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CAN PEDESTRIANS UNDERSTAND THE NEW PARTICLES or WHO NEEDS THE OKUBO-ZWEIG-IIZUKA RULE *

Harry J. Lipkin

Weizmann Institute of Science, Rehovot, Israel
Argonne National Laboratory, Argonne, Illinois 60439
Fermi National Accelerator Laboratory†, Batavia, Illinois 60510

## I. INTRODUCTION

What is so special about the new particles and why is there such excitement? So many old particles have been known for a long time that the discovery of an additional particle has created very little excitement. An unbiased observer from another field would conclude "seen one particle, seen them all." However, the new particles were immediately seen to be peculiar and interesting because they are very narrow states at high excitation and nobody still understands why these states are so narrow. The only argument given supporting the narrowness is based on the Okubo-Zweig-Iizuka rule, but nobody understands the OZI rule even for the old particles, where many interesting open questions still remain. There must be interesting physics in this rule worthy of further theoretical and experimental investigation. The major part of these talks is devoted to interesting questions regarding the theoretical validity and possible experimental tasks of the OZI rule.

In trying to explain to some of my nuclear colleagues why these new particles are so exciting and interesting, I first tell them they are narrow resonances that appear at much too high excitation for their narrowness. The nuclear physicist says, "We know about those things. We have them too; isobaric analog[1] states". Then I explain that they were completely unexpected. No one thought that they were going to be there. He says, "Of course. Theorists didn't expect isobaric analog resonances either. But as soon as they were found, they had the explanation".

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Then I say, "That's the difference. As soon as the new particles were found, all the theorists came out with explanations, but all of them are wrong. And they still don't understand why the particles are so narrow." As soon as the new particles were found theorists dug into their old files and tried to show that their old theories really predicted these particles. One theorist actually quoted the reference to an old paper where he claimed he had predicted these particles. One of our nasty graduate students actually looked up the reference and gleefully circulated the abstract around the department. The abstract said that this was the only paper explaining the new weak neutral currents without requiring the existence of new particles for which there was no experimental evidence. The status of the new particles is well described by the following quotation:

> "I have no data as yet. It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts."
>
> ....A. Conan Doyle
> 'The Adventures of Sherlock Holmes'

Niels Bohr developed his model of the atom on the basis of the experimental data of the Balmer series. Much hard work by many people was then needed to get modern quantum mechanics started. But today's theorists are trying to develop a theory for the new particles as beautiful as modern quantum mechanics when they don't even have the Balmer series.

During the time that this talk was being prepared and even during the Erice school itself, new data were being accumulated indicating that the new particles were those theoretically predicted by the charm scheme in which an additional fourth "charmed" quark[2] is added to the three conventional members of the quark triplet. All this charm spectroscopy was known long before the discovery of the new particles. Searches for charm have been suggested for a long time. Yet nobody suggested that SLAC search for very narrow resonances in electron-positron annihilation in the 3-GeV range. It is instructive to examine why nobody suggested such a search.

Vector meson states constructed from a charmed quark-antiquark pair were predicted long before the discovery of the new particles but they were not expected to have narrow widths. Since their decays into ordinary uncharmed states was known to violate the Okubo[3]-Zweig[4]-Iizuka[5] rule, these states were expected to be narrower than conventional uncharmed states at this mass. But there was no reliable theory underlying the OZI rule and its breaking and no theoretical calculation predicting the strength of OZI violation. The only clue was experimental OZI violating decays into nonstrange hadrons of mesons consisting of a strange quark-antiquark pair e.g., the $\phi \to \rho\pi$ decay. These indicated that OZI suppression factors were one order of magnitude, possibly two but certainly not more. This would still leave a large width for a state at 3 GeV with many open channels. Such a state would not easily be seen as a resonance in electron-positron annihilation.

Thus even if the charm model is correct, one crucial step is missing in the description of the new particles and responsible for the failure to predict their discovery. This missing link is understanding the OZI rule and why the suppression factor is very much larger for the new particles than for forbidden old-particle transitions. This question is still open and considered in detail in these talks. Some indications, but no conclusive answers, are given, but answers should not be expected from this talk. They say that when one asks a Jew a question he answers by answering with another question and when he is asked why he always answers a question by asking another question, he answers "why not?". I shall raise many questions in this talk but I shall not answer them. Instead I will raise more questions. I hope that pursuing the answers to these questions will lead to even more interesting questions and to a better understanding of hadron physics.

## II. THE SU(6) BANDWAGON

How can pedestrians understand the new particles when we still have so much trouble understanding the old particles? We still don't understand why the old hadron spectrum has been fit very successfully by an SU(6) symmetry scheme[6] which suggests that hadrons are built from elementary objects called quarks with spin 1/2 and three flavors. If hadrons are made of quarks, and the forces are independent of charge, strangeness and spin, all of these six states are equivalent and transformations among them generate an SU(6) symmetry. Particles can then be classified into SU(6) multiplets. The lowest-lying mesons and baryons fit very beautifully into two SU(6) supermultiplets, the baryons in a 56-plet, the mesons in a 35-plet and a singlet. Since the SU(6) scheme was proposed more new evidence has been found for additional 56-plets, 35-plets and 70-plets. Rosner's review[7] at the 1974 London conference listed the known SU(6) multiplets as a "Michelin Guide" in which he gave four stars, three stars, and two stars to the multiplets, depending on how well they were established experimentally. But where are the quarks?

Now we have a new SU(6), which I call the Sicilian SU(6) because it was invented by a Sicilian (with some help from another island). Arima and Iachello[8] suggest that we should not stop with the 56-plet obtained by putting three basic building blocks having six states in a totally symmetric configuration. Why not try the

252-plet or 1287-plet, obtained by using five or eight building blocks? Arima and Iachello have played the standard game of building an SU(6) supermultiplet degenerate in the symmetry limit, removing the degeneracy by using a simple ansatz for symmetry breaking and obtaining a mass formula which they compare with experiment. Figure 2.1 shows a typical hadron spectrum obtained in this way from the 1287-plet. Figure 2.2 shows a comparison with experiment of the lowest states in the 252-plet and the spectrum of hadrons with baryon number 170 and electric charge 68. Further spectra and comparisons with experiment are shown in Figs. 2.3 and 2.4.

This SU(6) bandwagon is very amusing: Particle physicists build particles from a fundamental building block with six possible states and introduce an SU(6) symmetry. But nobody has found any quarks, and more data are needed to see whether this symmetry is really there and particles are really made out of quarks.

Now the nuclear physicists have jumped on the SU(6) bandwagon. If particles can be made out of a sextet of objects that are not really there, maybe nuclei are too. But the Arima-Iachello nuclear SU(6) model is not based on elementary fermion quarks. Their building blocks have six states, but they are bosons, one with spin zero and one with spin two. Everyone knows that there are no elementary bosons in the nucleus, but the agreement with experiment shown in Figs. 2.2, 2.3 and 2.4 is just as impressive as the quark model fits to particle data. Perhaps the bosons in the nucleus are just as real or unreal as the quarks in the particles.

Why do nuclei look like composite systems of $S = 0$ and $S = 2$ bosons which nobody has seen? Why do hadrons look like composite systems of spin-1/2-three-flavored quarks which nobody has seen? Are these bosons or quarks confined? Are they in a bag? Or are they simply not there? Perhaps there is an underlying substructure which makes nuclei behave as if they were made of bosons in certain experiments and makes hadrons behave as if they were made out of quarks.

But bosons in the nucleus is not really as crazy as it sounds. We know that nuclei are made of neutrons and protons. If they pair in some fashion they may behave somehow like bosons. Before the BCS theory there were suggestions that Bose condensation of electron pairs was responsible for superconductivity. Then BCS showed how a proper treatment gave not only the properties that look like bosons but also the important differences between fermion-pairs and real bosons. So maybe there is something in this boson model of the nucleus and we are waiting for the right theoretical description.

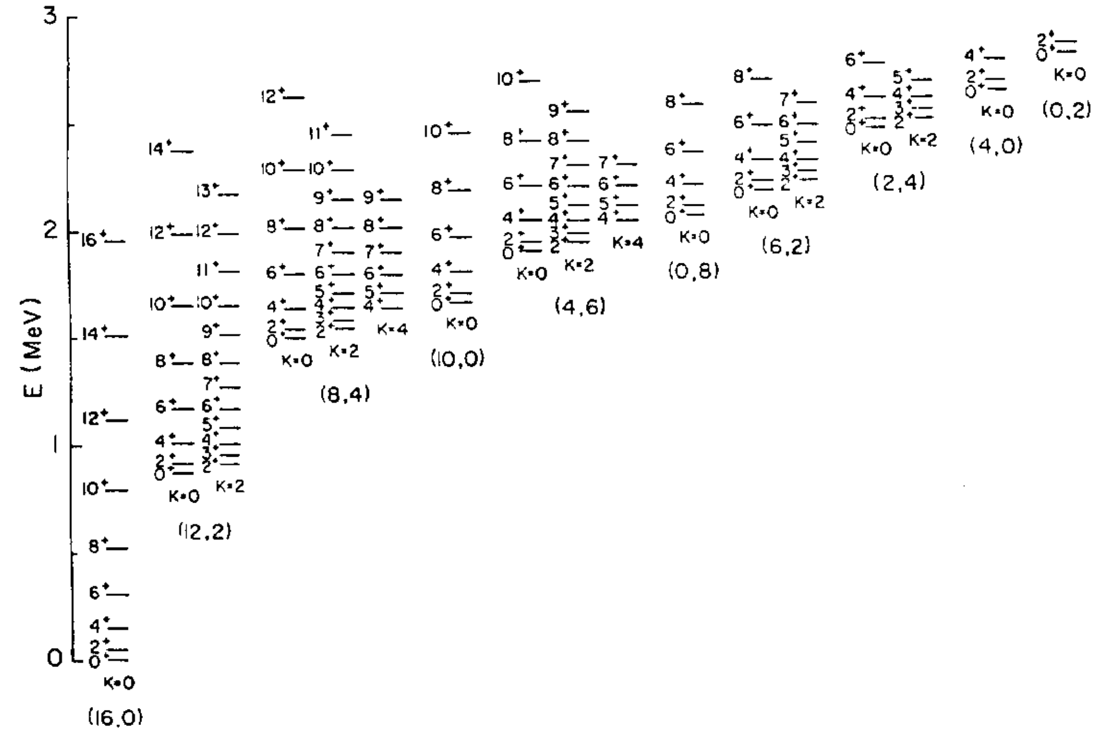



Fig. 2.1. Typical hadron spectrum from 1287-plet.

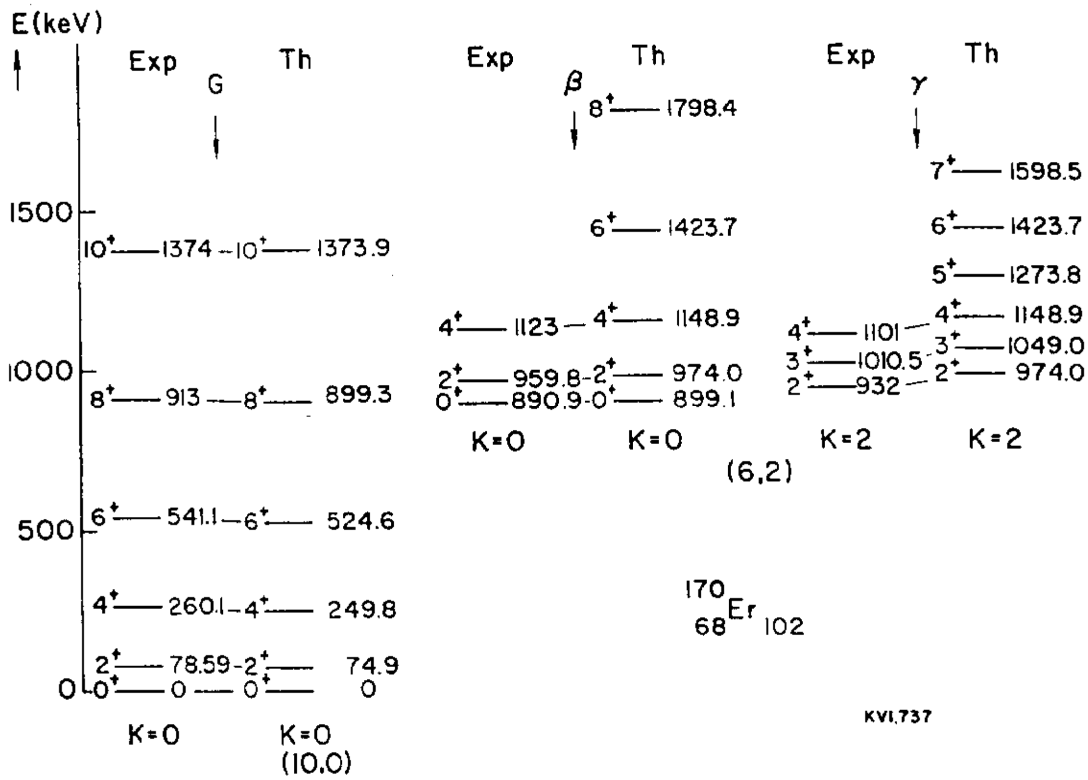



Fig. 2.2. Comparison of 252-plet spectrum with experiment.

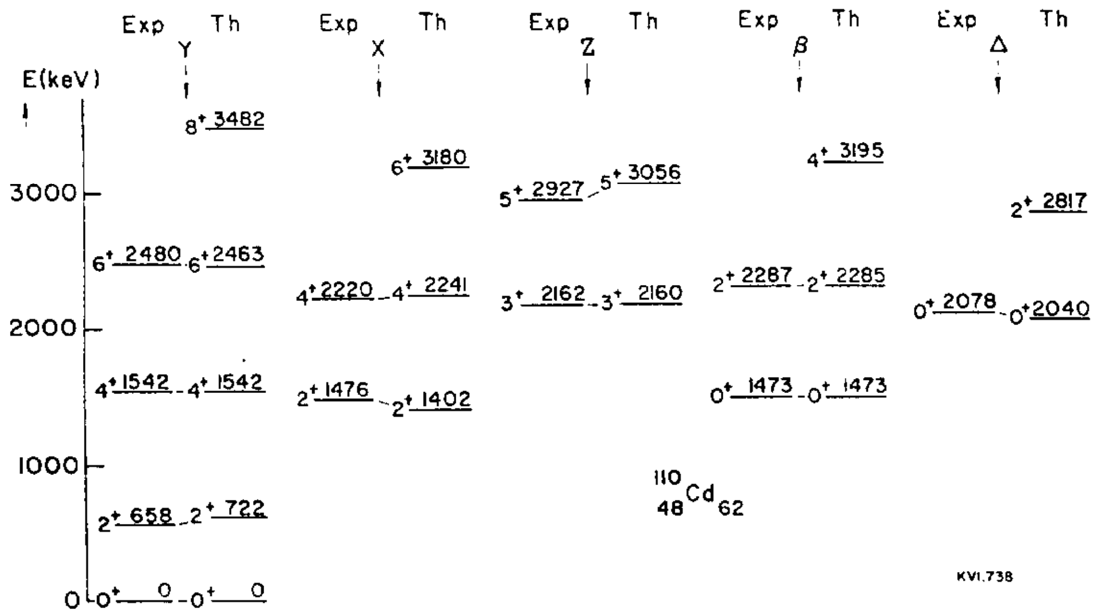



Fig. 2.3. Boson model fit to the $^{110}$Cd vibrational spectrum.

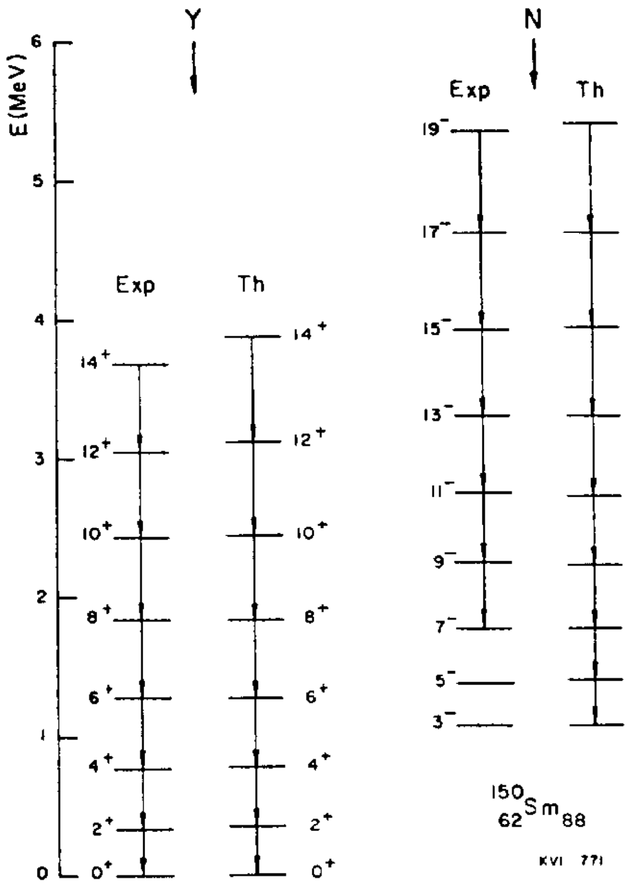



Fig. 2.4. Boson model fit to the $^{150}$Sm vibrational spectrum.

Let us now discuss the nuclear SU(6) and Figs. 2.1—2.4 in slightly more detail. Figure 2.1 shows a typical SU(6) supermultiplet with the energy spectrum obtained when SU(6) is broken by a particular type of boson-boson interaction that preserves the subgroup SU(3). The spectrum looks very much like that found in the nuclear collective model with a ground-state rotational band and beta and gamma vibrations. The states are labelled by the quantum numbers of the SU(3) classification. Data are fitted in a large group of nuclei in the rare earth and transuranium regions where these rotational and vibrational spectra occur. The example of $^{170}$Er shown in Fig. 2.2 has experimental energy levels which fit two SU(3) multiplets which are just the lowest two multiplets found in a single SU(6) supermultiplet.

In particle physics, the SU(3) subgroup of SU(6) gives a good classification for baryon states, but the SU(4) group works better for mesons. For nuclei the SU(3) subgroup of SU(6) gives a good description of rotational nuclei; while the SU(5) subgroup works better for vibrational nuclei. Figures 2.3 and 2.4 show typical vibrational spectra and some fits to vibrational nuclei.

