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THEORETICAL REVIEW OF STRANGE AND NONSTRANGE MESONS

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I. INTRODUCTION

Modern SU(6) quark-model spectroscopy began in the middle sixties and exploded at the 1966 Berkeley Conference where an overwhelming number of quark model contributions were received and the discussion was expanded to double the normal time. The rapporteur's review was given by Dalitz.[1] It is amusing to recall some of the points made then after over ten years.

One of the issues being resolved was the nature of excitations above the lowest-lying 56 and 35 supermultiplets. The evidence favored the orbital-excitation model[2] which obtained states of angular momentum higher than 3/2 without requiring states of exotic isospin, hypercharge and baryon number. Models which added more quarks to get higher SU(6) multiplets were gradually dropped. Today, after years of oblivion, the multiquark exotic states are back, and the lowest four-quark states, as pointed out by De Rujula[3] and Leith[4] do not have exotic quantum numbers at all. Another issue mentioned was the general confusion about mass formulas and strangeness splittings in SU(3) multiplets. This confusion is still with us today, and even more so now that we have charm. At the same time I suggested that the strange axial vector mesons would have peculiar interesting properties including mixing and interference,[5] and that a simple mixing mechanism[6] would naturally give a 45° mixing angle which decoupled one state from the $K^*\pi$ decay mode and decoupled the other from $K\rho$. This mixing model was dropped after simple quark model and $SU(6)_W$ calculations showed that polarization effects destroyed the mixing. Some years later we learned that naive $SU(6)_W$ fails to give proper polarization predictions. Now we hear from Leith[4] that the two axial vector mesons have been found and seem to have this kind of mixing where each state is decoupled from one of the two dominant decay modes.

II. STRANGENESS AND SPIN MASS SPLITTINGS

An open problem in hadron spectroscopy is how to describe the regularities in mass splittings occurring in hadron multiplets and supermultiplets. The basic difficulty in deriving any formula for masses from symmetry breaking is the absence of an underlying theory. Simple formulas are obtained by postulating simple transformation properties of the symmetry breaking, e.g. octet splitting for SU(3) which gives the Gell-Mann-Okubo mass formula. But there is no theory to tell whether the formula applies to linear masses, quadratic masses, some exotic power of the mass, the S-matrix, or to "reduced" matrix elements with certain kinematic factors removed. The original folklore suggested linear mass formulas for baryons and quadratic formulas for mesons. These gave good agreement with experiment for SU(3) and SU(6) mass formulas. But the quark model gave results which related baryon mass splittings to meson mass splittings, in particular, the naive assumption that the difference between strange and nonstrange quarks relates meson and baryon splittings as well as mesons and baryons among themselves. Within the meson and baryon supermultiplets these quark model relations are equivalent to SU(6) relations. But between mesons and baryons they give something new, which agrees with experiment when linear masses are used. The situation was summarized at the 1966 Berkeley conference[1] by the "crazy mass formula"

$$K - \pi \overset{Q}{=} K^*-\rho \overset{L}{=} \Sigma^* - \Delta \overset{L,Q}{=} \Xi - \Sigma\ , \qquad (2.1)$$

where the L above the equality implies that linear masses should be used and the Q above the equality implies that quadratic masses should be used.

While there are many ways to derive some of these equalities, no credible model includes both the linear and quadratic relations involving the same vector meson mass splitting. But the experimental agreement with the crazy formula is sufficiently impressive to suggest that it cannot be wholly accidental.

The discovery of charm allows a similar formula to be written for the charmed states by simply replacing all strange quarks in (2.1) by charmed quarks. The result is

$$D - \pi \overset{Q}{=} D^* - \rho \overset{L}{=} C_1^* - \Delta = \qquad , \qquad (2.2)$$

where the last equality is left open since the doubly charmed baryon analogous to the Ξ has not yet been found. This formula also agrees with experiment, as shown in Table 2.1. Thus changing a nonstrange quark in the ρ to a strange or to a charmed quark produces a linear mass shift which is equal to that produced by the corresponding change of a quark in the Δ, while the shift in squared mass is equal to that produced by the corresponding quark change in the pion.

An interesting relation between the spin splittings of the masses of strange and nonstrange baryons was given by Federman, Rubinstein and Talmi[7] in 1966

$$(1/2)(\Sigma + 2\Sigma^* - 3\Lambda) = \Delta - N. \qquad (2.3)$$

Experimentally the left and right hand sides of this relation are 307 and 294 MeV, which is rather good agreement. This relation follows from the assumption that the mass differences are due to two-body forces which are spin dependent. The right hand side is just (3/2) the difference between the interaction of two nonstrange quarks in the triplet and singlet spin states when these quarks are bound in a nonstrange baryon. The left hand side is the same difference for a nonstrange quark pair bound in a hyperon (the particular linear combination chosen causes the contribution from the strange quark interaction to cancel out). The experimental agreement indicates that the assumptions of two-body forces and SU(6) spin couplings in the wave functions are good approximations.

Here again the relation can be extended to charm by replacing strange quarks everywhere with charmed quarks.

$$(1/2)(C_1 + 2C_1^* - 3C_o) = \Delta - N \qquad (2.4)$$

Since the present experimental information on charmed baryons[8] gives a mass of 2260 for the C_o and a mass of 2500 for a broad peak interpreted to be the unresolved C_1-C_1^* combination, it is convenient to rewrite eq. (2.4) as

$$(C_1 + 2C_1^*)/3 = C_o + (2/3)(\Delta - N). \qquad (2.5)$$

The left hand side is a weighted average of the C_1 and C_1^* masses, which can be roughly approximated by the value 2500 MeV for the unresolved peak. The left hand side is 2456 MeV, which is in reasonable agreement. So the spin interactions of the ordinary u and d quarks in charmed hadrons are the same as in nucleons and hyperons.

We see that charm really behaves very much like strangeness, and that we don't understand it either!

TABLE II.1 Experimental Tests of Crazy Mass Formula
a) Strangeness Splittings

	$K-\pi$	$\overset{Q}{=}$	$K^*-\rho$	$\overset{L}{=}$	$\Sigma^*-\Delta$	$\overset{L,Q}{=}$	$\Xi-\Sigma$
ΔM(GeV)	0.35 GeV		0.12		0.15		0.12
ΔM^2(GeV)^2	0.22		0.20				

b) Charm Splittings

	$D-\pi$	$\overset{Q}{=}$	$D^*-\rho$	$\overset{L}{=}$	$C^*-\Delta$
ΔM(GeV)	1.72		1.23		1.26 (if $M_C*=2.5$)
ΔM^2(GeV)^2	3.3		3.4		

III. ARE THERE EXOTIC HADRONS?

3.1 Naive Exotics and Saturation

A simple-minded quark model suggests that the quark-antiquark interaction is attractive in all states, because bound states are found as mesons with all the quantum numbers allowed for the $q\bar{q}$ system. Then if two positive pions are brought together, there should be a strong attraction between the quark in one pion and the antiquark in the other to produce a doubly charged bound state with I=2 below 300 MeV. Since no such exotic bound state or resonance has been found the naive model fails and some saturation mechanism is needed to explain the absence of naive exotics around the dipion mass.

The presently accepted colored quark model with forces from exchange of an octet of colored gluons provides a saturation mechanism in which the $q\bar{q}$ and 3q states behave like neutral atoms.[9] Different parts of the bound state wave function attract and repel an external particle and the net force exactly cancels. Thus theory and experiment now agree on the absence of naive exotics. But the possibility exists of higher exotics. Molecular-type exotics in which attraction results from spatial polarization of one hadron by another have been considered, but crude calculations indicate that the force is insufficient to produce binding.[9,10] Rosner[11] has postulated the existence of exotics from the point of view of finite energy sum rules and duality. This approach has been carried further by other theorists and experiments have been suggested in a search for exotics by baryon exchange processes. There may be some evidence for such states at the conference.[12]

So far there is no evidence for exotic mesons with masses below 2 GeV. This has been taken as evidence against the $qq\bar{q}\bar{q}$ configuration for low-lying states. Although $qq\bar{q}\bar{q}$ states without exotic quantum numbers also exist, these were not taken seriously as possible configurations for the known states, because there was no good theoretical reason why such states should be present and their exotic partners should be absent. But now there seems to be evidence that the low-lying 0^{++} nonet is indeed such a $qq\bar{q}\bar{q}$ state,[4] and there are new convincing theoretical reasons why only states with nonexotic quantum numbers are seen.[13]

3.2 Color-Spin (Magnetic) Exotics and the Flavor Antisymmetry Principle

Recently Jaffe[13] has suggested the existence of exotics bound by the "magnetic-type" spin dependent forces arising naturally in the colored-quark-gluon (QCD) models. The prediction rests on much more general grounds than the specific M.I.T. bag model used in Jaffe's original derivation. The essential physical input is that the N-Δ mass difference is much larger than the binding energy of the deuteron:

$$M_\Delta - M_N \quad >> \quad M_n + M_p - M_d \tag{3.1}$$

where n, p and d denote neutron, proton and deuteron, not quarks, and this equation shows that there are problems of ambiguities in both the pnλ and uds notations.

