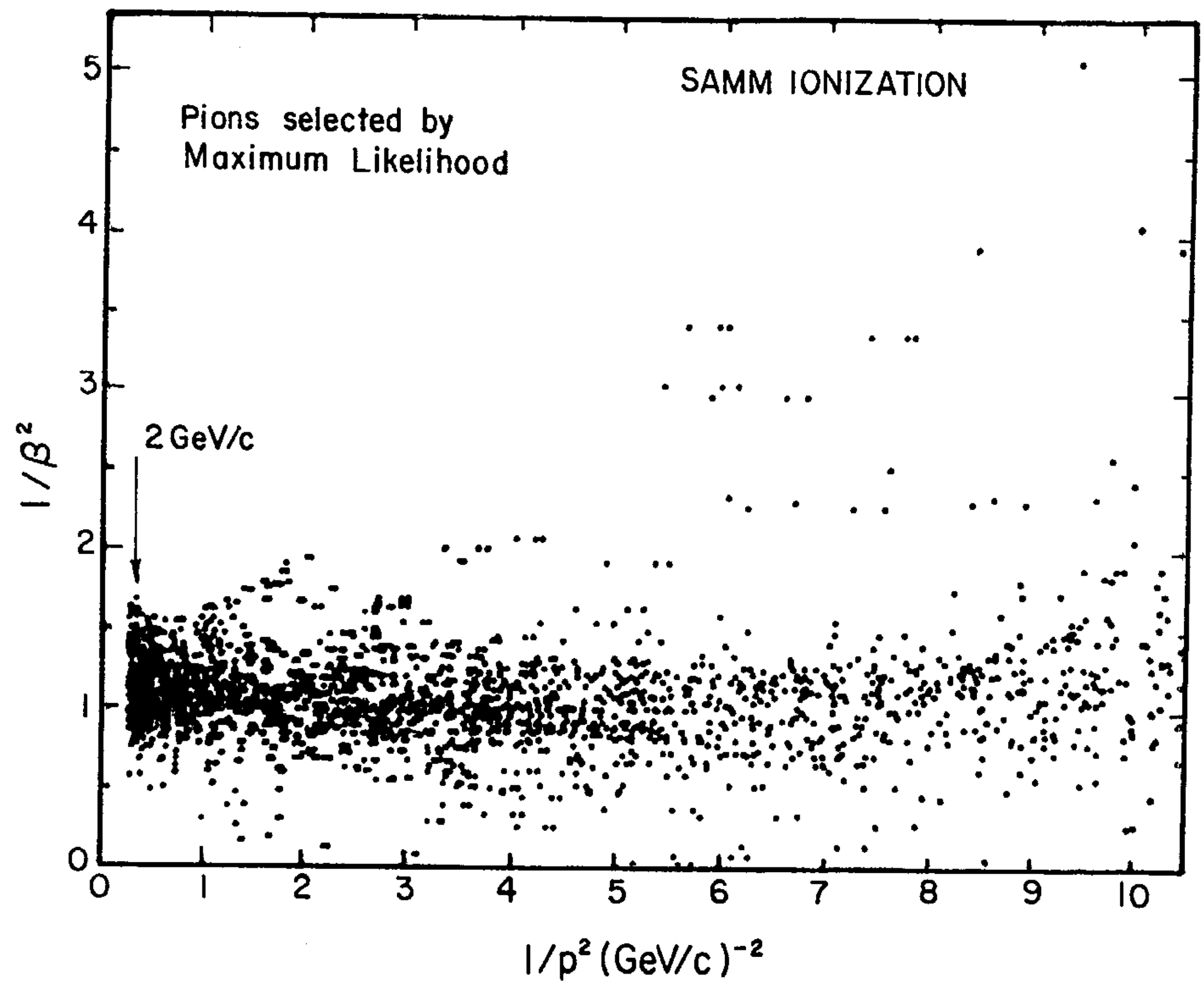

Fig. 2

Scatter plot of $1/\beta^2$ vs $1/p^2$ for all tracks selected as pions by maximum likelihood.

