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Three and Higher Order Rapidity Distributions in pp Collisions at 200 GeV/c

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ABSTRACT

Three-particle rapidity distributions have been studied for different charged multiplicities in pp interactions at 200 GeV/c. Appropriately defined semi-inclusive three-particle correlations are found to be sufficiently small, leading us to conclude that the three-particle density can be expressed in terms of one- and two-particle densities. For events with more than four charged particles, a superposition approximation for the three-particle density holds surprisingly well. This approximation enables us to express all higher order rapidity distributions in terms of one- and two-particle distributions alone.

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1. Introduction

After the first observations at the CERN Intersecting Storage Rings (ISR) of a strong, short range rapidity correlation among the products of pp collisions[1], considerable effort, both experimental and theoretical, has been devoted to this field. There now exists an abundant amount of literature[2] about two-particle correlations in a center-of-mass energy interval $\sqrt{s}$ = 5 to 60 GeV. A convenient, yet powerful framework, consistent with existing data on inclusive and semi-inclusive two-particle correlations has emerged according to which particles are assumed to be produced in uncorrelated, independent low multiplicity clusters[3]. Simple assumptions about the manner in which these clusters decay, together with their mean charged multiplicity, are then sufficient to explain the correlations observed in multiparticle final states. It is clear that a study of three and higher order distributions is the next natural step towards understanding the dynamics of correlations, in general, and the role these clusters play in multiparticle production, in particular. However, few experimental studies of three-particle correlations have been made so far.[4] In this report we present results on three-particle distributions using a sample of 6,889 inelastic events from 200 GeV/c proton-proton interactions obtained in an exposure of the 30-inch hydrogen bubble chamber-wide gap spark chamber hybrid system[5] at Fermilab. Further details[6] about the data used in this study and results on two-particle correlations have already been published.[7]

2. Three-Particle Correlations and their Approximate Representations

In the following we integrate over all transverse momenta and label a particle's position in phase space by its rapidity alone. One-, two- and

$C_n(0,0,0)$ is quite small compared to, for example, $\frac{\rho_n(0,0,0)}{n(n-1)(n-2)}$, for all charged multiplicities studied. In fact, we have found $C_n(1,2,3)$ to be quite small over the entire rapidity range $-3 \leq y_1, y_2, y_3 \leq 3$. This important observation is in complete agreement with the results of Ref. 4.

The smallness of the three particle correlation C_n implies that the semi-inclusive three particle densities can be expressed in terms of the one- and two-particle densities. Setting $C_n(1,2,3) = 0$ in equation (4) and solving for $\rho_n(1,2,3)$ gives:

$$\begin{aligned}\rho_n(1,2,3) &= \frac{(n-1)(n-2)}{n^2}\ \rho_n(1)\ \rho_n(2)\ \rho_n(3) \\ &+ (n-1)(n-2)\ [\rho_n(1)\ C_n(2,3) + \rho_n(2)\ C_n(3,1) \\ &+ \rho_n(3)\ C_n(1,2)]\ . \qquad (6)\end{aligned}$$

A detailed comparison of equation (6) with our data for different charged multiplicities, over the range $-3 \leq y_1, y_2, y_3 \leq 3$, is deferred to the next section.

In addition to equation (6), we have tried to find other more general ways of expressing $\rho_n(i,j,k)$ in terms of $\rho_n(i,j)$ and $\rho_n(k)$. It may be noted that questions of a similar nature have been well studied in the classical theory of fluids.[8] There one often employs a superposition approximation for the higher order distribution functions of molecules in a fluid in order to get a closed integral equation for the radial distribution function. According to this approximation, the correlation between a group of h particles can be reduced to a superposition of the correlations between smaller constituent particle groups. At what order such an approximation may be used

3. Comparison of Approximations for $\rho_n(1,2,3)$ with Data

In Figures 1(a-d) we show the three particle density function for 8-prongs, with y_1 fixed (within ±0.5 units) at -3.0, -2.0, -1.0 and 0.0 respectively, while y_2 and y_3 are allowed to vary over the range $-3 \leq y_2, y_3 \leq 3$. The density $\rho_n(1,2,3)$ for positive values of y_1 can be obtained by symmetry. This encompasses about 95% of the total phase space available for the three-particle semi-inclusive rapidity distribution considered here.

