



A TURNING POINT IN THE DEVELOPMENT OF QUANTUM MECHANICS AND THE EARLY YEARS OF THE MOSSBAUER EFFECT*

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It is interesting to hear about the exciting early days recalled by Professors Wigner and Wick. I learned quantum theory at a later period, which might be called a turning point in its development, when the general attitude toward quantum mechanics and the study of physics was very different from what it is today.

As an undergraduate student in electrical engineering in 1940 in the United States I found a certain disagreement between the faculty and the students about the "relevance" of the curriculum. Students thought a 4-year course in electrical engineering should include more electronics than a one-semester 3-hour course. But the establishment emphasized the study of power machinery and power transmission because 95% of their graduates would eventually get jobs in power. Electronics, they said, was fun for students who were radio hams but useless on the job market. Students at that time did not have today's attitudes and did not stage massive demonstrations and protests against the curriculum. Instead a few of us who wished to learn more interesting things satisfied all the requirements of the engineering school and spent as much extra time as possible listening to fascinating courses in the physics building. There we had the opportunity to listen to two recently-arrived Europeans, Bruno Rossi and Hans Bethe. In the long run this served us much better than any protest or demonstration. Even if we would have won a confrontation with the establishment our reward would only have been a number of additional courses in electronics, including vacuum-tube circuits, oscillators, amplifiers, detectors, transmitters and superheterodyne receivers. But we would have graduated the university without ever hearing of Maxwell's equations. We were much better off learning about Maxwell's equations and other exciting physics from Rossi, Bethe and the other professors in the Physics Department.

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After graduation the importance of electronics had become clear and I spent four years during World War II at the Radiation Laboratory at M.I.T. working on radar development. The contrast was very clear between physicists who had studied basic physics without much electronic circuitry and electrical engineers who had studied electronic circuits without learning Maxwell's equations. It was comparatively easy for the physicists to learn everything needed about circuitry and to plunge immediately into the new domains of pulsed circuits, ultra-high frequencies and microwaves. But electrical engineers had great difficulty understanding how a signal could travel through the air from one piece of an amplifier to another without going through wires. I was once sent to help a well-known electronics firm debug a prototype radar receiver and told the experienced technicians and engineers that a wire soldered at one point on the chassis should be moved to another point a quarter of an inch away. They were sure that I was crazy, because all points on the chassis were ground and all grounds were equivalent. But rather than argue they made the change and expected to laugh at me afterwards. When all the peculiar parasitic oscillations disappeared after the wire was moved they thought it was black magic.

Many such experiences revealed the importance of a broad education in basic fundamentals and the weakness of specialization. Neither professors nor students know what will be relevant in five or ten years time. A narrow specialization in areas relevant at the moment is a sure route to obsolescence. Thus after the war I went to graduate school and decided that a study of the fundamentals of physics would be more useful than continuing study in engineering whatever my future career would be.

Today the engineers have learned this lesson and provide much broader backgrounds to students to help them adjust to a changing technology. But perhaps the physicists have slipped backwards into overspecialization. There is a tendency to train students to be highly sophisticated and specialized in narrow areas like group theory and symmetries, Regge poles, and gauge theory without knowing much about other areas of physics. Perhaps it is time for some "interdisciplinary" communication within physics.

The period 1946-48 was very unusual for the study of quantum mechanics, particularly at Princeton. This was just before the modern developments of quantum electrodynamics by Feynman, Schwinger and Tomonaga, just before the discovery of the pion, and before modern beta-decay experiments showed the true shape of the beta spectrum. The Princeton student in 1946 received the impression that everything was wrong with conventional physics and that a new revolution was needed. The Fermi theory of beta decay was wrong because experimentally-measured beta spectra did not agree

with theoretical predictions. The Yukawa meson theory of nuclear forces was wrong because the meson refused to interact with nuclei. All the elaborate attempts to explain why the meson captured in an atomic orbit decayed rather than being absorbed by its strong nuclear interaction did not work and looked to graduate students like grasping at straws to save a wrong theory.*

Quantum mechanics was presented to the student as a sick theory which had outlived its usefulness. It had been very exciting and successful in atomic physics but a new crisis had developed at smaller distances. All atomic physics calculations where quantum mechanics could give useful results had already been done. All attempts to use quantum mechanics for new phenomena did not work. We were told that the new theory needed to describe small-distance phenomena would be as different from ordinary quantum mechanics as quantum mechanics was from classical mechanics. We were not even taught to use quantum mechanics in the domains where the old quantum mechanics would still be expected to work such as nuclear structure and solid-state physics. These complicated many-body systems were considered much too difficult to treat with simple models. Instead, we heard about statistical models for nuclei.

