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The Case Against the Pseudoscalar Nonet and SU(4) Symmetry in Meson Spectroscopy*

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ABSTRACT

The standard mixing recipe for pseudoscalar nonets and SU(4) 15-plets is questioned and the necessity for more complicated mixing including radially excited configurations is demonstrated. Experimental tests for the validity of simple nonet mixing are discussed, with present evidence from sum rules in favor of an alternative description in which the η is described by conventional mixing but the η' has additional mixing of radially excited states. Results are ambiguous because sum rules depend on additional assumptions beyond mixing like the OZI rule. More conclusive tests are suggested involving predictions for the decays of the new particles into channels containing the η and η' which assume only simple nonet mixing and SU(3), with no further assumptions.

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The Okubo nonet ansatz[1] has been remarkably successful for vector and tensor meson classifications and couplings and led to standard recipes for mixing angles, masses and couplings. The nonet scheme has been extended to the 16-plet with SU(4) symmetry[2] to include new particle states made from charmed quarks. But troubles arise in the treatment of pseudoscalar mesons. This paper suggests that the conventional approach is unjustified and inconsistent for the pseudoscalars.

The inconsistency is most easily seen by examining possible mixing between new charmonium states and old states, commonly attributed to annihilation of the charmed quark-antiquark pair into vector gluons[3] and creation of a light quark-antiquark pair. But the usual procedure of mixing only the lowest configuration of the light quark spectrum has no justification. Most models predict the existence of many excited light-quark states with the same quantum numbers; e.g. as radial excitations in quark models or as daughters in Regge pole and dual models. The experimental discovery of the ψ' and its production in e^+e^- annihilation show the presence in the charmonium spectrum of such excitations with couplings to the one photon intermediate state comparable to the ground state coupling. The light quark spectrum should have similar excited states with transition matrix elements to photon or gluon intermediate states comparable to those of ground state configurations and much closer in mass to the new particles.

The use of only ground state configurations in mixing calculations is thus unjustified for systems involving both new and old particles where the annihilation mechanism is important. We now examine the difference between this case and that of the old vector and tensor mesons where ground state mixing has been very successful in order to specify explicitly the conditions under which mixing in the truncated ground state subspace is justified. We then apply these criteria to the pseudoscalar mesons.

Two crucial conditions are satisfied in cases where ground state mixing succeeds: 1) A reasonably good SU(3) symmetry limit exists with the lowest singlet and octet states nearly degenerate and well separated from higher excited states; 2) The dominant symmetry-breaking interaction has only a very weak spatial dependence and does not mix states with orthogonal wave functions.

The vector and tensor mesons are well described by a nonet symmetry limit with nine degenerate mesons with the symmetry broken only by a mass difference between strange and nonstrange quarks. The nonet degeneracy suggests treatment of symmetry breaking by conventional degenerate perturbation theory; i.e. removing the degeneracy by diagonalizing the symmetry-breaking interaction in the subspace of the degenerate unperturbed states. The absence of radial dependence in the dominant symmetry-breaking interaction, the mass difference,causes the mixing matrix elements between the ground and radially excited states to vanish because of the orthogonality of the radial wave functions. Even if the radial wave functions are not exactly orthogonal because of differences in potentials and masses between strange and nonstrange quarks the overlap integral between a nodeless ground state wave function and an excited state wave function with nodes remains small.

In the mixing of old and new particles the 16-plet symmetry limit is not good because the mass difference is too large and spreads the 16-plet of ground state wave functions over a mass range which includes many other radially excited states with the same quantum numbers. If the only interaction breaking the 16-plet symmetry is the mass term, the radial dependence argument still holds and SU(4) gives a good description of a mass spectrum with ideal mixing and couplings which satisfy the Okubo-Zweig-Iizuka[4] rule. This may well apply to the vector states including the ψ. But this argument breaks down as soon as other symmetry-breaking interactions are included which have radial dependence. Annihilation and pair-creation via the gluon intermediate state destroys the simple mixing scheme because no overlap integral between initial and final states appears in the transition matrix element, while a strong radial dependence appears in the operator which emphasizes the behavior of the wave function near the origin in both initial and final states.

For the pseudoscalar mesons an intermediate situation exists. Nonet symmetry is not good. There is a wide spread of pseudoscalar meson masses. Although the lowest nine states are not spread to the point where they overlap with radial excitations, the mass difference between the η' and the η is probably not much smaller than that between the η' and the lowest radially excited pseudoscalar. Radial mixing is thus not obviously ruled out. The dominant feature of the mass splittings is not the quark mass difference but an SU(3)-invariant interaction which splits the singlet from the octet and shifts the center-of-gravity of the η and η' far above the kaon mass. (A quark mass term leaves the center of gravity of the two isoscalar states degenerate with the strange meson, as in the vector and tensor nonets). The nature of this interaction which splits the singlet and octet is unknown. If it comes about via diagrams involving annihilation into gluons, it could well mix in radially excited wave functions into the unitary singlet.