## III. THE SEARCH FOR NEW DEGREES OF FREEDOM

What are these new particles? They indicate some new degree of freedom, but what is it? At the Palermo conference, Cabbibo[9] presented the charm approach as analogous to the search for the planet Neptune where other data on irregularities in the orbit of Uranus had indicated something must be there. My guide to any search is two key questions[10]: 1) who needs it? and 2) who cares if it is not found? In the case of the planet Neptune anybody who believed Newton's descriptions of the motions of the planets knew that something had to be there. It would have been very serious if nothing was found to produce the observed irregularities in the orbit of Uranus. But if all the new additional particles that are suggested by new theories are not found the theorists will find new excuses for their absence and change the theory a bit to explain it. As a guide to the search for new degrees of freedom, it is instructive to recall the search for a higher symmetry[11,12] that eventually turned out to be SU(3). It begins with isospin, which is SU(2) and strangeness, which is U(1). The correct higher symmetry SU(3) which included SU(2) and U(1), was found by an eight-year journey, in which all possible wrong symmetries were tried first. When they finally found SU(3) they called it the eight-fold way because it took them eight years to find it.

Why did it take so long? Why did they try everything else because before they tried SU(3) instead of noting that $SU(2) + U(1) \subset SU(3)$ is as simple as $2+1 = 3$. Physicists are not stupid; the reason that they could not see that 2 + 1 = 3 was because they did not know that they had two. In those days isospin was believed to be a rotation in a three-dimensional space like ordinary spin, described by isospin operators similar to angular momentum operators. The natural candidates for a higher symmetry to include isospin rotations were rotations in four, five, six, seven and eight dimensions. None of these worked because the algebra of the group of three dimensional rotations is accidentally isomorphic to the algebra of two-dimensional unitary transformations and isospin is really SU(2), not O(3). The two-dimensional Hilbert space of proton and neutron states and the transformations of protons and neutrons into one another have no relation to any physical three-dimensional space.

Beyond three dimensions there is no longer this isomorphism between rotations and unitary transformations. Thus theorists could not get anywhere by extrapolating what they already had. They had to learn something new, but they did not realize it. At the Princeton Institute for Advanced Study many now famous theoretical particle physicists did not bother going to Guilio Racah's famous lectures on Group Theory and Spectroscopy [13] because they did not think unitary groups had anything to do with particle physics.

Now the pendulum has swung to the other direction. We know all about SU(n). We begin with n basic building blocks and define unitary transformation among these objects to make an SU(n) symmetry. Now that something beyond SU(3) is needed, theorists play the same games with SU(n) instead of rotations: SU(4), SU(5), SU(6) and so on. If we keep it up, we will get to SU(ROSENFELD) where n is the number of entries in the Rosenfeld table of particles and all particles are classified in the fundamental representation.

This reminds me of an explanation I heard from a physicist who works in the field of controlled thermonuclear reactions about the difference between CTR and particle accelerators. The particle physicist builds an accelerator. It works and he is happy and does some physics with it. After a while he realizes that to progress further he needs a bigger accelerator. He gets money to build it, builds it, it works and he is happy and does some physics. After a while he realizes that he needs an even bigger accelerator, etc. etc. The CTR man builds a machine which does not work and he is unhappy. Then he decides that if he had more money he could build a bigger machine that might work. He gets the money, builds it, it does not work and he is unhappy. He then decides again that if he had more money he could build a bigger machine which might work etc. etc.

The quark model[14] started with the idea that everything is made from three fundamental building blocks. The experimentalists looked for the quarks and did not find them. So the theorists said maybe there are more quarks. It started from three, it has gone up to four, nine, twelve, etc. The current popular colored six-quark model has 18 quarks. But still nobody is finding them. So, perhaps what we need is not to keep adding more of the things that we know, maybe we need to learn something new. In other words, maybe the quark is something like three-dimensional isospace, a useful realization of the symmetry at a certain stage which enables us to do calculations very nicely. But it freezes our intuition in the wrong direction and thus hides the new things that we may have to learn to advance to the next stage.

## IV. THE PRESENT STATUS OF THE NEW PARTICLES AND THE OZI RULE

Let me now briefly review the status of the new particles. They are narrow resonances discovered at SLAC as peaks in the cross section for $e^+ - e^-$ annihilation into hadrons and at Brookhaven as peaks in the mass spectrum of $e^+ - e^-$ pairs in nucleon-nucleon collisions produced with hadrons. In the SLAC experiment the mass of the $\psi$ is about 3000 MeV, the instrumental resolution is about 1 MeV, and the width of the $\psi$ is even smaller. The peak is already smeared by the instrumental resolution by an order of magnitude, and would be smeared further by magnet drift unless the magnetic field is kept stable to better than 1 part in 3000. Such a narrow resonance was not expected by theory and there is no point in getting that much stability in all the apparatus if it is not needed. So the particles were discovered by accident. The first J or $\psi$ that was discovered had a mass of 3100, the same quantum numbers as the photon (spin 1, odd parity and odd charge conjugation), and decays into hadrons. Soon afterwards, a whole family of particles were found having the same quantum numbers as the photon. This does not mean that most of the new particles have the same quantum numbers as the photon; it is just that an $e^+ - e^-$ colliding beam experiment excites most strongly those states having the same quantum numbers as the photon.

Once these are found one asks what they might be; something completely new, a new kind of weak or semi-weak boson, a new kind of quark-antiquark pair, having a new quantum number like charm, or some hadron with a new internal degree of freedom. With too many possibilities and not enough experimental data, we can try to look for general properties and draw conclusions which are not too model dependent. It seems fairly clear that the production is electromagnetic, because it all fits together with what is known of electromagnetic production via a single virtual photon. The particles have the quantum numbers of the photon— if they are produced by some other mechanism there is no reason to pick out $1^-$ states in particular.

The most peculiar property of these particles is the inconsistency between their production and decay. They are produced electromagnetically by $e^+ - e^-$ collisions presumably through a photon, and with a production cross section comparable to that for the ordinary vector mesons $\rho$, $\omega$ and $\phi$. But they decay into hadrons with a very narrow width. The characteristic widths for vector mesons are 100 MeV for strong decays like $\rho \to 2\pi$ and 1 MeV for electromagnetic decays like $\omega \to \pi\gamma$. This is consistent with the picture that 100 MeV is a strong width, and electromagnetic widths are first order in $\alpha$ and down by a factor of 100. The $J/\psi$ decay into hadrons is of the order of 100 kilovolts while decays into hadrons $+\gamma$ are very small. Thus both hadronic and electromagnetic decays are down at least three orders of magnitude from the expected decays of an ordinary hadron.

Relatively large electromagnetic production and suppressed decays into hadrons suggest that the $J/\psi$ might not be a hadron at all, but a composite state of some new kind of fermion-antifermion pair, coupled to the photon because it has electric charge, but not coupled to hadrons. That does not work either, because the coupling to hadrons is actually too strong. This coupling is conveniently measured by a quantity called R defined by the relation[15]

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} . \tag{4.1}$$

Production of $\mu^+\mu^-$ pairs is known from quantum electrodynamics and experiments agree with QED predictions. Thus the $\mu^+\mu^-$ cross section provides the scale for measuring other processes. In parton models where hadron production occurs via the production of an intermediate state of a quark-antiquark pair by the virtual photon, the ratio R depends on the number of different kinds of quarks and their electric charges[16]

$$R = \sum_i Q_i^2 \tag{4.2}$$

where $Q_i$ is the charge of the quark of type i and the sum is over all types of quarks. Experiments show that R in the vicinity of the $J/\psi$ but off resonance is of the order of two to five, but inside the resonance it rises to 25.

The $J/\psi$ is produced electromagnetically with a strength comparable to the $\rho$, $\omega$ or $\phi$. It is more strongly coupled than the photon to the hadrons but decays much more slowly than it should. So in some sense its coupling to hadrons is too small but it is also too big.

I conclude this review with a more detailed picture of why this narrowness problem is such a nuisance. There are two problems. The widths for both electromagnetic and strong decays are too narrow. We first consider the electromagnetic problem. The paradox is that no simple selection rule can forbid the decay of $J/\psi$ into something else plus a photon. Any selection rule based on a conservation law that forbids electromagnetic decay also forbids electromagnetic production. We know that in $e^+ - e^-$ annilihation both the $J/\psi$ and $\omega$ are produced with comparable cross sections. Thus the transition matrix elements for the two processes are comparable

$$\langle\psi|J_{em}|0\rangle \sim \langle\omega|J_{em}|0\rangle \qquad (4.3)$$

where $J_{em}$ denotes the electromagnetic current. Thus the $J/\psi$ cannot have a peculiar eignevalue of a new quantum number, which is conserved in electromagnetic interactions. Therefore, we cannot forbid by symmetry the decay of a $J/\psi$ to a photon and some hadron states having the same quantum numbers as the vacuum for all conserved quantities. Since 3100 MeV can make many pions there are many such states. But experimentally we know that the sum of the squares of these matrix elements over all possible states is still very much smaller than the squared matrix element for the $\omega \to \pi\gamma$ decay. That is the electromagnetic trouble.

For the strong decays, a selection rule is possible. There are "generalized color models", with color symmetry in which all ordinary hadrons are color singlets.[14,17] If the $\psi$ is not a color singlet, its decay into ordinary singlet hadrons is forbidden, but the photon can excite it because the photon need not be a color singlet. But in all such models the electromagnetic trouble is still there because the $\psi$ is allowed to decay into color singlet hadrons plus a photon. Thus generalized color models solve the strong decay trouble but cannot explain the electromagnetic trouble.

There is a dynamical selection rule which may be relevant known as the Okubo-Zweig-Iizuka rule. We know from ordinary hadron physics that the $\phi \to \rho\pi$ and $f' \to 2\pi$ decays are suppressed. The initial states of both decays contain a strange quark-antiquark pair and the final states contain no strange quarks. Some dynamical principle suppresses the transition in which a strange quark-antiquark pair annihilates and only nonstrange quarks are produced. This suggests that if the $\psi$ is a new kind of quark-antiquark pair like a charmed pair there might be a similar principle preventing a charmed pair from disappearing. But this selection rule can only hold in Born approximation . It cannot be rigorous in higher order because a succession of transitions which satisfy the selection rule can produce a forbidden transition. For example both the $\phi$ which consists only of strange quarks and the $\omega$ which consists only of nonstrange quarks are coupled to the $K\bar{K}$ state. So the $\phi$ can go to a nonstrange final state via the intermediate $K\bar{K}$ and $\omega$ states.

Thus the basic question of why the new particles are so narrow is still not understood. There are all kinds of hand waving explanations that may be right but are still unconvincing. It may very well be that the real structure in the new particles is completely different from anything that is being considered today.

## V. THE OKUBO-ZWEIG-IIZUKA RULE

One of the principal open problems in trying to understand the old as well as the new particles is the Zweig rule, or the Okubo-Zweig-Iizuka rule, as it is now commonly called. The Okubo ansatz,[3] which antedated not only Zweig[4] and Iizuka[5] but even the quark model, applied to nonet couplings and gave all results for the three-meson vertex later obtained by Zweig and Iizuka from quark diagrams. The quark-line rules of Zweig and Iizuka define one possible generalization of the Okubo ansatz for four-point functions and more complicated vertices, but this generalization is not unique. Okubo has pointed out other possible generalizations[18] that may be relevant to experiment. Previous papers have separated the "cookbook rules" of Zweig and Iizuka and the Okubo ansatz, since the validity of the Okubo ansatz for three-meson couplings is experimentally well established, whereas the particular ZI generalization to more complicated vertices has not yet been conclusively tested. These notes follow the present common usage of giving Okubo proper credit for his pioneering work by using the name OZI rule. This leaves some ambiguity in its definition for four-point and higher couplings as discussed in detail below.

### 5.1. Pedagogical Examples and Some Basic Questions

The OZI rule has entered the folklore of particle physics without any clear theoretical understanding or justification. At the present time nobody really understands it, and anyone who claims to should not be believed. Investigating the OZI rule for the old particles raises many interesting questions[19,20] which may lead to a better understanding of strong interactions as well as giving additional insight into the experimentally observed suppression of new particle decays attributed to the OZI rule. We follow an iconoclastic approach emphasizing embarrassing questions with no simple answers which might lead to fruitful lines of investigation.

The most common applications of the OZI rule are selection rules forbidding the couplings of the $\phi$ and f' mesons to nonstrange mesons and nucleons

$$g_{\phi\rho\pi} \ll g_{\omega\rho\pi} \tag{5.1a}$$

$$g_{f'\pi\pi} \ll g_{f\pi\pi} \tag{5.1b}$$

$$g_{N\bar{N}\phi} \ll g_{N\bar{N}\omega} \tag{5.1c}$$

$$g_{N\bar{N}f'} \ll g_{N\bar{N}f} \tag{5.1d}$$

where the couplings on the left-hand sides of the inequalities are forbidden and those on the right-hand side are allowed. Since the selection rule is not exact, the degree of suppression is expressed quantitatively by comparing the corresponding forbidden and allowed couplings appearing in these inequalities.

The selection rules can also be formulated in terms of two-body reaction cross sections,

$$\sigma(\pi^- p \to \phi n) \ll \sigma(\pi^- p \to \omega n) \tag{5.1e}$$

$$\sigma(\pi^- p) \to f'n) \ll \sigma(\pi^- p \to fn) \; . \tag{5.1f}$$

Although these agree very well with experiment, no consistent theoretical or phenomenological model explains them without raising paradoxes and contradictions. There is also no theoretical indication of how good the selection rule should be in different processes; i.e. no description of the breaking mechanism.

A principal difficulty to be overcome in any theoretical formulation is that a succession of transitions all allowed by the OZI rule can lead to one which is forbidden. For example, all forbidden couplings (5.1) can proceed through an intermediate $K\bar{K}$ state via the following transition amplitudes observed experimentally and allowed by the OZI rule.

$$T(\phi \to K\bar{K}) \neq 0 \tag{5.2a}$$

$$T(f' \to K\bar{K}) \neq 0 \tag{5.2b}$$

$$T(K\bar{K} \to \rho\pi) \neq 0 \tag{5.2c}$$

$$T(K\bar{K} \to \pi\pi) \neq 0 \tag{5.2d}$$

$$T(K\bar{K} \to N\bar{N}) \neq 0 \; . \tag{5.2e}$$

All the selection rules can thus be broken by the following allowed higher-order transitions

$$\phi \to K\bar{K} \to \rho\pi \tag{5.3a}$$

$$f' \to K\bar{K} \to \pi\pi \tag{5.3b}$$

$$\phi \to K\bar{K} \to N\bar{N} \tag{5.3c}$$

$$f' \to K\bar{K} \to N\bar{N} \,. \tag{5.3d}$$

If the OZI rule holds only to first order in strong interactions, much greater violations are expected than those experimentally observed. Some mechanism for reducing these violations seems to be present. One possibility is a cancellation of the violating amplitudes (5.3) by other amplitudes, as occurs in the case of symmetry selection rules. This is described in detail below.

The essential features of many problems arising in applications of the OZI rule are illustrated in the following examples.

Consider the decays of vector mesons into two pseudoscalar mesons. The following decays are all allowed by the OZI rule and observed experimentally.

$$\Gamma(\rho \to 2\pi) \neq 0 \tag{5.4a}$$

$$\Gamma(K^* \to K\pi) \neq 0 \tag{5.4b}$$

$$\Gamma(\phi \to K\bar{K}) \neq 0 \,. \tag{5.4c}$$

The decay

$$\Gamma(\phi \to 2\pi) \approx 0 \tag{5.4d}$$

is forbidden by the OZI rule and experiments are consistent with zero decay rate. Thus, the VPP decays (5.4) all agree with the OZI rule.

But the decay

$$\Gamma(\omega \to 2\pi) \approx 0 \tag{5.4e}$$

is allowed by the OZI rule and is also observed experimentally to be very weak. This apparent contradiction is resolved by noting that the decays (5.4d) and (5.4e) are forbidden by G parity. The OZI rule is thus completely irrelevant to VPP decays. A similar situation obtains for tensor-vector-pseudoscalar decays, where the

$f' \to \sigma\pi$ decay forbidden by the OZI rule is also forbidden by G parity.

This example shows that experimentally-observed suppression of a transition forbidden by the OZI rule does not necessarily provide evidence for the validity of the rule. The transitions may be forbidden for other reasons.

The VPP example also raises two questions with interesting implications for the general case.

1. The Doubly Forbidden Question . Since both the $\omega \to 2\pi$ and $\phi \to 2\pi$ decays are forbidden by G parity but the $\phi \to 2\pi$ decay is also forbidden by OZI rule, is the doubly forbidden $\phi \to 2\pi$ decay weaker than the $\omega \to 2\pi$ decay?

2. The Higher Order Paradox. The scattering amplitude $T(K^+K^- \to \pi^+\pi^-)$ is allowed by both G parity and the OZI rule. Thus the $\phi \to 2\pi$ decay could take place as a two-step transition in which both steps are allowed

$$\phi \to K^+K^- \to \pi^+\pi^- \; . \tag{5.5a}$$

How is this transition inhibited in a theory of strong interactions where there is no small parameter to make second-order transitions weaker than first order?

The answers to these questions are simple in this trivial case and very illuminating for more interesting non-trivial cases.

1. Double Forbiddeness. There is no simple answer to this question. A transition already otherwise forbidden can be additionally suppressed by the OZI rule only if the dynamical process which breaks the other selection rule also respects the OZI rule. In the case of the VPP decays the G-parity selection rule is broken by electromagnetic transitions which violate the OZI rule. The OZI violating transition

$$\phi \to \gamma \to 2\pi \tag{5.4d}$$

and the OZI conserving transition

$$\omega \to \gamma \to 2\pi \tag{5.4e}$$

have couplings of the same order of magnitude. The OZI conserving transition (5.4e) may be favored by kinematic factors if the $\gamma \to 2\pi$ transition is dominated by the $\rho$ pole, but this has no simple connection to the OZI rule.

2. The Higher-Order Paradox. The answer is that the allowed transition (5.5a) is exactly cancelled by the transition

$$\phi \to K^o\overline{K^o} \to \pi^+\pi^- \quad . \tag{5.5b}$$

This cancellation is characteristic of transitions rigorously forbidden by a conservation law. The conservation law, in this case G conservation, must hold to all orders. G-violating contributions to the transition amplitude can arise from particular intermediate states which are not eigenstates of G. These contributions must be cancelled by other contributions in the sum over all intermediate states related to these states by the G transformation.

Note that this cancellation (5.5) does not occur in the non-trivial selection rules (5.1) allowed by G parity. The contributions to the higher-order transitions (5.3) from the $K^+K^-$ and $K^o\overline{K^o}$ intermediate states have the same phase and cannot cancel. Thus, any cancellation must come from some other state.

