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Peripheral Models

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Hadron-hadron collisions evolve through several forms at different center-of-mass energies. At very low energies, below about 2 GeV, elastic scattering is the most common occurrence. Except for combinations of particles with "exotic" quantum numbers (those not present in SU(3) singlets and octets of mesons or in singlets, octets, and decimets of baryons), resonant scattering is the norm. At c.m. energies exceeding about 2 GeV, inelastic scattering accounts for a major fraction of the total cross section. Over a significant range of energies most of the inelastic scattering leads to two (stable or unstable) particle final states. Multiparticle production dominates at c.m. energies above about 8 GeV.

The quasi-two body reactions which are preëminent in the intermediate energy regime are characteristically <u>peripheral</u>, in that the outgoing hadrons are sharply collimated about the forward or backward directions in the c.m. frame. A representative angular distribution is shown in Fig. 1. This article is concerned with the phenomenology of peripheral reactions.

It is convenient to describe the structure of the differential cross section in terms of the Lorentz-invariant Mandelstam variables as indicated in Fig. 2. The kinematic variables

$$s = (p_a + p_b)^2 = (p_c + p_d)^2 = (\text{c.m. energy})^2,$$

$$\left.\begin{aligned} t &= (p_a - p_c)^2 = (p_b - p_d)^2 \\ &\approx -\frac{s}{2}(1 - \cos\theta_{c.m.}), \\ u &= (p_a - p_d)^2 = (p_b - p_c)^2 \\ &\approx -\frac{s}{2}(1 + \cos\theta_{c.m.}), \end{aligned}\right\} \text{(invariant momentum transfer)}^2$$

are related by the constraint $s + t + u = m_a^2 + m_b^2 + m_c^2 + m_d^2$. According to the principle of crossing, a statement about the analyticity of the quantum mechanical scattering amplitude, a single analytic function of the kinematic invariants describes the three reactions

$a + b \rightarrow c + d$ (s-channel reaction),

$a + \bar{c} \rightarrow \bar{b} + d$ (t-channel reaction),

$a + \bar{d} \rightarrow \bar{b} + c$ (u-channel reaction).

The peripheral peaks in the differential cross section $d\sigma/dt$ occur as fixed-t (or fixed-u) structures which first appear in the resonance region and persist, largely unchanged, to the highest energies studied. These structures provide a means for deducing the range of the interactions responsible for inelastic reactions. Let us consider for illustration a fixed-t zero in the differential cross section for the scattering of spinless particles. The angular dependence for scattering in the ℓ-th partial wave is specified by the Legendre polynomial $P_\ell(\cos\theta_{c.m.})$ or, in terms of the Mandelstam variables, $P_\ell(1+2t/s)$. In order for the first zero of the Legendre polynomial to remain at a fixed position in t as the energy varies, the most prominent partial wave must vary as $\ell(s) \sim s^{1/2}$. On a geometrical view, if the maximum radius of interaction is fixed at a value R, the maximum c.m. orbital angular momentum (or equivalently, the highest partial wave excited) is given by $\ell_{max}(s) \propto R s^{1/2}$. It is appealing to infer that the most important partial waves for generating fixed-t structures are the most peripheral ones which arise in glancing collisions of hadrons. Indeed, the interaction radius implied by this sort of analysis is on the order of one fermi (10^{-13} cm), which is consistent with the known range of nuclear potentials.

The analogy of the Yukawa force suggests that the peripheral interaction be associated with the exchange of quanta of low mass. For example, the exchange of a spinless particle of mass μ yields an interaction of the form $e^{-\mu r}/r$ in position space, where r measures the separation of the colliding particles. This corresponds to a momentum space propagator proportional to $(t-\mu^2)^{-1}$. Thus a forward-peaked differential cross section, the shape of which is independent of s, arises naturally from a one-particle exchange interaction.

This intuitively reasonable association of particle exchange in the crossed channel with peripheral scattering turns out to be correct and underlies the theory of two-body reactions. Without important exception all occurrences and nonoccurrences of peripheral peaks can be understood in terms of the known [SU(3) multiplets of] mesons and baryons. The correspondence of forward peaks with t-channel exchanges and of backward peaks with u-channel exchanges is indicated in Fig. 3. As this discussion would suggest, it is useful to classify reactions according to the quantum numbers exchanged in the crossed channel. For c.m. energies around 3.25 GeV the peripheral peak cross sections are 100-300 μb. [1 μb. = 10^{-30} cm^2] for nonstrange meson exchange, 10 - 100 μb. for strange meson exchange, and 1 - 10 μb. for baryon exchange. Reactions which require the exchange of exotic quantum numbers, forbidden in the one-particle-exchange picture, have peripheral cross sections which do not exceed 1 μb.