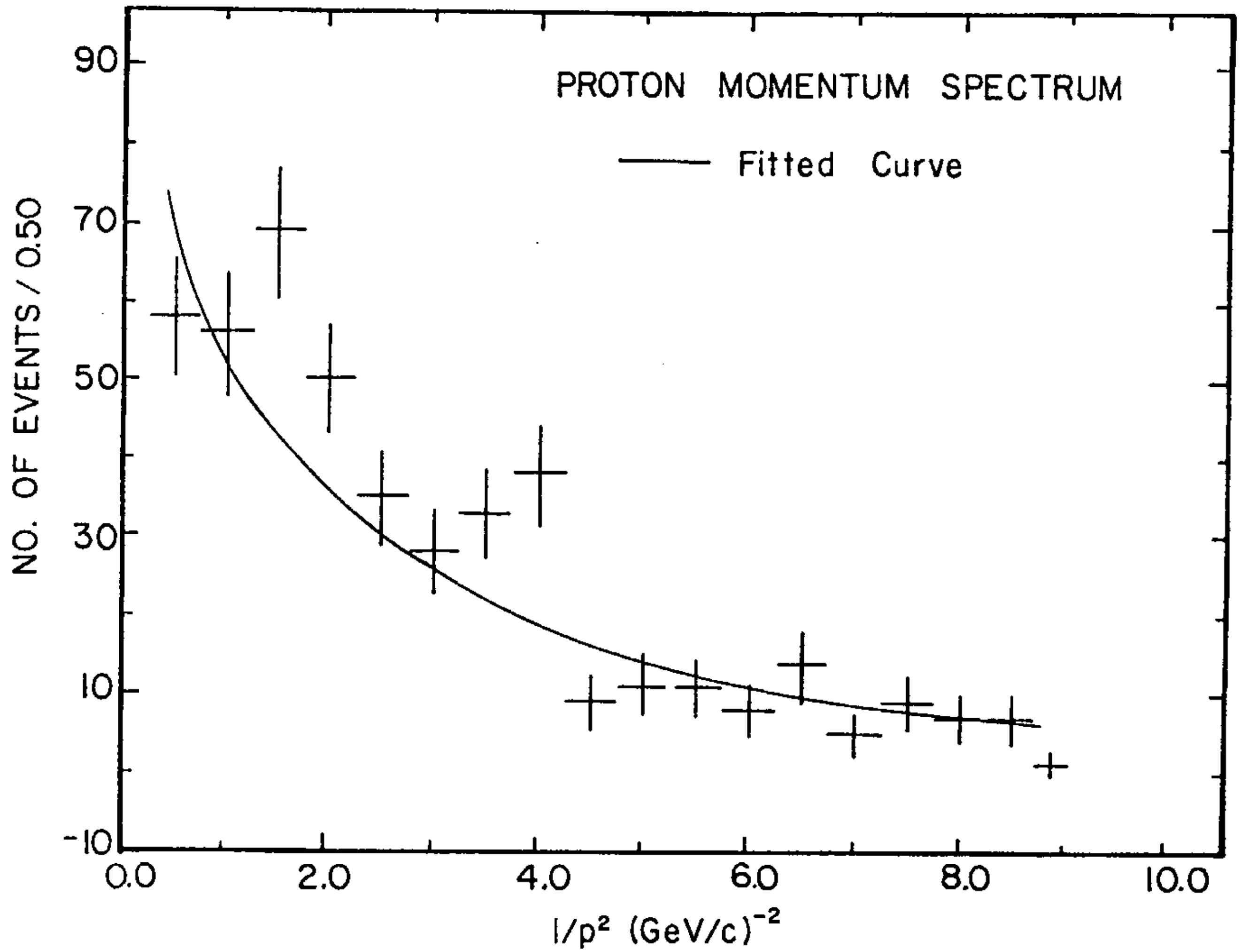

Fig. 3(a)

