Fermi National Accelerator Laboratory

FERMILAB-Conf-77/16-THY
January 1977

The Alexander... Zweig Rules and What Is Wrong With Pseudoscalar Mesons

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Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration

The rule forbidding $\phi \to \rho\pi$ and other stuff has been credited to a number of physicists in various combinations. To avoid arguments about credit, this paper refers to the A...Z rule and allows the reader to insert the names of all desired friends from Alexander[1] to Zweig.[2] Unfortunately the wide distribution of credit has introduced a fuzziness in the definition of the rule. All formulations forbid $\phi \to \rho\pi$, $f' \to \pi\pi$ and $J/\psi \to$ ordinary hadrons. But other predictions are not universal and depend upon whose formulation is used.

The first formulation was Okubo's nonet ansatz[3] for three-meson couplings. Okubo did not draw quark diagrams because the quark had not yet been invented, and did not treat baryons because the quark structure is essential to define the difference between the baryon and meson octets. The SU(3) analog of Okubo's meson ansatz would forbid the coupling of the ϕ and f' from the Σ which has hypercharge zero and occupies the same position in the baryon octet that the ρ and π occupy in the meson octet.[4,5]

The correct extension of Okubo's nonet ansatz to baryons requires at least a mathematical quark description of the baryons as constructed from three fundamental SU(3) triplets. This decouples the ϕ and f' from the nucleon which has strangeness zero. Strangeness, unlike hypercharge, is outside SU(3) and arises naturally only in a quark description where it counts the number of strange quarks. The baryon selection rule arises naturally in the quark-line-diagram descriptions of Zweig[2] and Iizuka,[6] where disconnected diagrams are forbidden. It can also be cast into Okubo's language by representing baryons as third rank tensors and forbidding the appropriate disconnected contractions of these tensors. But in Okubo's terminology a difference still remains between the simple nonet ansatz for mesons and its extension to baryons. The possibility exists that the breaking of the rule may be much stronger for one than for the other, because the breaking mechanisms may be different in the two cases. This is an interesting point which remains to be settled by experiment.

Many possibilities arise in more complicated vertices. The most naive Zweig-Iizuka formulation forbids all disconnected diagrams. The simplest extension of the Okubo ansatz forbids only hairpin diagrams, in which one external particle is disconnected from the rest. Two interesting open questions are whether disconnected diagrams which are not hairpin diagrams might be less forbidden than hairpin diagrams, and whether doubly disconnected diagrams are more forbidden than singly disconnected ones. Interesting cases of these ambiguities arise in simple decays of the J/ψ.

Consider, for example,

$$\psi' \rightarrow \psi + \pi^+ + \pi^- \quad . \tag{1a}$$

This decay is described by a disconnected diagram which is not a hairpin diagram. A similar diagram, rotated by 90° but with the same topology describes the crossed reaction

$$\psi + \pi^+ \rightarrow \psi' + \pi^+ \quad . \tag{1b}$$

This process is diffractive excitation of the ψ' in $\psi\pi$ scattering and is allowed by Pomeron exchange. The same topology describes elastic $\psi\pi$ scattering. There is no reason to forbid the process (1b) and one can question whether a process related to it by crossing is as forbidden as a process described by a hairpin diagram and which is not related to any allowed process by crossing, e.g.

$$\pi^- + p \rightarrow \phi + n \quad . \tag{2}$$

Another example is the difference between the two decays

$$J/\psi \rightarrow \omega\pi\pi \tag{3a}$$

$$J/\psi \rightarrow \phi\pi\pi \quad . \tag{3b}$$

Both processes involve hairpin diagrams and are forbidden in any formulation. But the decay (3b) is doubly disconnected, while the decay (3a) is only singly disconnected. Originally the experimental results seemed to indicate little difference between the two processes and that singly and doubly connected diagrams were equally forbidden. A possible theoretical description of such a relation was proposed by Okubo[7] as a natural extension of his ansatz. However, recent experiments[8] suggest that the doubly forbidden process (3b) is indeed very different from the singly forbidden one (3a), and that it proceeds via an intermediate state whose propagator violates the A...Z rule.

An instructive example of the "forbidden propagator" mechanism by which the disconnected process (3b) can take place is seen in the example of the decay of a high K^{*-} resonance into three kaons via an intermediate nonstrange resonance M^{o}

$$K^{*-} \rightarrow K^{-}M^{o} \rightarrow K^{-}K\overline{K} \quad , \tag{4}$$

where M^{o} is a resonance like the ρ^{o}, ω, f or A2 which consists only of nonstrange quarks.