The physics of eq. (3.1) is that the dominant spin-independent (color charge) forces which bind quarks into hadrons saturate at the $q\bar{q}$ and 3q states and the residual forces between color singlet hadrons is only of the order of 2 MeV like the deuteron binding energy. However, the spin dependent force responsible for the mass difference between the N and Δ is very much larger, of order 300 MeV. Thus if two hadrons are brought very close together so that the quarks in one can feel the interactions of the quarks in the other, there is only a very weak force if the wave functions of the individual hadrons are not changed. However, if the spins of the quarks are recoupled to optimize the spin dependent interactions between the quarks in different hadrons, binding energies of the order of 300 MeV are available and could give rise to bound exotics. In the quark-antiquark system, the ρ-π mass splitting shows that 600 MeV is gained by changing the spins from S=1 to S=0.

Jaffe has simply used the N-Δ and ρ-π mass splittings as input for the strength of the spin dependent interaction and calculated its effect in binding exotic configurations. Only one further ingredient is needed, the color dependence of the interaction. In color singlet $q\bar{q}$ and 3q systems, every $q\bar{q}$ pair is in a color singlet state and every qq pair is in the antisymmetric color triplet state. Exotic configurations, even if they are overall color singlets, can have some $q\bar{q}$ pairs in the color octet state and some qq pairs in the symmetric sextet state. The interactions in these states are not obtainable from observed masses, and are obtained by assuming that the color dependence of the interaction is that obtained from the spin-dependent part of the one-gluon exchange potential in QCD. Evidence supporting this interaction is the agreement with qualitative features of the low-lying hadron spectrum not obtained in any other way, in particular the sign of the N-Δ and Λ-Σ mass splittings.[14] With this form for the interaction, its contribution to the binding of exotic hadron states is easily calculated by the use of algebraic techniques.

One result of the algebraic derivation is simply expressed as the "flavor-antisymmetry principle." <u>The binding force between two quarks of different flavors in the optimum color and spin state is stronger than the binding force between two quarks of the same flavor</u>. Although the forces are assumed to be flavor-independent, their color and spin dependence appears as a flavor dependence because of the generalized Pauli principle. For maximum binding the state should be overall symmetric in color and spin together. Thus if the quarks are in the same orbit, and therefore symmetric in space, they must be flavor antisymmetric.

The flavor antisymmetry principle immediately leads to two very interesting qualitative predictions:

1. The lowest lying four quark states will not have exotic quantum numbers.[13]
2. The lowest lying four quark states which have both charm and strangeness[15] include exotics.

These predictions are simply derived by noting that a four body system must have two bodies with the same flavor if there are only three flavors. Since the flavor-antisymmetry principle requires the flavors of the quark pair and of the antiquark pair to be different in the lowest states, the two bodies with the same flavor must be a quark-antiquark pair. The flavor quantum numbers of this pair cancel one another and the quantum numbers of the system are those of the remaining pair and therefore not exotic. Prediction 1 gives a natural explanation for the absence of low-lying states with exotic quantum numbers, while allowing low-lying four-quark states with nonexotic quantum numbers. Jaffe has called such states "crypto-exotic". Prediction 2 follows from the observation that the flavor antisymmetry principle is easily satisfied with exotic quantum numbers when there are four flavors. Thus exotic states with both charm and strangeness may be found in the same mass range as the lowest F and F* mesons with both charm and strangeness.

We now examine some experimental implications of these two predictions.

3.3 Low-lying "Crypto-Exotics"

Experimental evidence seems to indicate that the lowest lying 0^{++} mesons are not the quark-antiquark p-states, as formerly believed, but are indeed four quark states, while the quark-antiquark 0^{++} states are up at higher mass together with the other p-wave excitations like the f^{0} and the A2 tensor mesons.[4] It is significant that the lowest states predicted by the colored-gluon exchange model form precisely a nonet of 0^{++} states without exotic quantum numbers. Further experiments will tell whether these states are indeed four-quark states and will establish the existence of higher states.

The four-quark states constructed with flavor antisymmetry have very different properties from the quark-antiquark states with the same quantum numbers. An isovector non-exotic, for example, is required to have the quark constitution like $(us\bar{d}\bar{s})$; it must have a strange quark-antiquark pair to avoid having two quarks or two antiquarks of the same flavor. Thus isovector four quark states will decay dominantly into modes containing strange quarks, $K\bar{K}$, $\pi\phi$, etc. This is very different from the decays of conventional quark-antiquark isovector states, like the A2, which decay into nonstrange channels like $\rho\pi$ without any inhibition. This property is masked in the 0^{++} isovector, the δ, because it is below the $\rho\pi$ and the $K\bar{K}$ thresholds and its dominant decay mode $\eta\pi$ is ambiguous because of mixing in the η of both strange and nonstrange components. But striking features in decay rates should be seen in the first four-quark isovector state which is above the $\rho\pi$ and the KK thresholds. An unusual decay pattern is seen for the tensor meson T_{π}^{s} with the quantum numbers of the A2 but which does not decay into $\rho\pi$ but rather into $K\bar{K}$, $K\bar{K}^{*}$ and $\eta\pi$ and for the axial vector meson A_{π}^{s} with the quantum numbers of the B, but with the $\phi\pi$ decay dominant and $\omega\pi$ forbidden. The $\phi\pi$ decay mode is particularly interesting, since it is forbidden for all normal quark-antiquark mesons by the OZI rule,[16] while perfectly allowed for four quark states. Thus a search for $\phi\pi$ resonances might be an interesting way to find four quark mesons.

3.4 Charm-Strange Exotics.

The four-quark states with four different flavors and the same color-spin couplings as the low-lying 0^{++} nonet constitute a set of charmed-strange scalar mesons which are expected to lie in the same mass range as the two-quark charmed-strange F mesons. These include exotic states whose quantum numbers differ from those of the F by having either the wrong sign of strangeness or the wrong isospin. The two types of states are denoted by F_I(udcs, etc. - wrong isospin) and F_S (udcs, etc. - wrong sign of strangeness). The "crypto-exotic" (uucs) four-quark state with the same quantum numbers as the F is denoted by F_x. The F_I can be considered as an Fπ or DK resonance or bound state, the F_S as a DK resonance or bound state, and the F_x as an excited F coupled to the DK channel. One way to see the relation of these exotics to the low-lying 0^+ nonet is to note that changing a charmed quark to a strange quark in the F_I and F_x gives a state in the 0^+ nonet, while F_S has no such charm-strange analog state. Rough estimates of their masses are near the DK threshold. If the F_s and F_x are below the DK threshold, as appears likely, they would be stable against strong decays and decay only weakly or electromagnetically.

Table III.1 lists these states with their quark structure, quantum numbers, dominant strongly coupled channels, possible weak and EM decay modes and their "charm-strange analog" states in the light quark spectrum, obtained by changing the charmed quark to a strange quark.

One way to see how spin-dependent forces can bind a charmed-strange exotic is by examining such an exotic configuration created by bringing together a D^+ and a K^- meson. The spin-dependent force between the $\bar{d}$ antiquark in the D^+ and the s quark in the K^- can be made stronger by recoupling the spins. In the D^+-K^- system these two spins are completely uncorrelated since both the D^+ and K^- have spin 0 and are spherically symmetric. Thus the $\bar{d}$ s system is a statistical mixture of triplet and singlet spin states, 75% S=1 like the K^* and 25% S=0 like the K. Modifying the wave function to give the spin coupling of the $\bar{d}$s system a larger S = 0 component produces additional binding on the mass scale of the 400 MeV K-K^* mass difference. A wave function 75% S = 0 and 25% S = 1 instead of vice versa would gain 200 MeV in binding. Since such recoupling of the $\bar{d}$ and s quark spins changes the spin couplings of each of these with the other quarks, the lowest configuration must minimize the total spin interaction energy of all pairs. The dependence of the interaction on color couplings must also be considered, and is treated by the use of the SU(6) color-spin algebra introduced by Jaffe.

The spin-dependent interactions of the charmed quark are much smaller than those of light quarks, as indicated by the small DD^* mass splitting relative to the $\rho\pi$ and KK^* splittings. This is also expected in QCD models, where the "magnetic" interaction of a quark is inversely proportional to its mass. This also suggests that the D^+K^- system will be bound, because recoupling the spin of the $\bar{d}$ anti-quark in a D^+ to a more favorable configuration with respect to the spins of the quark and antiquark in the K^- can only lose a small amount in the more unfavorable coupling of the $c\bar{d}$ system. The worst possible coupling can only lose the D-D^* mass difference.