An instructive example of a symmetry selection rule broken in higher order is the SU(3) and charge conjugation selection rule forbidding the transition[21]

$$e^+ + e^- \to \gamma \to K^o + \overline{K^o} \quad . \tag{5.6}$$

However, this reaction is observed experimentally near the mass of the $\phi$,

$$e^+ + e^- \to \gamma \to \phi \to K^o + \overline{K^o} \quad . \tag{5.7a}$$

All the individual transitions are allowed by charge conjugation and SU(3). If charge conjugation and SU(3) are exact symmetries this contribution (5.7a) to the amplitude (5.6) must be cancelled by other contributions. In the SU(3) limit the $\rho$ and $\omega$ are degenerate with the $\phi$ and the amplitude for the reaction (5.7a) is exactly cancelled by the amplitudes for the reactions

$$e^+ + e^- \to \gamma \to \rho^o \to K^c + \overline{K^o} \tag{5.7b}$$

$$e^+ + e^- \to \gamma \to \omega \to K^o + \overline{K^o}. \tag{5.7c}$$

In the real world the $\rho$, $\omega$ and $\phi$ are not degenerate and no such cancellation occurs at the $\phi$ peak. The SU(3)-violating reaction (5.6) thus occurs just as strongly as the corresponding SU(3) conserving transition to the $K^+K^-$ final state.

The implications of these examples for the OZI rule are clear. Higher-order OZI violations can be suppressed by cancellations from different immediate states. But such cancellations require a degeneracy of the relevant intermediate states. Without exact degeneracy the OZI rule will be broken like the SU(3) selection rule forbidding the transition (5.6). But no physical state is degenerate with the $K\bar{K}$ intermediate state occurring in the transitions (5.3). Thus these cancellations can at best be only approximate.

We now pose a number of interesting questions which have no simple answers today.[20]

1. What is the theoretical basis of the OZI rule? Can it be formulated with predictive power to give strengths of forbidden transitions at least on the phenomenological level? Could there be a description analogous to the Cabibbo description of strangeness violation in weak interactions where one or more parameters analogous to Cabibbo angles describe the relative strengths of OZI-conserving and OZI-violating transitions?

2. What is the experimental evidence for the OZI rule? How many of the so-called OZI selection rules also follow from other considerations like G-parity conservation and therefore do not really test the OZI rule? How much unprocessed or easily-available data could be used to test the OZI rule?

3. Can the OZI rule be exact in some symmetry limit like SU(3)? Can it be formulated as a conservation law?

4. How is the rule broken and by how much? Can the rule be kept exact for vertices with all the breaking introduced in properties of external particles and propagators for virtual states, as in the conventional SU(3) phenomenology?

5. Where does the rule apply? To baryons as well as mesons? To the not ideally mixed pseudoscalar nonet as well as ideally mixed nonets? To the new as well as the old particles? Is it better in some cases and worse in others? How is it formulated for multiparticle vertices?

6. What is the relation of the OZI rule to ideally mixed nonets? Can the breaking be described as entirely due to deviations from ideal mixing? This would allow the deviation angle to play the role of a Cabibbo angle in a phenomenological description with symmetric vertices and breaking only in mixing angles for physical particles.

7. A cascade of OZI-allowed transitions can produce a forbidden transition. How are these higher-order transitions suppressed?

8. How are the large suppression factors observed in the new particles explained? If they are entirely due to the OZI rule then the breaking must be much smaller than SU(3) breaking.

9. Are thresholds important? Higher-order transitions which violate the OZI rule are possible for the $\phi$ and f' decays via the physical $K\bar{K}$ state with both kaons on shell. But no such physical states exist for the $\psi$ and $\psi'$ decays, which are below the threshold of the analogous $D\bar{D}$ state. Will the OZI rule be better for the $\psi$ and $\psi'$ which cannot proceed by a cascade of allowed on-mass-shell transitions than it is for the old particles or the higher $\psi$'s above the $D\bar{D}$ threshold?

### 5.2. Symmetry, Dynamics, Mixing and the $K_1$-$K_2$ Analogy

There are two possible approaches to explaining the OZI selection rules (5.1), symmetry and dynamics. All these rules apply to processes involving mixed meson nonets, where the SU(3) breaking and mixing of singlet and octet states plays a crucial role. When the couplings and amplitudes for the processes (5.1) are expressed in terms of the unbroken SU(3) singlet and octet states, the two corresponding quantities are of the same order of magnitude and nothing vanishes.

$$g_{\omega_1 \rho\pi} \neq 0 \neq g_{\omega_8 \rho\pi} \tag{5.8a}$$

$$g_{f_1 \pi\pi} \neq 0 \neq g_{f_8 \pi\pi} \tag{5.8b}$$

$$g_{N\bar{N}\omega_1} \neq 0 \neq g_{N\bar{N}\omega_8} \tag{5.8c}$$

$$g_{N\bar{N}f_1} \neq 0 \neq g_{N\bar{N}f_8} \tag{5.8d}$$

$$\sigma(\pi^- p\to\omega_1 n) \neq 0 \neq \sigma(\pi^- p\to\omega_8 n) \tag{5.8e}$$

$$\sigma(\pi^- p\to f_1 n) \neq 0 \neq \sigma(\pi^- p\to f_8 n)\ . \tag{5.8f}$$

Thus the selection rules (5.1) apparently arise from a mysterious mixing mechanism which chooses the physical states to be just the right linear combinations of the SU(3) singlet and octet eigenstates so that the singlet and octet contributions to the processes (5.1) exactly cancel for one of the physical states. There must be a better way to understand this.

Each cancellation of singlet-octet contributions required for the selection rule depends upon two parameters not constrained by SU(3): (1) the ratio of the singlet and octet couplings, (2) the mixing angle of the meson nonet which determines the ratio of singlet to octet in the physical meson. These two parameters appear very different in character. A specific ratio of singlet-to-octet couplings suggests a higher symmetry beyond SU(3) which relates amplitudes unrelated by SU(3). A mixing of singlet and octet states to give physical states having different masses requires SU(3) breaking.

In the symmetry approach one looks for a higher symmetry which classifies the full nonet of mesons in a single supermultiplet and describes singlet and octet couplings by a single coupling. The mixing angles of the physical states could have a simple description if the symmetry-breaking mechanism breaks SU(3) but conserves another subgroup of the higher symmetry which does not commute with SU(3) and mixes SU(3) eigenstates. This other subgroup would then predict new conservation laws not found in SU(3) and could give the selection rules (5.1). One example is an SU(6) description[6] which breaks both SU(6) and SU(3) while conserving the subgroup SU(4) $\times$ SU(2). But such symmetry approaches have all had troubles.[14,22] The basic difficulty is that a conservation law rigorously forbidding the transitions (5.1) must either forbid the higher order transitions (5.3) or cancel them exactly with other transitions. They cannot be forbidden without also forbidding some of the experimentally observed amplitudes (5.2). They cannot be cancelled exactly without introducing additional channels whose intermediate states are exactly degenerate with the kaon pair states, and such degeneracies do not exist.

The dynamical approach looks for dynamical models which naturally give the mixing that decouples one eigenstate.

The neutral kaon system provides an instructive analogy for describing the mixing and selection rule problem and illustrates both the symmetry and dynamical approaches. We begin with the eigenstates $K^o$ and $\overline{K^o}$ of a symmetry, strangeness, that is broken in the decay process. Symmetry breaking determines new eigenstates which are linear combinations of the strangeness eigenstates and could be described by a mixing angle

$$|K_A\rangle = |K^o\rangle \cos\theta + \overline{|K^o\rangle} \sin\theta \tag{5.10a}$$

$$|K_B\rangle = -|K^0\rangle \sin\theta + |\overline{K^0}\rangle \cos\theta \ . \qquad (5.10b)$$

Both the $K^0$ and $\overline{K}^0$ states are coupled to the $2\pi$ decay channel. Consider the particular value of the mixing angle defined by the relation

$$\tan\theta = \frac{\langle 2\pi|T|\overline{K^0}\rangle}{\langle 2\pi|T|K^0\rangle} \ . \qquad (5.11)$$

Then we can write

$$\langle 2\pi|T|K^0\rangle = T\cos\theta \qquad (5.12a)$$

$$\langle 2\pi|T|\overline{K^0}\rangle = T\sin\theta \qquad (5.12b)$$

where T is defined by the relation

$$T = \{ |\langle 2\pi|T|K^0\rangle|^2 + |\langle 2\pi|T|\overline{K^0}\rangle|^2 \}^{1/2} . \qquad (5.12c)$$

Substituting Eqs. (5.10) into Eqs. (5.12) we obtain the transition matrix elements for the $2\pi$ decay in the $K_A,K_B$ basis

$$\langle 2\pi|T|K_A\rangle = T \qquad (5.13a)$$

$$\langle 2\pi|T|K_B\rangle = 0 \ . \qquad (5.13b)$$

Thus if both the $K^0$ and $\overline{K^0}$ are coupled to the $2\pi$ decay mode, a mixing angle can always be found which decouples one state from the $2\pi$ system. The physical mixing angle is chosen by diagonalization of the mass matrix. If experimentally one of the two kaon eigenstates is decoupled from the $2\pi$ system, one can turn the question around. Instead of asking why one of the eigenstates is decoupled from the $2\pi$ system one can ask why diagonalizing the mass matrix chooses the particular state decoupled from the $2\pi$ system to be an eigenstate, or why nature chooses the mixing angle given by Eq. (5.11). We examine two possible answers to this question, one based on symmetry and one based on dynamics.

1. Symmetry. If an additional symmetry remains unbroken it can give rise to the selection rule. For example, if CP is conserved in the kaon decays, then the eigenstates of the mass matrix must be eigenstates of CP and the mixing angle $\theta$ must

be equal to $\pm 45^\circ$. Since the $2\pi$ state is an eigenstate of CP the $2\pi$ decay is forbidden by CP conservation for the 1 kaon eigenstate of CP with the wrong eigenvalue.

2. Dynamics. If the symmetry is broken by the decay itself and there is no other symmetry breaking, the decoupling is automatic. Assume, for example, that only the $K \to 2\pi$ coupling breaks strangeness conservation. Then all breaking and mixing come from the loop diagram shown in Fig. 5.1a. The mass matrix is a $2 \times 2$ matrix whose elements are given by computing this loop diagram. But Eqs. (5.13) show that the loop diagram is diagonal in the $K_A, K_B$ basis, where all matrix elements vanish except the diagonal matrix element for the state $K_A$.

Thus we see that two completely independent mechanisms naturally decouple one of the eigenstates of the neutral kaon system from the $2\pi$ decay mode. If the conditions required for either mechanism hold exactly the decay of one neutral kaon state into two pions is forbidden. In the real world, both conditions are very good approximations but not exact. CP is conserved to a good approximation, but is still violated. The $2\pi$ decay mode is the dominant strangness-violating decay mode of the neutral kaon system but it is not the only decay mode. Additional loop diagrams involving other states contribute to the mass matrix. Thus, both conditions are slightly violated and the neutral kaon system contains a long-lived state which decays weakly into two pions.

## 5.3 A Dynamical Model for Selection Rules at the SU(3) Level

The first attempt to explain the selection rule (5.1a) took the point of view[23] of the $K_1$-$K_2$ analogy and searched for a dynamical symmetry breaking mechanism which naturally chose the decoupled state as an eigenstate of the mass matrix. The approach begins in the nonet symmetry limit where all nine states of the vector or tensor nonet are degenerate and breaks the symmetry by coupling to decay channels. This removes the degeneracy and mixes the two states via transitions through decay channels as shown by the loop diagrams of Fig. 5.1. If a single loop diagram gives the

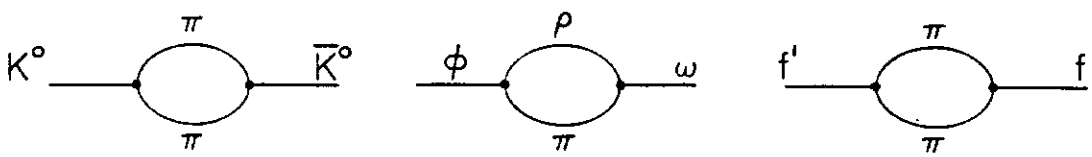



Fig. 5.1. Loop diagrams.

only contribution to the symmetry breaking the diagonalization of the mass matrix is trivial and leaves one of the two eigenstates decoupled from that decay channel. Thus, just as the $K_2$ is decoupled from the $2\pi$ channel the $\phi$ and f' are decoupled from the $\rho\pi$ and $\pi\pi$ channels if these are the only decay modes. The presence of other decay modes complicates the diagonalization but their effect is small if a single mode is dominant as in the case of the $2\pi$ mode in $K^0$ decay. The dominant decay mode then remains decoupled from one of the two eigenstates to a very good approximation.

For the $\phi$ and f' decays the final states involving the other members of the pseudoscalar and vector octets must be considered together with the $\rho\pi$ and $\pi\pi$ modes. However, in the SU(3) symmetry limit these loop diagrams conserve SU(3) and cannot mix singlet and octet states. Mixing can arise only from symmetry breaking. The conventional description of symmetry breaking assumes that the vertices in loop diagrams satisfy SU(3) symmetry and breaks SU(3) by using physical non-degenerate masses for particles in the propagators. Because the dominant breaking effect in the masses is the low mass of the pion relative to the K and $\eta$, the dominant symmetry breaking effects in the loop diagrams come from the $\rho\pi$ and $\pi\pi$ channels. Thus, a dynamical model in which the symmetry is broken by loop diagrams gives the meson selection rules (5.1a) and (5.1b). The particular model considered for the vector mesons gave a natural suppression of the $\phi \rightarrow \rho\pi$ decay and a mass formula which fit the vector meson masses.

The loop diagram model does not give the baryon selection rules (5.1c) and (5.1d) in any simple way. It also gives no indication why the particular linear combinations of singlet and octet states which satisfy the selection rules (5.1a) and (5.1b) should also happen to satisfy the baryon coupling selection rules (5.1c) and (5.1d). Furthermore although the mass formula obtained gave a non-trivial fit to the data, the very simple experimental mass spectrum with $\rho$-$\omega$ and f-A2 degeneracy and equal $\rho$ - $K^*$ - $\phi$ and A2 - $K^*$ - f' spacings is not obtained naturally in this model and is fit by adjusting a free parameter. Thus, despite its initial promise the loop diagram model does not provide a satisfactory description of the selection rules.

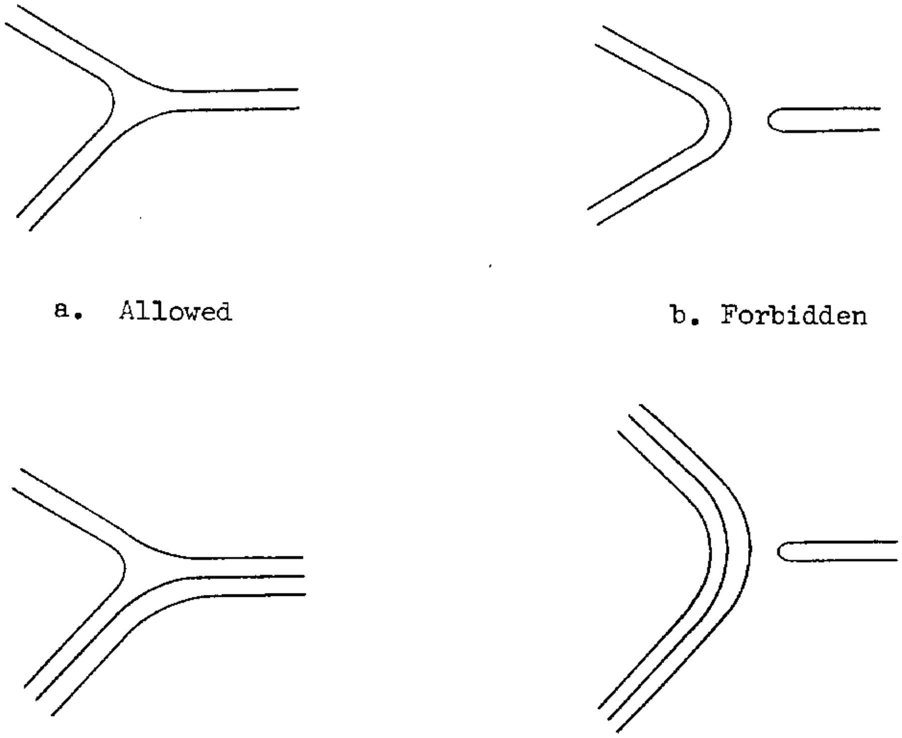
### 5.4 The Quark Line Selection Rules

A crucial mystery at the SU(3) level is the difference between the meson couplings (5.1a) and (5.1b) and the baryon couplings (5.1c) and (5.1d). Baryon selection rules having the same SU(3) couplings as the meson selection rules (5.1a) and (5.1b) would decouple the $\Sigma$ rather than the nucleon from the $\phi$ and f', since the $\Sigma$ and $\pi$ both have zero hypercharge and occupy corresponding positions in the baryon and meson octets. The simple unified statement of the selection rules (5.1) is in terms of strangeness, rather than hypercharge. But hypercharge is simple at the SU(3) level, while strangeness is not, since it depends on baryon number which is outside SU(3).

In the quark model the meson and baryon octets are very different because mesons are quark-antiquark pairs while baryons are three-quark states, and the very different SU(3) couplings arise naturally. There does not seem to be any simple description of the baryon selection rules (5.1c) and (5.1d) without invoking a quark-like structure for the baryons in which they are composed of three fundamental SU(3) triplets. The Zweig-Iizuka formulation with quark diagrams provides a simple unified description of the meson vertex selection rules (5.1a) and (5.1b), the baryon vertex selection rules (5.1c) and (5.1d), the choice as eigenstates of the mass matrix of just those particular linear combinations of singlet and octet which satisfy the selection rules, and the simple nonet mass spectrum.

The quark picture begins with a degenerate meson nonet and breaks the nonet degeneracy by a mass difference between strange and nonstrange quarks. This gives the so-called "ideal mixing" which chooses as eigenstates those mixtures of singlet and octet states corresponding to a pure strange quark-antiquark pair and a pure nonstrange quark-antiquark pair, and gives a simple mass formula with a mass splitting proportional to the number of strange quarks. The selection rule is simply stated by drawing quark line diagrams for the three-point vertex functions as in Figs. 5.2 and postulating that only the connected diagrams Fig. 5.2a and Fig. 5.2c are allowed while the disconnected diagrams Figs. 5.2b and Fig. 5.2d are forbidden. All the couplings (5.1) are forbidden since the $\phi$ and f' both consist only of strange quarks while the remaining particles consist only of nonstrange quarks. Since the quantum numbers of the quark remain the same on a given line, the strange quark lines begin and end on the $\phi$ or f' and are completely disconnected from the nonstrange quark lines. Thus, the couplings (5.1) are described by forbidden diagrams, Figs. 5.2b and 5.2d.