Figures 1(a-d) reveal: (i) In the region $-2 < y_{i,j,k} < 2$, $\rho_8(1,2,3)$ depends strongly on the pair rapidity differences $|y_i - y_j|$, while it is insensitive to the sums $|y_i + y_j|$. It has maxima at the positions $y_i = y_j$, and decreases rapidly with increasing $|y_i - y_j|$; (ii) On the other hand, in the region $|y_{i,j,k}| > 2$, $\rho_8(1,2,3)$ displays a strong dependence on $|y_i + y_j|$, due (at least in part) to energy-momentum conservation and diffraction effects. It will be shown that these features can be understood in terms of relatively better known two-particle correlation functions. In Figure 1 a set of straight lines have been drawn between the data points representing each of the approximations to $\rho_8(1,2,3)$. Equation (6) is shown by dashed lines, and equation (7) by solid lines. For convenience, equation (6) is shown only when it differs noticeably from equation (7). We observe that both approximations reproduce $\rho_8(1,2,3)$ remarkably well. Only when the rapidities of two or more particles are similar do we find some hint of disagreement at a few percent level. This observation is in accord with the idea that the superposition approximation breaks down at small distances.[8]

Figures 2(a-d) show the three-particle density function for 4-, 6-, 10-, and 12-prong events respectively in arbitrarily selected regions of

phase space. In all cases, but the 4-prongs, the agreement between the data and their approximate representation is good, apart from the minor deviations mentioned above. As is clear from Figure 2(a), both approximations fail completely in predicting the magnitude as well as the shape of $\rho_n(1,2,3)$ for events with four charged particles in the final state. This interesting observation implies the existence of genuine three-particle correlations in these events, a conclusion not totally unexpected since diffractive excitation is a dominant production mechanism at low multiplicities.

4. A Representation of the General h Particle Semi-inclusive Distribution Function

In the previous section we have verified the validity of the superposition approximation. We now use it to construct a general h particle semi-inclusive distribution function for an n particle final state. Let $y_1, y_2 \ldots y_n$ be the rapidities of these particles. We shall assume that the total differential cross section has the following representation:

$$\frac{d\sigma_n}{dy_1 dy_2 \ldots dy_n} = \prod_{i<j=1}^{n} f(y_i, y_j)\, F(Y) \,, \tag{8}$$

where $Y \sim \ln\sqrt{s}$. This form may be considered as a generalization of the multiperipheral model (MPM) since in equation (8) the correlations between all pairs of particles have been considered, instead of just the nearest neighbor terms usually considered by MPM. In the following we will attempt to calculate the basic two-particle function $f(y_1, y_2)$ introduced in equation (8) using single-particle and two-particle semi-inclusive cross sections as input and making use of the observation that the three-particle density satisfies the superposition approximation.

Using equation (8) we can easily derive the n-particle production cross section, and single-and two-particle semi-inclusive distribution functions;

$$\sigma_n = \frac{1}{n!} \int dy_1 dy_2 \ldots dy_n \prod_{i<j} f(y_i, y_j)\, F(Y), \tag{9}$$

$$\sigma_n \rho_n(1) = \frac{d\sigma_n}{dy_1} = \frac{1}{(n-1)!} \int dy_2 \ldots dy_n \prod_{i<j} f(y_i, y_j)\, F(Y), \tag{10}$$

and

$$\sigma_n \rho_n(1,2) = \frac{d\sigma_n}{dy_1 dy_2} = \frac{1}{(n-2)!} \int dy_3 dy_4 \ldots dy_n \prod_{i<j} f(y_i, y_j)\, F(Y) \tag{11}$$

Letting $f(y_i, y_j) = e^{-\chi(y_i, y_j)}$, and differentiating equation (11), we obtain

$$\frac{\partial \rho_n(1,2)}{\partial y_1} = - \frac{\partial \chi(y_1, y_2)}{\partial y_1} \rho_n(1,2) - \int dy_3 \frac{\partial \chi(y_1, y_3)}{\partial y_1} \rho_n(1,2,3), \tag{12}$$

where the three-particle density $\rho_n(1,2,3)$ is defined in a manner similar to equations (10) and (11). If we assume the superposition approximation, equation (7), for the three-particle density, equation (12) can be rearranged to give

$$\frac{\partial \chi(y_1, y_2)}{\partial y_1} = - \frac{1}{\rho_n(1,2)} \frac{\partial \rho_n(1,2)}{\partial y_1} - \frac{n(n-2)}{(n-1)^2\, \rho_n(1)\, \rho_n(2)} \int dy_j \frac{\partial \chi(y_1, y_j)}{\partial y_1} \frac{\rho_n(2,j)\, \rho_n(j,1)}{\rho_n(j)} . \tag{13}$$