Exact mass predictions for charm-strange exotics are difficult because of uncertainties in the model. Rough estimates are obtained by use of the charm-strange analogy, in which mass relations for systems involving strange quarks are assumed to hold when one strange quark is replaced by a charmed quark in each state. Examples of the success of this analogy are eqs. (2.1-2.6). While the theoretical basis of these mass relations is still not understood, in particular why linear masses work in some cases and quadratic masses work in others, the observation that whatever works for strange quarks also works for charmed quarks suggests that the analogy may be used to extrapolate relations from the systems with two strange quarks to systems with one strange quark and one charmed quark. We assume that the $\delta(970)$ is a four-quark exotic 0^+ state with the configuration $(q\bar{q}s\bar{s})$, where q denotes u or d light quarks, and note the inequalities

$$M(\eta) + M(\pi) < M(\delta) < 2M(K). \tag{3.2a}$$

Changing one strange quark to a charmed quark everywhere gives

$$M(F) + M(\pi) \overset{?}{<} M(\tilde{F}_p q\bar{q}c\bar{s}) < M(K) + M(D) \tag{3.2b}$$

where the question mark expresses the uncertainty due to mixing in the η, which is not a pure $s\bar{s}$ state and therefore not strictly the charm-strange analog of the F. Thus the statement that the δ is below the $K\bar{K}$ threshold and decays to $\eta\pi$ leads to the analog that the F should be below the DK threshold and might decay to the $F\pi$, but it might also be below this threshold.

We now consider the most interesting possibilities for decay modes and signatures for the different mass ranges: Note that the $F\pi$ decay is forbidden by isospin for strong decays of the F_x, the $F2\pi$ decay is forbidden by angular momentum and parity for all strong and electromagnetic decays and the $F3\pi$ channel is probably well above DK threshold.

1. <u>All States Above the DK Threshold:</u> Strong decays would be recognized as resonances in mass plots of the DK, $D\bar{K}$, $\bar{D}K$ and $\bar{D}\bar{K}$ systems. Decays in the $F\pi$ and $F3\pi$ mode would also be allowed for the F_I. Particularly striking signatures would be the double-strangeness decay modes

$$\tilde{F}_S \to D^+K^- \to K^-K^-\pi^+\pi^+ \tag{3.3a}$$

$$\overline{\tilde{F}}_S \to D^-K^+ \to K^+K^+\pi^-\pi^- \tag{3.3b}$$

2. States below the DK Threshold but Above Fπ. The F_I would still decay strongly to final states containing an F, but the F would decay electromagnetically and the F_S weakly. The $F_x \to F$ decay is a second order $0^+ \to 0^-$ transition with the emission of either two photons or no photon,

$$F_x^{\pm} \to F^{\pm} + 2\gamma. \tag{3.4a}$$

$$F_x^{\pm} \to F^{\pm} + \pi^o. \tag{3.4b}$$

There is also the first order radiative decay

$$F_x^{\pm} \to F^{\pm}\ \pi^o\gamma. \tag{3.4c}$$

An intermediate $F^*\gamma$ state could be present in the decay (3.4a).

Another possible decay for the F_x is into the $\tilde{F}_I$, if it is above the F_I. There would then be the cascade decay,

$$F_x^{\pm} \to \tilde{F}_I^{\pm} + (2\gamma \text{ or } e^+e^-) \to F^{\pm} + \pi^o + (2\gamma \text{ or } e^+e^-). \tag{3.4d}$$

In this $0^+ \to 0^+$ transition, the e^+e^- decay can go via a single photon and be of second order in α like the 2γ decay.

The Cabibbo favored weak decays of the $\tilde{F}_S$ would be to states of strangeness −2. States with two charged kaons would provide the best signature for identifying these states, since neutral kaons lose the memory of their strangeness by decaying in the K_L and K_S modes,

$$\tilde{F}_S \to K^-K^-\pi^+ + (\text{leptons and/or pions})^+ \tag{3.5a}$$

$$\overline{\tilde{F}}_S \to K^+K^+\pi^- + (\text{leptons and/or pions})^- \tag{3.5b}$$

Strong K^* signals might be expected in the $K^{\pm}\pi^{\mp}$ combinations, and there should be no D present in the final state. Decays to the four-body final states $KK\pi\pi$ might be the best signature, analogous to the decays (3.3) but without the intermediate DK state and with the possibility of one or two K^*'s. Another possible signature is in the two-body neutral decays,

$$\tilde{F}_S \to \overline{K^0K^0} \to K_SK_S \tag{3.6a}$$

$$\overline{\tilde{F}}_S \to K^oK^o \to K_SK_S. \tag{3.6b}$$

Although the final states have lost the memory of the double strangeness, the dominant CP-conserving decay remembers that the initial state is even under $\widetilde{CP}$. If this decay mode of the $\tilde{F}_S$ is important, it can lead to $\tilde{F}_S - \bar{\tilde{F}}_S$ mixing, as in the neutral kaon system.

3. States Below the DK and Fπ Thresholds but Above the F. The $\tilde{F}_I^{\pm}$ would decay electromagnetically by two photon emission

$$\tilde{F}_I^{\pm} \rightarrow F^{\pm} + 2\gamma. \tag{3.7}$$

The other charge states of the $\tilde{F}_I$ would decay weakly. The multibody decays would resemble the expected decays of the F with an extra pion, and the Cabibbo favored decays would be into states of zero strangeness. In addition there would be the two-body decays

$$\tilde{F}_I^{\pm\pm} \rightarrow \pi^{\pm}\pi^{\pm} \tag{3.8a}$$

$$\tilde{F}_I^0, \bar{\tilde{F}}_I^0 \rightarrow \pi^+\pi^-, \pi^o\pi^o, K^+K^-, K_SK_S. \tag{3.8b}$$

The exotic double-charge signature for the decay (3.3a) might be a useful indicator for this state.

The two-pseudoscalar decay modes (3.8b) are all states of even CP for a J = 0 final state. The two states $\tilde{F}^o$ and $\bar{\tilde{F}}_I^o$ can be expected to mix like the neutral kaons. If CP violation is neglected then the eigenstates will be CP eigenstates and the even state will have the decay modes (3.8b) while the odd CP state will not decay into two pseudoscalars and will decay to three or more in the nonleptonic modes and into semi-leptonic decay modes. Note that this $\tilde{F}_I^o - \bar{\tilde{F}}_I^o$ mixing will be much stronger than K^o-$\bar{K}^o$ mixing in a gauge theory, because it can go via exchange of two intermediate W bosons, with all vertices Cabibbo favored and no cancellation of the GIM type.

4. States Below the F. This is highly improbable, but if the $\tilde{F}$ is below the F, the F would now decay into the $\tilde{F}_I$ and the roles of the F and $\tilde{F}_I$ would be reversed.

Note that if the F-$\tilde{F}_I$ mass difference is less than the pion mass in either direction there will be a particle whose dominant decay mode is electromagnetic with the emission of a low mass photon pair or electron pair. The mass spectrum of the pair will be continuous, but its maximum must be less than the pion mass.

TABLE III.1 PROPERTIES OF CHARMED-STRANGE FOUR-QUARK MESONS

State	Quark Structure	(I,S,C)	Resonance or Bound State of	Possible Weak or EM Decays	CS Analog
$\tilde{F}_S^{\,o}$	$cs\overline{ud}$	(0,-1,+1)	D^+K^-	$K^-K^-\pi^+\pi^+$,K_SK_S	None
$\bar{\tilde{F}}_S^{\,o}$	$\overline{cs}ud$	(0,+1,-1)	D^-K^+	$K^+K^+\pi^-\pi^-$,K_SK_S	None
$\tilde{F}_I^{\,\pm\pm}$	$cu\overline{sd}$ and c.c	(1,±1,±1)	$D^\pm K^\pm$,$F^\pm\pi^\pm$	$\pi^\pm\pi^\pm$	$\delta^\pm$
$\tilde{F}_I^{\,o}$	$(cd\overline{su})$	(1,+1,+1)	D^oK^o,$F^+\pi^-$	$\pi^+\pi^-$,K^+K^-,K_SK_S	δ^-
$\bar{\tilde{F}}_I^{\,o}$	$(\overline{cd}su)$	(1,-1,-1)	$\overline{D^oK^o}$,$F^-\pi^+$	$\pi^+\pi^-\pi^o$,$K^+K^-\pi^o$	δ^+
$\tilde{F}_I^{\,\pm}$	$c\bar{s}(q\bar{q})_{I=1}$ and c.c.	(1,±1,±1)	$D^\pm K_S$,$F^\pm\pi^o$	$F^\pm\gamma\gamma$	δ^o
$F_x^{\,\pm}$	$c\bar{s}(q\bar{q})_{I=0}$ and c.c.	(0,±1,±1)	$D^\pm K_S$	$F^\pm\gamma\gamma$,$F^\pm\pi$	S^*