Identification of the object exchanged in the t-channel (or u-channel) as a specific particle carrying the requisite quantum numbers suggests that beam particle--(virtual) exchanged particle collisions provide access to the study of scattering from unstable targets. Pion-pion scattering has been studied extensively by this technique. The identification also has implications for the energy dependence of the cross section and for spin correlations. The latter have been of special importance in separating the contributions of different exchanges to one reaction and in indicating the form of refinements to the peripheral exchange picture. The implied energy dependence indicates a shortcoming of the elementary particle exchange model, as follows. To the exchange of a particle with spin J and mass M_J between spinless particles there corresponds an amplitude

$$A_J(s,t) = \frac{f_{Ja\bar{c}}(t)\, f_{J\bar{b}d}(t)}{t - M_J^2} \left[p_{a\bar{c}}\, p_{\bar{b}d}\right]^J P_J\,(\cos\theta_t),$$

where the form factors f_{Jij} depend only upon the invariant momentum transfer, $p_{a\bar{c}}$ and $p_{\bar{b}d}$ are the incident and outgoing momenta in the t-channel c.m. frame, and the t-channel scattering angle is given by $\cos\theta_t = (s-u)/4\, p_{a\bar{c}}\, p_{\bar{b}d}$. The origin of the threshold factors $(p_{a\bar{c}})^J$ and $(p_{\bar{b}d})^J$ is classical; the Legendre polynomial $P_J\,(\cos\theta_t)$ defines a spin-J object in the t-channel. Taking the large-argument limit $P_J(Z) \xrightarrow[|Z|\to\infty]{} Z^J$ appropriate for large values of s and small values of t, we arrive at the asymptotic form

$$A_J(s,t) \underset{\substack{s\to\infty,\\ t\text{ fixed}}}{\sim} \frac{f_{Ja\bar{c}}(t)\, f_{J\bar{b}d}(t)}{t - M_J^2} \left(\frac{s-u}{4}\right)^J ,$$

which implies that the amplitude for exchange of a spin-J object is proportional to s^J, independent of any form factor behavior. In a number of cases of interest, the predicted energy dependence of $d\sigma/dt \propto |A|^2/s^2$ conflicts with experiment. Furthermore, for spins $J > 1$ it violates Froissart's rigorous upper bound on the growth of the total cross section: $\sigma_{total} \leq \text{constant} \times \log^2 s$.

The Regge pole hypothesis (discussed at greater length in the article on Regge theory) provides an escape from this difficulty. It is known that for a given set of internal quantum numbers there exist several resonances with different spins and masses. For the exchange of an infinite sequence of resonances with spins given by $J(M^2) = \alpha_o + \alpha' M^2$, for example, the asymptotic behavior of the scattering amplitude is proportional to $s^{J(t)}$. This connection between the resonance spectrum and high-energy behavior of scattering amplitudes is in accord with all experimental observations.

The extension of the peripheral exchange picture to multiparticle production processes is known as the multiperipheral model. Although the multiperipheral model for particle production is less firmly established than the peripheral exchange picture for two-body scattering, many of its predictions have been verified. Thus the idea that two-body reactions are mediated by the exchange of (families of) particles in the crossed channel is basic to the understanding of high-energy collisions.

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FIGURE CAPTIONS

Fig. 1: Peripheral angular distribution typical of inelastic two-body reactions at intermediate energies.

Fig. 2: Two-body reaction kinematics.

Fig. 3: Three descriptions of two-body scattering. Below $E_{c.m.} \approx 2$ GeV, direct-channel formation of discrete resonances prevails, and angular distributions are characteristic of fixed J in the s-channel. At higher energies, the peripheral exchange picture is more economical. Forward scattering is driven by t-channel exchange; backward scattering by u-channel exchange.