There are two diagrams for this decay with final transitions from the $u\overline{u}$ and $d\overline{d}$ (or if you prefer $p\overline{p}$ and $n\overline{n}$)* components of M^{o}. The $p\overline{p}$ diagram (or $u\overline{u}$) is connected, obeys the A... Z rule and leads to the final state $K^{-}K^{+}K^{-}$. The $n\overline{n}$ (or $d\overline{d}$) diagram is disconnected, violates the A... Z rule and leads to the final state $K^{-}K^{o}\overline{K}^{o}$. Thus if the rule holds,

$$(K^{*-} \rightarrow K^{-}(u\overline{u}) \rightarrow K^{-}K^{+}K^{-}) \text{ is allowed} \tag{5a}$$

$$(K^{*-} \not\rightarrow K^{-}(d\overline{d}) \rightarrow K^{-}K^{o}\overline{K}^{o}) \text{ is forbidden} \quad . \tag{5b}$$

But if the resonance M^{o} has a definite isospin, the two transitions (5a) and (5b) must be equal by isospin invariance. Contradictions between the A... Z rule and isospin invariance are avoided if the nonstrange meson spectrum consists of degenerate isospin doublets, like ρ and ω or f and A2. In that case the transition (5a) proceeds via the particular coherent linear combination of isovector and isoscalar particles which has the quark

*Two notations are used commonly for quark flavors, u, d, s and p, n, λ. For old nuclear physicists like myself, who have resisted the u, d notation because we could never remember which way was "up" (in nuclear physics the neutron has isospin up because common stable nuclei have more neutrons than protons), there is a simple mnemonic. Simply write the words

up

down .

People who read from left to right naturally call these u and d. People who read from right to left, as we do in the Middle East, naturally call them p and n. This leads to the correspondence

$$u \leftrightarrow p$$

$$d \leftrightarrow n \quad .$$

The invariance of this transformation under 180° rotations is exhibited by turning this page upside down.

In this talk I bounce between both notations because I am still using old transparencies, in accordance with the well-known rule: "Old transparencies never die, they just fade away."

constitution $p\bar{p}$ (or $u\bar{u}$). The A... Z rule is thus intimately related to the existence of the isospin doublets found in ideally mixed nonets.

If the M^o in the transitions (5) is not a member of an isospin doublet, the A... Z rule is inconsistent with isospin invaraince. This is the case if M^o is a π^o, which has no degenerate isoscalar partner. The π^o cannot contribute to the reactions (5) but can appear as an exchanged particle in the analogous two-body scattering reactions

$$K^+ + K^- \rightarrow K^{*o} + \overline{K}^{*o} \tag{6a}$$

$$K^+ + K^- \rightarrow K^{*+} + K^{*-} \tag{6b}$$

$$K^o + K^- \rightarrow K^{*o} + K^{*-}. \tag{6c}$$

The charge exchange reaction (6a) is clearly allowed by the A... Z rule and can go by pion exchange. The amplitudes for the pion exchange contribution to the reactions (6b) and (6c) are uniquely related to the charge exchange amplitude (6a) by isospin invariance. But the reaction (6b) is allowed by the A... Z rule and the reaction (6c) is forbidden when only nonstrange quark exchange is considered. (The reaction (6c) is allowed by $s\bar{s}$ (or $\lambda\bar{\lambda}$) exchange but this is irrelevant to the present argument). The A... Z rule could be saved from inconsistency with isospin invariance if a contribution from isoscalar exchange degenerate with pion exchange cancelled the pion exchange contribution to the reaction (6c). But no such isoscalar exists. Thus violations of the A... Z rule might be expected in processes where pseudoscalar exchange plays a dominant role.

The above examples are only a few of the puzzles and paradoxes of the A...Z rule. A consistent theoretical derivation of the rule should resolve these, but no such derivation exists. The most promising approach to such a derivation at present seems to be in the framework of dual resonance models,[9] but there are many open unanswered puzzles in this approach as well. The situation may be summed up by the statement that nobody understands the A...Z rule and don't believe anyone who claims he does. A comprehensive review of these puzzles is given in reference 5.

A principal difficulty to be overcome in any theoretical formulation is that a succession of transitions all allowed by the A...Z rule can lead to one which is forbidden. For example, the forbidden couplings $\phi\rho\pi$ and $f'\pi\pi$ can proceed through an intermediate $K\bar{K}$ state via the following transition amplitudes observed experimentally and allowed by the A...Z rule.