Quantum mechanics was taught with the emphasis on the fundamentals. Our aim was not to learn to use it but to find out what was wrong with it, to find the troubles and inconsistencies which would provide clues to the new theory for small distances. The spirit of this period is seen in David Bohm's book which grew out of lectures given at Princeton. It is also interesting that the most outstanding student of that period at Princeton, Arthur Wightman, has devoted his subsequent research efforts to investigating fundamentals rather than applying the theory to the very wide area of physical phenomena opened to quantum mechanics after a short time.

Today, 30 years later, quantum mechanics is still going strong. The basic theory has not undergone any drastic revolution and is exactly the same as what I studied in graduate school. New mathematical techniques have been developed to avoid the difficulties which seemed insuperable to us and new models and applications now provide a rich field for applications.

*The answers were that the experimentalists had first discovered the wrong meson, the muon, which indeed had nothing to do with nuclear forces, and that the beta-decay experiments were wrong. Much effort these days seemed to go into performing wrong (or incorrectly-interpreted) experiments or in inventing theories to explain wrong experiments. As a graduate student I did some of both.

The great breakthroughs which opened up quantum mechanics as an exciting field for work in all areas occurred in a series of developments around 1950. The modern quantum electrodynamics of Feynman, Schwinger and Tomonaga enabled the calculation of all processes in QED without ambiguities or infinities. The development of the nuclear shell and collective models enabled simple descriptions of nuclei in which quantum mechanics played an essential role. The discovery of the pion, the use of pion beams to study pion-nucleon scattering, and the discovery of Δ led to a new domain of particle physics where basic ideas of quantum mechanics were again applied in many ways. Solid-state physics and quantum optics with semiconductors, superconductors, masers, and lasers opened up an enormous new area for the application of quantum mechanics. Very soon after I finished my studies the teaching of quantum mechanics changed drastically. It was presented as the basis and key tool for all applications of modern physics.

But this explosion of quantum physics was accompanied by a splitting of physics into specialized areas. I learned physics from professors like Bethe and Wigner who were at home in all areas and made important contributions to atomic, nuclear, particle, solid-state, and astrophysics. However, the generation of students trained by these people already shows specialization. The students of the great men who participated in the early days of quantum mechanics include outstanding physicists who have made important contributions in many areas but who have difficulty in communicating with one another because of their specialization. Today at the Erice school we have heard Steve Weinberg give a series of lectures translating the achievements of statistical mechanics into a language that can be understood by field theorists in particle physics. Professors Wigner and Wick understand both languages but nearly all of today's students need a translator.

In the education of graduate students today, the quantum mechanics course is the last opportunity for all physicists to study together before specialization. Quantum mechanics should be presented from a broad point of view that can enable the student to understand developments in fields outside his own area and facilitate his moving to a different area at some time in the future. Simple fundamentals can exhibit the power and generalities of the basic physics without over-specialization and complicated formalism.

As one example of how quantum mechanics provides much physics from a few simple ideas, let us consider some elementary problems in the theory of identical particles. In the simple Schroedinger description of a two-particle system the wave function has the form $\psi_1(x)\psi_2(y)$ and describes the amplitude for particle 1 being at point x and particle 2 at point y . There is also another wave function $\psi_1(y)\psi_2(x)$ which describes particle 1 being at point y and particle 2 being at point x . For two identical

particles like two electrons, there is no difference between particle 1 and particle 2 and these two wave functions must describe the same physical state.