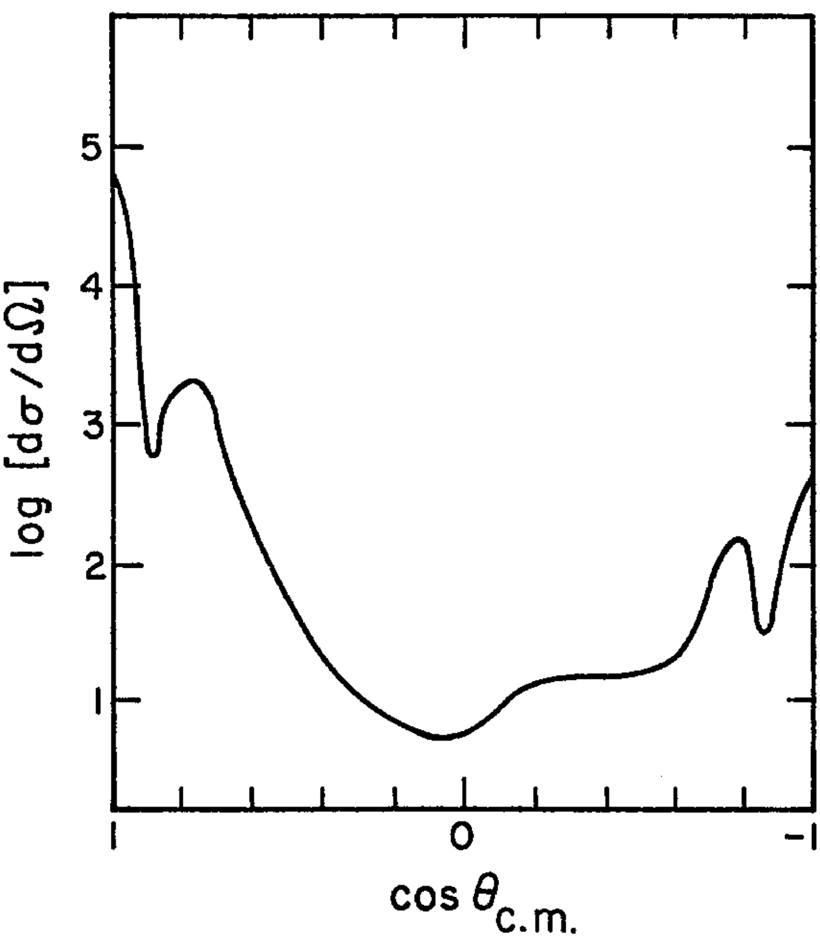

Fig. 1

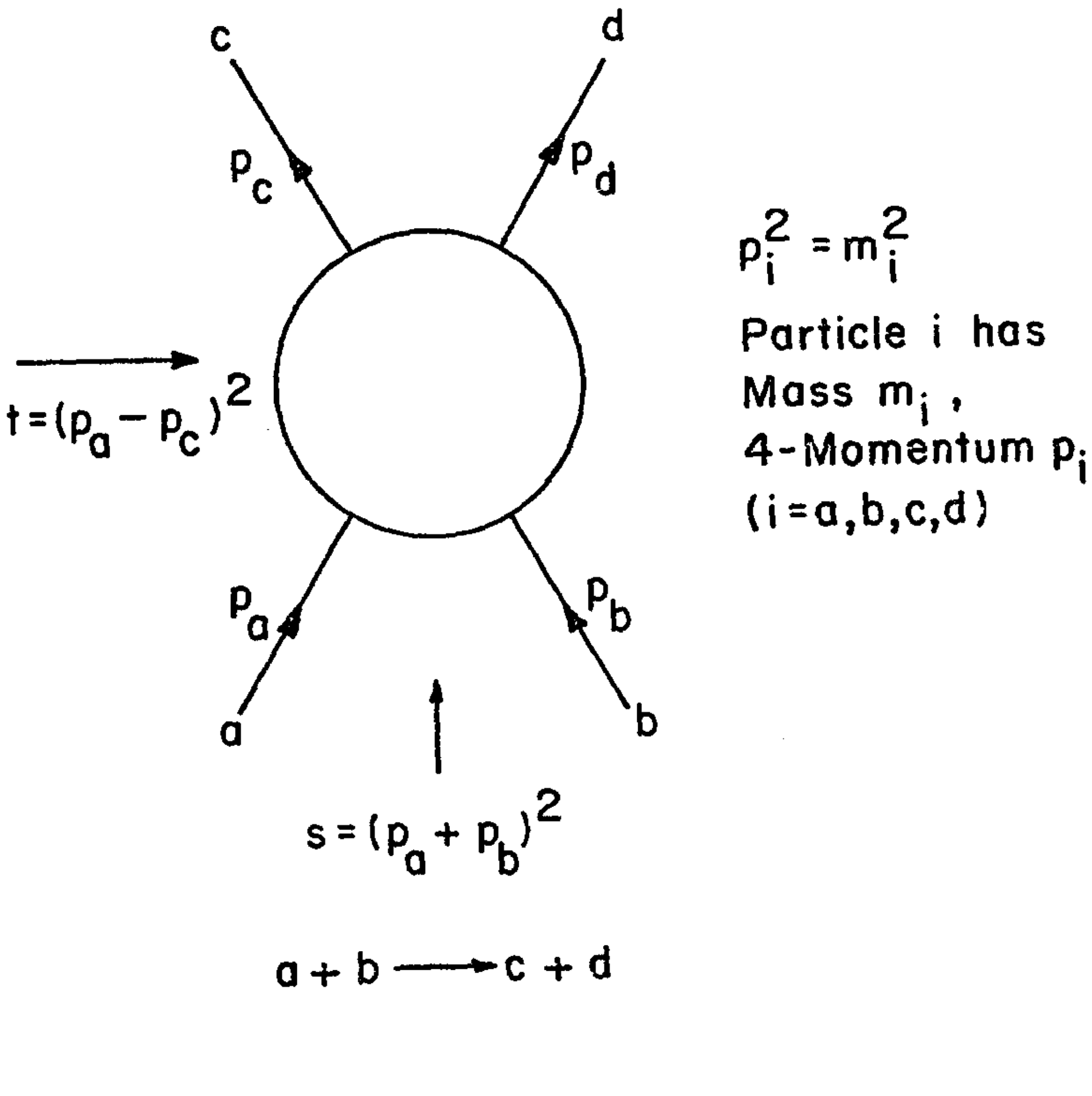
c
d
p_c
p_d
$p_i^2 = m_i^2$
Particle i has
Mass m_i ,
4-Momentum p_i
$(i = a,b,c,d)$
$t = (p_a - p_c)^2$
p_a
p_b
a
b
$s = (p_a + p_b)^2$
$a + b \longrightarrow c + d$