IV. THE STRANGE AXIAL VECTOR MESONS (Q'S)

Leith has reported on the experimental situation of the Q mesons,[4] with the rather surprising result that one state seems to be decoupled from the $K^*\pi$ decay mode while the other is decoupled from the ρK decay mode.[17] A mixing of the SU(3) eigenstates with a mixing angle of 45° could produce such a decoupling, and arises naturally in a dynamical model considered ten years ago and forgotten. We review the essential features here.[5]

We denote the strange members of the A_1 and B octets by Q_A and Q_B respectively. The dominant decay modes $K^*\pi$ and ρK are allowed for both Q_A and Q_B states. In the limit of SU(3) symmetry, conserved "parities" G_u and G_v analogous to G parity can be defined by replacing isospin by U spin or V spin in the definition of G parity. The neutral and charged Q's are eigenstates of G_u and G_v respectively. However, the charged ρ and π mesons are not eigenstates of either of these parities, just as the K mesons are not eigenstates of G parity. Thus there is no selection rule forbidding $K^*\pi$ and ρK final states for either of these decays. If the Q_A and Q_B are produced coherently in some experiment, they contribute coherently to the ρK and $K^*\pi$ final states.

If SU(3) is broken, G_u and G_v parities are not conserved. There can then be mixing, analogous to $\omega\phi$ mixing, between the Q_A and Q_B states, even though G parity remains conserved and prevents mixing of the corresponding non-strange states. However, there is no ideal mixing angle determined by quark masses, as in the $\omega\phi$ case, because the Q_A and Q_B have the same quark constituents and are not mixed by a mass term. Some other SU(3) breaking mechanism is needed to produce the observed mixing. One possibility is discussed below.

Consider the decay of the mixed states

$$|Q_1\rangle = \cos\theta|Q_A\rangle + \sin\theta|Q_B\rangle \tag{4.1a}$$

$$|Q_2\rangle = -\sin\theta|Q_A\rangle + \cos\theta|Q_B\rangle \ , \tag{4.1b}$$

where θ is the mixing angle.

For the $K^*\pi$ and ρK decay modes the branching ratio is unity in the SU(3) limit except for differences in kinematic (phase space) factors for the two final states. However, because the two octets have opposite charge conjugation behavior, the A_1-octet decay is described with D-coupling and the B-octet decay with F-coupling. The relative phases of the $K\rho$ and $K^*\pi$ decay amplitudes are thus opposite for the two cases

$$\langle K\rho|Q_A\rangle = \langle K^*\pi|Q_A\rangle \tag{4.2a}$$

$$\langle K\rho|Q_B\rangle = -\langle K^*\pi|Q_A\rangle \ . \tag{4.2b}$$

the decay amplitudes for the mixed states (4.1) are then

$$\langle K^*\pi|Q_1\rangle = \cos\theta\langle K^*\pi|Q_A\rangle + \sin\theta\langle K^*\pi|Q_B\rangle \tag{4.3a}$$

$$\langle K\rho|Q_1\rangle = \cos\theta\ \langle K^*\pi|Q_A\rangle - \sin\theta\langle K^*\pi|Q_B\rangle \tag{4.3b}$$

$$\langle K^*\pi|Q_2\rangle = -\sin\theta\ \langle K^*\pi|Q_A\rangle + \cos\theta\langle K^*\pi|Q_B\rangle \tag{4.3c}$$

$$\langle K\rho|Q_2\rangle = -\sin\theta\ \langle K^*\pi|Q_A\rangle - \cos\theta\langle K^*\pi|Q_B\rangle \ . \tag{4.3d}$$

Eqs. (4.3) show that for any mixing with a real phase, the effect for one eigenstate is to enhance the $K^*\pi$ decay mode and suppress the $K\rho$, and vice versa for the orthogonal eigenstate. For $\theta = 45°$ one eigenstate is completely decoupled from $K^*\pi$ and the other from $K\rho$ if the amplitudes $\langle K^*\pi|Q_A\rangle$ and $\langle K^*\pi|Q_B\rangle$ are equal.

A dynamical mechanism which naturally leads to this mixing is the SU(3) breaking in decay channels originally introduced to explain $\omega\phi$ mixing before SU(6) and the quark model. The states Q_A and Q_B are coupled to one another via their decay channels $K^*\pi$ and $K\rho$.

$$|Q_A\rangle \leftrightarrow |K^*\pi\rangle \leftrightarrow |Q_B\rangle \tag{4.4a}$$

$$|Q_A\rangle \leftrightarrow |K\rho\rangle \leftrightarrow |Q_B\rangle \tag{4.4b}$$

In the SU(3) symmetry limit, the two transitions (4.4a) and (4.4b) exactly cancel one another and produce no mixing. This cancellation no longer occurs when SU(3) breaking introduces kinematic factors arising from the mass difference between the two intermediate states. These suppress the strength of the transition (4.4b) via the higher mass $K\rho$ intermediate state relative to the transition (4.4a) via $K^*\pi$.

The simple analysis of the transitions (4.4a) and (4.4b) gives 45° mixing for the eigenstates if $\langle K^*\pi|Q_A\rangle = \langle K^*\pi|Q_B\rangle$. This decouples the two states from $K^*\pi$ and $K\rho$ respectively. However, a more careful analysis shows that two partial waves are present in the decay, s-wave and d-wave, and the result is very sensitive to the relative amplitudes and phases of the s and d waves. In particular, for the ratio of s to d wave amplitudes predicted by the naive $SU(6)_W$ quark model, the transitions (4.4) vanish and cannot produce mixing, because the Q_A is coupled only to vector meson states with transverse polarization and the Q_B is coupled only to longitudinally polarized states. For this reason the mechanism (4.4) for mixing was dropped.

Now that the $SU(6)_W$ predictions are known not to agree with experiment,[18] particularly in the closely related polarization predictions for B and A_1 decays, and the experimental data are consistent with pure s-wave for the Q decays, the mixing mechanism (4.4) should perhaps again be considered. However, a more realistic calculation would consider the coupled channels $K^*\pi$ and $K\rho$ through the resonance region, with phase space factors changing within the resonances because of the proximity to threshold.

V. NEW FEATURES OF HIGH MASS SPECTROSCOPY

As both the spectroscopy of old and new mesons moves into the mass range of several GeV, methods for discovering and verifying the existence of new states are changed very much by the large number of open decay channels. The signature for the detection of a resonance is a particular decay mode and gives a signal proportional to the branching ratio which is only 0.1%-1%. The signal to noise is thus crucial in high mass spectroscopy.

It is useful to define a figure of merit $F(P,T)$ for the production of particle P, by observing a characteristic T of the final state which may either be used as a trigger or as a signature for picking out events. The trigger T may be either the full final state like the electron pair in the decay of the J, or one of the particles produced inclusively in the decay such as a single muon. The figure of merit is defined by the relation

$$F(P,T) = \sigma(P+X) \cdot BR(T)/\sigma(T+X) \tag{5.1}$$

where $\sigma(P+X)$ and $\sigma(T+X)$denote the cross sections for inclusive production of the particle P and the trigger T in the reaction under consideration and BR(T) denotes the branching ratio for the appearance for the trigger T in the decay of the particle P.

Examination of Eq. (5.1) shows that the optimization of the figure of merit may best be achieved by finding a trigger T with low inclusive production. The characteristics of the signal appearing in the numerator will not be changed very much by choosing a different trigger or a different production mechanism. However, the denominator may be reduced by a large factor by choosing a trigger for which the background is low. Possibilities for improving F(P,T) by reducing the noise seem to be more favorable than by enhancing the signal. We examine three possible approaches to noise reduction.

1) Production of a low noise signal. The signal can be produced by a mechanism which naturally has a low background, as in the production of the Ψ as a very narrow resonance in e^+e^- annihilation.

2) A low noise signal signature. An exclusive decay channel can be found which has a low production background as in the detection of the J particle by its leptonic decay mode. The particular case of ϕ signatures is of interest.

3) Use of background signature. Since many partial waves in the background can appear at the high mass available and only a few

in the signal, the background may have a characteristic structure which enables cuts in selected kinematic regions of the multiparticle phase space to reduce the noise by a large factor.