Momentum spectrum of the protons

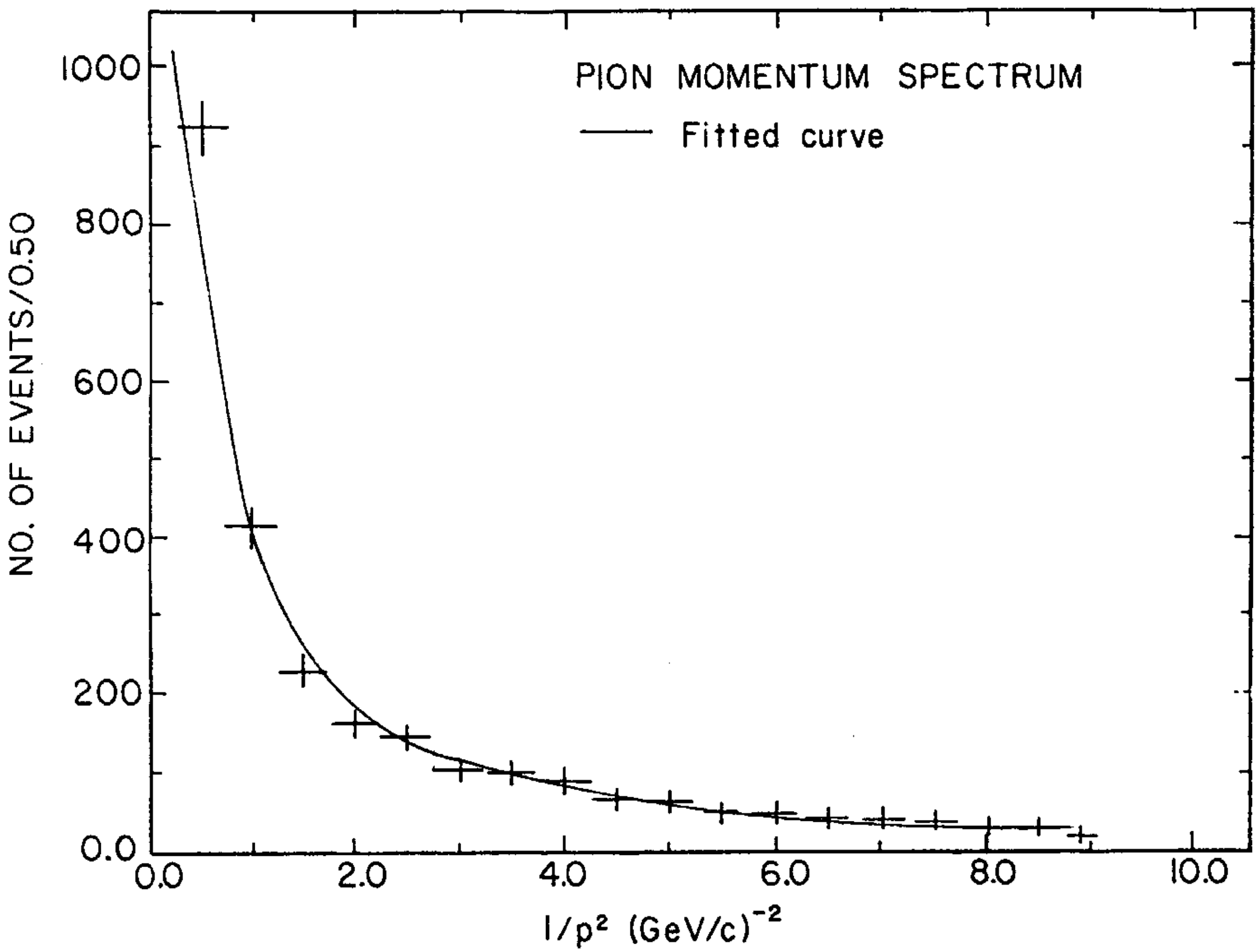

Fig. 3(b)

Pions with the fitted curves superimposed.