This ambiguity can be avoided by using a notation from field theory which does not introduce unphysical labels 1 and 2 for the two particles but simply describes the state in which one particle is at point x and one at point y . We can use this notation without the formal apparatus of field theory and define creation operators $a^\dagger(x)$ and corresponding destruction operators $a(x)$ in the usual way. A one-particle state is written as a creation operator acting on the vacuum. The state with one particle at x and a second particle at y is written as the product of two creation operators acting on the vacuum. One ambiguity remains because the state can be written with either of the operators operating first and the other second. Since these two wave functions describe the same physical state, we require them to differ only by a phase factor and obtain the relation

$$a^\dagger(x)a^\dagger(y)|0\rangle = \pm a^\dagger(y)a^\dagger(x)|0\rangle. \quad (1)$$

This specifies a commutation or anti-commutation relation for the creation operators and leads to the two well-known possibilities of Bose and Fermi statistics.

At this stage we have simply described familiar phenomena in a particular notation. But now that the notation and commutation rules are defined we can continue into new areas. For example the question whether a two-particle bound state like a deuteron is a boson or a fermion now has a clear answer with no further ambiguities. A straightforward calculation tells how a two-particle bound state behaves. The operator which creates it out of the vacuum is uniquely defined and its commutator or anti-commutator with its Hermitean conjugate tells whether it satisfies Bose or Fermi statistics.

The most general two-particle state having total momentum q can be written

$$\sum_k g_k a_{k+q}^\dagger a_{-k}^\dagger |0\rangle \equiv B_q^\dagger |0\rangle \quad (2)$$

where a_k^\dagger creates a particle with momentum k and g_k is the wave function for the relative motion in momentum space. Spin is neglected in this discussion because it plays no essential role and can be easily incorporated by introducing appropriate additional indices. The operator B_q^\dagger thus creates this particular two-particle bound state out of the vacuum. Let the operators a_k^\dagger be Fermion operators satisfying the Fermi anti-commutation relation. The

commutator of the creation and destruction operators for the two-particle states is easily calculated and shows that the two-particle bound states are almost but not quite bosons. They satisfy commutation relations like those of bosons but with an additional correction term.

$$[B_q, B_{q'}^\dagger] = \delta_{qq'} + \text{correction} . \quad (3)$$

The correction term is seen to be small as long as the density of particles is low; i.e. as long as the probability that two bound pair wave functions overlap is very small. This expresses the obvious physics that when two Fermion pairs are too close together they cannot behave like bosons because the constituent Fermions in different pairs must satisfy the exclusion principle. They cannot be placed into the same quantum state or in any state where there is an appreciable overlap of the two wave functions.

Suppose that the wave function (2) describes a bound two-particle state which is held together by a two-body interaction. The total interaction energy which we denote by $-V_{\text{pair}}$, is obtained by calculating the expectation value of the potential V in the wave function (2),

$$-V_{\text{pair}} = \langle 0 | B_q V B_q^\dagger | 0 \rangle \equiv \sum_{k''k'''} \langle k''+q, -k'' | V | k'+q, -k' \rangle g_{k''}^* g_{k'} \quad (4)$$

where the interaction potential V is expressed in terms of its matrix elements between plane wave states.

Let us now consider the three-particle state obtained by adding a Fermion with momentum k' to a bound-pair state with zero momentum

$$\begin{aligned} |k', B_0\rangle &\equiv a_{k'}^\dagger B_0^\dagger |0\rangle = a_{k'}^\dagger \sum_k g_k a_k^\dagger a_{-k}^\dagger |0\rangle \\ &= a_{k'}^\dagger \sum_{k \neq \pm k'} g_k a_k^\dagger a_{-k}^\dagger |0\rangle, \end{aligned} \quad (5a)$$

since

$$a_{k'}^\dagger a_{k'}^\dagger = 0. \quad (5b)$$

This equation shows an interesting physical effect of the Fermi statistics. Because two Fermions cannot occupy the same state the addition of a Fermion in the state k' removes the term in the pair wave function which has one particle in the state k' . This change modifies the interaction energy since all contributions from matrix elements of V involving the state k' are lost. This effect is expressed quantitatively by calculating the interaction between the two members of the bound pair. We neglect the interaction of the third particle with them. Then

$$\langle k', B_0 | V | k' B_0 \rangle = -V_{\text{pair}} + \epsilon \quad (6a)$$

where

$$\epsilon = \sum_k \langle k', -k' | V | k, -k \rangle + \text{c. c.} \quad (6b)$$

and we neglect the dependence of ϵ on k' . A complete description also considers the kinetic energy as well as the potential energy but the basic physics of the problem is not changed by disregarding the kinetic energy and calculating only expectation values of the interaction.