However, exactly the same meson selection rules (5.1a) and (5.1b) are obtainable from other approaches[14,19,20] without invoking connected and disconnected quark diagrams. A mathematically equivalent description for the three-meson coupling was first proposed by Okubo as a nonet coupling ansatz. The same selection rule was obtained by Alexander et al.[24] from the Levin-Frankfurt approach[25] in which any hadron transition involves a change in the state of only one active quark in the hadron while the remaining quarks are spectators. If pion emission is described as a single



a. Allowed    b. Forbidden

c. Allowed    d. Forbidden

Fig. 5.2. Quark diagrams for three-point functions.

quark transition only nonstrange quarks can emit pions and conserve isospin. Thus, a state like the $\phi$ or f' which contains only strange quarks cannot decay by pion emission. This argument can be stated more precisely in the language of PCAC and the Melosh transformation.[26] The PCAC prescription relates the amplitude for a pionic decay to the matrix elements of the axial charge operator $Q_5$ between the initial state and the final state remaining after pion emission. The Melosh prescription postulates that $Q_5$ is a single quark operator. Since a single quark operator cannot change a strange quark-antiquark pair into a nonstrange pair,

$$\langle \phi | Q_5 | 0 \rangle = 0 \tag{5.14a}$$

$$\langle f' | Q_5 | \pi \rangle = 0 \; . \tag{5.14b}$$

The PCAC prescription then gives the selection rules (5.1a) and (5.1b).

None of these alternative approaches for the meson selection rules (5.1a) and (5.1b) are applicable to the baryon selection rules (5.1c) and (5.1d). The baryon couplings are much more complicated than meson couplings at the phenomenological SU(3) level. The octet three-meson coupling is constrained by charge conjugation to be pure F and pure D depending on the charge conjugation properties of the mesons. The singlet meson coupling is forbidden when the octet coupling is pure F. For baryons no such restrictions exist. The D/F ratio is a free parameter and the singlet coupling is always allowed even if the octet is pure F. Furthermore, an additional spin degree of freedom is present in the baryon coupling. There are two independent couplings corresponding to helicity flip and nonflip at the baryon vertex.

For the three meson couplings the OZI rule follows automatically from G conservation for those channels where charge conjugation invariance requires the antisymmetric F type SU(3) coupling. For example, the $\phi \to \pi\pi$ decay is forbidden while $\phi \to K^*\bar{K}$ is allowed. For cases like (5.1a) and (5.1b) where the symmetric D type octet coupling is required and the singlet is allowed one additional constraint on the couplings is needed to obtain the selection rules; namely a particular value for the ratio of the singlet octet couplings. The correct value of this singlet to octet ratio is naturally obtained from the Okubo-Levin-Frankfurt and Melosh approaches.

For the baryon case, the singlet to octet coupling ratio needed to obtain the selection rules (5.1c) and (5.1d) depend upon the D/F ratio. These ratios are different for the spin flip and nonflip transitions. Thus very complicated constraints on the couplings are needed in order to obtain the baryon selection rules. The only simple description so far has been the Zweig-Iizuka formulation with connected and disconnected quark diagrams.

The baryon selection rules (5.1c) and (5.1d) thus provide the best unambiguous tests of the ZI rule and its breaking. One can envision a hierarchy in which the rule holds best when several mechanisms reinforce one another. The smallest breaking effects would occur in pionic meson transitions where the PCAC derivation with eqs. (5.14) still holds while the other mechanisms which give rise to the ZI rule in the other cases are broken. The strongest breaking effects would occur in the baryon vertex and meson vertices not involving pions would lie somewhere between these two extremes.

The extension to more complicated vertices of the quark-line selection rules for three-point functions is not unique, as has been pointed out by Okubo.[18] The forbidden diagrams of Figs. 5.2b and 5.2d can be characterized either as "disconnected diagrams," which allow external particles to be separated without breaking quark lines, or as "hairpin diagrams" in which one external meson has its two lines connected together rather than joining lines to other particles. Forbidding all disconnected diagrams and forbidding all hairpin diagrams are equivalent for three-point functions. However, for four-point and higher functions there are disconnected diagrams which are not hairpin diagrams, multiply disconnected diagrams and diagrams with more than one hairpin, as shown in Fig. 5.3. Without some fundamental theory for the OZI rule it is not clear whether disconnected diagrams which are not hairpin diagrams are forbidden just as much as hairpin diagrams, and whether multiply disconnected diagrams or multiple hairpin diagrams are forbidden more than corresponding single diagrams.

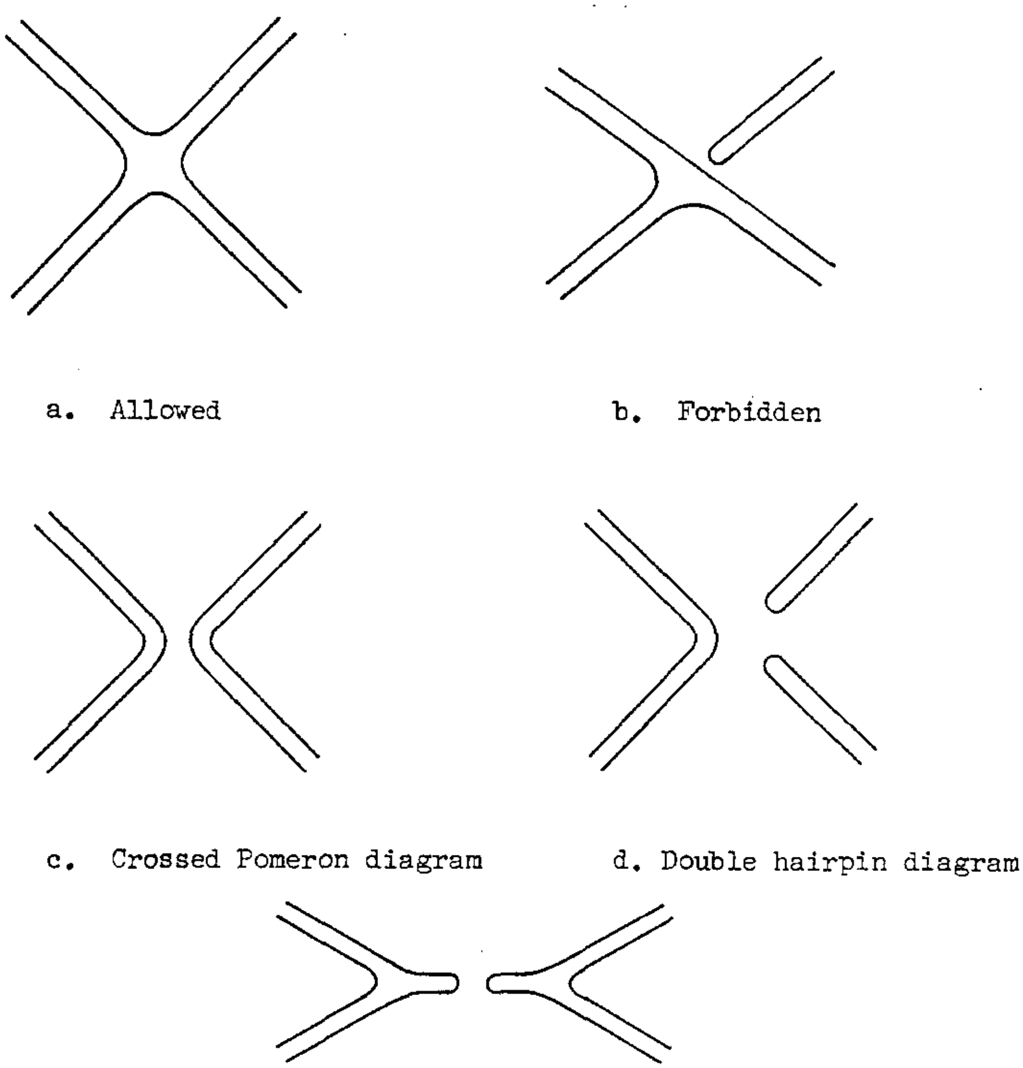



Fig. 5.3. Quark diagrams for three-point functions.

The four-point function of Fig. 5.3a is clearly allowed and that of Fig. 5.3b is clearly forbidden by any version of the OZI rule. But Figs. 5.3c and 5.3d are ambiguous. The simplest example of a disconnected diagram which is not a hairpin diagram is shown in Fig. 5.3c and could describe the process $\pi\pi \to \phi\phi$, or elastic $\phi\pi$ scattering in the cross channel. The elastic amplitude must have a Pomeron contribution, as in $\phi N$ scattering. This "crossed Pomeron diagram" is forbidden in the Zweig-Iizuka formalism but is not forbidden if only hairpin diagrams are forbidden.

Double hairpin diagrams like that of Fig. 5.3d can occur when charm and strangeness are both present, because the two hairpins could describe a strange and a charmed quark-antiquark pair. One example of a process described by such a diagram is the decay $J/\psi \to \phi\pi\pi$ in the charm model for the $J/\psi$. In this model all decays of the $J/\psi$ into normal hadrons are forbidden by the OZI rule in any formulation because one hairpin is required for the $J/\psi$. Whether the two-hairpin decays are more forbidden or not has been questioned.[18] The most recent experimental results on the $\phi\pi\pi$ decay mode suggest that such decays are indeed more forbidden,[27] but further data are necessary before any gendral conclusions can be drawn.

One possible approach to the generalization of the OZI rule to more complicated vertices is to build everything from three-point functions which satisfy the OZI rule. Inconsistencies arise in this approach because of the higher order paradox in which combinations of OZI-allowed transitions can produce an OZI-forbidden transition, as in Eqs. (5.3). There is also the question of possible OZI violations in the propagators of particles appearing as internal lines in the diagrams, as indicated in Fig. 5.3e. These points are discussed in detail below.

## 5.5 The Higher Order and Unitarity Paradoxes

We now examine in more detail the violation of the OZI rule by the transitions (5.3) in which two OZI-conserving amplitudes (5.2) combine to produce an OZI-violating transition. All these transitions are from an initial state containing only strange quarks to a final state containing only nonstrange quarks via the intermediate state of a kaon pair. The kaon plays a crucial ambivalent role. Since it contains one strange and one nonstrange quark, it couples equally to strange and nonstrange systems and can go either way. A kaon pair state contains one strange and one nonstrange quark-antiquark pair. It can therefore be created from a strange pair by the creation of a non-strange pair or vice versa. The kaon pair state thus links two kinds of states between which transitions are forbidden by the OZI rule.

The quark diagram Fig. 5.4 for the forbidden transition (5.3b) illustrates the essential features of the paradox. Viewed as a single topological diagram it is indeed disconnected and can be separated into two disconnected hairpin diagrams. But when it is separated into two individual transitions, each half is connected. Connecting the two diagrams together results in a topological disconnected diagram because of the twist in the quark and antiquark lines in the kaon intermediate state.

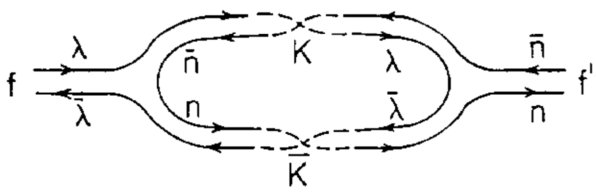



Fig. 5.4. OZI Violation via twisted diagram

Thus, to save the OZI rule the connection of allowed diagrams by a "twisted propagator" must somehow be forbidden. But a twisted propagator has physical meaning only if there is additional information in a kaon pair state to specify "which way it is twisted"; i.e., whether it originally came from a strange or a nonstrange system. Some memory of the origin of the pair is necessary to prevent the nonstrange decay of a pair which originated in a strange system. But a physical kaon pair state has no such memory. A kaon pair produced from a nonstrange system is indistinguishable from a pair produced from a strange system.

The transition (5.3) thus must exist in any consistent scheme which incorporates the OZI rule. Saving the rule requires additional transitions via other intermediate states which exactly cancel these amplitudes. Such cancellations do in fact occur in dual resonance models where twists in diagrams denote changes of the relative phase of the contributions of such intermediate states. But the degeneracy requirement discussed above poses difficulties. This is discussed in detail below.

The same higher order paradox appears in an S-matrix formulation as unitarity violation. Consider for example D wave $\pi\pi$ and $K\bar{K}$ scattering in the vicinity of the f' pole treated as a system with two coupled channels. The OZI selection rule (5.1b) requires the f' to appear as a pole only in the $K\bar{K}$ channel but not in the $\pi\pi$ channel. But the eigenstates of the $2 \times 2$ S matrix are not the $\pi\pi$ and $K\bar{K}$ states but mixtures of the two, because the $\pi\pi \rightarrow K\bar{K}$ transition amplitude (5.2d) does not vanish. Thus, both eigenstates of the S matrix contain a $\pi\pi$ component and any pole appearing in the S matrix must have a nonvanishing coupling to the $\pi\pi$ channel. Complete decoupling of a pole from the $\pi\pi$ channel is possible only when the $\pi\pi$ and $K\bar{K}$ channels are completely decoupled from one another.

This can be seen explicitly by writing the unitarity equation

$$\mathrm{Im}\langle K\bar{K}|T|\pi\pi\rangle = \langle K\bar{K}|T^{+}|K\bar{K}\rangle\langle K\bar{K}|T|\pi\pi\rangle + \langle K\bar{K}|T^{+}|\pi\pi\rangle\langle\pi\pi|T|\pi\pi\rangle \ . \tag{5.15}$$

Since there are only two channels the unitarity sum has only two terms. If the OZI rule holds and the f' pole is decoupled from the $\pi\pi$ channel, the f' pole appears only in a single term in Eq. (5.15), namely the first term in the right-hand side. Thus the OZI rule is inconsistent with unitarity in this two-channel model.

The way out of this paradox is to include more than the $\pi\pi$ and $K\bar{K}$ channels. Additional channels can introduce new terms in the unitarity sum of Eq. (5.15) which can cancel the term with the f' pole and enable the decoupling of the f' from the $\pi\pi$ channel without violating unitarity. Again the paradox is resolved by canceling the transitions via some other set of intermediate states.

Some symmetry scheme or dynamical model is needed to choose which additional intermediate states cancel the $K\bar{K}$ contribution. There are several possibilities. SU(3) symmetry suggests that the full pseudoscalar octet be included with the additional $\eta\eta$ intermediate state. Nonet symmetry requires the $\eta'\eta'$ and $\eta\eta'$ states as well. SU(6) symmetry suggests that vector and pseudoscalar mesons be treated together with the inclusion of intermediate states involving vector mesons. Duality and exchange degeneracy suggest that the vector and tensor mesons which lie on degenerate trajectories must be included together. Exactly how these conflicting suggestions can be resolved is an open question.

## 5.6 A Simple SU(3) Model

We now show how the higher order and unitarity paradoxes can be resolved in a simple way in the framework of SU(3) symmetry by including the $\eta\eta$ channel together with the $K\bar{K}$ channel. The model is not relevant to the physical particles, but the manner in which it avoids the difficulty of the higher order transition (5.3b) and the associated problems of the unitarity of the S-matrix is instructive. In the nonet symmetry limit there is always a particular linear combination of the two isoscalar tensor mesons for which the couplings of the two components to the $2\pi$ channel cancel one another exactly. We consider a model in which a small SU(3) symmetry breaking with appropriate properties splits the masses and leaves the decoupled states as an eigenstate.

We denote by $f_1$ and $f_8$ the two isoscalar tensor mesons classified in the SU(3) symmetry limit in the singlet and octet representations. We assume an unmixed octet of pseudoscalar mesons. There are three possible two-pseudoscalar decay modes for these tensor mesons namely, $\pi\pi$, $K\bar{K}$ and $\eta\eta$. The branching ratios for the $f_1$ and $f_8$ into these three decay modes are determined uniquely by SU(3), but the relative strengths of the $f_1$ and $f_8$ couplings are not determined

$$\sqrt{8/3}\ \langle\pi\pi|f_1\rangle = \sqrt{8}\ \langle\eta\eta|f_1\rangle = \sqrt{2}\ \langle K\bar{K}|f_1\rangle = \sqrt{\gamma_1} \tag{5.16a}$$

$$\sqrt{5/3}\ \langle\pi\pi|f_8\rangle = -\sqrt{5}\ \langle\eta\eta|f_8\rangle = -\sqrt{5}\ \langle K\bar{K}|f_8\rangle = \sqrt{\gamma_8} \tag{5.16b}$$

where $\gamma_1$ and $\gamma_8$ are reduced total widths for the $f_1$ and $f_8$ with phase space factors removed

$$\gamma_1 = |\langle\pi\pi|f_1\rangle|^2 + |\langle\eta\eta|f_1\rangle|^2 + |\langle K\bar{K}|f_1\rangle|^2 \tag{5.17a}$$

$$\gamma_8 = |\langle\pi\pi|f_8\rangle|^2 + |\langle\eta\eta|f_8\rangle|^2 + |\langle K\bar{K}\ f_8\rangle|^2. \tag{5.17b}$$

If $\gamma_1 \neq \gamma_8$, the states $f_1$ and $f_8$ have different lifetimes and cannot be mixed, as mixed states would not have simple exponential decays. To allow mixing of $f_1$ and $f_8$ in the symmetry limit we set their widths equal

$$\gamma_1 = \gamma_8. \tag{5.18}$$

The states $f_1$ and $f_8$ are now degenerate and any linear combinations can be chosen to give a basis of states with simple exponential decays. We choose a basis in which one state is completely decoupled from the $2\pi$ channel. From Eqs. (5.16) and (5.18) this basis is

$$|f\rangle = \sqrt{\frac{5}{13}}|f_1\rangle + \sqrt{\frac{8}{13}}|f_8\rangle \tag{5.19a}$$

$$|f'\rangle = \sqrt{\frac{8}{13}}|\ f_1\rangle - \sqrt{\frac{5}{13}}|f_8\rangle\ . \tag{5.19b}$$

We now assume that the symmetry breaking chooses these states as eigenstates of the mass matrix for some mysterious reason. The partial widths for the various decays of these states are given by

$$\langle\pi\pi|f\rangle = \sqrt{(39/40)\gamma} \tag{5.20a}$$

$$\langle\eta\eta|f\rangle = -\sqrt{(9/520)\gamma} \tag{5.20b}$$

$$\langle K\bar{K}|f\rangle = \sqrt{(1/130)\gamma} \tag{5.20c}$$

$$\langle\pi\pi|f'\rangle = 0 \tag{5.21a}$$

$$\langle\eta\eta|f'\rangle = \sqrt{(4/13)\gamma} \tag{5.21b}$$

$$\langle K\bar{K}|f'\rangle = \sqrt{(9/13)\gamma}. \tag{5.21c}$$

The state denoted by f' is decoupled from the $2\pi$ channel by construction. But the mixing angle described by Eqs. (5.19) is very different from the ideal mixing angle of the quark model. Note, however, that Eqs. (5.20) and (5.21) lead to the result

$$\langle(3K\bar{K} + 2\eta\eta)|f\rangle = 0 \tag{5.22a}$$

$$\langle(2K\bar{K} - 3\eta\eta)|f'\rangle = 0 \tag{5.22b}$$

$$\langle f'|K\bar{K}\rangle\langle K\bar{K}|f\rangle + \langle f'|\eta\eta\rangle\langle\eta\eta|f\rangle = 0. \tag{5.23}$$

Equation (5.23) shows that the higher order transition amplitude (5.3b) is canceled exactly by the analogous transition via the $\eta\eta$ intermediate state. Equations (5.22a) and (5.22b) show that the f and f' are coupled to two orthogonal linear combinations of the $K\bar{K}$ and $\eta\eta$ channels. The particular linear combination (5.20a) which is decoupled from the f is the eigenstate of the S-matrix whose amplitude has the f' pole. This pole does not appear in the other eigenstates of the S-matrix, which are decoupled from the f', namely, the $2\pi$ channel. Thus the selection rule (5.1a) can be rigorous without difficulties from the higher order transitions (5.3b) or the unitarity of the S matrix.