$$T(\phi \to K\bar{K}) \neq 0 \tag{7a}$$

$$T(f' \to K\bar{K}) \neq 0 \tag{7b}$$

$$T(K\bar{K} \to \rho\pi) \neq 0 \tag{7c}$$

$$T(K\bar{K} \to \pi\pi) \neq 0 \; . \tag{7d}$$

The selection rules can thus be broken by the following allowed higher-order transitions

$$\phi \rightarrow K\bar{K} \rightarrow \rho\pi \tag{8a}$$

$$f' \rightarrow K\bar{K} \rightarrow \pi\pi \quad . \tag{8b}$$

If the A... Z rule holds only to first order in strong interactions, much greater violations are expected than those experimentally observed. Some mechanism for reducing these violations seems to be present. One possibility is a cancellation of the violating amplitudes by other amplitudes, as occurs in the transitions (5) where the violating amplitudes from isoscalar and isovector meson states M^o must cancel one another to preserve the selection rule. This requires a degeneracy of the intermediate states. For an analogous cancellation to be effective for the transitions (8) an additional intermediate state degenerate with the $K\bar{K}$ state is required. But no such state exists in the spectrum of physical particles. Thus the cancellation can only be approximate and hold in some higher symmetry limit where other states are degenerate with the $K\bar{K}$ state.

The kaon plays a crucial ambivalent role in the A... Z forbidden transitions (8) between a state containing only strange quarks and a state containing only nonstrange quarks. Since the kaon contains one strange and one nonstrange quark, it couples equally to strange and nonstrange systems and can go either way. A kaon pair state contains one strange and one nonstrange quark-antiquark pair. It can therefore be created from a strange pair by the creation of a non-strange pair or vice versa. The kaon pair state thus links two kinds of states between which transitions are forbidden by the A...Z rule. The quark diagram for the forbidden transition (8) illustrates the essential features of the paradox. Viewed as a single topological diagram it is indeed disconnected and can be separated into two disconnected hairpin diagrams. But when it is separated into two individual transitions, each half is connected. Connecting the two diagrams together results in a topological disconnected diagram because of the twist in the quark and antiquark lines in the kaon intermediate state.[5]

Thus, to save the A...Z rule the connection of allowed diagrams by a "twisted propagator" must somehow be forbidden. But a twisted propagator has physical meaning only if there is additional information in a kaon pair state to specify "which way it is twisted"; i.e., whether it originally came from a strange or a nonstrange system. Some memory of the origin of the pair is necessary to prevent the nonstrange decay of a pair which originated in a strange system. But a physical kaon pair state has no such memory. A kaon pair produced from a nonstrange system is indistinguishable from a pair produced from a strange system.

In dual resonance models twisted diagrams are forbidden because of cancellations from contributions from exchange degenerate Regge trajectories. The intermediate states in dual resonance quark diagrams do not represent physical particles but Reggeons, and these always occur in exchange degenerate pairs with opposite behavior under charge conjugation. Twisted diagrams always include cancelling contributions from exchange degenerate trajectories. This description is valid, however, only for diagrams where all internal lines can be interpreted as Reggeons. This clearly does not hold for the transitions (8) where the intermediate state can occur as physical particles on their mass shell, and the corresponding exchange degenerate trajectories have no such states. This problem has been considered[9, 10] as a possible mechanism for the breaking of the A... Z rule in the particular case of the reaction $\pi^- p \rightarrow \phi n$.

We thus see that two types of degeneracies are required for the particle spectrum in order to avoid the breaking of the A... Z rule by the propagators of intermediate states. Nonet degeneracy with ideal mixing is required so that the $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ states which are not eigenstates of isospin and SU(3) can be eigenstates of the mass operator. Exchange degeneracy is required to avoid breaking of the rule by twisted propagators. Whenever these degeneracies are not exact, troubles with the A... Z rule can be expected. The reader is referred to reference 5 for further consideration of these effects.

The use of the quark model to determine mixing angles of neutral mesons from experimental data on neutral meson production processes was first suggested by G. Alexander.[1] This work also included the first derivation of the A... Z rule for four point functions, based on the Levin-Frankfurt additive quark model[11, 12] in which every hadron transition is assumed to involve only one active quark with all remaining quarks behaving as spectators. Hairpin diagrams are naturally forbidden in this model, since they involve two active quarks in the same hadron. Thus the prediction that ϕ production is forbidden in πN reactions was immediately obtained. Also obtained were a number of other predictions which have since been shown to be in very good agreement for vector meson production. These include the prediction of no exotic t-channel exchanges and some sum rules and equalities which are listed below.

At the time that the paper of Alexander et al.[1] was presented for publication, the quark model was ridiculed by the particle physics establishment and the paper was rejected by a referee who dismissed the quark model as nonsense. It was resubmitted for publication and finally accepted after new experimental data[13] supported predictions made in the paper before the data was known. Today papers based on the single-quark-and-spectators transition are covered by mentioning the magic name of "Melosh", since no referee will admit that he really doesn't understand what this Melosh transformation[14] is all about. Meanwhile many of the sum rules and equalities of Alexander et al.[1] were rederived[12, 15] by people who do not like quarks and give them other names to avoid calling them quark model sum rules which they really are. Reviewing the experimental results over the past decade shows striking agreement with these quark model relations for all processes of vector meson production and strong disagreement with relations for processes of pseudoscalar meson production, particularly for relations involving η' production.[4, 13, 15, 16] We suggest that an appropriate conclusion from these results is that the quark model description indeed holds for these processes, but that something is wrong with the pseudoscalars,[17] particularly the η'.