The change in the interaction energy (6) does not come from the interaction of the pair with the third particle but rather because the third particle occupies a particular state and makes it unavailable to the pair. If the wave function (2) is an eigenfunction of the Schroedinger equation for the particular potential (4) the values of the coefficients g_k have been chosen to minimize the energy. Any modification of these coefficients gives a state with higher energy. The particular modification produced by adding an additional particle, namely to eliminate one term in the sum, is a special case of this kind of change and increases the energy by an amount ϵ .

Consider now the addition of two particles with momenta k' and k'' to the bound pair wave function

$$|k', k'', B_0 \rangle \equiv a_{k'}^\dagger a_{k''}^\dagger a_{B_0}^\dagger |0 \rangle = \sum_{\substack{k \neq \pm k' \\ k \neq \pm k''}} a_{k'}^\dagger a_{k''}^\dagger g_k a_k^\dagger a_{-k}^\dagger |0 \rangle. \quad (7)$$

For this case two terms in the bound state wave function are suppressed by the addition of the two particles if $k' \neq -k''$. The interaction energy is

$$\langle k', k'', B_0 | V | k', k'', B_0 \rangle = -V_{\text{pair}} + 2\epsilon \text{ if } k' \neq -k''. \quad (8)$$

But if $k' = k''$ only one term in the bound-state wave function is suppressed

$$|k', -k', B_0 \rangle \equiv a_{k'}^\dagger a_{-k'}^\dagger B_0^\dagger |0\rangle = \sum_{k \neq \pm k'} a_{k'}^\dagger a_{k''}^\dagger g_k a_k a_{-k} |0\rangle, \quad (9)$$

and the interaction energy is given by

$$\langle k', -k', B_0 | V | k', -k', B_0 \rangle = -V_{\text{pair}} + \epsilon. \quad (10)$$

Let us now construct a wave packet of states of the form (7) in which the two additional particles are in a bound-pair wave function with momentum q

$$|B_q B_0 \rangle \equiv |B_q^\dagger B_0^\dagger |0\rangle. \quad (11)$$

The interaction energy for each of the pairs is given by Eq. (8) because the additional pair always closes off two states. If we neglect the interaction between pairs, which could be shown to be small, the total interaction for the state (11) is

$$\langle B_q B_0 | V | B_q B_0 \rangle = -2V_{\text{pair}} + 4\epsilon = 2[-V_{\text{pair}} + 2\epsilon], \text{ if } q \neq 0. \quad (12)$$

However, if both bound pairs have the same total momentum the interaction energy of each pair is given by Eq. (10). Each pair in this state sees two additional particles with their momenta correlated to suppress only one term in the pair wave function

$$\langle B_q B_q | V | B_q B_q \rangle = -2V_{\text{pair}} + 2\epsilon = 2[-V_{\text{pair}} + \epsilon]. \quad (13)$$

This result, Eqs. (12) and (13), shows a remarkable physical feature of overlapping bound pair wave functions. If the

two bound pairs are moving with exactly the same momentum their interaction energy is lower by a finite amount than if they are moving with different momenta. There is an energy gap between the states where both pairs have exactly the same momenta and other states where both pairs have different momenta, even very slightly different momenta.