We now consider the conventional quark model formulation using ideal mixing and the OZI rule. This case is very different from the above SU(3) treatment where the cancellation (5.23) of the higher order transition amplitude (5.3b) depends crucially on the description of the $\eta$ as a member of an unmixed octet. No such cancellation occurs in the case of an ideally mixed pseudoscalar nonet with couplings given by the OZI rule. We denote the pseudoscalar state consisting of a nonstrange quark-antiquark pair by $\eta_n$ and the state of a strange quark-antiquark pair by $\eta_s$. The OZI rule then immediately gives

$$\langle\eta_n\eta_n|f'\rangle = \langle\eta_s\eta_s|f\rangle = \langle\eta_s\eta_n|f\rangle = \langle\eta_s\eta_n|f'\rangle = 0 \quad (5.24a)$$

$$\langle\eta_n\eta_n|f\rangle = \sqrt{2}\,\langle\eta_s\eta_s|f'\rangle = \sqrt{1/3}\,\langle\pi\pi|f\rangle$$

$$= \langle K\bar{K}|f\rangle = \sqrt{1/2}\,\langle K\bar{K}|f'\rangle\ . \quad (5.24b)$$

Thus if the coupling of the tensor mesons to the pseudoscalar nonet is described by the OZI rule with ideal mixing, there is only one pseudoscalar channel coupled to both f and f', the $K\bar{K}$ channel, and no additional channel is available to cancel the transition (5.3b).

For a more realistic treatment we consider pseudoscalars which are not ideally mixed,

$$\eta' = \eta_n \cos\theta_p + \eta_s \sin\theta_p \quad (5.25a)$$

$$\eta = -\eta_n \sin\theta_p + \eta_s \cos\theta_p \quad (5.25b)$$

where $\theta_p$ is the angle of deviation from ideal mixing. In this basis Eqs. (5.24a) and (5.24b) become

$$\frac{\langle\eta'\eta'|f\rangle}{\cos^2\theta_p} = \frac{\langle\eta\eta|f\rangle}{\sin^2\theta_p} = \frac{\sqrt{2}\langle\eta'\eta'|f'\rangle}{\sin^2\theta_p} = \frac{\sqrt{2}\langle\eta\eta|f'\rangle}{\cos^2\theta_p} =$$

$$= \frac{\langle\eta'\eta|f'\rangle}{\sin\theta\cos\theta} = -\frac{\langle\eta'\eta|f\rangle}{\sqrt{2}\sin\theta\cos\theta} = \frac{1}{\sqrt{3}}\langle\pi\pi|f\rangle$$

$$= \langle K\bar{K}|f\rangle = \frac{1}{\sqrt{2}}\langle K\bar{K}|f'\rangle. \quad (5.26)$$

Equations (5.26) give the transition amplitudes for the case where the couplings to ideally mixed states satisfy SU(3) symmetry and the OZI rule, but the physical pseudoscalar states are not ideally mixed.

From Eqs. (5.24b)

$$\langle f'|\eta\eta\rangle\langle\eta\eta|f\rangle = \frac{\sin^2(2\theta_p)}{8}\langle f'|K\bar{K}\rangle\langle K\bar{K}|f\rangle \tag{5.27a}$$

$$\langle f'|\eta'\eta'\rangle\langle\eta'\eta'|f\rangle = \frac{\sin^2(2\theta_p)}{8}\langle f'|K\bar{K}\rangle\langle K\bar{K}|f\rangle \tag{5.27b}$$

$$\langle f'|\eta\eta'\rangle\langle\eta\eta'|f\rangle = -\frac{\sin^2(2\theta_p)}{4}\langle f'|K\bar{K}\rangle\langle K\bar{K}|f\rangle. \tag{5.27c}$$

If the result (5.27) is summed over all channels with equal weighting the total contribution of $\eta$ and $\eta'$ decay modes vanishes, as expected from Eq. (5.24a). A finite contribution appears under the more realistic assumption that the $\eta\eta$ channel is dominant and others are neglected. However, this contribution has the same phase as the contribution from the $K\bar{K}$ intermediate state and cannot produce a cancellation. It is also much smaller than the $K\bar{K}$ contribution.

## 5.7 The Selection Rule in $SU(6)_W$ Symmetry

An early derivation of the selection rule forbidding the $\phi \to \rho\pi$ decay was based on $SU(6)_W$ symmetry.[6,28] This selection rule holds in any model which satisfies $SU(6)_W$. It is therefore of interest to examine the higher order transition (3a) and see how $SU(6)_W$ operates in this case. In $SU(6)_W$ the $K^*(890)$ is in the same super-multiplet with the kaon, and higher order transitions via all possible intermediate states involving one or two $K^*$ mesons must also be considered. In the approximation where K and $K^*$ are degenerate and all vertices are related by $SU(6)_W$ the contributions from the different K and $K^*$ intermediate states all cancel and the decay $\phi \to \rho\pi$ is still forbidden. This cancellation is simply described by a conservation law. In the decay (5.1a) the outgoing $\rho$ and the initial $\phi$ must all be in the same polarization state with $S_z = \pm 1$, since angular momentum and parity conservation forbids the transition for the states with $S_z = 0$. The transition from the initial state to the intermediate state of two strange mesons conserves separately the total W spins of strange and nonstrange quarks. The initial $\phi$ state has strange W-spin 1 and nonstrange W spin zero. Thus each strange meson in the intermediate state is a coherent linear superposition of a K and a $K^*$ which is an eigenstate of the z component of the spin of the strange quark or antiquark, with the eigenvalue required to conserve the strange W spin. The spin directions of the two strange quarks are parallel. Thus, they cannot annihilate and conserve strange quark W spin.

The $SU(6)_W$ selection rule is thus follows simply from the conservation of the z component of the strange quark spin.

However, the transition (5.3a) is forbidden in a much simpler way by $SU(6)_W$, which also illustrates a basic weakness in $SU(6)_W$, which holds only for colinear processes. In $SU(6)_W$ all momenta are in the z direction and a $K\bar{K}$ state must have $S_z = J_z = 0$. Thus the $S_z = 0$ polarization state of the initial $\phi$ decays into the $K\bar{K}$ channel and the $S_z = \pm 1$ state decays into the $\rho\pi$ channel and angular momentum conservation forbids transitions between the two states. But this argument holds for the transition (5.3a) only if the momentum of the kaon pair in the intermediate state is in the same direction as the momentum of the final $\rho\pi$ state. Consider for example the decay of a $\phi$ in the state with $S_x = 0$ into a $\rho\pi$ final state with momentum in the z direction. The transition of a $\phi$ into an $\omega$ via an intermediate $K\bar{K}$ state with momenta in the x direction is allowed by $SU(6)_W$ because the $\phi$ and $\omega$ states with $S_x = 0$ are both states with $W = 0$ with respect to the x axis. However, the state of the $\omega$ with $S_x = 0$ is a linear combination of states with $S_z = \pm 1$ for which the $\rho\pi$ decay in the z direction is allowed.

The $SU(6)_W$ argument does not hold for the f' decay, because the outgoing pions are spinless and there can be no correlation between the quark spins of the final state and the quark spins in the initial state. In the decay (5.1b) a component in the initial f' wave function has the quark and antiquark spins antiparallel, and therefore has $W = 0$. They are allowed to annihilate without any angular momentum transfer, and the transition from the $W = 0$ component of the final two-pion state is allowed by $SU(6)_W$. Thus, $SU(6)_W$ cannot be used to give a general derivation of the OZI rule.

## 5.8 Ideal Mixing and Symmetry Cancellations

The peculiar role of ideal mixing and higher symmetries in the OZI rule is illustrated by the following example:

Consider the decay of a high $K^{*-}$ resonance into three kaons via an intermediate nonstrange resonance $M^0$

$$K^{*-} \rightarrow K^- M^0 \rightarrow K^- K\bar{K}, \tag{5.28}$$

where $M^0$ is a resonance like the $\rho^0$, $\omega$, f or A2 which consists only of nonstrange quarks.

There are two quark line diagrams for this decay shown in Fig. 5.5, using the $p\bar{p}$ and $n\bar{n}$ components of $M^0$. The $p\bar{p}$ diagram is connected, obeys the OZI rule and leads to the final state $K^-K^+K^-$. The $n\bar{n}$ diagram is disconnected, violates the OZI rule and leads to the final state $K^-K^o\bar{K}^o$. Thus if the OZI rule holds,

$$(K^* \to K^-M^o \to K^-K^+K^-) \text{ is allowed} \qquad (5.29a)$$

$$(K^* \to K^-M^o \to K^-K^o\bar{K}^o) \text{ is forbidden .} \qquad (5.29b)$$

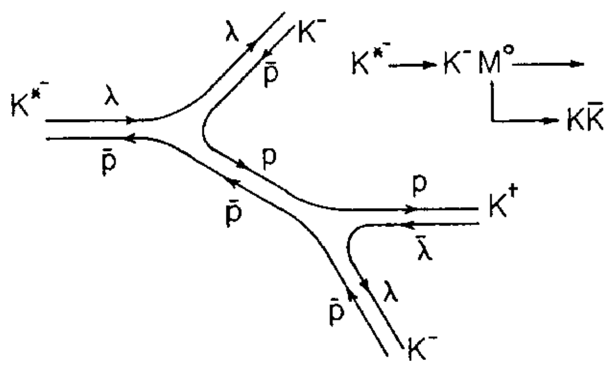



Fig. 5.5a. Allowed $K^{*-}$ into charged mode.

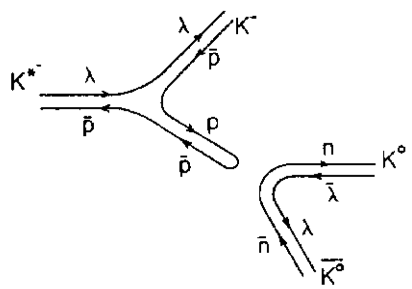



Fig. 5.5b. Forbidden $K^{*-}$ decay into neutral mode.

But if the resonance $M^o$ has a definite isospin, the two transitions (5.29a) and (5.29b) must be equal by isospin invariance. Contradictions between the OZI rule and isospin invariance are avoided if the nonstrange meson spectrum consists of degenerate isospin doublets, like $\rho$ and $\omega$ or f and A2. In that case the transition (5.29a) proceeds via the particular coherent linear combination of isovector and isoscalar particles which has the quark constitution $p\bar{p}$. The OZI rule is thus intimately related to the existence of the isospin doublets found in ideally mixed nonets.

If the $M^{o}$ in the transitions (5.29) is not a member of an isospin doublet, the OZI rule is inconsistent with isospin invariance. This is the case if $M^{o}$ is a $\pi^{o}$, which has no degenerate isoscalar partner. Although the $\pi^{o}$ cannot appear as a physical resonance in the reactions (5.29) because of its low mass, it can appear as an exchanged particle in the analogous two-body scattering reactions

$$K^{+} + K^{-} \to K^{*o} + \bar{K}^{*o} \qquad (5.30a)$$

$$K^{+} + K^{-} \to K^{*+} + K^{*-} \qquad (5.30b)$$

$$K^{o} + K^{-} \to K^{*o} + K^{*-}. \qquad (5.30c)$$

The quark diagrams for these reactions are shown in Fig. 5.6 .

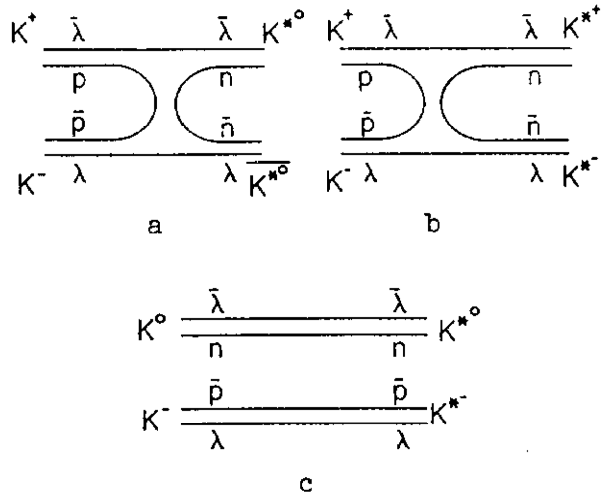



Fig. 5.6. Reactions allowed by pion exchange.

The charge exchange reaction (5.30a) is clearly allowed by the OZI rule and can go by pion exchange. The amplitudes for the pion exchange contribution to the reactions (5.30b) and (5.30c) are uniquely related to the charge exchange amplitude (5.30a) by isospin invariance. But the reaction (5.30b) is allowed by the OZI rule and the reaction (5.30c) is forbidden when only nonstrange quark exchange is considered. (The reaction (5.30c) is allowed by $\lambda\bar{\lambda}$ exchange but this is irrelevant to the present argument). The OZI rule could be saved from inconsistency with isospin invariance if a contribution from isoscalar exchange degenerate with pion exchange cancelled the pion exchange contribution to the reaction (5.30c). But no such isoscalar exists. Thus violations of the OZI rule might be expected in processes where pseudoscalar exchange plays a dominant role.

### 5.9 Cancellations and Degeneracies in Quark Line Models

Figure 5.7 shows the essential piece of the diagrams of Figures 5.5 and 5.6 which break the OZI rule, a transition between a $p\bar{p}$ and $n\bar{n}$ pair. This diagram exists in the propagator of any neutral isovector meson and will lead to OZI breaking unless it is cancelled by a contribution from a degenerate isoscalar partner. The diagrams of Fig. 5.4 describe the higher order transitions (5.3) which violate the OZI rule as the connection of two allowed diagrams by a twisted pair of lines. The essential piece of this diagram which breaks the OZI rule is shown in Fig. 5.8. This is a transition which interchanges the quark and antiquark lines.

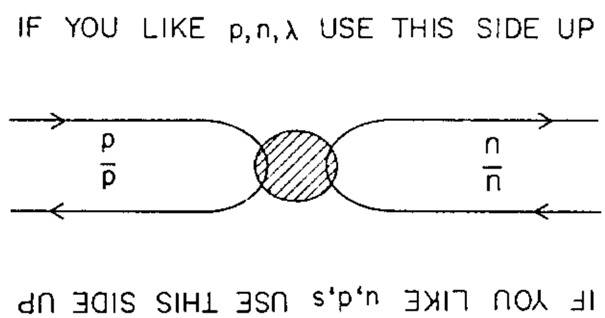



Fig. 5.7. OZI-Violating diagram required by isospin.

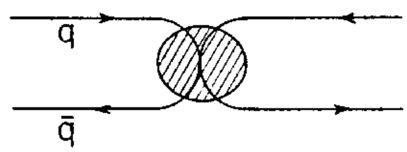



Fig. 5.8. OZI-Violating diagram required by charge conjugation.

The two basic OZI-violating diagrams, Fig. 5.7 and Fig. 5.8 have a very similar structure. Figure 5.7 describes a transition between two states related by isospin. It occurs naturally in the propagator of any particle which is an isospin eigenstate. Figure 5.8 describes a transition which interchanges quark and antiquark and occurs naturally in the propagator of any particle which is an eigenstate of charge conjugation. The OZI violation implied by the diagram of Fig. 5.7 can be avoided by an additional degeneracy of isoscalar and isovector particles. This allows the choice of a basis of states which are not isospin eigenstates and whose propagators do not include the diagram of

Fig. 5.7. Similarly, the OZI violation implied by Fig. 5.8 can be avoided by an additional degeneracy of particle states which are even and odd under charge conjugation. This allows the choice of a basis which are not eigenstates of charge conjugation and whose propagators do not include the diagram of Fig. 5.8. For the case where the quark and antiquark in Fig. 5.8 do not have the same internal symmetry quantum numbers, the relevant transformation is not charge conjugation but an appropriate combination like G parity of charge conjugation and an internal symmetry transformation. The conclusions are the same.

We can now specify the additional degeneracies essential for the validity of the OZI rule in higher order in models described by quark line diagrams. The states described by the left-hand and right-hand sides of the diagrams of Fig. 5.7 and Fig. 5.8 must be physical eigenstates which can propagate unchanged and in particular can avoid undergoing transitions indicated by Fig. 5.7 and Fig. 5.8. These states are eigenstates of quark number having a well-defined quark composition (either $p\bar{p}$ or $n\bar{n}$, but not a linear combination of them) and are linear combinations of states even and odd under charge conjugation (either $\bar{p}p$ or $p\bar{p}$ but not the linear combination of them which is a charge conjugation eigenstate). These states required by the OZI rule are not eigenstates of SU(3) and its isospin subgroup nor of charge conjugation. They can be physical eigenstates only if additional degeneracies are present beyond those imposed by these symmetries.

There must be ideal mixing of the SU(3) singlet and octet states so that the $\lambda\bar{\lambda}$ state remains an eigenstate and the $p\bar{p}$ and $n\bar{n}$ eigenstates which go into one another under isospin transformations are degenerate. When processes are described in terms of the isospin eigenstates as in Fig. 5.5 the amplitude for the forbidden diagram vanishes because of a cancellation between the contributions involving degenerate isoscalar and isovector states.

Charge conjugation degenerate doublets are required to eliminate the OZI violation due to twisted diagrams like Fig. 5.8. In duality and dual resonance models this degeneracy appears in Regge trajectories, rather than in individual particle states as exchange degeneracy of trajectories having opposite signature. In these formulations the two states on the left-hand and right-hand sides of Fig. 5.8 represent two different linear combinations of even signature and odd signature trajectories with equal magnitude and opposite phase. When the OZI-violating higher-order transitions (5.3) are described in the conventional basis using states which are eigenstates of charge conjugation the violating diagrams cancel one another in pairs involving states which behave oppositely under charge conjugation.