The relevant sum rules are the charge exchange sum rule (CHEX)

$$\sigma(\pi^- p \to \pi^o n) + \sigma(\pi^- p \to \eta n) + \sigma(\pi^- p \to \eta' n) = \sigma(K^+ n \to K^o p) + \sigma(K^- p \to \bar{K}^o n), \quad (9a)$$

and the strangeness exchange sum rule (SEX)

$$\sigma(K^- p \to \eta Y) + \sigma(K^- p \to \eta' Y) = \sigma(K^- p \to \pi^o Y) + \sigma(\pi^- p \to K^o Y) \ . \qquad (9b)$$

These sum rules hold for any meson nonet and do not make any assumption about the mixing angle, except for the conventional description of the η and η' as two orthogonal linear combinations of pure SU(3) singlet and octet states defined in terms of a single mixing angle. For the case of ideal mixing, as in the vector mesons the two sum rules each split into two equalities, CHEX becomes

$$\sigma(\pi^- p \to \phi n) = 0 \qquad (10a)$$

which is just the A... Z rule, and substituting (10a) into (9a) gives

$$\sigma(\pi^- p \to \rho^o n) + \sigma(\pi^- p \to \omega n) = \sigma(K^+ n \to K^{*o} p) + \sigma(K^- p \to K^{*o} n) \ . \qquad (10b)$$

With ideal mixing SEX becomes

$$\sigma(K^- p \to \omega Y) = \sigma(K^- p \to \rho^o Y), \qquad (11a)$$

$$\sigma(K^- p \to \phi Y) = \sigma(\pi^- p \to K^{*o} Y) \ . \qquad (11b)$$

The relation (11a) is seen also to be a consequence of the A. . . Z rule for the meson vertex. The incident K^- contains no d or $\bar{d}$ quarks or antiquarks and therefore the production of a $d\bar{d}$ pair is forbidden. The ρ^o and ω are therefore produced via the $u\bar{u}$ component which is a linear combination of the two with equal weight.

If there is no mixing, which is a rough approximation for the pseudoscalar mesons, the charge exchange sum rule simplifies to

$$\sigma(\pi^- p \to \pi^o n) + 3\sigma(\pi^- p \to \eta_8 n) = \sigma(K^+ n \to K^o p) + \sigma(K^- p \to \bar{K}^o n) \ , \qquad (12a)$$

$$\sigma(\pi^- p \to \eta_8 n) = 2\sigma(\pi^- p \to \eta_1 n) \ . \qquad (12b)$$

All the vector meson relations (10) and (11) are in excellent agreement with experiment. However, the pseudoscalar meson relations (9) and (12b) are in strong disagreement. The relation (12a) agrees with experiment if the η is assumed to be pure octet. This suggests that the conventional picture in which there is small mixing may be valid for the η, but that something is wrong with the η', and it is wrong in the direction that the η' has an inert piece in the wave function which does not contribute to the sum rules (9) and (12).

We suggest that there is indeed an additional piece in the η' wave function and that it is a radially excited configuration. This leads to a re-examination of the standard mixing folklore and the discovery that it is completely unjustified.[17] In a formulation which begins with unperturbed singlet and octet states in the SU(3) symmetry limit, there is no reason to assume that SU(3) symmetry breaking should admix only the lowest ground states of the singlet and octet spectra. This may work for the tensor and vector mesons, where the entire nonet seems to be degenerate in the SU(3) symmetry limit and the dominant breaking of nonet symmetry is by a quark mass term. The degeneracy suggests the use of degenerate perturbation theory which diagonalizes the symmetry breaking interaction in the space of the degenerate unperturbed states. The mass term has no radial dependence and would not mix ground state and radially excited wave functions which are orthogonal and would have a zero overlap integral.

For the pseudoscalars where there is a large singlet-octet splitting in the SU(3) symmetry limit there is no reason to use degenerate perturbation theory and mix only ground state wave functions. Furthermore, the singlet-octet splitting can only be produced by an interaction which violates the A... Z rule because it is not diagonal in the quark basis and mixes $s\bar{s}$ with $u\bar{u}$ and $d\bar{d}$. The accepted mechanism for such A... Z violation in the pseudoscalars is annihilation of the quark-antiquark pair into gluons and the creation of another pair. Here there is no reason to restrict the pair creation to the ground state configuration. There is no overlap integral between the two $q\bar{q}$ states, as the intermediate gluon state does not remember which radial configuration it came from. If the annihilation process depends primarily on the value of the $q\bar{q}$ wave function at the origin, then all radially excited configurations couple with equal strength for wave functions from a confining linear potential.