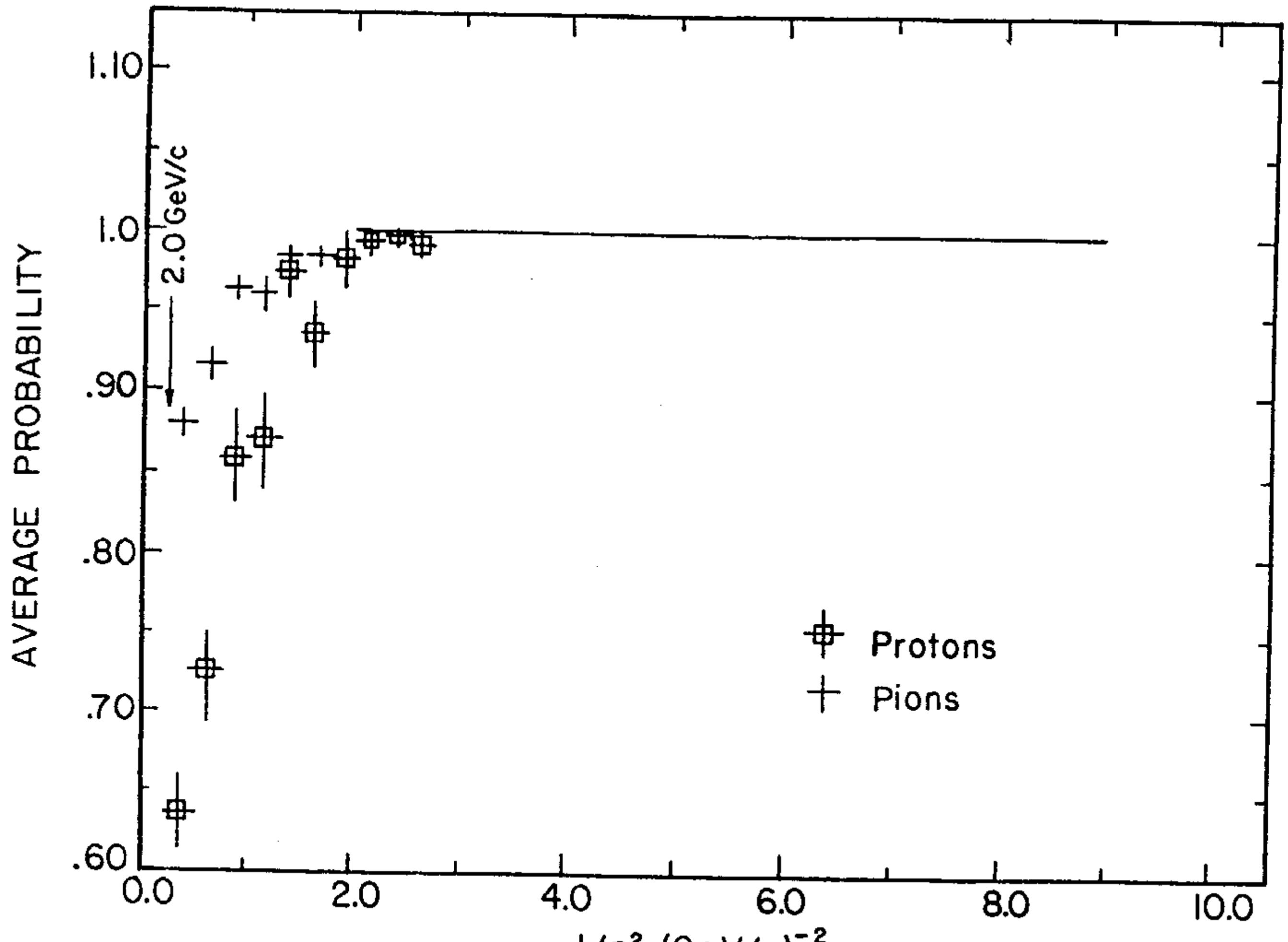

Fig. 4

Average probability as a function of $1/p^2$.