The peculiar relation between ideal mixing, exchange degeneracy and the OZI rule was noted in early treatments of duality using finite energy sum rules.[29] Two possible mechanisms for the breakdown of the necessary cancellations are immediately evident upon closer examination of the diagrams of Fig. 5.7 and Fig. 5.8.

The diagram of Fig. 5.7 will occur and break the OZI rule whenever a propagator appears for a state which is not ideally mixed, such as a pseudoscalar meson. One can expect the OZI rule to be violated in processes where there is a strong contribution from pseudoscalar exchange.

The cancellation of the diagram of Fig. 5.8 must break down because charge conjugation doubling exists only for Regge trajectories and not for individual physical states.[30] Thus, even if exchange degeneracy is exact the OZI-violating diagram of Fig. 5.8 is cancelled only in the kinematic region where Reggeization is a good approximation; i.e., where the scattering amplitude is well described by a Regge exchange rather than by one or two resonances.[31] This is clearly not the case for the higher-order transitions (5.3a) and (5.3b) where the mass is above the threshold for the intermediate $K\bar{K}$ state but below threshold for all other states on the kaon trajectory and its exchange degenerate partner. In general, one might say that contributions from high momentum intermediate states could be described in the Regge approximation and the desired cancellations from exchange degenerate pairs could occur. But at low momenta, where the large mass difference between individual resonances on exchange degenerate trajectories is significant, such cancellations should not be expected.

## 5.10 Quantitative Estimate of OZI Violation

Let us now attempt to estimate the violation of the OZI selection rule (5.1b) resulting from the higher order transition (5.3b). We consider the f-f' system analogous to the $K_1$-$K_2$ system and diagonalize a 2 × 2 mass matrix. We assume that the dominant portion of the mass splitting comes not from the loop diagram but from a quark mass term. We therefore use the ideally mixed basis in which the quark mass term is diagonal. The loop diagram contributes both a real part and an imaginary part to the mass matrix.

The real part is dominated by high momenta. There is no justification for considering only the contribution of the $K\bar{K}$ state rather than all states on the K and $K^*$ Regge trajectories. Exchange degeneracy and Regge arguments suggest some kind of cancellation in these diagrams. But it is very difficult to estimate these cancellations quantitatively. One possibility is to consider the entire set of states on Regge trajectories and use duality and dual resonance models in order to calculate the contributions. Such calculations are beyond the scope of this paper.

The imaginary part, however, is dominated by the f and f' poles. It is therefore reasonable as a first approximation to consider only the $K\bar{K}$ intermediate state and neglect the contributions of higher strange meson resonances. Estimates of the imaginary part of the mass matrix are obtained by using the experimental partial widths for the decays. We therefore neglect the real part, which we cannot calculate anyway, and calculate the contribution of the imaginary part to the OZI violation. This gives a lower bound since the contribution of the real part cannot cancel that of the imaginary part.

In this formulation the violation of the OZI rule comes about as a result of a deviation from ideal mixing produced by the loop diagram. The relevant parameter which characterizes the mixing is the ratio of half the width of the f' to the f-f' mass splitting. For $\Gamma(f' \to K\bar{K}) \approx 40$ MeV and $m_{f'} - m_f = 240$ MeV this ratio is small and consistent with the small observed violation. This also implies that we can treat the deviation from ideal mixing as a first-order perturbation. Let us write

$$|f'\rangle = \cos\theta_{f'}|f_I'\rangle + \sin\theta_{f'}|f_I\rangle$$

where the subscript I denotes the ideally mixed states and $\theta_{f'}$ is the deviation of the f' mixing angle from ideal mixing. This is not necessarily equal to the f mixing angle because the eigenstates of a complex mass matrix are not necessarily orthogonal. However, only $\theta_{f'}$ is needed to calculate the OZI rule violation. In first-order perturbation theory $\sin\theta_{f'}$ is given by

$$\sin\theta_{f'} = \frac{\pi\langle f|T|K\bar{K}\rangle\langle K\bar{K}|T|f'\rangle\rho_F(f')}{\hbar(M_{f'} - M_f)} \tag{5.31a}$$

where $\rho_F(f')$ denotes the density of final $K\bar{K}$ states at the mass of the f'. This can be expressed in terms of the experimental width of the f' and the ratio of f and f' transition matrix elements

$$\sin\theta_{f'} = \frac{\Gamma(f' \to K\bar{K})}{2(M_{f'} - M_f)} \; \frac{\langle f|T|K\bar{K}\rangle}{\langle f'|T|K\bar{K}\rangle} \, . \tag{5.31b}$$

The amplitude of the OZI violating transition is then given by

$$\langle 2\pi|T|f'\rangle = \sin\theta_{f'}\,\langle 2\pi|T|f\rangle = \frac{\Gamma(f'\to K\bar{K})}{2(M_{f'}-M_f)}\;\frac{\langle f|T|K\bar{K}\rangle\langle 2\pi|T|f\rangle}{\langle f'|T|K\bar{K}\rangle}\,. \qquad (5.31c)$$

To evaluate the expressions (5.31) we introduce the SU(3) relation between the transition amplitudes

$$\langle 2\pi|T|f\rangle = \sqrt{3}\,\langle K\bar{K}|T|f\rangle = \sqrt{3/2}\,\langle K\bar{K}|T|f'\rangle. \qquad (5.32)$$

We now obtain

$$\sin\theta_{f'} = \frac{\Gamma(f'\to K\bar{K})}{2\sqrt{2}(M_{f'}-M_f)} \qquad (5.33a)$$

$$|\langle 2\pi|T|f'\rangle|^2 = \frac{3\,\Gamma(f'\to K\bar{K})^2}{16(M_{f'}-M_f)^2}|\langle K\bar{K}|T|f'\rangle|^2\,(p_\pi/p_k)^5 \qquad (5.33b)$$

where the d-wave phase space factor $(p_\pi/p_k)^5$ is introduced to account for the difference between pion and kaon momenta in the decay of the f'. Substituting the experimental values $\Gamma(f'\to K\bar{K}) = 40$ MeV, $M_{f'} = 1514$ MeV, $M_f = 1270$ MeV we obtain $\sin\theta_{f'} = 0.06$ and

$$\left|\frac{\langle 2\pi|T|f'\rangle}{\langle K\bar{K}|T|f'\rangle}\right|^2 = 0.005\,(p_\pi/p_k)^5 = 0.02. \qquad (5.34)$$

This result is in qualitative agreement with experiment.[32] Obtaining a better approximation is difficult because there are too many uncertainties in the additional effects which must be taken into account. In addition to the contribution from the real part there are additional contributions from channels like $\eta\eta$, $\eta'\eta'$ which have large uncertainties because of the deviation from ideal mixing. In the limit of ideal mixing these channels give no contribution as shown by Eqs. (5.24) since there is no state which is coupled both to the f and to the f'. However, the physical pseudoscalar mesons are approximately equal mixtures of $\eta_n$ and $\eta_s$

and all three physical states contribute to the transition. The three contributions cancel exactly in the nonet symmetry limit where $\eta$ and $\eta'$ are degenerate and the ratio of the singlet and octet couplings is given by the OZI rule. For the physical nondegenerate and nonideally mixed states the cancellation no longer occurs and the magnitude of the contribution is very sensitive to the mixing angle and to possible changes in the singlet to octet ratio. Thus, there does not seem to be a serious possibility of obtaining a better approximation than Eq. (5.34).

An alternative approach to including the higher order transition (5.3b) is to use an S-matrix formalism and calculate unitarity corrections. In the most naive approximation results similar to (5.34) are obtained. However, attempts to include more channels run into the same difficulties discussed above for the $\eta$ and $\eta'$.

Our analysis of the $f' \rightarrow \pi\pi$ decay suggests that the observed violation of the OZI rule is due to the higher order transition via the open $K\bar{K}$ channel, and that there is no effective mechanism available for cancelling the contribution of this channel. The effect is small because it is characterized by a small parameter, the ratio of half the width of the $f'$ to the $f$-$f'$ mass difference. Whether the smallness of this parameter has any deep theoretical significance is not clear at this point. Two fundamental quantities having the dimensions of mass appear in this ratio, the characteristic width of strong decays and the mass difference between strange and nonstrance particles or between the strange and nonstrange quark. Both these quantities are generally considered to be of the order of 100 MeV and the ratio of the two is then of order unity.

However, in the particular case of the $f'$ decay there are several factors of 2 present which conspire to provide a factor which is an order of magnitude. Two factors of 2 arise because the relevant parameters are half of the width and twice the energy difference between strange and nonstrange quarks. The width of the $f'$ is 40 MeV rather than 100 MeV. As long as there is no fundamental theory which predicts the ratio of the strangeness mass difference to strong decay widths, there can be no explanation for why the numerical value of the parameter which characterizes OZI violations is small in this particular case.

It is amusing that the selection rule holds both in the limit of very small and very large values for this ratio of the widths to the mass splitting. In the other extreme case of the large value the mixing of the two states is completely dominated by the decay process and leads to the decoupling of one state via the $K_1$-$K_2$ mechanism.

In intermediate cases, where the mixing is not dominated by the decay process the $K_1$-$K_2$ mechanism still has some effect in reducing OZI violation from higher-order transitions. In the above analysis of mixing produced via the intermediate $K\bar{K}$ state in f' decay contributions from the intermediate $\pi\pi$ state were neglected. In first order these contributions vanish because the unperturbed f' does not couple to the $\pi\pi$ channel. This approximation is justified in this case by the small magnitude of the results (3.31-3.34). But when the mixing is larger and higher order effects must be considered the contribution of the $\pi\pi$ intermediate state introduces an effective "restoring force" opposing the mixing by the $K\bar{K}$ intermediate state and opposing the increased violation of the OZI rule.

A qualitative estimate of this restoring force effect is obtainable by introducing all the two-pseudoscalar meson intermediate states into the calculation. We assume that the $2 \times 2$ mass matrix M has the form:

$$M = \Lambda m + L_s + L_\pi \tag{5.35}$$

where $\Lambda m$ is a quark mass term diagonal in the ideal mixing basis as in the naive mixing model, $L_s$ represents the contribution of the loop diagram of Fig. 5.1c but with all octet pseudoscalar mesons included in the intermediate state and full SU(3) symmetry, and $L_\pi$ denotes an additional contribution from the $\pi\pi$ intermediate state resulting from SU(3) symmetry breaking. We do not have values for the strengths of these terms from first principles, but give each one a strength parameter and see how the OZI rule is affected by their variation. In particular, we shall see that the $L_\pi$ term indeed has the effect of a restoring force reducing OZI violation.

If $L_s = L_\pi = 0$, the only contribution to the mass matrix comes from $\Delta m$ and gives ideal mixing and the OZI rule. If $L_s \neq 0$ but $L_\pi$ is kept zero; i.e. SU(3) symmetry in the loop diagram, there is no longer ideal mixing, and the OZI violating decay $f' \to \pi\pi$ occurs. However, if $\Delta m$ and $L_s$ are fixed and the symmetry breaking term $L_\pi$ is turned on, the OZI violation is reduced by this "restoring force". Table 5.1 shows values of the OZI violation, expressed as the ratio of the forbidden and allowed decay rates, $\Gamma(f' \to \pi\pi)/(\Gamma(f \to \pi\pi)$ for different values of $L_\pi$ and $L_s$. The value of $L_s$ is given in arbitrary units, and $L_\pi$ is given in units of the $\pi\pi$ contribution to $L_s$; i.e. $L_\pi = 1$ means that the contributions from the $\pi\pi$ intermediate state in $L_\pi$ and $L_s$ are equal. Table 5.1 shows that restoring force effects can be quite appreciable. The significant quantity is the change in $\Gamma(f' \to \pi\pi)/(\Gamma(f \to \pi\pi)$ with increasing $L_\pi$. For example, when the loop diagram contribution is sufficiently large to give a 31%

violation of the OZI rule the addition of a symmetry breaking $\pi\pi$ loop of equal magnitude to the SU(3) symmetric $\pi\pi$ contribution reduces the violation to 7%.

Table 5.1

Effect of $K_1 - K_2$ Restoring Force Mechanism

Values of $\Gamma(f' \to \pi\pi)/\Gamma(f \to \pi\pi)$ as Functions of $L_s$ and $L_\pi$

| $L_s$ \ $L_\pi$ | 0 | 1/2 | 1 | 3/2 | 2 |
|---|---|---|---|---|---|
| 1 | .04 | .03 | .02 | .016 | .012 |
| 2 | .11 | .06 | .04 | .03 | .02 |
| 3 | .18 | .09 | .05 | .035 | .02 |
| 7 | .31 | .14 | .07 | .04 | .03 |

## 5.11 Experimental Tests of the OZI Rule

The OZI rule has been found to be in qualitative agreement with experiment for selection rules forbidding the production of the $\phi$ and $f'$ mesons.

Two types of further experimental information are needed as a guide to theoretical understanding of the underlying dynamics: 1) Quantitative results on the magnitude of the OZI-violating transitions with systematic comparisons of different processes, 2) Tests of the OZI rule for nonets like the pseuco-scalar nonet which are not ideally mixed.

If production of the $\phi$ and $f'$ from nonstrange systems arises primarily from a small admixture of a nonstrange quark-antiquark pair into the wave function, as in the example of section 5.10, then all OZI-violating transitions will be expressible[19] in terms of the angle $\theta_{f'}$ and the analogous angle $\theta_\phi$

$$\frac{g^2_{\phi\rho\pi}}{g^2_{\omega\rho\pi}} = \frac{g^2_{N\bar{N}\phi}}{g^2_{N\bar{N}\omega}} = \frac{\sigma(\pi N \to \phi X)}{\sigma(\pi N \to \omega X)} = \frac{\sigma(NN \to \phi X)}{\sigma(NN \to \omega X)} = \frac{\sigma(K^- p \to Y\phi)_{back}}{\sigma(K^- p \to Y\omega)_{back}} = \tan^2\theta_\phi \tag{5.36a}$$

$$\frac{g^2_{f'\pi\pi}}{g^2_{f\pi\pi}} = \frac{g^2_{N\bar{N}f'}}{g^2_{N\bar{N}f}} = \frac{\sigma(\pi N \to f'X)}{\sigma(\pi N \to fX)} = \frac{\sigma(N\bar{N}\to f'X)}{\sigma(N\bar{N}\to fX)} = \frac{\sigma(K^- p\to Yf')_{back}}{\sigma(K^- p\to Yf)_{back}} = \tan^2\theta_f \tag{5.36b}$$

where X denotes any single or multiparticle hadron state which contains no strange particles, Y denotes any neutral hyperon or hyperon resonance and the subscript back denotes backward neutral meson production by baryon exchange.

If on the other hand, the violation in a given process comes from a specific higher order transition appropriate for that particular process, the relations (5.36) will not hold. One example of such a higher order transition which has been considered in detail is[30,31]

$$\pi^- p \to K^{*o}\Lambda \to \phi n \tag{5.37a}$$

$$\pi^- p \to K^{*o}\Lambda \to \omega n \ . \tag{5.37b}$$

The unitarity paradox has been formulated for the box diagrams corresponding to these reactions and has been shown to be related by SU(3) symmetry to the reactions

$$K^- p \to \rho^o\Lambda \to \pi^+\Sigma^- \tag{5.38a}$$

$$K^- p \to \rho^o\Lambda \to \pi^-\Sigma^+ . \tag{5.38b}$$

The transition (5.37a) is forbidden by the OZI rule; the transition (5.38a) is forbidden in standard models because it involves an exotic exchange. Yet both steps in these second-order processes are allowed and are simply related respectively to the corresponding allowed processes (5.37b) and (5.38b). The relations between corresponding allowed and forbidden processes have been written as a unitarity sum involving the intermediate states in the box diagram

$$\mathrm{Im}\ T_{AB} = \sum_i T_{Ai}\ T_{iB} \tag{5.39}$$

where A, B and i denote initial, final and intermediate states for any of the reactions (5.37) and (5.38). Note that for any intermediate state i, the right-hand sides of Eq. (5.39) are either equal for corresponding allowed and forbidden processes or differ only by a symmetry coefficient. The paradox is resolved by requiring cancelations in the unitarity sum for the forbidden processes, which are seen in quark-line formations to be represented by twisted diagrams analogous to Fig. 5.4.

Without a detailed model for all possible intermediate states, it is impossible to estimate how good the cancellations are. However, SU(3) symmetry has been used to relate the two forbidden processes, so that OZI breaking can be predicted with experimental data on exotic exchange amplitudes used an input.[30] Estimates of OZI violation have also been made by this box-diagram mechanism for low energies where the lowest mass intermediate state can be assumed to be dominant without cancellations from other channels.

Experimental results indicate that in some cases the mixing mechanism giving rise to Eqs. (5.36) are valid, while in other cases the box diagram description (5.37) may hold.[32,33] Further experiments would be of great interest.

Interference experiments have been useful in detecting OZI-violating transitions, since these measure a small amplitude directly, rather than the square. The forbidden f' production amplitude has been observed as interference with the tail of the allowed f amplitude in the reaction[32]

$$\pi^- p \to M^o n \to K^+ K^- n \qquad (5.40a)$$

where $M^o$ denotes either the f or f'. Similar f-f' interference effects between the tail of an allowed f peak and a forbidden f' amplitude could be seen in the reactions

$$K^- p \to M^o \Lambda \to \pi^+ \pi^- \Lambda \qquad (5.40b)$$

$$K^- p \to \Lambda M_o \to \Lambda K^+ K^- \quad \text{(backward)}\ . \qquad (5.40c)$$

Comparison of the reactions (5.40a) and (5.40b) would be interesting since they are very similar, with the roles of the production and decay of the f' interchanged. The production of the f' is forbidden by the OZI rule in reaction (5.40a) and the decay allowed, while the decay is forbidden and the production allowed in (5.40b). The reaction (5.40c) tests the forbidden baryon vertex, rather than the forbidden meson vertex and would give insight on the relation between OZI violations for the two cases.

Further tests of the OZI rule are now possible[11,18] in the decays of the new particles into final states containing the $\phi$ and f'. These also enable tests of the generalizations of the OZI rule to multiparticle vertices and can settle questions regarding the crossed Pomeron and multiply forbidden diagrams of Fig. 5.3.