An alternative description of the pseudoscalar mesons begins with nonet symmetry badly broken but SU(3) still reasonably good, as indicated by the approximate validity of the Gell-Mann-Okubo mass formula for the lowest eight pseudoscalars. There are two unperturbed SU(3) spectra, octet and singlet, each with a ground state and radial excitations. The lowest set of states is the ground state configuration of the unitary octet, next comes the ground state singlet, then the first radially excited octet, etc. The SU(3) symmetry-breaking interaction is treated as a perturbation, but mixing cannot be limited to the lowest nine states. As a first approximation, one might try "nearest-neighbor mixing" in which the mixing into each state is dominated by the other states closest in mass. This gives the conventional description of the η with only ground state singlet and octet configurations, as the first radially excited singlet is rather far away in the mass spectrum. The η' would differ from the conventional description by mixing the ground state singlet configuration with both the ground and the radially excited octet states. The η could thus be described by the standard mixing-angle formalism as a singlet-octet mixture. But the η' would no longer be the orthogonal state described by the same mixing angle; it would have a piece with a different wave function.

This analysis is not easily carried further without a detailed model, and no convincing model is available at present, particularly for light quarks where the asymptotic freedom arguments sometimes used to justify charmonium spectroscopy do not hold. However, general features of the mixing of radial excitations can be tested experimentally. We therefore consider such experimental tests.

The use of neutral meson production processes for experimental tests of naive mixing and the determination of mixing angles was first suggested by Alexander.[5] The sum rules derived by Alexander et al for charge exchange and strangeness exchange processes have provided the basis for all subsequent comparisons with experiment.

$$\sigma(\pi^- p \to \pi^o n) + \sigma(\pi^- p \to \eta n) + \sigma(\pi^- p \to \eta' n)$$

$$= \sigma(K^+ n \to K^o p) + \sigma(K^- p \to \bar{K}^o n), \tag{1a}$$

$$\sigma(K^- p \to \eta Y) + \sigma(K^- p \to \eta' Y) = \sigma(K^- p \to \pi^o Y) + \sigma(\pi^- p \to K^o Y). \tag{1b}$$

Analogous sum rules were derived for vector and tensor mesons. The original derivation was based on the quark model for high energy scattering without assuming SU(3) for vertices. The only assumption used beyond naive mixing is that the transition between the incident meson state to the final neutral meson is described by a single quark operator with the other quark remaining a spectator. Subsequent derivations with SU(3) symmetry[6] together with other assumptions often unspecified have left the misleading impression that some sum rules were better than others although all are derivable from the same sets of assumptions. The results at present can be summarized with the conclusion that excellent agreement is obtained for all sum rules involving vector mesons, while disagreement is found for sum rules involving pseudoscalar mesons[7] except for one peculiar relation.[8] If SU(3) symmetry is assumed and the η is assumed to be a pure octet state, the contribution of the η' can be eliminated from the charge exchange sum rule (1a) and agreement with experiment is obtained. In all other cases the disagreement is in the direction of too small a contribution from η and η' production to any sum rule.

In examining the disagreements to determine which assumption has gone wrong, the quark model derivation gives the clearest insight. This "single quark" assumption has been so successful in all other mesonic transitions[9,10] that it seems reasonable to attribute the failure of the sum rule to the breakdown of naive mixing. The same is true for the SU(3) derivations which avoid mentioning quarks, but bring in the quark model through the back door by assuming no exotic exchanges and the OZI rule. It is hard to see why either the single quark assumption, absence of exotic exchanges or the OZI rule should break down in this case and hold so well everywhere else, including the exactly analogous sum rules for vector meson production. However, this conclusion is not rigorous and an alternative test without additional assumptions is desirable.

If the sum rules fail because of breakdown of naive mixing, the data indicate the trouble to be mainly in the η'. The success of the SU(3) charge exchange sum rule which tests the non-strange part of the wave function suggests that the non-strange part of the η belongs in the same octet as the pion and kaon. The η', however, must have an inert piece in the wave function which does not contribute to the sum rule; e.g. a piece with the radial dependence of a radial excitation. In the quark model derivation this additional piece does not contribute to the transition because of its small overlap with the incident meson wave function. This discussion can provide guide lines for the kinds of experimental disagreements to be expected in experimental tests of naive mixing which do not involve the quark model or the OZI rule.