Thus there is considerable reason to suspect that the trouble with the pseudoscalar meson sum rules is in admixture of a radially excited wave function into the η'. One might expect the η to be purer because the SU(3) flavor octet state does not couple to gluons which are singlets and because it is the lowest state, far in mass from the nearest SU(3) singlet radial excitation. The η', on the other hand is sitting in between the ground state and first radially excited octet states and would be expected to mix with both. Note that mixing of the octet ground state and first radially excited octet state by an SU(3)-symmetric potential need not be considered because it is merely a change in the radial wave function. This mixing can be transformed away by choosing a new radial basis (i. e. a slightly different potential) for which the modified ground wave function in the original basis is the exact ground state in the new basis.

With no further theoretical results or suggestions regarding the treatment of the pseudoscalars, let us turn to experiment and see how experimental data on the new particles can provide additional clues and useful information. The outstanding open question is the existence of the charmonium ($c\bar{c}$) pseudoscalar, the η_c, its mass and properties. At the time of this conference it is seen only at DESY in its $\gamma\gamma$ decay mode.[18] There is therefore interest in observing the hadronic decay modes. The $p\bar{p}$ mode, which was first reported and then faded away, is irrelevant. There is no reason to expect such a mode to be strong. The $c\bar{c}$ is expected to decay into normal hadrons by annihilation of the $c\bar{c}$ in an A... Z violating transition, presumably via gluons, into a pseudoscalar state of light quarks. One would expect the coupling of this state to be similar to the coupling of the known pseudoscalars, the η and η'. The simplest hadronic decay channels would be $\pi A2$, ηf^{o}, $K\bar{K}^{*}$, $\pi\delta$ and $\eta\epsilon$.

The difficulty in observing these hadronic states arises from the presence of neutrals which are not easily detected and the difficulty of distinguishing photons from π^{o}'s. Of these decay modes the $\pi A2$ may be the best for experimental observation. This decay can be expected to be relatively strong, because the $A2-\eta-\pi$ coupling is known to be strong from the A2 decay. It may be observable even with unidentified neutrals because of the peculiar kinematics of the decay

$$\psi \to \gamma\eta_c \to \gamma\pi A2 \to \gamma 4\pi \quad . \tag{13}$$

This decay populates a peculiar region of phase space for four charged pions and a neutral which may be useful even if the neutral is not identified as a γ. The photon has a momentum of about 300 MeV, while the pion which is not in the A2 has a high momentum varying between 1 and 1.2 GeV depending upon the angle between the pion momentum and the momentum of the photon. This angular distribution should be isotropic in the center of mass system of the η_c, since it has spin zero.

The large disparity between the photon and pion momenta can serve to distinguish between $\gamma\pi^+\pi^+\pi^-\pi^-$ and $\pi^o\pi^+\pi^+\pi^+\pi^-$ events. The $\pi\pi A2$ final state is required by isospin invariance to have a symmetrical distribution in phase space for the two pions, and the probability that the charged pion has 1 - 1.2 GeV momentum while the neutral has only 300 MeV can be expected to be small. Furthermore, exact relations following from isospin invariance can be used to estimate this background from data in other regions of phase space and to subtract the background.

Consider the decay

$$\psi \rightarrow \pi\pi A2 \quad . \tag{14}$$

Isospin invariance requires the two pions to be in a state of total isospin one, coupled to the isospin one of the A2 to give an overall isospin of zero. This isospin coupling gives a unique relation between the different possible charge states where one pion has a momentum of 0.3 GeV and the second has a momentum of 1.1 GeV. Thus it is possible to define linear combinations of branching rations which must vanish for the decay (14) and can serve as background subtractions; e.g.

$$\Delta = W(0,+,-) + W(0,-,+) - \tfrac{1}{2}\left[W(+,0,-) + W(-,0,+) + W(+,-,0) + W(-,+,0)\right] = 0 \tag{15}$$

where $W(q_1, q_2, q_3)$ denotes the branching ratio for a state with a 300 MeV pion with charge q_1, a 1.1 GeV pion with charge q_2 and an A2 with charge q_3. The first two states in eq. (15) appear as background in the same region of phase space as the desired decay (15) while the remaining four are in a completely different region.