Fig. 2

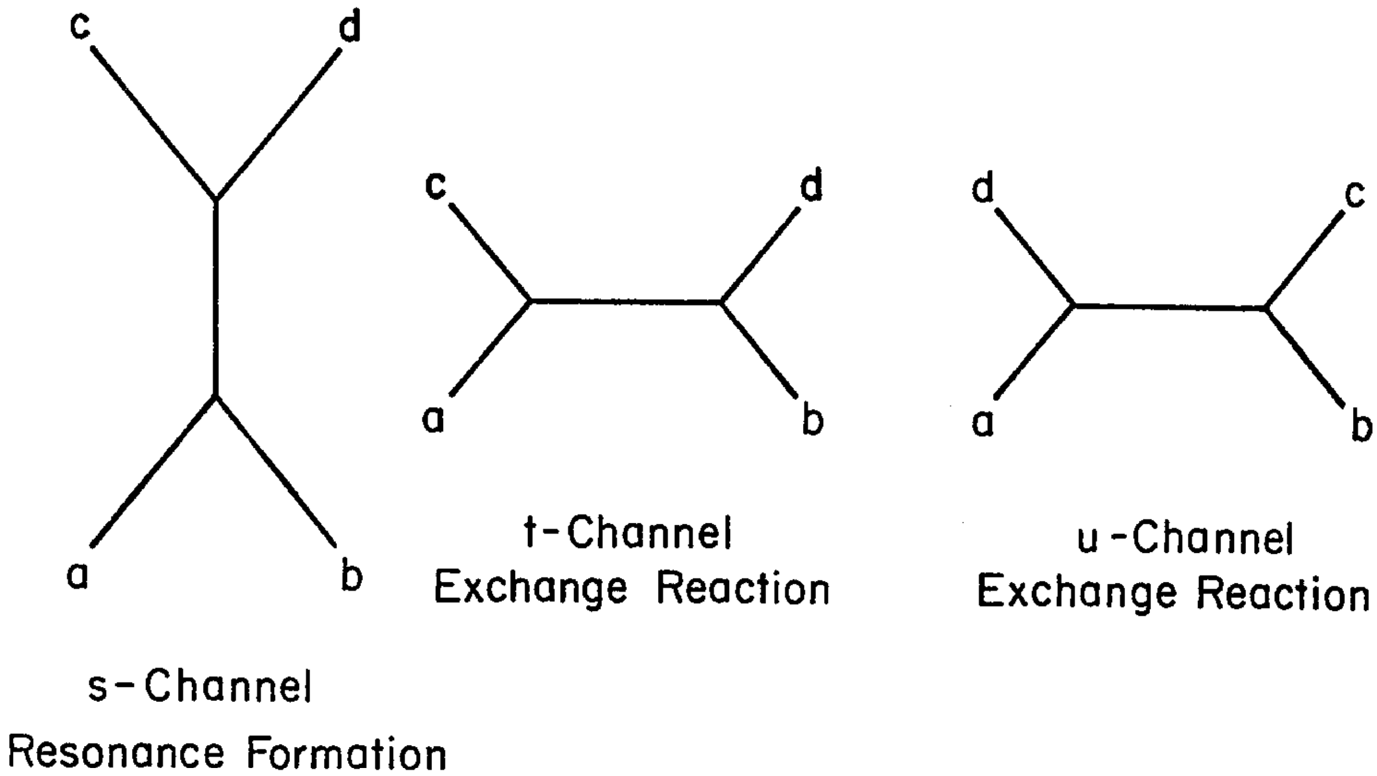
c
d
a
b
s-Channel
Resonance Formation
c
d
a
b
t-Channel
Exchange Reaction
d
c
a
b
u-Channel
Exchange Reaction

Fig. 3