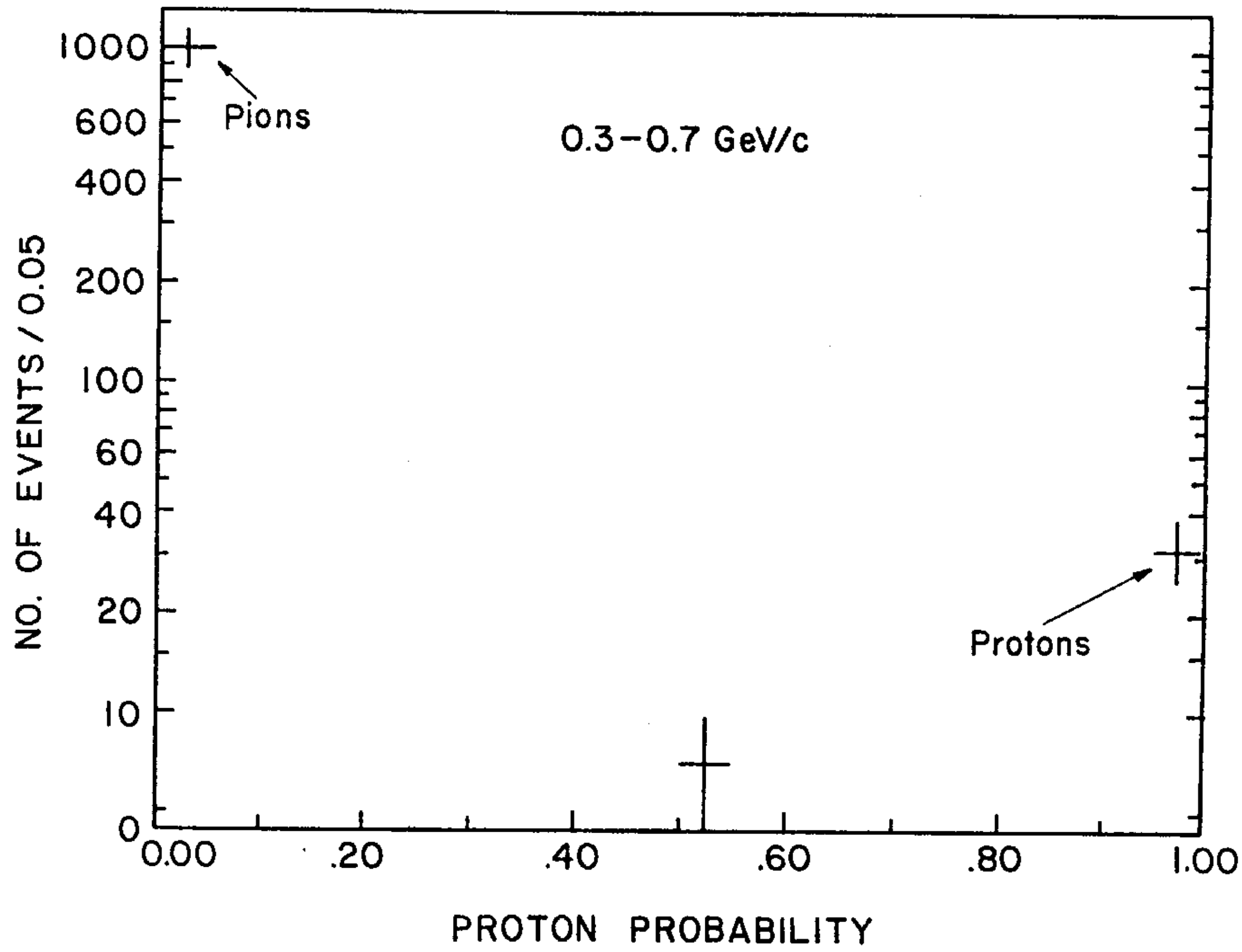

Fig. 5(a)

Proton probability distributions for all tracks in the momentum range 0.3-0.7 GeV/c.