The new particles at high mass which are SU(3) singlets and decay into channels including the η and η' enable tests of the SU(3) properties of these states, without the additional assumption of the OZI rule or the quark model for reactions. Predictions based on SU(3) symmetry and standard nonet mixing can be compared with experiment. If disagreement is found, it would provide the best evidence so far against naive mixing of ground state configurations only for pseudoscalar mesons.

Decays into the vector-pseudoscalar channel have been observed for the ψ and measurements for the $\phi\eta$, $\phi\eta'$, $\omega\eta$ and $\omega\eta'$ final states can determine whether the simple naive mixing picture is valid for these states. These four decay amplitudes are expressed in terms of the two invariant SU(3) amplitudes which we denote by A_8 and A_1 respectively for the octet-octet and singlet-singlet final states. The octet-octet amplitude A_8 is determined from other decays such as $\rho\pi$ and $K^*\bar{K}$, and the validity of SU(3) symmetry is verified by the relation between these two. Thus if the naive mixing description for the η and η' is valid, the four decay rates for final states involving the η and η' are expressed in terms of three unknown real parameters; the pseudoscalar mixing angle and the magnitude and relative phase of the singlet-singlet amplitude A_1. One predicted relation between these four rates can therefore be obtained which depends only on SU(3) and on the assumption of naive mixing; i.e., that the mixing is confined to a 2×2 space of singlet and octet states. If in addition to SU(3) the OZI rule is assumed to hold for the decay, the magnitude and relative phase of A_1 are determined and two additional relations are obtained. Thus, a study of the vector-pseudoscalar decay modes can test the validity of SU(3) symmetry, the validity of the OZI rule and the validity of the naive nonet mixing model for the pseudoscalars.

We now list the predictions explicity. We begin with those obtained from the assumptions of SU(3) symmetry and naive mixing without the OZI rule. Ideal mixing for the ω and ϕ is assumed, since known deviations are small. The one equality obtained is the sum rule:

$$2\Gamma(\psi \to \phi\eta) + 2\Gamma(\psi \to \phi\eta') = \Gamma(\psi \to \rho^+\pi^-) + \Gamma(\psi \to \omega\eta) + \Gamma(\psi \to \omega\eta'), \tag{2a}$$

where Γ denotes the reduced width without phase space corrections. In addition the following relations are obtained,

$$\Gamma(\psi \to \omega\eta) + \Gamma(\psi \to \omega\eta') = \Gamma(\psi \to \rho^+\pi^-)\,(1 + 2|A_1/A_8|^2)/3 \tag{2b}$$

$$\Gamma(\psi \to \phi\eta) + \Gamma(\psi \to \phi\eta') = \Gamma(\psi \to \rho^+\pi^-)\,(2 + |A_1/A_8|^2)/3\ . \tag{2c}$$

These relations give testable inequalities without additional assumptions, since the quantity $|A_1^2/A_8^2|$ is positive definite.

The validity of SU(3) symmetry for the decay is tested independent of the nonet mixing assumption by the relation which does not involve any mixed mesons

$$\Gamma(\psi \to \rho^+\pi^-) = \Gamma(\psi \to K^{*+}K^-) \quad . \tag{3}$$

This relation is found to be roughly in agreement with experiment, and suggests that SU(3) can be used.

When the OZI rule is assumed, Eqs. (2) hold with $A_1/A_8 = 1$ and the following additional relation is obtained:

$$\Gamma(\psi \to \omega\eta) = \Gamma(\psi \to \phi\eta') . \tag{4a}$$

This can be combined with the realtions (2) to obtain other simple relations

$$\Gamma(\psi \to \phi\eta) = \Gamma(\psi \to \omega\eta') \tag{4b}$$

$$\Gamma(\psi \to \omega\eta) + \Gamma(\psi \to \phi\eta) = \Gamma(\psi \to \rho^+\pi^-) \tag{4c}$$

$$\Gamma(\psi \to \omega\eta') + \Gamma(\psi \to \phi\eta') = \Gamma(\psi \to \rho^+\pi^-) . \tag{4d}$$

The relations (4b-d) are not linearly independent of the previous relations. However, if some relations disagree with experiment and others agree, these different combinations can furnish clues to determine what has gone wrong. For example. if the OZI rule holds and naive mixing breaks down for the η' but still holds for the η, then relation (4c) which involves only the η might agree with experiment while other relations like (2) and (4d) which involve the η might not. If there is an inert piece in the η' wave funtion, the right hand sides of (4a) and (4b) and the left hand side of (4d) would all be suppressed by the same factor.

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