Consider an experiment in which four charged pions are observed and the missing mass indicates an additional neutral particle which may be either a photon or a π^o. If all of these events are substituted into the relation (15) including those where the neutral is a photon as well as a π^o, the pion contribution must vanish, in view of the relation (4). For this case the quantity Δ will measure

$$\Delta = W(\gamma, +, -) + W(\gamma, -, +) - W(+, \gamma, -) - W(-, \gamma, +) \ . \tag{16}$$

This is just the difference between the desired decays (13) and decays in which the final state has the momenta of the photon and pion interchanged; e.g. a 300 MeV pion and a 1.1 GeV photon. The background of π^o events is completely eliminated by isospin invariance. The quantity (16) can then be plotted as a function of the mass of the πA2 system to see if it exhibits a peak at 2.8 GeV. Note that the values of 0.3 and 1.1 GeV for momenta were picked just for example. In actual practice the quantity will be defined for a boson of charge q_1 and momentum k_1 and a boson of charge q_2 and momentum k_2 and the boson is either a photon or a pion.

Further information on the properties of the pseudoscalar mesons can be obtained from the decays of new particles into channels including the η and η'. Analysis of the decays involves SU(3) symmetry and the A...Z rule as well as properties of the pseudoscalars. But sufficient data are available to enable comprehensive tests of all these assumptions. We first list some SU(3) predictions discussed in reference 17. We begin with those obtained from the assumptions of SU(3) symmetry and naive mixing without the A... Z rule. Ideal mixing for the ω and ϕ is assumed, since known deviations are small. The one equality obtained is the sum rule:

$$2\Gamma(\psi \to \phi\eta) + 2\Gamma(\psi \to \phi\eta') = \Gamma(\psi \to \rho^+\pi^-) + \Gamma(\psi \to \omega\eta) + \Gamma(\psi \to \omega\eta') , \quad (17a)$$

where Γ denotes the reduced width without phase space corrections. In addition the following relations are obtained,

$$\Gamma(\psi \to \omega\eta) + \Gamma(\psi \to \omega\eta') = \Gamma(\psi \to \rho^+\pi^-)(1 + 2|A_1/A_8|^2)/3 \quad (17b)$$

$$\Gamma(\psi \to \phi\eta) + \Gamma(\psi \to \phi\eta') = \Gamma(\psi \to \rho^+\pi^-)(2 + |A_1/A_8|^2)/3 \quad . \quad (17c)$$

These relations give testable inequalities without additional assumptions, since the quantity $|A_1^2/A_8^2|$ defined in ref. 17 is positive definite.

The validity of SU(3) symmetry for the decay is tested independent of the nonet mixing assumption by the relation which does not involve any mixed mesons

$$\Gamma(\psi \to \rho^+\pi^-) = \Gamma(\psi \to K^{*+}K^-) \quad . \quad (18)$$

When the A... Z rule is assumed, Eqs. (2) hold with $A_1/A_8 = 1$ and the following additional relation is obtained:

$$\Gamma(\psi \to \omega\eta) = \Gamma(\psi \to \phi\eta') \quad . \quad (19a)$$

This can be combined with the relations (2) to obtain other simple relations

$$\Gamma(\psi \to \phi\eta) = \Gamma(\psi \to \omega\eta') \tag{19b}$$

$$\Gamma(\psi \to \omega\eta) + \Gamma(\psi \to \phi\eta) = \Gamma(\psi \to \rho^+\pi^-) \tag{19c}$$

$$\Gamma(\psi \to \omega\eta') + \Gamma(\psi \to \phi\eta') = \Gamma(\psi \to \rho^+\pi^-) . \tag{19d}$$

The relations (19b - d) are not linearly independent of the previous relations. However, if some relations disagree with experiment and others agree, these different combinations can furnish clues to determine what has gone wrong. For example, if the A... Z rule holds and naive mixing breaks down for the η' but still holds for the η, then relation (19c) which involves only the η might agree with experiment while other relations like (17) and (19d) which involve the η might not. If there is an inert piece in the η' wave function, the right hand sides of (19a) and (19b) and the left hand side of (19d) would all be suppressed by the same factor.

We now consider the possibility of SU(3) breaking. A number of SU(3) predictions have been shown to be in qualitative agreement with experiment in the observed branching ratio of J/ψ decays. However there is also evidence for appreciable SU(3) symmetry breaking. We introduce here a simple and intuitively attractive symmetry breaking mechanism which provides a consistent description of several very different breaking effects.

We assume that the decay of the J/ψ into a final state containing ordinary hadrons proceeds via an A...Z-rule violating annihilation of a charmed quark-antiquark pair and the creation of a light quark-antiquark pair. SU(3) symmetry requires the amplitudes for the production of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pairs to be equal from a unitary singlet state. We assume that the symmetry is broken by suppressing the production of strange quark pairs,

$$<u\bar{u}|Z|c\bar{c}> = <d\bar{d}|Z|c\bar{c}> = <s\bar{s}|Z|c\bar{c}>/(1-\zeta) \tag{20}$$

where Z denotes the operator describing the A...Z-violating transition and ζ is the parameter specifying the suppression of strange quark production. The suppression of strange particle production is a well-known experimental effect,[19] and it is therefore reasonable to attribute SU(3) symmetry breaking to a mechanism which reduces strange quark production. We do not consider the dynamical origin of the suppression factor ζ, but rather investigate the implications of this type of breaking on observed decay branching ratios.