5.1 Production of Low Noise Signal

The production of a new particle with a very low background is possible for a narrow s-channel resonance whose cross section is very much enhanced over the background in a narrow energy region. This approach can be used only for the production of resonances having the quantum numbers available in the initial state. It is particularily suitable for the production of vector meson resonances in electron-positron annihilation.

For states which do not have the quantum numbers of the photon or of the meson-baryon, nucleon-nucleon or nucleon-antinucleon system, some possibilities exist for production via the decays of states which do have these quantum numbers; e.g. in the production of the positive parity charmonium states by radiative decay of the ψ' and the production of charmed particle pairs by the decays of higher vector resonances.

For states not easily produced in this way and available only in inclusive production there is no simple mechanism for reducing the multiparticle background by choice of a particular production mechanism. This applies to most cases of hadronic resonance production, as in J production where no one production mechanism seems to be superior by any large factor.

5.2 Low Noise Triggers and ϕ signature spectroscopy

The triggers which have low inclusive production cross section in normal hadronic processes include photons and leptons produced by electromagnetic interactions. These are suppressed by powers of α relative to hadron production. Some examples are the lepton pairs used as the signature for the discovery of the J particle, the photons used as a signature to discover even parity charmonium states produced by the decay of the ψ' and the two-photon and multiphoton channels used for the possible detection of the pseudoscalar mesons.

In addition to these electromagnetic triggers which have already been used successfully, particles like the ϕ and f' which are suppressed by the OZI rule[19] in nonstrange hadron reactions might be used successfully. These appear as signatures for states whose branching ratios into decay channels involving ϕ and f' are not suppressed by significant factors over other decays. ϕ signature spectroscopy looks attractive for states decaying into a ϕ because inclusive ϕ production without kaons is forbidden for nucleon-nucleon and pion-nucleon reactions and the background should be small. Typical suppression factors observed experimentally for ϕ production are a factor of 500 below ω production in pion-nucleon reactions[20] at 6 GeV/c or a factor of 100 below pion production at Fermilab energies.[21] The ϕ is easily detected in the $K^+ K^-$ decay mode at high energies because the Q of the decay is so low that both kaons will pass together in the same arm of a spectrometer and will not trigger a Cerekov detector set for pions.[17] An even smaller background would be expected in $\phi\phi$ spectroscopy for states expected to decay into two ϕ's. Examples of such states are isoscalar bosons even under charge conjugation which have the structure of a quark-antiquark pair, either strange, charmed, or some new heavy quark.

"Strangeonium" states of a strange quark-antiquark pair are allowed by the OZI rule to decay into $\phi\phi$ and should have a comparatively strong branching ratio. Such strangeonium states are of general interest since no such states above the ϕ or f' are well known. Our present knowledge of charmonium spectroscopy is at present much better than strangeonium because the low noise electromagnetic signature of lepton pairs and photons enables charmonium to be seen much more easily. Even if $\phi\phi$ spectroscopy does not lead to the discovery of any new charmonium or "x-onium" states made from heavy quarks of type x, the development of strangeonium spectroscopy would add to our understanding of hadron dynamics.

The ϕ decay of charmonium or x-onium is singly forbidden by OZI or other quark line rules and is therefore on the same footing as all other hadronic decays which are also at least singly forbidden. Estimates of the $\phi\phi$ branching ratios for these particles are of the order of 0.1%, which is probably only a small factor below the $\rho\rho$ branching ratio. The $\phi\phi$ background should be very much lower than the $\rho\rho$ background and therefore can provide a fruitful trigger for such states.* The most interesting of such states at present are the pseudoscalar states of charmonium or of the new heavier quarks if they are there.

Single ϕ spectroscopy would be useful also in observing decays of higher strange resonances such as K^*, Λ^*, Σ^*, Ξ^*, * and ϕ^* which could decay into lower resonances with the same quantum numbers by ϕ emission above the threshold. Nonstrange baryon resonances at high masses have been observed by the technique of pion-nucleon phase shift analysis. ϕ spectroscopy may enable the discovery of corresponding resonances with different quantum numbers not accessible to phase shift analysis.

States like the $F^{\pm}$ meson containing both charm and strangeness might be observed by the decay into a ϕ and a pion or lepton pair. The $\phi\pi$ decay mode might also be useful in the search for the exotic four-quark states discussed in section III.

The $\phi\pi$ decay mode is particularly interesting in searches for new objects, because $\phi\pi$ decay is forbidden by the OZI rule for <u>any</u>

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*One estimate for $(x\bar{x})_{c=+} \to \phi\phi$, is based on the analogy with $\psi \to \phi\eta$ which also involves annihilation of a heavy quark pair and creation of two strange quark pairs. Another is based on the analogy $\psi \to \rho\pi$ and $(x\bar{x}) \to \rho\rho$ and used SU(3) to relate $\phi\phi$ to $\rho\rho$.

boson constructed from a quark-antiquark pair. Thus resonances in the $\phi\pi$ system indicate either a new object like a four-quark system, an OZI-violating strong decay of a conventional boson, or a weak decay into a system containing a strange quark-antiquark pair.

A partial list of states which might be detected by ϕ signature spectroscopy are

Single ϕ spectroscopy:

$$K^* \rightarrow K + \phi \tag{5.2a}$$

$$\Lambda^* \rightarrow \Lambda + \phi \tag{5.2b}$$

$$\Sigma^* \rightarrow \Sigma + \phi \tag{5.2c}$$

$$\Xi^* \rightarrow \Xi + \phi \tag{5.2d}$$

$$\Omega^* \rightarrow \Omega^- + \phi \tag{5.2e}$$

$$\phi^* \rightarrow K + \bar{K} + \phi \tag{5.2f}$$

$$F^\pm \rightarrow \pi^\pm + \phi \tag{5.2g}$$

$$F^\pm \rightarrow \text{leptons} + \phi \tag{5.2h}$$

$$\tilde{F}^o_I \rightarrow \pi^o + \phi \tag{5.2i}$$

$$A^s_\pi \rightarrow \pi^o + \phi \tag{5.2j}$$

ϕ-ϕ spectroscopy

$$\eta_c \rightarrow \phi + \phi \tag{5.3a}$$

$$\text{charmonium } (c\bar{c})_{C=+} \rightarrow \phi + \phi \tag{5.3b}$$

$$\text{strangeonium } (s\bar{s})_{C=+} \rightarrow \phi + \phi \tag{5.3c}$$

$$\text{x-onium } (x\bar{x}, \text{ where x is a new heavy quark})_{C=+} \rightarrow \phi + \phi \tag{5.3d}$$

Above 3 GeV the possibility of observing 3ϕ decay arises. Vector meson states like the ψ' and other higher members of the ψ family can decay into three vector mesons. The dominant 3V final state would be $\omega\rho\rho$ but 3ϕ would be of the same order of magnitude in the SU(3) symmetry limit. The 3ϕ state would have a unique signature and a very low background.

The use of ϕ triggers can thus lead to various kinds of interesting physics. The first step is the understanding of ϕ production itself, by examining the other particles produced along with the ϕ and looking for ϕx resonances. Understanding the mechanisms for ϕ production can provide insight into models for particle production, even if no new phenomena or resonances are found. But chances are that some part of the production will be due to decays of higher resonances, and at this stage any resonance with a ϕ-decay mode is interesting.

5.3 Background signatures

The signal to noise ratio can be improved by the alternative approach of characterizing peculiar signatures for the background in order to enable its removal from the signal. This approach is based on the fundamental difference between the spectroscopies of the high mass resonances and old low-lying resonances. The conventional low-lying resonances show up as peaks in cross sections with particular decay angular distributions against a comparatively smooth and structureless background. At high mass the background may have a more striking and easily identified structure than the signal.

High mass resonances are states of low angular momentum decaying primarily into multi-particle channels. Their decays reflect the low angular momentum by containing very few parital waves all having relatively low angular momentum. The background on the other hand can have very large angular momenta and a sharp structure in momentum and angular distributions are present in the signal. A small portion of the multi-particle phase space could include a very large portion of background events. In this case the signal to noise ratio would be improved by a cut excluding this small volume of phase space. The exact kind of cut to be effective depends on the individual case and could be most easily decided by examining the background and looking for its most striking features.

Consider for example the search for a new particle in a particular four-particle decay channel by looking for peaks in the mass spectrum, e.g. looking for a charmed baryon decaying into $\Lambda 3\pi$. The problem is how to use the angular distributions of these four particlesin the center-of-mass system of the four particle cluster (hopefully the rest system of the new particle) as a means of distinguishing between signal and background. Three axes are relevant for examining the angular distributions, (1) the direction of the incident beam momentum, (2) the direction of the momentum of the four-particle clusters, and (3) the normal to the production plane. Signatures which characterize the new particle appear most clearly in angular distributions with respect to the direction of the momentum of the four-particle cluster or with respect to the production plane. But signatures for the noise will show up in angular distributions with respect to the incident beam direction.