This treatment is easily extended to calculate the interaction energy of a state having n bound pairs with the same momentum.

$$\langle 0 | (B_q)^n V (B_q^\dagger)^n | 0 \rangle = n [-V_{\text{pair}} + (n-1)\epsilon], \quad (14)$$

since each of the n pairs has $n - 1$ terms in its wave function suppressed. If we change this wave function by putting one of the pairs into a different momentum state q' , the interaction energy becomes

$$\langle 0 | (B_q)^{n-1} B_{q'} V B_{q'}^\dagger (B_q^\dagger)^{n-1} | 0 \rangle = (n-1)(-V_{\text{pair}} + n\epsilon) + (V_{\text{pair}} + 2(n-1)\epsilon) \quad (15)$$

$$= n [-V_{\text{pair}} + (n-1)\epsilon] + 2(n-1)\epsilon.$$

The energy gap between the state where all particles have the same momentum and the state where one pair has a slightly different momentum is proportional to the number of pairs. This is the energy gap responsible for the stability of the persistent current in the BCS theory of superconductivity. A large number of bound electron pairs all with the same bound-state wave function and all carrying a tiny momentum q carry a finite electric current. The electrical resistance mechanism responsible for the decay of a normal current scatters electrons out of the direction of the current. In a normal conductor this scattering requires no appreciable energy. The energy is even lowered if the kinetic energy of the electron is decreased. For a current carried by bound pairs of the type described by Eqs. (14) and (15) a finite energy is required for any scattering process which moves one pair out of the state having the same momentum as all the other pairs. Moving all the pairs simultaneously from a state of momentum q to a state of zero momentum does not change the interaction energy and lowers the total energy by the decrease in kinetic energy. However, the normal mechanism of electrical resistance can only scatter one electron at a time and must therefore go through very high energy intermediate states before reaching the ground state in which all pairs are moving with zero momentum. Thus the transition must go through a very high potential barrier which stabilizes the current.

The energy gap described by the difference between Eqs. (14) and (15) contains the basic physics of the stability of a superconducting current in the BCS description. This basic physics is easily presented very early in an elementary quantum mechanics course and can give the student some insight into the application of quantum theory to physical problems.

The history of the Mössbauer effect presents an interesting example of the type of "interdisciplinary difficulty" which arises when a new phenomenon is discovered whose description requires elementary ideas from several normally separated areas of physics. In this case, elementary intuition in both nuclear and solid-state physics was needed. The history of the Mössbauer effect is conveniently summarized in Table I. There was a long period in which the effect could have been found if anyone had thought of looking for it but nobody did. The early iridium age refers to the period of Mössbauer's first experiments which used an iridium isotope. When the effect was discovered by Mössbauer, then a graduate student, nobody paid attention to it. During the next period, Mössbauer's paper was known and discussed but very few people believed the results. After several other groups had repeated Mössbauer's experiment with iridium, the effect was believed but considered to be a rather unimportant curiosity which would not have any application. The big explosion into many areas of physics began with the use of the isotope ^{57}Fe for the Mössbauer effect, where it was used to find Zeeman and hyperfine splittings of nuclear spectral lines, to measure the gravitational red shift, and enabled a variety of studies in atomic, nuclear, solid-state, chemical and molecular physics.

TABLE I. History of Mössbauer effect.

Period	Comments
Prehistoric (before 1958)	Could have been found but wasn't
Early iridium age	Found but not known
Middle iridium age	Known but not believed
Late iridium age	Believed but not important
Iron age	WOW!!!!

For this audience the Mössbauer effect is most conveniently described in terms of form-factor physics. Consider for example the diffractive excitation of a K^* resonance by coherent scattering of a kaon beam on the nucleus ^{12}C .



This process results from an elementary scattering process on a single nucleon



In the impulse approximation generally valid for this type of process, the transition amplitude for the process (16a) in the nucleus is given by summing the transition amplitudes for the scattering process on free nucleons (16b) and multiplying them by form factors which describe the probability amplitude for momentum transfer to the nucleus as a whole without nuclear excitation or breakup. The transition probability from an initial state A to a final state B for a bound system in the impulse approximation is expressed in terms of the corresponding transition probability for free systems as

$$W_{A \rightarrow B} (\text{Bound}) = W_{A \rightarrow B} (\text{Free}) \cdot |F_q|^2 \quad (17a)$$

where F_q is the form factor of the bound state for momentum transfer q and is given by the expectation value of the operator $e^{i\vec{q} \cdot \vec{r}}$ in the bound-state wave function, where r is the coordinate of the bound particle making the transition

$$F_q = \langle \text{Bound} | e^{i\vec{q} \cdot \vec{r}} | \text{Bound} \rangle \quad (17b)$$