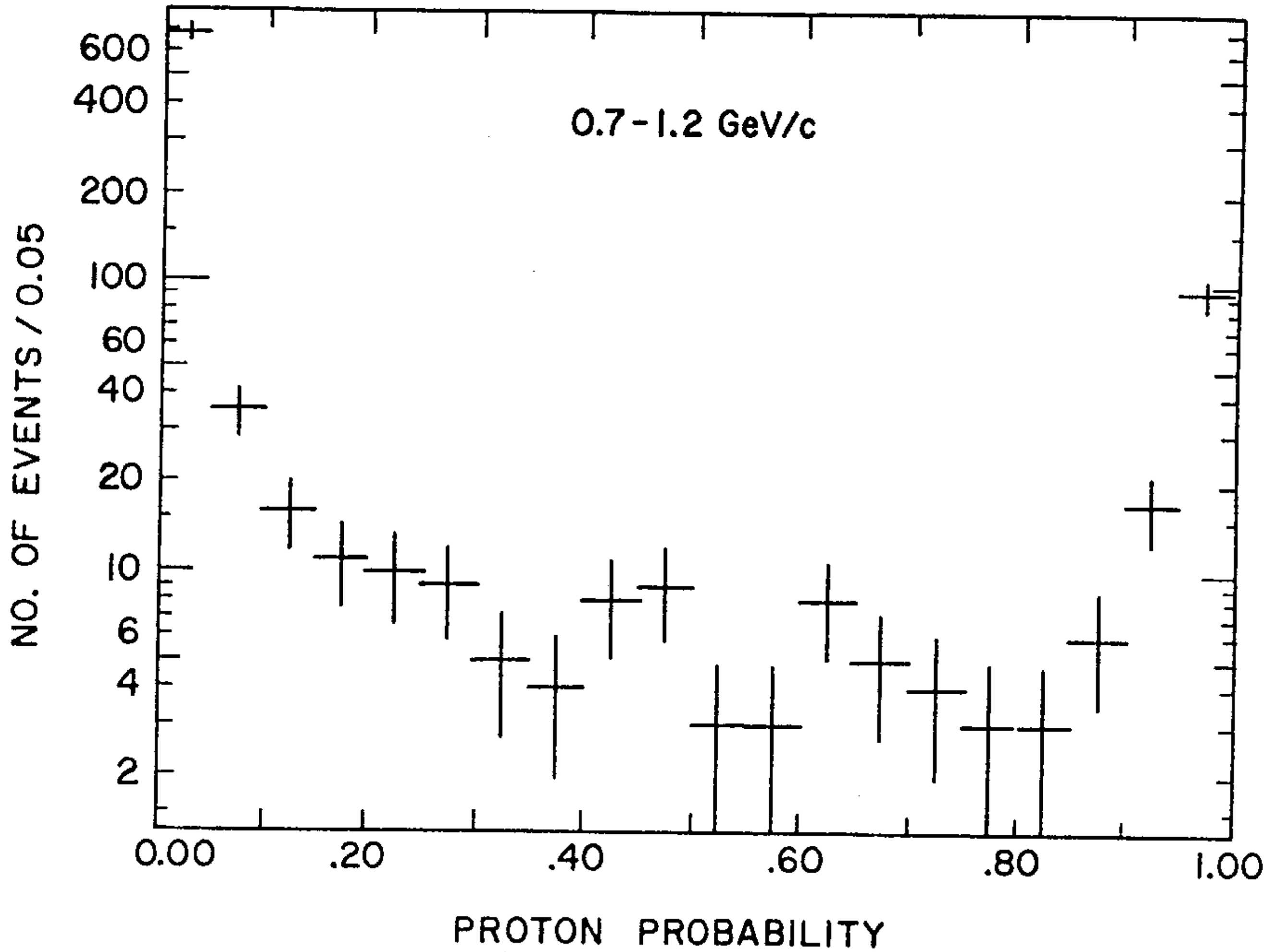

Fig. 5(b)

Proton probability distributions for all tracks in the momentum range 0.7-1.2 GeV/c.