We consider three SU(3) predictions: 1) The selection rule forbidding the PP, VV, TT and PT final states, 2) The predicted equality of the K^+K^{*-} and $\rho^+\pi^-$ decay modes, and 3) The predicted ratio of the $\eta\gamma$ and $\eta'\gamma$ final states, for which the decay into the octet component η_8 is forbidden by SU(3) if the $c\bar{c}$ pair first emits the photon and then turns into a light pseudoscalar meson via the transition (20). The justification of this assumption is discussed below.

We first consider the selection rule forbidding the PP etc. states. This is conveniently parametrized by examining the ratio of the forbidden K^+K^- and allowed K^+K^{*-} decay modes. Both transitions involve the disappearance of a $c\bar{c}$ pair and the creation of a $u\bar{u}$ pair and an $s\bar{s}$ pair. We assume that the first pair is created by the transition (20) and consider the possibility of a similar additional SU(3) breaking in the second process. However, the results are qualitatively unaffected by considering only the breaking given by eq. (20).

Two diagrams are seen to contribute to the transitions to the K^+K^- and K^+K^{*-} final states, one in which the $u\bar{u}$ pair is created first and one in which the $s\bar{s}$ is created first. In the SU(3) limit these two contributions are seen to be equal in magnitude and to have a relative phase depending upon the behavior of the corresponding octets under charge conjugation. Thus the two diagrams exactly cancel one another for the K^+K^- final state and add constructively for the K^+K^{*-} final state. This gives the selection rule. When SU(3) is broken by the mechanism (20) the cancellation no longer holds and the selection rule is violated. This is expressed quantitatively by the relation

$$\frac{A(K^+K^-)}{A(K^+K^{*-})} = \frac{(1-\zeta')\langle u\bar{u}|Z|c\bar{c}\rangle - \langle s\bar{s}|Z|c\bar{c}\rangle}{(1-\zeta')\langle u\bar{u}|Z|c\bar{c}\rangle + \langle s\bar{s}|Z|c\bar{c}\rangle} \cdot \frac{\langle K^+K^-|u\bar{u}\rangle}{\langle K^+K^{*-}|u\bar{u}\rangle} \qquad (21)$$

where A(f) denotes the amplitude for the J/ψ decay into the final state f, $\langle f|u\bar{u}\rangle$ denotes the amplitude for the decay of the $u\bar{u}$ state into the final state f and ζ' is a strange quark suppression factor analogous to ζ in eq. (20) describing SU(3) symmetry breaking in the transition to the final state, defined by the relation

$$<f|u\bar{u}> = \pm<f|s\bar{s}>(1 - \zeta') \tag{22}$$

where the phase is - for forbidden processes and + for allowed processes.

Substituting eq. (20) into eq. (21) then gives

$$|A(K^+K^-)/A(K^+K^{*-})|^2 = \left[(\zeta - \zeta')/(2 - \zeta - \zeta')\right]^2 R(K, K^*) \tag{23}$$

where $R(K, K^*)$ is a factor of order unity expressing the ratio of PP and VP widths when both are allowed by SU(3)

$$R(K, K^*) = |<K^+K^-|u\bar{u}>/<K^+K^{*-}|u\bar{u}>|^2 \quad . \tag{24a}$$

If symmetry breaking in the second process is neglected and ζ' is set equal to zero,

$$|A(K^+K^-)/A(K^*K^{*-})|^2 = (\zeta/2 - \zeta)^2 R(K, K^*) \text{ if } \zeta' = 0 \quad . \tag{24b}$$

Eqs. (4) show that the forbidden decay is no longer zero in the presence of SU(3) symmetry breaking, but that <u>it still remains very small even if the breaking is appreciable</u>. Note, for example that $\zeta = 1/2$, which is appreciable suppression of the strange quark production, the K^+K^- suppression factor is 1/9 if $\zeta' = 0$. For $\zeta = 1/4$ the K^+K^- suppression factor is 1/49.

The $\rho\pi$ decay goes via two diagrams analogous to the $K\overline{K}^*$ decay, but no strange quarks are involved. In both cases the two contributions add constructively to give

$$|A(K^+K^{*-})/A(\rho^+\pi^-)|^2 = \left(1 - \frac{\zeta + \zeta'}{2}\right)^2 \quad . \tag{25}$$

Here no additional factor analogous to (24b) is necessary because both final states are in the same SU(3) multiplet and the ratio is determined by SU(3).