In many cases of interest this is simply expressed in terms of the mean square radius of the wave function of the bound particle.

$$|F|^2 \approx e^{-\langle q^2 r^2 \rangle} \quad (17c)$$

There is also an important kinematical difference between the scattering process (16a) on a nucleus and the process (16b) on a free nucleon. Because of the difference in masses the kinetic energy of recoil is different in the two cases. Thus for momentum transfer $\hbar q$ the recoil kinetic energies for the processes (16a) and (16b) are given by

$$\text{K. E. } ({}^{12}\text{C}) = (\hbar q)^2 / 24 M_N \quad (18a)$$

$$\text{K. E. } (N) = (\hbar q)^2 / 2 M_N \quad (18b)$$

where M_N is the nucleon mass. The energy of the outgoing K^* is thus different in the two cases.

$$E_{K^*}({}^{12}\text{C}) = E_K - (\hbar q)^2 / 24 M_N \quad (19a)$$

$$E_{K^*}(N) = E_K - (\hbar q)^2 / 2 M_N \quad (19b)$$

Thus in the scattering process on the complex nucleus the outgoing K^* has a higher energy than the corresponding process on the free nucleon.

The above discussion applies in general to many processes where momentum is transferred to a member of a bound system and the impulse approximation holds. Electron scattering by nucleons in a nucleus, light scattering by electrons bound in an atom, beta decays of complex nuclei, neutron scattering and absorption by nuclei bound in crystals and ordinary x-ray diffraction scattering from atoms bound in crystals are some examples of such processes. In all of them expressions analogous to the Eqs. (17-19) occur.

Mössbauer studied the resonance emission and absorption of gamma rays from nuclei bound in a crystal. When a nucleus emits a photon of wave vector q the nucleus recoils with momentum

$$\hbar q = E_\gamma / C \quad (20)$$

where E_γ is the energy of the gamma ray. The kinetic energy of recoil for the case analogous to (16a) where the whole crystal recoils and takes up the momentum is given by

$$K.E. = (\hbar q)^2 / 2 M_{\text{crystal}} \sim 0. \quad (21a)$$

This is negligible since the mass of the crystal is infinite for all practical purposes.

When the recoil momentum is taken by a single nucleon, the free recoil energy conventionally denoted by R is given by

$$R = (\hbar q)^2 / 2 A M_N = (E_\gamma / c)^2 / 2 A M_N = E_\gamma \cdot \frac{E_\gamma}{2 A M_N c^2} \quad (21b)$$

where A is the atomic number of the nucleus. For a typical case like the iridium isotope originally studied by Mössbauer $A \approx 200$, $E_\gamma \approx 10^5$ eV and $R \approx 1/40$ eV. This might seem negligible since a photon with an energy of 10^5 eV should hardly notice a loss of $1/40$ eV. But this is misleading for resonance scattering processes in which a photon emitted from one nucleus is absorbed by the inverse transition in another nucleus of the same kind. For such absorption to occur, the energy loss due to recoil must not shift the energy outside the resonance; i.e. the recoil energy loss must not be much greater than the natural line width. Natural line widths for nuclear gamma rays are of the order of 10^{-3} – 10^{-7} electron volts. Thus the recoil energy loss (21b) is very large and a photon which has suffered this energy loss in emission will not produce the inverse transition in another nucleus.

However, the thermal energy kT at room temperature is $1/40$ eV. Thus the Doppler shift due to thermal motion at room temperature can compensate for the recoil energy loss. Thermal energy produces a line much broader than the natural line width and allows resonance absorption to occur.

Consider now the spectrum of photons emitted by a nucleus bound in a crystal. There are two kinds of transitions, those in which the single nucleus recoils with the energy R and those in which recoil momentum is taken up by the whole nucleus while the photon carries the full energy of the nuclear transition. The probability for the latter to occur is given by the square of the form factor

$$|F_q|^2 = e^{-\langle q^2 r^2 \rangle}. \quad (22a)$$

If the nucleus in the crystal is moving in a harmonic oscillator potential with angular frequency ω the form factor at zero temperature when the system is in its ground state is given by

$$|F_q|^2_{T=0} = \exp(-\hbar^2 q^2 / 2 AM_N \hbar\omega) = \exp(-R/\hbar\omega). \quad (22b)$$

The energy $\hbar\omega$ is a characteristic lattice energy related to the Debye temperature of the crystal and is of order thermal energy.