Background from uncorrelated particles whose mass happen accidently to fall in the desired range should have angular distributions with respect to the incident beam direction similar to those for single-particle inclusive productions. They should be peaked in the forward and backward directions with a rapidly falling cutoff in transverse momentum. Background events could show forward-backward asymmetry or a tendency to be concentrated in cones forward and backward relative to the direction of the incident beam. The signal from decay of a D meson of spin zero should show a completely isotropic angular distribution with respect to any axis. Particles of non-zero spin might have some anisotropy in their angular distributions if they are polarized in production. But these will involve only low order spherical harmonics and will not concentrate large numbers of events in a small region of phase space. Thus a cut eliminating events in which one or more particles appear within a narrow cone forward and/or backward with respect to the incident beam direction could reduce the background considerably with a neglible effect upon any signal coming from the decay of a low angular momentum state.

As an example consider a four particle decay into a baryon and three pions of a state produced by a high energy accelerator beam hitting a fixed target. This state appears as a four particle cluster with a low mass in the several GeV region but with total laboratory momentum in the 100 GeV range. In the center-of-mass system of the cluster the momenta of the baryon and of the pions are all small and of the same order of magnitude. In the laboratory the baryon has a much larger momentum than the pions because of the effect of the mass on the Lorentz transformation. If the baryon is not a proton and cannot be a leading particle the inclusive momentum distribution for the baryon and the pions can be expected to be very different in the relevant ranges. In particular the momentum distribution for high momentum hperon or anti-hyperons could be falling rapidly in this region while the momentum distribution for relatively low momentum pions could be rising. This would appear in the center-of-mass system for the multi-particle cluster as baryons being preferentially emitted backward and pions preferentially emitted forward. Cutting out events in which <u>all</u> pions are in the forward hemisphere would thus appreciably reduce the background, but would only remove one eigth of the signal. Using a cone instead of a hemisphere would interferere even less with the signal and still substantially reduce the background.

VI. MIXING OF PSEUDOSCALAR MESONS

The conventional mixing description seems to be in both experimental and theoretical trouble for the pseudoscalar mesons. The η and η' do not behave like orthogonal mixtures of a single SU(3) singlet and a single SU(3) octet. More complicated mixing is indicated perhaps requiring inclusion of radially excited states as well as ground state configurations.[22,23]

The use of the quark model to determine mixing angles of neutral mesons from experimental data on neutral meson production processes was first suggested by G. Alexander.[24] This work, based on the Levin-Frankfurt additive quark model[25,26] in which every hadron transition is assumed to involve only one active quark with all remaining quarks behaving as spectators, presented a number of predictions which have since been shown to be in very good agreement with experiment. These include the first derivation of the

A...Z rule for four point functions, as the prediction that ϕ production is forbidden in πN reactions since the process requires two active quarks in the same hadron. Also obtained were the prediction of no exotic t-channel exchanges and some sum rules and equalities which are listed below. Analysis of a decade of experimental data show a consistent pattern of good agreement with all predicted relations for processes of vector meson production and strong disagreement with relations for processes of pseudoscalar meson production, particularly for relations involving η' production. We suggest that an appropriate conclusion from these results is that the quark model description indeed holds for these processes, but that something is wrong with the pseudoscalars, particularly the η'.

The relevant sum rules are the charge exchange sum rule (CHEX)

$$\sigma(\pi^- p \to \pi^o n) + \sigma(\pi^- p \to \eta n) + \sigma(\pi^- p \to \eta' n)$$
$$= \sigma(K^+ n \to K^o p) + \sigma(K^- p \to \overline{K}^o n) \ , \tag{6.1a}$$

and the strangeness exchange sum rule (SEX)

$$\sigma(K^- p \to \eta Y) + \sigma(K^- p \to \eta' Y) = \sigma(K^- p \to \pi^o Y) + \sigma(\pi^- p \to K^o Y) \tag{6.1b}$$

These sum rules hold for any meson nonet and do not make any assumption about the mixing angle, except for the conventional description of the η and η' as two orthogonal linear combinations of pure SU(3) singlet and octet states defined in terms of a single mixing angle. For the case of ideal mixing, as in the vector mesons the two sum rules each split into two equalities, CHEX becoms

$$\sigma(\pi^- p \to \phi n) = 0 \tag{6.2a}$$

which is just the A...Z rule, and substituting (6.2a) into (6.1a) gives

$$\sigma(\pi^- p \to \rho^o n) + \sigma(\pi^- p \to \omega n) = \sigma(K^+ n \to K^{*o} p) + \sigma(K^- p \to K^{*o} n). \tag{6.2b}$$

With ideal mixing SEX becomes

$$\sigma(K^- p \to \omega Y) = \sigma(K^- p \to \rho^o Y) \ , \tag{6.3a}$$

$$\sigma(K^- p \to \phi Y) = \sigma(\pi^- p \to K^{*o} Y) \quad . \tag{6.3b}$$

The relation (6.3a) is seen also to be a consequence of the A...Z rule for the meson vertex. The incident K^- contains no d for d quarks or antiquarks and therefore produced via the $u\overline{u}$ component which is a linear combination of the two with equal weight.

If there is no mixing, which is a rough approximation for the pseudoscalar mesons, the charge exchange sum rule simplifies to

$$\sigma(\pi^- p \to \pi^o n) + 3\sigma(\pi^- p \to \eta_8 n) = \sigma(K^+ n \to K^o p) + \sigma(K^- p \to \overline{K}^o n) \quad ,(6.4a)$$

$$\sigma(\pi^- p \to \eta_8 n) = 2\sigma(\pi^- p \to \eta_1 n) \quad . \qquad (6.4b)$$

All the vector meson relations (6.2) and (6.3) are in excellent agreement with experiment. However, the pseudoscalar meson relations (6.1) and (6.4b) are in strong disagreement. The relation (6.4a) agrees with experiment if the η is assumed to be pure octet. This suggests that the conventional picture in which there is small mixing may be valid for the η, but that something is wrong with the η', and it is wrong in the direction that the η' has an inert piece in the wave function which does not contribute to the sum rules (6.1) and (6.4).

More recent evidence of trouble in the $\eta - \eta'$ system comes from data presented at this conference on neutral meson production in $K^- p$ reactions at 4.2 GeV/c. The previously observed trouble with the SEX sum rule (6.1b) is confirmed with higher statistics. In addition there may be difficulty with backward production. Okubo[27] has pointed out that conventional mixing predicts that the ratio of η' to η production must be a universal constant in all processes where there are no active strange quarks,

$$\frac{\sigma(A + B \to \eta' + X)}{\sigma(A + B \to \eta \ + X)} = K = \frac{|\langle\eta'|\eta_{ns}\rangle|^2}{|\langle\eta|\eta_{ns}\rangle|^2} \qquad (6.5)$$

where A, B and S do not contain strange quarks and η_{ns} denotes the particular linear combination of η and η' which contains only nonstrange quarks, the analog of the physical ω. Okubo finds a value of K=0.5±0.25 by analysis of a large number of processes. But recent experiments[28] give

$$K = (2.0\pm0.68)/(1.27\pm0.39) \qquad (6.6)$$

for the ratio of η' to η production in the backward direction in $K^- p \to \Lambda\eta$ or $\Lambda\eta'$. If this is a baryon exchange process, the coupling of the η and η' to nonstrange baryons should also go via the η_{ns} component and the processes should satisfy Okubo's universality relation (6.5). The fact that K > 1 for this case whereas K < 1 for all meson exchange processes investigated by Okubo might suggest a difference between meson exchange and baryon exchange, if the discrepancy of less than two standard deviations proves to be statistically significant. This is again consistent with the description of the η as having a piece in the wave function which does not contribute to the meson exchange sum rules because of poor overlap with the wave function of the incident meson. For baryon exchange there is no such overlap integral and the additional piece could have a sizeable contribution, giving a higher η' production cross section relative to the η than in meson exchange reactions.

We suggest that there is indeed an additional pice in the η' wave function and that it is a radially excited configuration. This leads to a re-examination of the standard mixing folklore and the discovery that it is completely unjustified![12] In a formulation which begins with unperturbed singlet and octet states in the SU(3) symmetry limit, there is no reason to assume that SU(3) symmetry breaking should admix only the lowest ground states of the singlet and octet spectra. This may work for the tensor and vector mesons, where the entire nonet seems to be degenerate in the SU(3) symmetry limit and the dominant breaking of nonet symmetry is by a quark mass term. The degeneracy suggests the use of degenerate perturbation theory which diagonalizes the symmetry breaking interaction in the space of the degenerate unperturbed states. The mass term has no radial dependence and would not mix ground state and radially excited wave functions which are orthogonal and would have a zero overlap integral.