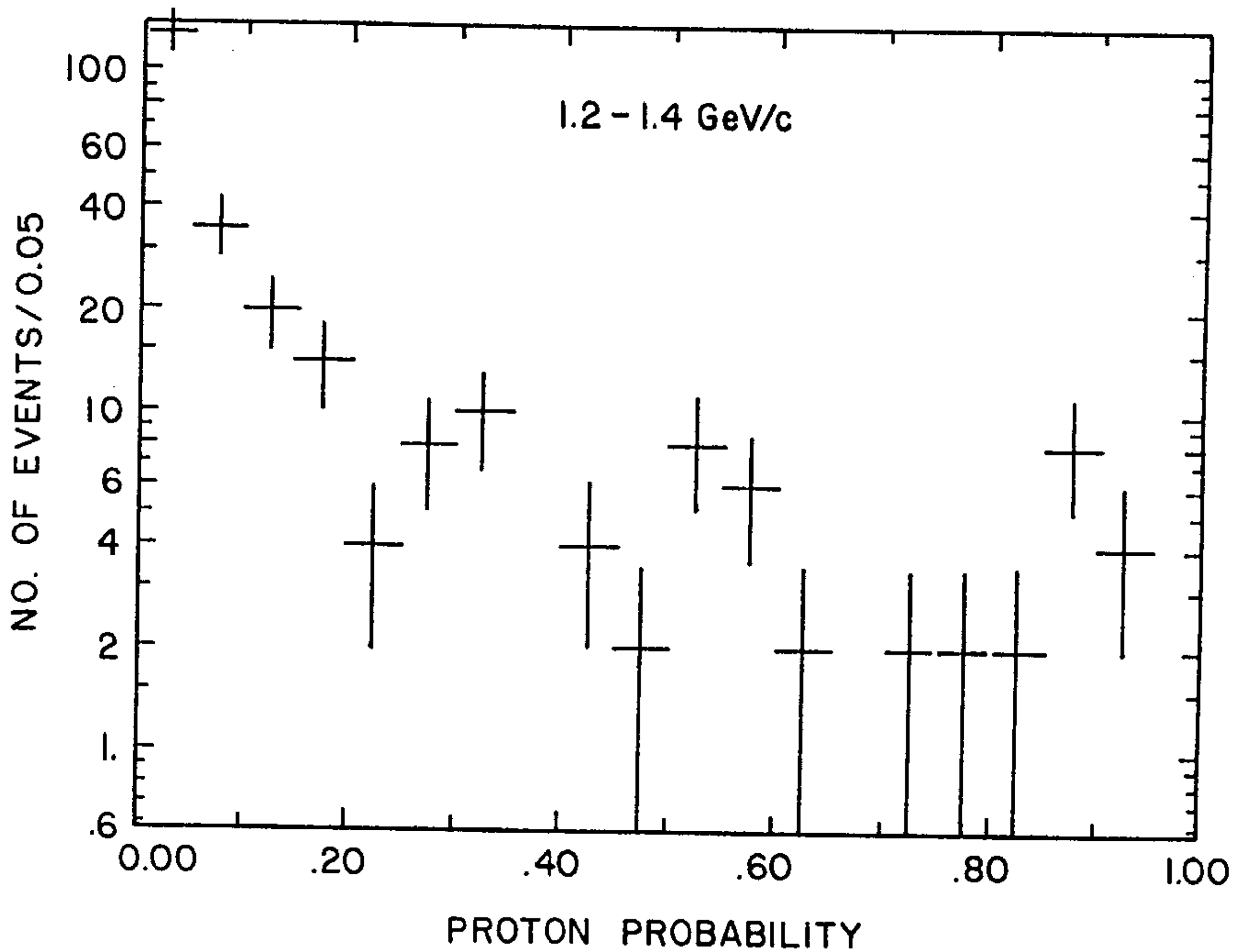

Fig. 5(c)

Proton probability distributions for all tracks in the momentum range 1.2-1.4 GeV/c.