This result has a very simple physical interpretation. If the recoil energy R is small compared with the lattice energy $\hbar\omega$, it is easy for the crystal to absorb the momentum and the form factor F_q is large. But if R is large compared to $\hbar\omega$ the crystal cannot absorb the momentum and F_q is smaller.

For Mössbauer's iridium experiments, R and $\hbar\omega$ were of the same order of magnitude and the exponent in the expression (22b) was of order unity. However, because the form factor depends exponentially on the gamma ray energy and the mean square radius, very large variations are obtained from small variations in these parameters. For example, with $A = 50$ and $E_\gamma = 1$ MeV the exponent in Eqs. (22) is -400 and the probability that the gamma ray is emitted with its full energy is negligible. For such cases where the form factor is very small the momentum transfer is nearly always given to the single nucleon and the gamma ray emission line is displaced by the energy loss R given in Eq. (21b), a finite temperature this line has a width of the order of the thermal energy.

For the particular case chosen by Mössbauer the form factor at room temperature was quite small and the bulk of the emission spectrum appeared in the vicinity of the energy $E_\gamma - R$ with a width of the order of the thermal energy and giving a spectrum whose tail extended out beyond the energy E_γ as shown in Fig. 1a. When the temperature is lowered to liquid nitrogen temperature two effects occur. The thermal distribution becomes narrower because of the lower thermal energy and the tail of the spectrum out at the energy E_γ is sharply reduced. However, at the lower temperature the form factor (22a) has increased and there is an additional component of the spectrum corresponding to the transition with no energy loss due to recoil. This appears as a very sharp peak with the natural line width at the energy E_γ , as shown in Fig. 1b.

The same argument applies to the absorption of a gamma ray by a nucleus bound in a crystal. There are two kinds of absorptive transitions, one in which the recoil momentum of the photon is absorbed by the whole crystal and one in which the single nucleus which absorbs the photon takes up all the recoil. The

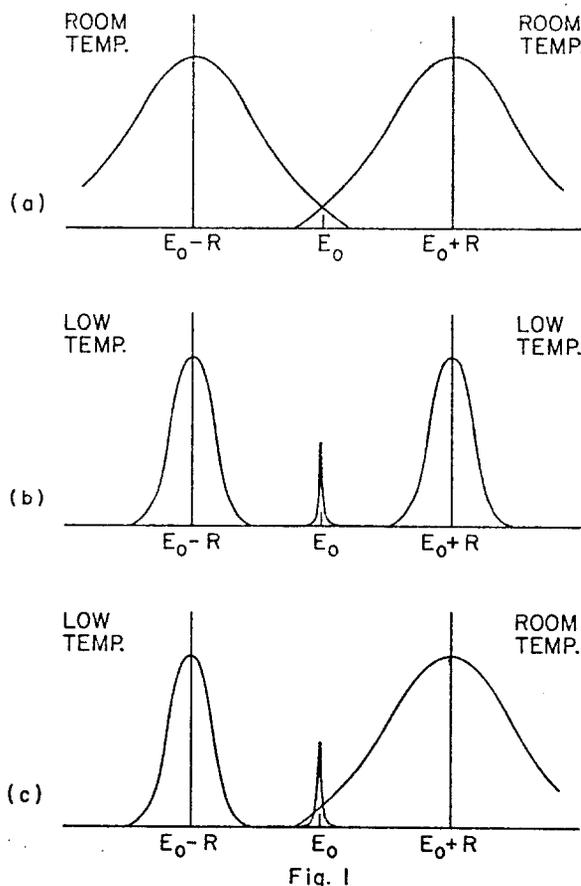


Fig. 1. Emission and absorption spectra for Mossbauer experiment.

recoil energy loss for these two cases are again given by Eqs. (21). Resonance absorption occurs when the incident photon has sufficient energy to supply both the excitation energy of the nuclear state and the required recoil energy. Thus the absorption spectrum is a mirror image of the emission spectrum about the energy E_γ . There is a small spike with the natural line width at the energy E_γ corresponding to the transition in which the whole crystal takes up the recoil and which is proportional to the form factor (22). There is a thermal distribution centered at $E_\gamma + R$ with a width depending upon the temperature, as shown in Fig. 1.