This result is seen to be much more sensitive to SU(3) breaking than the selection rule (23), because the breaking is linear in ζ rather than quadratic. For $\zeta = 1/2$, $\zeta' = 0$ the ratio (25) is 9/16 while for $\zeta = 1/4$, the ratio (25) is 49/64. Thus a symmetry breaking which reduces the ratio (25) from the SU(3) predicted value of unity by a factor of almost 2 keeps the forbidden transition suppressed by an order of magnitude, while a smaller breaking which reduces the ratio (25) to 75% of its predicted value only allows the forbidden transition to go with a strength of 2% of the allowed transitions. Note also that this effect is enhanced by any additional symmetry breaking expressed by setting $\zeta' \neq 0$. Such breaking reduces the strength of the forbidden transition (23) and increases the SU(3) symmetry breaking in the ratio (25).

Radiative decays can proceed via the conventional unitary octet component of the photon, which couples to ordinary light quarks or by the unitary singlet component which couples to charmed quarks. The octet component contributes to the $\pi^{o}\gamma$ and $\eta_8\gamma$ decays, while the singlet component contributes to the $\eta_1\gamma$ decay. Since the experimentally observed $\pi^{o}\gamma$ decay is much weaker than the observed $\eta\gamma$ and $\eta'\gamma$ decays, we neglect the contribution of the octet component of the photon and consider only the singlet. This is also consistent with the picture in which the A... Z-violating transition (20) is stronger for a pseudoscalar state, where it can go via a two-gluon intermediate state, than for a vector state where three gluons are required.

In this approximation the η and η' decays can only go via the η_1 state in the SU(3) symmetry limit. However, symmetry breaking via the mechanism (20) can also introduce a contribution from the η_8 state. Using eq. (20) we obtain:

$$\frac{A(\eta_8)}{A(\eta_1)} = \frac{\Sigma_q \langle\eta_8|q\bar{q}\rangle\langle q\bar{q}|Z|c\bar{c}\rangle}{\Sigma_q \langle\eta_1|q\bar{q}\rangle\langle q\bar{q}|Z|c\bar{c}\rangle} = \frac{(1/\sqrt{6})(\zeta)\langle u\bar{u}|Z|c\bar{c}\rangle}{(1/\sqrt{3})(3-\zeta)\langle u\bar{u}|Z|c\bar{c}\rangle} \quad . \tag{26a}$$

For the physical η and η' states rotated by a mixing angle θ, this becomes

$$\frac{A(\eta\gamma)}{A(\eta'\gamma)} = \left[\tan\theta + (\zeta/\sqrt{2})(3-\zeta)\right]/\left[1 - (\tan\theta)(\zeta)/\sqrt{2}(3-\zeta)\right]. \tag{26b}$$

The two terms in the numerator of the right hand side of eq. (26b) express respectively the contributions of the singlet-octet mixing and the SU(3) symmetry breaking of eq. (20). The two effects interfere constructively for positive values of the mixing angle, which corresponds to the <u>reduction</u> in the strange quark composition of the η.

With Isgur's mixing angle,[20] which corresponds to equal amounts of strange and nonstrange quarks in both mesons with opposite phase,

$$\frac{A(\eta\gamma)}{A(\eta'\gamma)} = \frac{\sqrt{2} - 1 + \zeta}{\sqrt{2} + 1 - \zeta} = \frac{0.41 + \zeta}{2.41 - \zeta} \quad . \tag{26c}$$

For the values of ζ of 1/2 and 1/4 considered above, the values of the ratio (26c) obtained are 1/2.1 and 1/3.2 respectively, which are the right order of magnitude to fit the available data.

Quantitative predictions from eqs. (23 - 26) should not be taken too seriously, because of the ambiguities in the value of ζ' and uncertainties regarding pseudoscalar mixing. However, the qualitative agreement of the strange quark suppression mechanism (20) in determining the order of magnitude of the three symmetry breaking effects is encouraging.

It is a pleasure to thank Hinrich Meyer for discussions of DESY results during the conference which motivated some of the analysis of new particle decays actually carried out during the conference and included in this text. Discussions with Harald Fritzsch at the conference are also gratefully acknowledged. Recent work by Fritzsch and Jackson[21] discusses pseudoscalar mesons and SU(3) symmetry breaking from a very different point of view, using QCD-motivated calculations of radiative decays with conventional mixing and arriving at an SU(3) symmetry breaking qualitatively similar to the ad hoc breaking introduced in eq. (20) of this paper. Combining the Fritzsch-Jackson picture with the idea of radial mixing and the applications to strong interaction processes treated in this paper might provide a consistent description of a large body of hadronic phenomena. Unfortunately the Fritzsch-Jackson preprint was received after this work was completed.

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