Equation (13) is a simple integral equation for the function $\frac{\partial \chi(y_1, y_2)}{\partial y_1}$. After replacing the integral over y_j by a sum over discrete values of y_j on the

right hand side, it is easily reduced to an algebraic equation. Using data for single-particle and two-particle semi-inclusive densities, a solution for $\frac{\partial\chi(y_1,y_2)}{\partial y_1}$ is readily obtained. Remembering that $\chi(y_1,y_2)$ is necessarily a symmetric function of y_1 and y_2, one can determine it up to a constant if $\frac{\partial\chi}{\partial y_1}(y_1,y_2)$ is known. Knowing $\chi(y_1,y_2)$ then enables us to write a general hth order semi-inclusive distribution function using equation (8).

In Figures 3(a-d), we present the results for the function $\chi(y_1,y_2)$ obtained using single- and two-particle density functions for 6-, 8-, 10-, and 12-prong events respectively. The solid curves in the figures represent the function $\chi(y_1,y_2)$ averaged over the four multiplicities considered here. The normalization for $\chi(y_1,y_2)$ has been fixed by requiring $\chi(0,0)=0$. It is clear from these figures that values for the function $\chi(y_1,y_2)$ obtained from different multiplicities are consistent with each other within the statistical uncertainties involved in the data and in the approximation employed to solve equation (13). A noticeable and interesting feature of $\chi(y_1,y_2)$ is that it possesses a rather broad minimum for $y_1=y_2$ in the range $-2 < y_1, y_2 < 2$. This is clearly a reflection of the short range order exhibited by two-particle correlation functions.

We have verified that the function $\chi(y_1,y_2)$ obtained as above, together with equation (8) reproduces correctly the one-, two- and three-particle semi-inclusive densities to a fair degree of accuracy. As an example, we show in Figure 4, $\rho_8(y_1)$, $\rho_8(y_1,y_2=0)$ and $\rho_8(y_1,y_2=0, y_3=0)$ calculated according to equation (8) using the n-averaged $\chi(y_1,y_2)$. Agreement between the calculations (solid curve) and the data (closed triangles) is satisfactory. This completes the construction of a general hth order semi-inclusive distribution function for an n particle final state with the knowledge of one- and two-particle spectra only.

5. Conclusions

In conclusion, we have found that for $n > 4$, the three-particle density function can be given simply in terms of the one- and two-particle densities. We have described and tested the validity of two independent approximations for $\rho_n(1,2,3)$. These reproduce the data fairly well over all phase space, with small deviations occurring when the rapidities of two or more particles are similar. Using one of these approximations, we have constructed and solved an example to predict all higher order distribution functions in terms of a basic two-particle function, $\chi(y_1,y_2)$. The functions obtained from different multiplicity data are consistent with each other within the statistical uncertainties of the data and the method employed to solve the integro-differential equation. We have verified that this function, $\chi(y_1,y_2)$, together with equation (8) reproduces correctly the one-, two- and three-particle semi-inclusive densities.

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Table 1. The semi-inclusive three-particle correlation C_n for $y_1 \approx y_2 \approx y_3 \approx 0$ (within ±0.5 units) for different charged multiplicities n.

n	No. of Events Used	$\rho_n(0)/n$	$\frac{\rho_n(0,0)}{n(n-1)}$	$\frac{\rho_n(0,0,0)}{n(n-1)(n-2)}$	$C_n(0,0,0)$
4	1880	0.117±.003	0.0188±.0010	0.0032±.0003	-0.0002±.0005
6	1944	0.181±.004	0.0384±.0008	0.0095±.0002	0.0005±.0006
8	1454	0.216±.004	0.0494±.0008	0.0114±.0002	-0.0003±.0007
10	1024	0.233±.005	0.0553±.0008	0.0142±.0002	-0.0006±.0010
12	587	0.280±.006	0.0816±.0010	0.0246±.0003	-0.0000±.0014

FIGURE CAPTIONS

Figure 1. The three-particle rapidity density function for 8-prong events, with y_1 fixed (within ±0.5 units) at (a) -3.0, (b) -2.0, (c) -1.0, and (d) 0.0. The dashed (solid) curves represent the approximation of Equation (6) (Equation (7)).