Tests of the OZI rule are more complicated for production of mesons which are not ideally mixed like the pseudoscalars because there is no clear selection rule forbidding the production of any physical particle. The pseudoscalar state analogous to the $\phi$ or f' whose production is forbidden by the OZI rule in $\pi$p reactions is a linear combination of the $\eta$ and $\eta'$. However, the OZI rule for neutral meson production can also be tested by other relations which are not selection rules. Such relations were first derived by Alexander et al.[24] together with the selection rule forbidding $\phi$ production in $\pi$p reactions. An SU(3) rotation of the selection rule forbidding $\phi$ production with incident pions leads to the observation that an incident $K^-$ which contains no n-type quarks or antiquarks cannot produce the $(n\bar{n})$ vector meson state. Since this state is not a physical meson but a linear combination of the $\rho^0$ and $\omega$ states, the selection rule is expressible as an equality between $\rho^0$ and $\omega$ production amplitudes, and similarly for the tensor mesons

$$\sigma(K^-p \to \omega Y) = \sigma(K^-p \to \rho^0 Y), \tag{5.41a}$$

$$\sigma(K^-p \to fY) = \sigma(K^-p \to A_2 Y). \tag{5.41b}$$

An additional relation is also obtained from the additive quark model for each case,

$$\sigma(K^-p \to \phi Y) = \sigma(\pi^-p \to K^{*0} Y), \tag{5.42a}$$

$$\sigma(K^-p \to f' Y) = \sigma(\pi^-p \to K^{*0} Y). \tag{5.42b}$$

In the quark model derivation, these relations follow from the requirement that the meson transition be described as a single quark transition with the other quark being a spectator. All meson transitions in Eqs. (5.42) have the same quark transition $\bar{p} \to \bar{\lambda}$ and differ only in the quantum numbers of the spectator quark, which is a $\lambda$ on the left-hand side and an n on the right-hand side. The same relations (5.42) arise in Regge exchange models which assume SU(3) symmetry, no exotic exchanges and the OZI rule.

The relations (5.41) and (5.42) hold for any meson nonet but are not directly applicable to the pseudoscalar mesons because they assume ideal mixing. However, they can be combined to give a sum rule which holds independent of mixing angle and can be applied to the pseudoscalars as well

$$\sigma(K^-p \to \omega Y) + \sigma(K^-p \to \phi Y) = \sigma(K^-p \to \rho^o Y) + \sigma(\pi^-p \to K^{*o}Y), \tag{5.43a}$$

$$\sigma(K^-p \to fY) + \sigma(K^-p \to f'Y) = \sigma(K^-p \to A_2^o Y) + \sigma(\pi^-p \to K^{*o}Y). \tag{5.43b}$$

$$\sigma(K^-p \to \eta Y) + \sigma(K^-p \to \eta' Y) = \sigma(K^-p \to \pi^o Y) + \sigma(\pi^-p \to K^o Y). \tag{5.43c}$$

Additional sum rules independent of mixing angle were also obtained by Alexander et al. for pseudoscalar meson production,[24]

$$\sigma(\pi^-p \to \pi^o n) + \sigma(\pi^-p \to \eta n) + \sigma(\pi^-p \to \eta' n)$$
$$= \sigma(K^+n \to K^o p) + \sigma(K^-p \to \bar{K}^o n), \tag{5.44a}$$

$$\sigma(\pi^+p \to \pi^o\Delta^{++}) + \sigma(\pi^+ + p \to \eta\Delta^{++}) + \sigma(\pi^+p \to \eta'\Delta^{++})$$
$$= 3\sigma(K^-p \to \bar{K}^o\Delta^o) + \sigma(K^+p \to K^o + \Delta^{++}). \tag{5.44b}$$

Analysis of experimental data at 3.9 GeV/c shows striking agreement with experiment for the relations (5.41a) and (5.42a) for fector meson production but strong disagreement with experiment[34] for the sum rule (5.43c) for pseudoscalar meson production where the left-hand side is 705 ± 91 μb and the right-hand side is 1121 ± 59 μb. There are also troubles with the sum rule (5.44a). The validity of the OZI rule for the pseudoscalar mesons thus remains unsettled. There is also the possibility that the conventional nonet classification does not apply to pseudoscalars. One possible explanation for the disagreement of the sum rules with experiment is to assume that the OZI rule holds, that the $\eta$ is well represented by a state which is mainly a member of the same octet that contains the pion and kaon, but that the $\eta'$ is a more complicated mixture involving radially excited octet configurations.[35]

Since the selection rule forbidding the production of the $(n\bar{n})$ states leads to equality of amplitudes for $\rho^0$ and $\omega$ production and similarly for f and A2, the equalities (5.41) include the phase detectable in $\rho\omega$ and $fA_2$ interference experiments and found to agree with the predictions.[19] This interference provides another test of the OZI rule in reactions like (5.41) where the production of the $(n\bar{n})$ state is forbidden; e.g.

$$e^+e^- \to K^+K^-\pi^+\pi^-, \quad \psi,\psi' \to K^+K^-\pi^+\pi^-, \tag{5.45a,b}$$

$$(q\bar{q}) \to K^+K^-\pi^+\pi^-, \quad e^+e^- \to K^+K^-(K\bar{K}), \tag{5.45c,d}$$

$$\psi,\psi' \to K^+K^-(K\bar{K}), \quad (q\bar{q}) \to K^+K^-(K\bar{K}), \tag{5.45e,f}$$

where $(q\bar{q})$ denotes any quark-antiquark meson state, including new particles. In (5.45a-c) the $\pi^+\pi^-$ mass spectrum should show the characteristic "peak-dip" $\rho\omega$-interference pattern, constructive on the low-energy side of the $\omega$ peak and destructive on the high-energy side. In the reactions (5.45d-f) the $K\bar{K}$ spectrum in the $fA_2$ region should show interference constructive in the $K^+K^-$ decay mode and destructive in the $K^0\bar{K}^0$ mode.[11] Since the relative magnitudes and phases of the amplitudes follow from the OZI rule independent of kinematics, data at different energies and from several reactions can be combined to improve statistics.

### 5.12 Applications of the OZI Rule to the New Particles

The narrow widths of the new particles are attributed to the OZI rule, but there has been no reliable quantitative estimate of these widths from any theoretical model. There have been attempts to estimate the OZI violation for the new particles by using experimental data from the old particles as input. However, the analysis of the $f' \to \pi\pi$ decay in section 5.10 shows that this is unjustified. There is no simple way to apply these results to new particles which are states of new heavy quark-antiquark pairs. For these states the channel analogous to the $K\bar{K}$ channel involves pairs of mesons each containing one new heavy and one ordinary light quark. These channels are closed for the decays of the lowest-lying new meson states made of heavy quark-antiquark pairs. Thus, there is no possible higher order decay to hadron states made of light quarks via an intermediate state on the mass shell. The higher-order paradox arises only in virtual transitions to states of pairs of mesons carrying charm or some new quantum number and which must be off their mass shell. The calculation of these effects requires some underlying field theory as well as a knowledge of the spectrum and of the couplings of all these new particles in order to determine the effectiveness of various cancellation mechanisms. There is no simple estimate analogous to the one above for the $f' \to \pi\pi$ decay which is dominated by the on shell transitions. There is no way to extrapolate the observed violations in the $f'$ and $\phi$ decays to these other states by assuming a dependence on masses, since the effect observed in the $\phi$ and $f'$ cases can be entirely attributed to open channels on shell which do not exist for the new particles. Such mass extrapolations could give upper limits for OZI violation under the reasonable assumption that the effects of the off shell transitions must be smaller than the observed violations in the $f'$ and $\phi$ cases. However, such extrapolations are very risky since the contributions of these off shell transitions are probably very sensitive to subtle cancellation mechanisms which may be very different for the old and the new particles.

Thus the experimental observation that the OZI rule must be much better for the new particles than for the old particles leads to no contradictions. But there is also no simple way to estimate the widths of the new particles from theoretical models for OZI breaking.

Another application of the OZI rule to the new particles has been in estimates of charmed particle production in experiments searching for these particles. It has been suggested that charmed particles might be found in associated production with the $J/\psi$ because the production of the $J/\psi$ without charmed particles violates the OZI rule.[36] This argument is incorrect, as can be seen from the analogous case of the $\phi$ and strangeness. Kaons are not frequently found in associated production with the $\phi$, even though the production of the $\phi$ without accompanying kaons in pp collisions is forbidden by the OZI rule. That $\phi$ production is dominated by OZI violating mechanisms is still consistent with the OZI rule.[19]

The OZI rule can be tested only by comparing corresponding pairs of processes as in Eq. (5.36). Other comparisons are misleading, such as comparing $\phi$ production with and without kaon pairs in NN reactions. A proper comparison of these processes involves other dynamical considerations. This can be seen by writing

$$\frac{\sigma(NN) \to \phi + K + \bar{K} + X)}{\sigma(NN \to \phi + X)} = \frac{K \cdot S}{Z}, \tag{5.46}$$

where X contains no strange particles and the parameters K, S and Z are defined as

$$Z \equiv \frac{\sigma(NN \to \phi + X)}{\sigma(NN \to \omega + X)} , \tag{5.47a}$$

$$S \equiv \frac{\sigma(NN \to \omega + K + \overline{K} + X)}{\sigma(NN \to \omega + X)} , \tag{5.47b}$$

$$K \equiv \frac{\sigma(NN \to \phi + K + \overline{K} + X)}{\sigma(NN \to \omega + K + \overline{K} + X)} . \tag{5.47c}$$

The parameter Z is the ratio of a corresponding pair of OZI-violating and OZI-conserving processes and is small in any model which suppresses OZI-violating transitions. The parameter S is also small because experiment shows that it is hard to produce kaon pairs. The quantity K is of the order unity since it relates two processes allowed by the OZI rule and differing only by the interchange of two members of the same vector nonet. The value of the ratio (5.40) is thus not determined by the OZI rule and requires additional dynamical input. It depends upon which of the two small quantities S and Z, is smaller, i.e. whether it is harder to violate the OZI rule or to produce a pair of strange particles. The available data indicate that the strange-particle-production factor S overwhelms the OZI-violation factor Z and that $\phi$ production is dominated by the OZI-violating transition without kaons[37] up to energies of at least 24 GeV.

A similar argument holds for the production of $J/\psi$ particles with and without pairs of charmed particles in a charmonium model. By analogy with Eqs. (5.46) and (5.47)

$$\frac{\sigma(NN \to J/\psi + D + \overline{D} + X)}{\sigma(NN \to J/\psi + X)} = \frac{K_c \cdot Z_c}{C} , \tag{5.48}$$

where X can now contain strange particles but no charmed particles and the parameters $K_c Z_c$ and C are defined by

$$Z_c \equiv \frac{\sigma(NN \to J/\psi + X)}{\sigma(NN \to \omega + X)} , \tag{5.49a}$$

$$C \equiv \frac{\sigma(NN \to \omega + D + \overline{D} + X)}{\sigma(NN \to \omega + X)} , \tag{5.49b}$$

$$K_c \equiv \frac{\sigma(NN \to J/\psi + D + \overline{D} + X)}{\sigma(NN \to \omega + D + \overline{D} + X)} \qquad (5.49c)$$

Again the ratio (5.48) depends on which is the smaller of two small quantities $Z_c$ and C; i.e. whether it is harder to violate the OZI rule or to produce a pair of charmed particles. In addition the factor $K_c$ which is of order unity in the SU(4) symmetry is probably also small. Present experiments suggest here also that charmed particle production is suppressed more than OZI violating transitions and that the $J/\psi$ is produced primarily without charmed particles. This can also be seen in a model like Eq. (5.36) with $Z_c$ a universal factor describing the suppression of all OZI-violating transitions involving a charmed quark-antiquark pair. A value of about $10^{-3}$—$10^{-4}$ for $Z_c$ is obtained by comparing the width of the OZI-violating decay of the J into normal hadrons with expected widths of about 100 MeV for OZI-conserving decays. The observed total cross sections for $J/\psi$ and $\omega$ production are consistent with this value of $Z_c$ and Eq. (5.49a).

If the OZI rule were exact and all OZI-violating processes had zero cross section, all production of particles like the $\phi$ and the $J/\psi$ would be via OZI-conserving reactions. However, once the OZI rule is broken the OZI-violating transitions are proportional to small but finite suppression factors as in Eqs. (5.47a) and (5.49a). Whether a given production process is dominated by OZI-conserving or OZI-violating transitions depends upon whether the OZI-conserving transition is suppressed more strongly than the OZI-suppression factor by a different dynamical mechanism, as in the reactions (5.46) and (5.48).

## VI. WHY ARE NARROW CONTINUUM STATES INTERESTING?

The new particles promise new exciting opportunities for interesting research. To understand why they are so interesting it is instructive to compare them with the isobaric analog states. There are many common features. The isobaric-analog states were understood very shortly after their discovery, but they are still very interesting and have opened a new field in nuclear physics. For the same reason the new particles will still be very interesting even after we understand their structure. There is much discussion about these new particles and whether they should be called $\psi$ or J. I see that Italians know all about the Psi (Italian socialist party).

Narrow states in the continuum are interesting because they combine the best features of the ordinary low-lying states and the ordinary continuum states. In any non-trivial spectroscopy there are models which are not exact but useful because they are reasonably good for the low-lying states where configurations are simple. At higher excitation the models get worse; the configurations are much more complicated, the density of states is higher, and there is much more mixing. The interactions neglected in the models become more unimportant at higher excitation, where the level density becomes greater and there are many configurations close together and easily mixed. Thus there is no simple theoretical model for the wave functions high in the continuum. On the other hand, high continuum states are very convenient for experiment because they have many open channels, can be excited in many ways, and provide a very rich source of experimental data. The isobaric analog states give the best features of both worlds. They are up in the continuum and have many possible open channels, many ways of excitation, and many things to study. But they also have a very simple structure very much like the low-lying states and simple theoretical models can be used. Any model good for the low-lying states is also just about as good for the narrow states in the continuum. This then opens a wide field for experimental tests and investigations of the various models.

How do we know that narrow states in the continuum have a simple structure? We know the structure of the isobaric analog resonances and can answer this question in detail. But the answer is similar for the new particles even though we do not understand their structure. In both cases we know that the structure of the states must be simple because they are simply produced. The isobaric analog state is strongly produced by a nuclear charge-exchange reaction at zero momentum transfer on a nuclear ground state; i.e. by changing a neutron into a proton without changing its momentum. Any state strongly produced by this very simple operation must differ from the ground state only by a simple elementary excitation and cannot be a state with twelve particles excited.

The new particles are made very strongly by electron-positron annihilation through one virtual photon. The transition matrix element connects the particle state with a vacuum by the operator of the electromagnetic current and is appreciable only if the particle has a very simple structure. For example, in simple quark or parton models, the electromagnetic current operator creates only single quark-antiquark pair. This tells us that the single pair part of the wave function must be appreciable in any strongly produced state. If it were mainly a state of thirty quarks and thirty antiquarks it would not have a very large matrix element for a single photon transition from the vacuum.

## VII. WHAT ARE STRANGENESS AND BARYON NUMBER?

The new charm degree of freedom[2] provides a new quantum number like electric charge, strangeness and baryon number. But understanding charm is difficult when we still do not understand the old internal degrees of freedom.[38] We have some understanding of the role of electric charge in particle interactions and dynamics even though we do not understand why electric charge is quantized and universal. But our understanding of baryon number and strangeness is much weaker. There is no theory like quantum electrodynamics in which baryon number or strangeness appear as coupling constants defining the strengths of interactions. There is no formula analogous to the Rutherford formula for Coulomb scattering describing the dependence of strong interaction scattering on baryon number and strangeness.

A few phenomenological models and symmetries like the quark model and SU(3) symmetry give rough descriptions of the dependence of total cross sections on baryon number and strangeness. But these descriptions are highly inadequate and the difference between mesons and baryons and between strange and non-strange hadrons are not really understood. Furthermore, many of the models developed work in only one area of hadron physics and are incompatible with models used in other areas. For example, the quark model used in describing hadrons strong interactions is not the same as the quark model used in weak interactions.

Consider, for example, the description by conventional models of the difference between pion and kaon wave functions. The quark model says that both are made from a quark-antiquark pair.[14] But weak interaction quarkists explain the ratio of the $\pi \to \mu + \nu$ and $K \to \mu + \nu$ decay requiring the wave functions at the origin to be very different as described by Weisskopf-Van-Royen[39] formula

$$\frac{|\psi_K(0)|^2}{|\psi_\pi(0)|^2} = \frac{M_K}{M_\pi} . \tag{7.1}$$

Strong interaction quarkists say that the difference between pion and kaon wave functions is measured by the difference between their scattering cross sections on nucleons. These differ by less than 20%. Recent data at high energies show that $\pi p$ and Kp differential cross sections approach equality with increasing momentum transfer. This suggests equality within 20% of the mean square radii of pion and kaon wave functions and nearly identical short distance behavior, in sharp contrast with the weak quarkist Eq. (7.1) .

The very precise experimental data[40] now available on pion, kaon and nucleon total cross sections give us some information about the difference between the interactions of strange and nonstrange particles with matter. Careful examination of the data show that this difference is very interesting but also very puzzling and not really understood.[41] Instead of the conventional plot of total cross sections versus laboratory momentum on a logarithmic scale, we show the systematics in a more interesting plot (Fig. 7.1) with a square root scale rather than a logarithmic scale for $P_{lab}$ and with the total cross section multiplied by $\sqrt{P_{lab}}$. This is equivalent at these high energies to a plot against center-of-mass momentum of the imaginary part of the forward amplitude obtained from the total cross section by the optical theorem. Theoretical reasons why the curve of 7.1 is so much simpler than the standard plot follow from a two-component description of the cross sections with a Regge component varying as $s^{-1/2}$ and a pomeron component varying slowly as a function of energy. A more detailed discussion

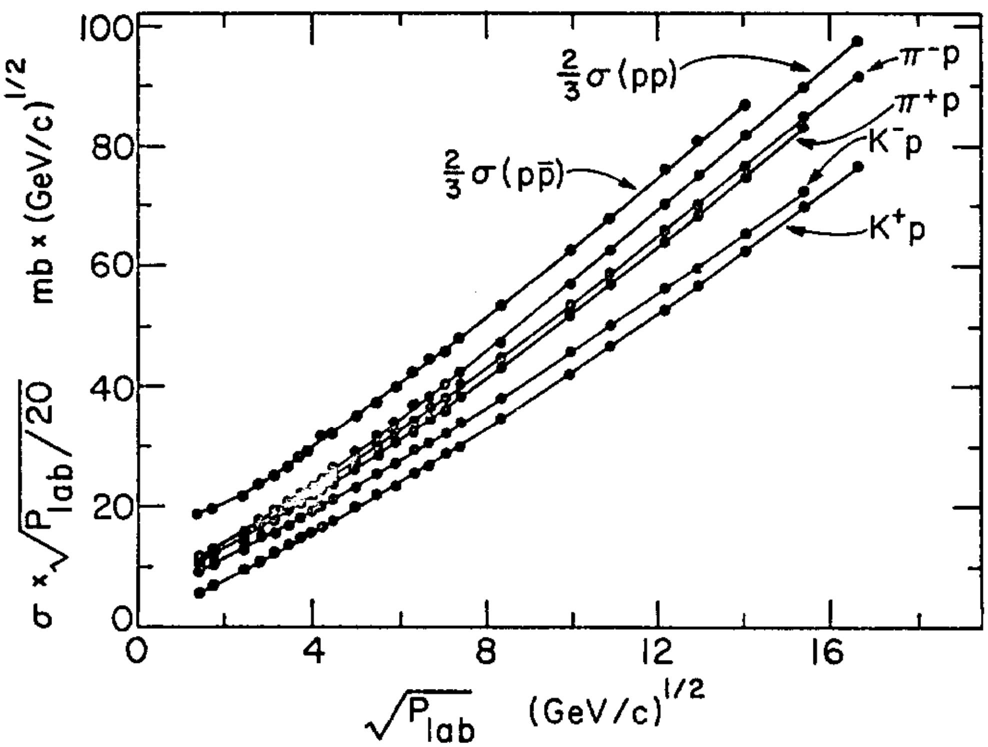



Fig. 7.1. $\sigma_{tot}\sqrt{P/20}$ vs. $\sqrt{P}$. Nucleon cross sections multiplied by 2/3.

is given elsewhere.[42] For our purposes this particular plot shows very clearly that there is a difference between strange and non-strange particles and that there are puzzles not explained by the quark model.