For the pseudoscalars where there is a large singlet-octet splitting in the SU(3) symmetry limit there is no reason to use degenerate perturbation theory and mix only ground state wave functions. Furthermore, the singlet-octet splitting can only be produced by an interaction which violates the A..Z rule because it is not diagonal in the quark basis and mixes $s\bar{s}$ with $u\bar{u}$ and $d\bar{d}$. The accepted mechanism for such A..Z violation in the pseudoscalars is annihilation of the quark-antiquark pair into gluons and the creation of another pair. Here there is no reason to restrict the pair creation to the ground state configuration. There is no overlap integral between the two $q\bar{q}$ states, as the intermediate gluon state does not remember which radial configuration it came from. If the annihilation process depends primarily on the value of the $q\bar{q}$ wave function at the origin, then all radially excited configurations couple with equal strength for wave functions from a c confining linear potential.

Thus there is considerable reason to suspect that the trouble with pseudoscalar meson sum rules is in admixture of a radially excited wave function into the η'. One might expect the η to be purer because the SU(3) flavor octet state does not couple to gluons which are singlets and because it is the lowest state, far in mass from the nearest SU(3) singlet radial excitation. The η', on the other hand is sitting in between the ground state and first radially excited octet states and would be expected to mix with both. Note that mixing of the octet ground state and first radially excited octet state by an SU(3)-symmetric potential need not be considered because it is merely a change in the radial wave function. This mixing can be transformed away by choosing a new radial basis (i.e. a slightly different potential) for which the modified ground wave function in the original basis is the exact ground state in the new basis.

VII. MESON SPECTROSCOPY AND THE CHARGE OF THE QUARK

It may be possible to measure the charge of the quark by observing the couplings of neutral mesons to the two-photon channel. It is difficult in normal spectroscopy and deep inelastic electron scattering to measure the color dependence of the quark charge because all observed hadron states are color singlets and are therefore completely symmetric in color space. If the charge of a red quark is different from that of a blue quark with the same flavor, this can be observed only if the red and blue directions in color space are defined. With an apparatus completely symmetric in color space, no preferred direction can be defined and all quantitites measured are averages over all colors.

Models like the Han-Nambu model[29] are constructed to make color averages of all matrix elements of the electromagnetic current exactly equal to these of the colored fractionally charged model. The difference between the currents of the two models has no color singlet component and its color average vanishes. Let us write

$$J_{em} = J_G(8_f, 1_c) + \Delta J, \tag{7.1}$$

where J_G is the electromagntic current in Greenberg's colored quark model[2] with the fractional charges of the Gell-Mann-Zweig quark model, and the arguments $8_f, 1_c$ denote that this current transforms under flavor and color like an octet and singlet respectively. In the Han-Nambu model, ΔJ is a flavor singlet and color octet. Thus

$$J_{HN} = J_G(8_f, 1_c) + \Delta J(1_f, 8_c) \ . \tag{7.1b}$$

The matrix elements of ΔJ thus all vanish between color singlet states, and ΔJ is unobservable in any measurement described by such a matrix element.[26,30]

The electric charges of the quarks have the same structure as the current operators. Thus the charge of a quark of flavor f and color c in the Han-Nambu model is given by

$$Q_{HN}(f,c) = Q_G(f) + \Delta Q(c) \tag{7.1c}$$

where Q_G depends only on flavor and is independent of color, and ΔQ depends only on color. We thus obtain for the color averages of Q and Q^2,

$$\langle Q_{HN}(f)\rangle_c = Q_G(f) \tag{7.1d}$$

$$\langle Q_{HN}(f)^2\rangle_c = Q_G(f)^2 + \langle \Delta Q(c)^2\rangle_c = Q_G(f)^2 + (2/9) \tag{7.1e}$$

where the value 2/9 is obtained by substituting the numerical values of $Q_{HN}(f,c)$.

As long as ΔJ has no observable effects, it is impossible to distinguish between the integrally and fractionally charged models. There are two possible approaches to the observation of ΔJ: 1) by observing states which are not color singlets, 2) by observing matrix elements of operators which are quadratic in J_{em} between color singlet states. Since states which are not color singlets have a presumably high excitation threshold to explain the failure to observe them to date, we consider the possibility of detecting ΔJ below threshold by measurements on color singlet states of operators quadratic in ΔJ.

One important case where effects of color have been observed in a second order electromagnetic transition is in the decay $\pi^o \to \gamma\gamma$. Decays of this type of meson into two photons are assumed to be described by a triangle diagram.[32] We consider all possible decays of common mesons which have allowed two-photon decays, namely the pseudoscalar and tensor mesons:

$$\pi^o \to 2\gamma \tag{7.2a}$$

$$\eta \to 2\gamma \tag{7.2b}$$

$$\eta' \to 2\gamma \tag{7.2c}$$

$$f^o \to 2\gamma \tag{7.2d}$$

$$A2 \to 2\gamma \tag{7.2e}$$

$$f' \to 2\gamma \tag{7.2f}$$

The contribution of the triangle diagram for each of these decays is obtained by summing the diagrams for different quark flavors with the appropriate weighting factors for each meson. The transition matrix element for this diagram with a quark of flavor f is proportional to the square of the quark charge and is given by

$$M_T(f) = M \sum_c [Q(f,c)]^2/\sqrt{H_c} = M \quad \langle Q(f)^2 \rangle_c \ , \tag{7.3a}$$

where M is the reduced transition matrix element which contains the dependence on all degrees of freedom except color. Substituting from eq. (7.1e) into eq. (7.3a), we obtain the relation between the transition matrix elements in the Greenberg model and the Han-Nambu model, denoted by $M_G(f)$ and $M_{HN}(f)$.

$$M_{HN}(f) = \sqrt{N_c} M \quad [Q_G(f)^2 + \langle \Delta Q^2 \rangle_c] \tag{7.3b}$$

$$M_{HN}(f) = M_G(f) \ [1 + (2/9)/\ Q_G(f)^2] \quad . \tag{7.3c}$$

The expressions (7.3) can be written for the specific cases of u,d and s quarks in the following convenient form,

$$M(u) = 3M\ (4+2\kappa)/9 \tag{7.4a}$$

$$M(d) = M(s) = \sqrt{3}\ M(1+2\kappa)/9 \tag{7.4b}$$

where κ is a parameter describing the deviation from the fractionally charged colored quark model and we have set $N_c=3$ $M_G(f)$ is given by setting $\chi = 0$ in eqs. (4) and $M_{HN}(f)$ is given by setting $\kappa = 1$. Intermediate values of κ are also of interest as will be shown below.

For the decay of a π^o which is a coherent linear combination of a $u\bar{u}$ and $d\bar{d}$ state with equal magnitude and negative phase, the transition matrix element is proportional to the difference $M_T(u) - M_T(d)$. This difference is seen to have the same value in both models.

$$M_T(u)_G - M_T(d)_G = 1/\sqrt{3} = M_T(u)_{HN} - M_T(d)_{HN} \tag{7.5a}$$

A similar equality holds for the decay of the η_8 which is the eighth component of an octet and depends upon the linear combination

$$M_T(u)_G + M_T(d)_G - 2M_T(s) = 1/\sqrt{3} = M_T(u)_{HN} + M_T(d)_{HN} - 2M_T(s)_{HN}. \tag{7.5b}$$

The statistical factor $\sqrt{N}$ in eq. (3) has been used as evidence in favor of color in the experimental value of the $\pi^o \to \gamma\gamma$ decay rate. However, eqs. (7.5) show that it is impossible to distinguish between fractionally and integrally charged models with this decay or the decay of the isoscalar unitary octet meson. This is also evident from the form of ΔJ in eq. (7.1b), which is a flavor singlet. Squaring ΔJ gives an operator which has a color singlet component and is observable in the space of color singlet states. But because it is a flavor singlet, it cannot contribute to the decay of a flavor octet state.

For the decay of a flavor singlet meson, the two models give different results,

$$M(1_f) = (1/\sqrt{3}) \sum_f M(f) = M\cdot(2/3)\cdot(1+\kappa) \tag{7.6}$$

Unfortunately this difference is not easily checked experimentally. The physical η' meson is a mixture of singlet and octet and is at such a high mass that the PCAC derivation for the absolute rate of the π^o decay is unreliable. Kinematic factors resulting from the η-η' mass difference confuse any comparison of the two rates.