Figure 2. The three-particle density function for (a) 4-, (b) 6-, (c) 10- and (d) 12-prong events. In each case y_1 is fixed within ±0.5 units of 0.0. The dashed (solid) curves represent the approximation of Equation (6) (Equation (7)).

Figure 3. The two-particle function $\chi(y_1,y_2)$ obtained using single- and two-particle density functions for (a) 6-, (b) 8-, (c) 10-, and (d) 12-prong events. The curve represents the χ function based on the average of 6-12 prong events.

Figure 4. One-, two-, and three-particle density functions for 8-prong events. The calculated values are shown by solid lines and data by closed triangles. For the data, $y_2 = 0.0 \pm 0.5$ for (b) and $y_2 = y_3 = 0.0 \pm 0.5$ for (c). Errors on the data points are less than the size of the triangles.

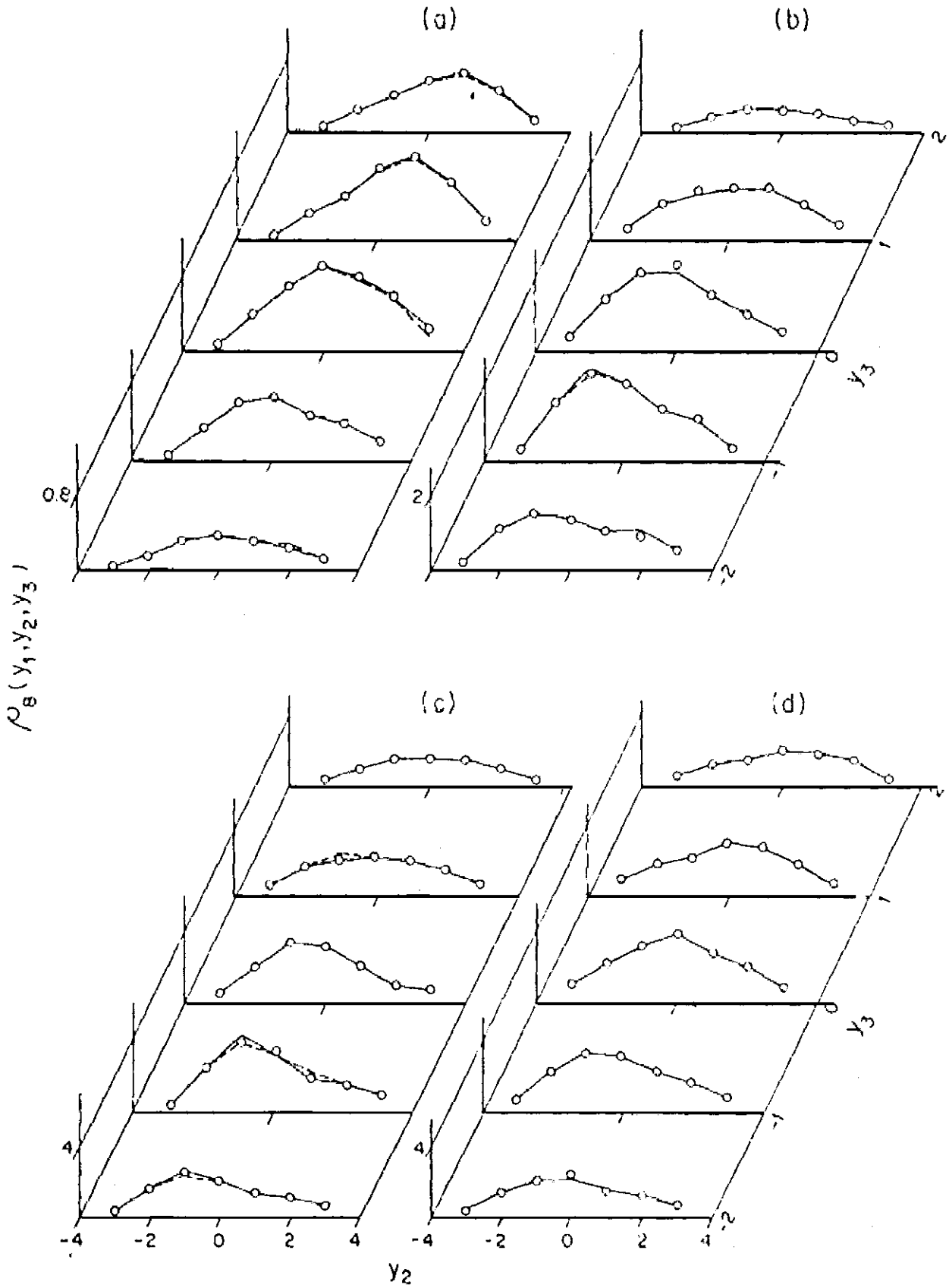

Figure 1

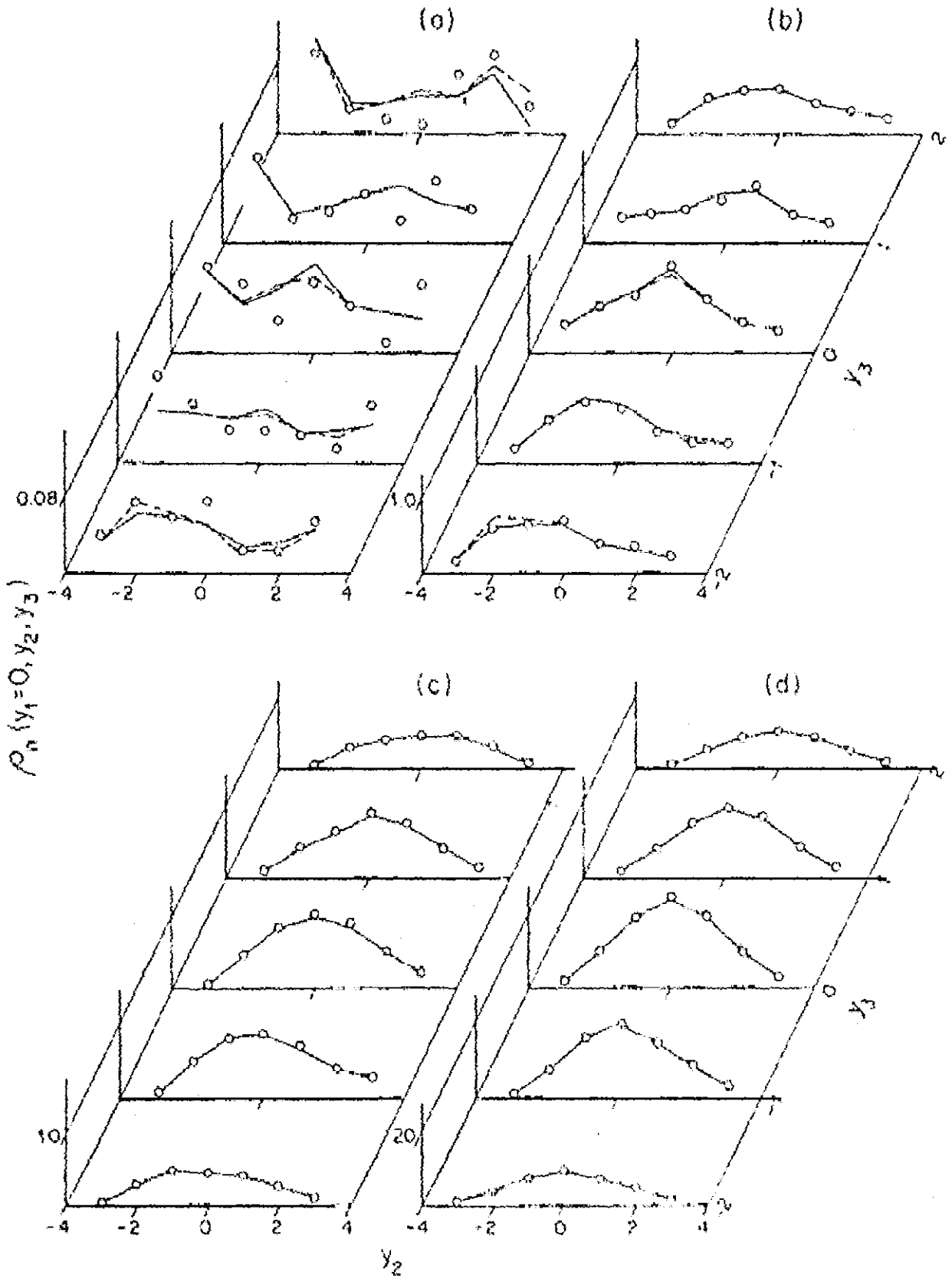

Figure 2

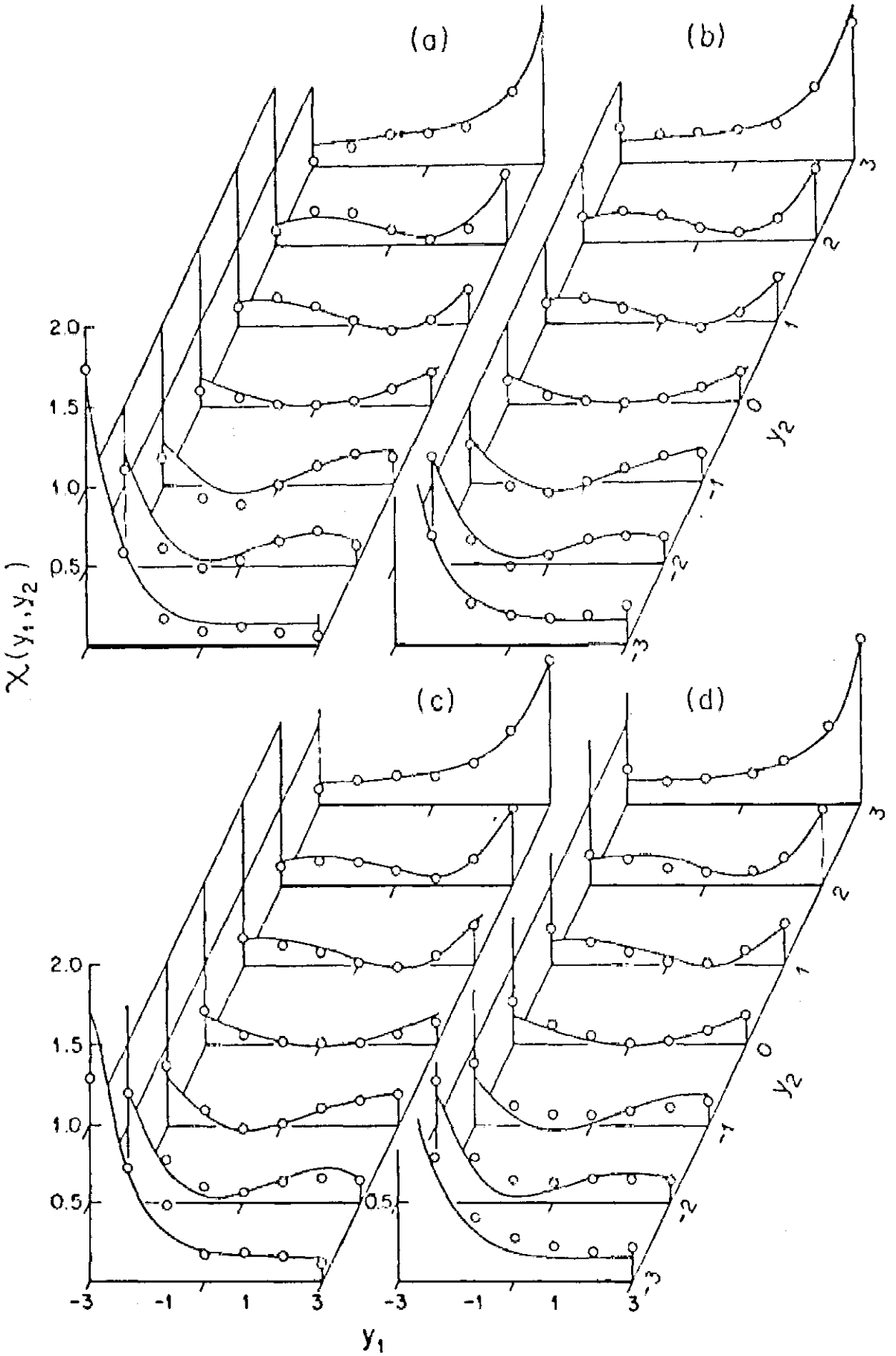

Figure 3

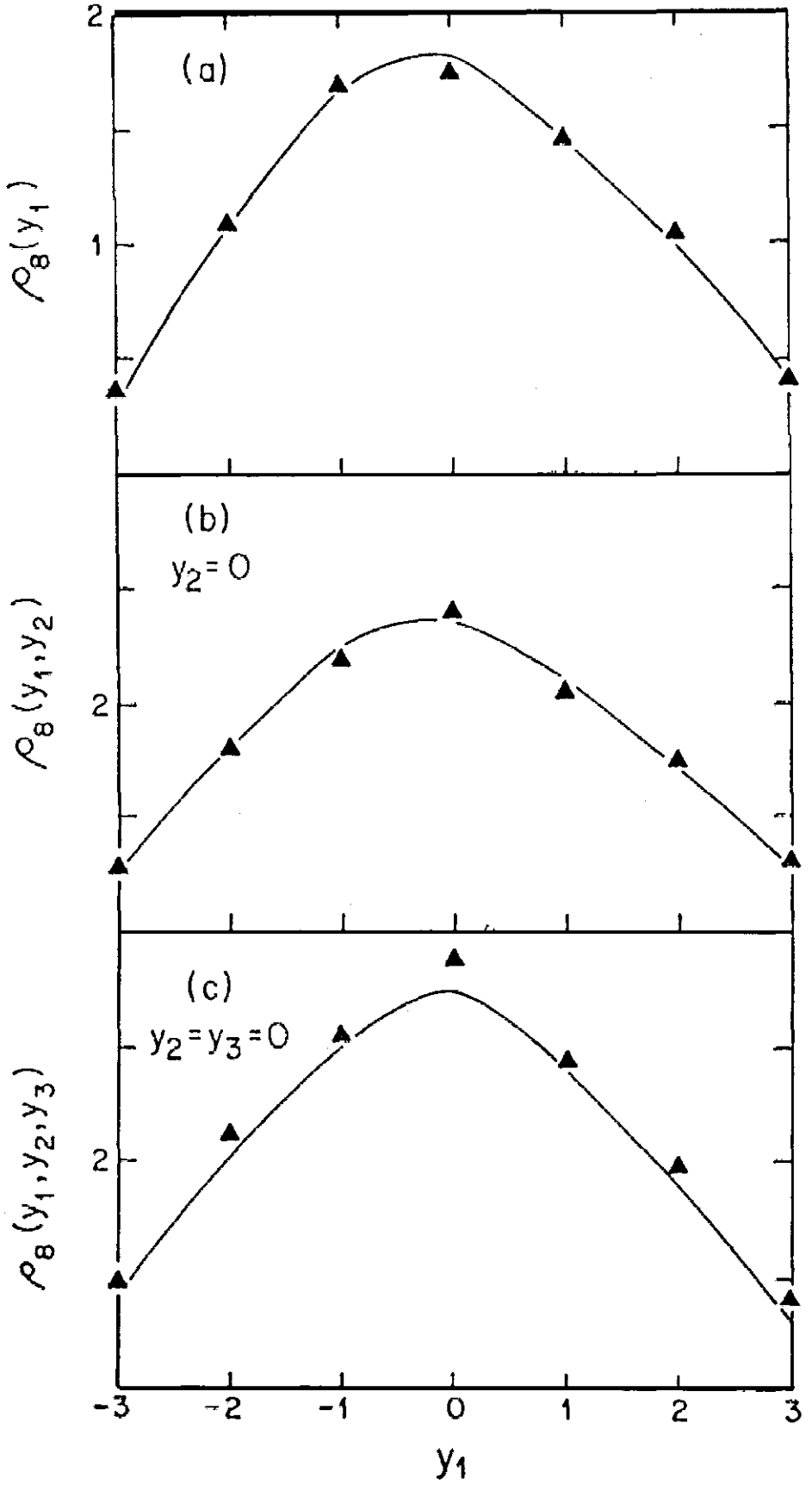

Figure 4