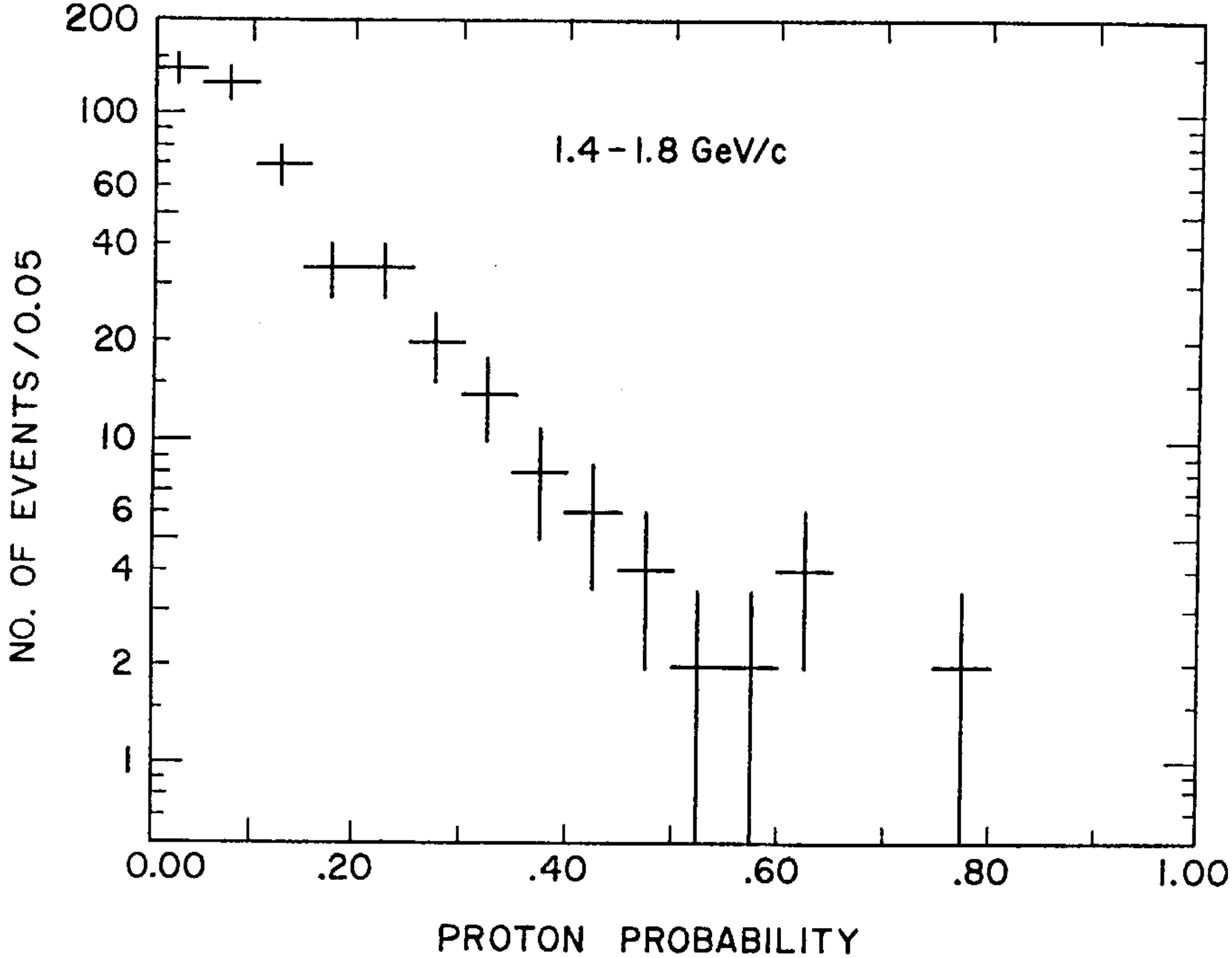

Fig. 5(d)

Proton probability distributions for all tracks in the momentum range 1.4-1.8 GeV/c.