In Fig. 7.1 the nucleon-nucleon and nucleon-antinucleon cross sections are multiplied by a factor 2/3. The six quantities plotted are just those predicted to be equal asymptotically in the simple quark model with the pomeron component an SU(3) singlet coupled equally to pions and kaons and coupled to mesons and baryons by simple quark counting prescriptions. Figure 7.1 shows that these cross sections are indeed all equal at the 20% level. However, beyond this approximation of "seen one hadron, seen them all" the difference between the $\pi p$ and the pp cross sections is seen to be strangely similar to the difference between the $\pi p$ and Kp cross sections. The difference between mesons and baryons seems to be similar to the difference between nonstrange and strange mesons.

This regularity is shown more precisely by examining linear combinations of cross sections which have no Regge component and are therefore conventionally assumed to be pure pomeron. The $K^+p$ and pp channels are exotic and have no contribution from the leading Regge exchanges under the common assumption of exchange degeneracy. The following linear combinations of meson-nucleon cross sections are constructed to cancel the contributions of the leading Regge trajectories

$$\sigma(\phi p) = \sigma(K^+p) + \sigma(K^-p) - \sigma(\pi^-p) \tag{7.1a}$$

$$\Delta(\pi K) = \sigma(\pi^-p) - \sigma(K^-p). \tag{7.1b}$$

Figure 7.2 shows these two quantities on the conventional plot of cross section versus $P_{lab}$ on a log scale.

$\sigma(\phi p)$ as defined by Eq. (7.1a) is the quark model expression for $\sigma(\phi p)$; i.e., the cross section for the scattering of a strange quark-antiquark pair on a proton. The very simple energy behavior of this quantity as seen in Fig. 7.2 is striking. It shows a monotonic rise beginning already at 2 GeV/c. That total cross sections rise at high energies was first noticed by Serpukhov data from 20—50 GeV/c, but the older data at lower energies already show this rising behavior in $\sigma(\phi p)$. If anyone has suggested something particularly fundamental about this cross section for strange quarks on a nucleon before the Serpukhov data were available and concluded that its rising cross section indicated that all cross sections would eventually rise he would naturally

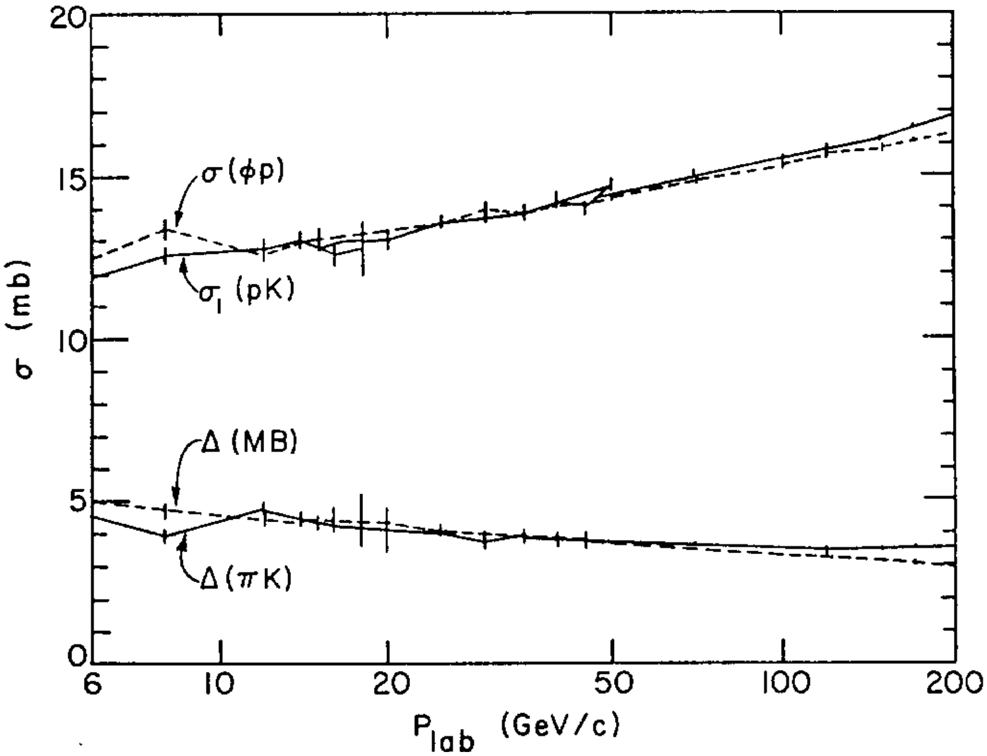



Fig. 7.2. Plots of Eqs. (7.1) and (7.2).

have been disregarded as crazy. But now that the whole picture up to 200 GeV/c is available we may conclude that there is indeed something simpler and more fundamental about the cross sections for strange quarks on a proton target. Understanding this simpler behavior may help us to understand the more complicated energy behavior of the other cross sections.

The quantity $\Delta(\pi K)$ defined by Eq. (7.1b) represents the difference in the scattering of a strange particle and a nonstrange particle on a proton target. In the quark model this is the difference between the scattering of a strange quark and a nonstrange quark on a proton target after the leading Regge contributions have been removed. This difference between strange and nonstrange also has a very simple energy behavior, decreasing constantly and very slowly (less than a factor of 2 over a range $P_{lab}$ of two orders of magnitude). So far there is no good explanation for why strange and nonstrange mesons behave differently in just this way.

Since the two quantities (7.1) have no contribution from the leading Regge trajectories they represent something loosely called the pomeron. However, their energy behaviors are different from one another and also from that of the quantities $\sigma(K^+p)$ and $\sigma(pp)$ which should also be "pure pomeron." However the following linear combinations of $\sigma(K^+p)$ and $\sigma(pp)$ have exactly the same energy behavior as the meson-baryon linear combinations (7.1)

$$\sigma_1(pK) = \frac{3}{2}\,\sigma(K^+p) - \frac{1}{3}\,\sigma(pp) \tag{7.2a}$$

$$\Delta(MB) = \frac{1}{3}\,\sigma(pp) - \frac{1}{2}\,\sigma(K^+p). \tag{7.2b}$$

These quantities are also plotted in Fig. 7.2.

The equality of the quantities (7.2) and the corresponding quantities (7.1) suggest that the pomeron, defined as what is left in the total cross sections after the leading Regge contributions are removed by the standard prescription, consists of two components, one rising slowly with energy and the other decreasing slowly. The coefficients in Eq. (7.2) were not picked arbitrarily but were chosen by a particular model. In this model the rising component of the total cross section is assumed to satisfy the standard quark model recipe exactly.

$$\sigma_R(Kp) = \sigma_R(\pi p) = \frac{2}{3}\,\sigma_R(pp) = \frac{2}{3}\,\sigma_R(Yp) = \frac{2}{3}\,\sigma_R(\Xi p), \tag{7.3a}$$

where Y denotes a $\Lambda$ or $\Sigma$ hyperon. The falling component has been assumed to satisfy the following relation

$$\sigma_F(Kp) = \frac{1}{2}\,\sigma_F(\pi p) = \frac{2}{9}\,\sigma_F(pp) = \frac{1}{3}\,\sigma_F(Yp) = \frac{2}{3}\,\sigma_F(\Xi p). \tag{7.3b}$$

This particular behavior is suggested by a model in which the correction to a simple quark-counting recipe comes from a double exchange diagram involving a pomeron and an $f$ coupled to the incident particle.[41]

We thus see unresolved problems in the total cross-section data associated with the questions of what is the difference between strange and nonstrange particles and what is the nature of the pomeron. Note that Eq. (3.1b) defines the difference between the scattering of a nonstrange quark and a strange quark while Eq. (7.2b) can be interpreted as the difference between the scattering of a quark in a baryon and a quark in a meson. The fact that the strange-nonstrange difference and the meson-baryon difference are equal and have the same energy behavior over such a wide range is a puzzle which may be explained by pomeron-f double exchange but may also indicate something deeper.

The cross section differences $\Delta(\pi K)$ and $\Delta(MB)$ are both predicted to vanish in the simple model where the pomeron is an SU(3) singlet which is coupled to the quark number and all the curves of Fig. 7.1 are equal. The deviation from the additive quark model ratio of 2/3 for meson to baryon scattering appears as a finite value of $\Delta(MB)$, the deviation of the pomeron coupling from an SU(3) singlet appears as a finite value of $\Delta(\pi K)$. One might ask whether both these decreasing quantities approach zero at high energies, so that the simple model would be valid in asymptopia.

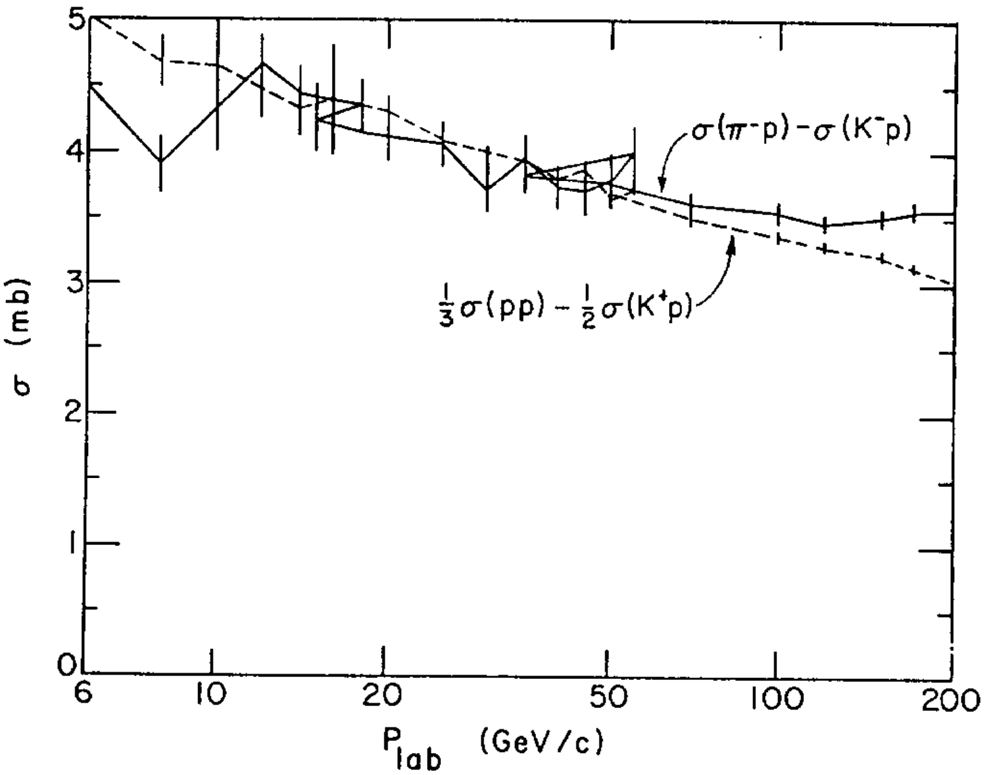



Fig. 7.3. Plots of $\Delta(\pi K)$ and $\Delta(MB)$ on an expanded scale.

A close look at the experimental plots on an expanded scale of $\Delta(\pi K)$ and $\Delta(MB)$ in Fig. 7.3 reveals a small difference in the behavior at the highest energies. The curve for $\Delta(\pi K)$ seems to be leveling off above 50 GeV/c, while that for $\Delta(MB)$ continues decreasing monotonically. This trend seems to continue in the one or two additional points available up to 280 GeV/c which are not plotted. Additional data up to 400 GeV/c should determine whether $\Delta(\pi K)$ has definitely stopped decreasing and is approaching a constant, while $\Delta(MB)$ is decreasing. If this is the case, then the additive quark model becomes good at high energies while pions and kaons continue to look different even at asymptopia and the SU(3) relation never becomes good. The equality observed between these two differences over the 6—200 GeV/c range and described by a two-component Pomeron ultimately breaks down at higher energies.

A search for similar systematics in elastic hadron scattering differential cross section data has led to new surprises and paradoxes.[38] With only differential cross section data available and no detailed amplitude analysis, it is convenient to define the quantity[41,43]

$$S(Hp) = [\frac{d\sigma}{dt}(\bar{H}p) + \frac{d\sigma}{dt}(Hp)]^{1/2} \tag{7.4}$$

where $H$ is any hadron. This quantity $S(Hp)$ is assumed to give a good approximation for the Pomeron contribution to the Hp scattering amplitude. With this assumption the simple additive quark model prediction that $\Delta(MB) = 0$ becomes

$$S(\pi p) = (2/3)S(pp) \tag{7.5a}$$

when we use $S(\pi p)$ to represent a typical meson baryon cross section. The assumption that the Pomeron is a SU(3) singlet predicts $\Delta(\pi K) = 0$ and

$$S(\pi p) = S(Kp). \tag{7.5b}$$

The two relations (7.5a) and (7.5b) describe the dependence of the scattering amplitude on baryon number and strangeness, respectively. The two component Pomeron model which relates the deviations from the two predictions (7.5a) and (7.5b) predicts the weaker sum rule

$$S(\pi p) = \frac{1}{2} S(Kp) + \frac{1}{3} S(pp). \tag{7.6}$$

The experimental data[44] show that the weaker sum rule (7.6) is in much better agreement with experiment than the additive quark model prediction (7.5a). However, the SU(3) prediction which is not very good at $t = 0$ becomes better at larger values of t and becomes much better than the two component Pomeron prediction (7.6) or the additive quark model prediction (7.5a). Two examples of this comparison with experiment are given in Table 7.1. The same qualitative features are present in all the data.

Table 7.1

Tests of Additive Quark Model (AQM), Two-Component Pomeron (P2) and SU(3) Relations Between Differential Cross Sections. RHS/LHS of Eqs. (7.5a), (7.5b) and (7.6).

| P = 100 GeV/c | | | | P = 175 GeV/c | | | |
|---|---|---|---|---|---|---|---|
| t (GeV/c) | AQM (7.5a) | P2 (7.6) | SU(3) (7.5b) | t (GeV/c) | AQM (7.5a) | P2 (7.6) | SU(3) (7.5b) |
| 0.0 | 1.2 | 1.0 | 0.84 | 0.0 | 1.1 | 0.97 | 0.84 |
| -0.08 | 1.0 | 0.95 | 0.86 | -0.08 | 0.98 | 0.92 | 0.85 |
| -0.16 | 0.94 | 0.91 | 0.88 | -0.16 | 0.89 | 0.88 | 0.86 |
| -0.24 | 0.85 | 0.87 | 0.90 | -0.24 | 0.81 | 0.84 | 0.88 |
| -0.32 | 0.78 | 0.85 | 0.92 | -0.32 | 0.74 | 0.81 | 0.89 |
| -0.40 | 0.71 | 0.83 | 0.94 | -0.40 | 0.68 | 0.79 | 0.90 |
| -0.48 | 0.66 | 0.81 | 0.97 | -0.48 | 0.63 | 0.77 | 0.92 |
| -0.56 | 0.61 | 0.80 | 1.0 | -0.56 | 0.58 | 0.76 | 0.93 |
| -0.64 | 0.56 | 0.80 | 1.0 | -0.64 | 0.54 | 0.74 | 0.95 |
| -0.72 | 0.53 | 0.80 | 1.1 | -0.72 | 0.50 | 0.73 | 0.96 |
| -0.80 | 0.50 | 0.80 | 1.1 | -0.80 | 0.47 | 0.72 | 0.98 |

The comparison with experiment of relations (7.5a) and (7.6) does not really add any new qualitative information. It is summed up by the observation that at the optical point the relation (7.5a) is not very good and the relation (7.6) is much better and that baryon-baryon cross sections decrease much more rapidly with t than meson-baryon cross sections. The behavior at the optical point is expected from the similar behavior of total cross sections. The high t behavior is expected since naive additive quark model predictions (7.5a) and (7.6) neglect differences between meson and baryon wave functions. These differences introduce additional form factors into the scattering amplitudes, which cause baryon amplitudes to decrease more rapidly with increasing t than meson amplitudes.

However, the improvement of the relation (7.5b) with increasing t comes as a complete surprise. One can ask why pions and kaons should look more alike[45] at high t than at low t. One might also ask whether the two are really approaching equality or whether there will be a cross over and that still at higher t the amplitude will differ in the opposite direction.

We thus seem to see a peculiar systematics in which the additive quark model becomes good at $\underline{t}$ = 0 and high $\underline{s}$ but not at high $\underline{t}$, the SU(3)-symmetric pomeron becomes good at high t, but not at t = 0, even at high $\underline{s}$, and the two-component pomeron description holds at t = 0 and s between 6 and 200 GeV/c, where there are discrepancies in both the additive quark model and the SU(3)-symmetric pomeron. Further data on total cross sections at higher energy, differential cross sections at higher momentum transfer, and hyperon total and differential cross sections everywhere will show whether these puzzling features are really in the data, and will provide clues for our understanding of the differences between strange and nonstrange particles and between mesons and baryons.

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but data from all three experiments have been used to test relations (7.5) and (7.6) with similar results. The help of all three groups in making data available before publication and computing preliminary tests of relations (7.5) and (7.6) is gratefully acknowledged, and in particular correspondence and discussions with D.D. Yovanovitch, R. Diebold, D. Leith and J. Mikenberg.

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