Thus, although $P \to \gamma\gamma$ decays appeared to have matrix elements quadratic in ΔJ which would distinguish between the two models, this is not feasible in practice. The situation looks somewhat better for an ideally mixed nonet, like the tensor mesons, where the mass degeneracy between nonstrange isoscalar and isovector states causes all kinematic factors to drop out in the ratio of the decay rates. For the ideally mixed f^o and f' decays.

$$M(f^o) = (1/\sqrt{2})\ [M(u)+M(d)] \tag{7.7a}$$

$$M(A2) = (1/\sqrt{2}\ \ [M(u)-M(d)] \tag{7.7b}$$

$$M(f') = \qquad M(s)\ . \tag{7.7c}$$

Substituting eqs. (7.4) into eqs. (7.7) gives

$$M(f^o)/M(A2)/M(f') = [(5/3)+(4/3)\kappa]/1/[(\sqrt{2}/3)+(2\sqrt{2}/3)\ \kappa] \tag{7.8}$$

Since the decay rates are proportional to the squares of the matrix elements, the ratio of the f^o to A2 decay rates is predicted to be 9 in the Han-Nambu model in comparison with 25/9 in the fractionally charged model. Furthermore, the f' decay rate is predicted to be larger than the A2 decay rate by a factor of 2 in the Han-Nambu model and lower by a factor of 2/9 in the fractionally charged model. These appear to be large observable effects.

Additional possibilities of observing the differences between the matrix elements (7.7a) and (7.7b) arise in coherent production of the f^o and A2 resonances by two photons in the reaction[33]

$$e^+e^- \to e^+e^- + \gamma\gamma \to e^+e^- + M^o \to e^+e^- + PP \qquad (7.9)$$

where M^o denotes a neutral nonstrange meson which is a coherent linear combination of f^o and A2 and PP denotes a state of two pseudoscalar mesons. In the approximation where the f^o and A2 are degenerate, SU(3) symmetry and the OZI Rule give the following results for the relative cross sections for the production of different PP states:

$$M(K^+K^-)/M(K^o\overline{K}^o)/\ M(\pi^+\pi^-) = (4+2\kappa)/(1+2\kappa)/(5+4\kappa) \qquad (7.10)$$

where the transition matrices M_T must be squared and multiplied by appropriate kinematic factors to obtain the observed cross sections. The kinematic factors should be identical for the charged and neutral kaon final states but may be somewhat different for the two-pion state. The results (7.10) are easily obtained by observing that the f^o and A2 are linear combinations of the $u\bar{u}$ and $d\bar{d}$ states and that charged kaon pairs are produced only via the $u\bar{u}$ state, neutral kaons only via the $d\bar{d}$ state and pion pairs only via the even G f^o state.

The relations (7.10) are derived under the assumption that the f^o and A2 are degenerate and have the same width. Calculations of the charged and neutral kaon pair mass spectra show that the qualitative features of eq. (7.10) remain when the masses and widths of the physical particles are introduced and each resonance decay is parametrized by a Breit-Wigner curve. In addition a strong interference effect appears in the region between the f' and the A2 from the overlapping of the tails of the resonances. With the Han-Nambu model these interference effects should be somewhat different, and might be used to distinguish between the two models.

The relations (7.8) and (7.10) look very promising for distinguishing between the two models if the simple triangle diagram describes the decay and its transition matrix element is given by equation (7.3). However, there are doubts about the validity of this description for the tensor mesons.

Suppose the triangle diagram of fig. 2 is interpreted as the successive emission of two photons. Then an intermediate state exists of a quark-antiquark pair with the quantum numbers of the photon, a vector meson state which is either a color singlet and flavor octet or flavor singlet and color octet. The transition matrix element computed from this diagram must include a propagator for the intermediate state. From eqs. (7.1) it is apparent that J_G appears in diagrams with color singlet intermediate states and ΔJ appears in diagrams with color octet intermediate states. If color octet states have a high threshold, diagrams with color octet intermediate states will be suppressed by propagators relative to diagrams with color singlet intermediate states. Thus the effect of the color threshold will reduce the contribution of the terms depending upon ΔJ below the values given by eqs. (7.3), (7.4), (7.6) and (7.8). Since the contribution of ΔJ is seen from eq. (7.3b) to be positive definite and to be given in eqs. (7.4), (7.6) and (7.8) by the term proportional to κ, the reduction of the contributions from ΔJ are expressed quantitatively by reducing the value of κ from unity in these relations.

For the case of the pseudoscalar meson decays, the existence of the axial anomaly allows the transition matrix element to be expressed by a triangle diagram dominated by high momenta where color thresholds are hopefully no longer important. For other cases where there is no anomaly, there is no reason to expect this dominance by high momenta in intermediate states and color threshold effects can be important. Unfortunately, the large deviation from ideal mixing makes the use of pseudoscalar decay rates difficult for distinguishing between the two models. The ideal mixing of the tensor nonet gives simple predictions, but these may be rendered useless by color threshold effects.

The effects in eqs. (7.8) and 7.10) are so large that they may still be observable even with an appreciable reduction from the propagators of the color octet states. The parameter κ will have a value $(m_1/m_8)^2$, where m_1 and m_8 are the masses of the color singlet and color octet intermediate states which are dominant in the transitions. If κ is between 10^{-1} and 10^{-2}, there may still be a possibility of observing these effects. For example, if κ=2%, there will be an 8% increase in the ratio of the two-photon decay widths of the f' and A2, a 3% increase in the ratio of the widths of the f and A2, and a 3% decrease in the production ratio of charged to neutral kaon pairs over the predictions of the fractionally charged quark model. Thus even if the effects are small, they appear as uniquely related discrepancies from the predictions of the fractionally charged model in three different ways.

VIII. SU(6) REVISITED

The SU(6) quark model is still going strong after over a decade. There have also been some new and peculiar applications of SU(6). Arima and Iachello have applied SU(6) to complex nuclei and obtained remarkable fits to nuclear spectra and properties by classifying nuclear states in enormous representations of SU(6). But although SU(6) is used to describe systems built from elementary building blocks with six states, the blocks used by Arima and Iachello[34] are not spin 1/2 quarks with three flavors, they are bosons with spin zero and spin two. But nobody has ever seen these bosons, and one wonders why nuclei behave as if they were made of bosons which are unobservable. Are they confined in a bag, where they never get out? Or are they simple mathematical representations of an underlying more complex structure? More recent work indicated that the bosons are not elementary objects but are a mathematical representation of nucleon pairs, which everyone expects to find in a nucleus. Perhaps the quarks and the quark-spin-flavor SU(6) are also mathematical representations of something more fundamental. Who knows?

I conclude with some poems[35] written in 1965 by modifying A.A. Milne's "Now We Are Six" to "Now We Are SU(6)". Many of these seem to be topical after twelve years.

I. Spin independence, Unitary Spin, Statistics and All That

I think I am a multiplet, I haven' any "L"
My spin and relativity do not mix very well
Perhaps I'm just a set of points on a diagrom
I'm feeling rather funny and I don't know <u>what</u> I am

BUT

<u>Round</u> about
And <u>round</u> about
And round about I go--
All around the table
The Table of Clebsch-Gordan coefficients.

II. A Word About Quarks

I think I am a boson, transforming like a pair.
I think I am an antiquark,
Bound to another antiquark
Bound to another antiquark that isn't really there...

SO

<u>Round</u> about
And <u>round</u> about
And <u>round</u> about and <u>round</u> about
And <u>round</u> about
And <u>round</u> about
I go

III. Quark Spins, Spin Conservation, and Good and Bad Selection Rules

I think I am a hexagon, or maybe just a square
I think I am a symmetry
Behind another symmetry
Behind another symmetry that isn't really there.

SO

Round about
And round about
And round about
I spin.

IV. Relativity, Groups, etc.

I think I am a little group, I don't know how to tell
I haven't got the properties that satisfy Michel
Perhaps I'm permutations on a Young diagram.
I'm feeling rather funny and I don't know what I am

SO

Round about
And round about
And round about and round about
And round about
And round about
I go

V. I-Spin, U-Spin, V-all Spin the Quark Spins

Clebsch Clebsch
Gordan Gordan
I-Spin U-Spin Vee
Took great
Care of his Physicist
Though only (SU) three
Clebsch Clebsch
Said to his Physicist
"Physicist" he said, said he:
"You must never go down to the end of the calculation,
If you don't go down with me"
..............from "When We Were Very "

VI. SU(6) Numerology and Do-It Yourself Multiplets.

Oh, whenever the Professor
Got into a mess, or
Felt himself sulky and sad,
He would murmur and murmur
Until he felt firmer
This curious rhyme which he had:

Two eights are one eight eight
Ten ten twenty-seven
When it's done,

Just for fun
    Try SU eleven.
Three sixes are fifty-six
    Symmetrize the three
If you need more
Use SU four,
    And then it's time for tea.

****

I guess it's time for Shelly Glashow's talk.

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