Mössbauer's original motivation in performing his experiment was to study the effect of the thermal distribution. At that time no one considered the contribution of the spike E occurring when the whole crystal takes up the recoil. For the case studied by Mössbauer there is an appreciable overlap of the thermal distributions at room temperature and some resonance absorption should be observed, as shown in Fig. 1a. However, when either the

source or the absorber is cooled to liquid nitrogen temperature this overlap decreases sharply as shown in Fig. 1c, and the amount of absorption is expected to decrease. From experimental observation of this decrease, the natural line width and the lifetime of the nuclear state can be calculated. This was the original purpose of the work. As long as either the source or the absorber is at room temperature the contribution of the recoilless spike at energy E_γ is negligible because the overlap of one recoilless spike with the thermal distribution is very small. Thus calculations based only on the thermal distribution give a good value for the natural line width. However, when both the source and the absorber are at liquid nitrogen temperature the two recoilless spikes have a large overlap and recoilless transitions provide the dominant contribution to the absorption. The two thermal distributions overlap even less than when either source or absorber was cooled and the other left at room temperature. But the absorption instead of being even less when both are cooled than when one is cooled suddenly became greater. This was completely unexpected by Mössbauer and any of his professors because no one was aware of the existence of the recoil-free transition. The complete theory of the effect had been worked out a long time before in a celebrated paper by Willis Lamb. Professor Hans Jensen advised Mössbauer to examine Lamb's paper to take account of peculiar crystal effects. After Mössbauer read and understood Lamb's paper he devised a very simple test of whether the recoil-free transition was responsible for his effect. Since the line width was very narrow the resonance absorption could be destroyed by giving either the source or the absorber a small Doppler shift. A phonograph turntable had sufficient velocity to provide an adequate Doppler shift and was used by Mössbauer to demonstrate that he indeed had this effect.

I first learned about the Mössbauer effect in 1959 at the University of Illinois. Professor Hans Frauenfelder sent us Mössbauer's paper from Europe and suggested that it might be interesting. I read it and realized that I did not know enough solid-state physics to understand it. But at Illinois there were many solid-state experts and we consulted one of them. His first reaction was to ask "Who is this fellow Mössbauer? Does anybody know him? Is he reliable? Give me a few days to think about it." Shortly afterwards he told us that he had looked again at the Mössbauer effect and found that it was all perfectly all right. But his first reaction had been that it was completely crazy.

The reason why solid-state physicists immediately reacted this way and thought that Mössbauer's experiment was crazy is seen in Eq. (22b). The form factor is the exponential of the ratio of two quantities. One, R is the recoil energy produced by a nuclear gamma ray. The other $\hbar\omega$ is a characteristic lattice energy. Everyone knows that characteristic nuclear energies are many orders of

magnitude larger than characteristic lattice energies. An effect proportional to a negative exponential of the ratio of a nuclear energy to a lattice energy would be expected to be unreasonably small. The one point missed by solid-state physicists is that the recoil energy of a nucleus which has emitted a photon of 100 keV is not of the order 100 keV but only $1/40$ eV as shown by Eq. (21b). The recoil energy is not equal to the energy of the gamma ray but is reduced by a factor which is the ratio of the energy of the gamma ray to twice the rest energy of the nucleus. This extremely tiny ratio brings down the recoil energy to the order of lattice energies.

The nuclear physicists on the other hand did not understand the elementary solid-state physics involved in calculating expressions like Eqs. (22) for the form factors. They also were misled by the intuitive picture a nucleus emitting 100 kilovolts of energy must have a large enough recoil to knock it out of the crystal.

At Illinois I easily learned the amount of solid-state physics necessary for understanding the Mössbauer effect and became a solid-state expert for the nuclear physicists because I knew a little bit of solid-state physics and spoke their language. I also learned enough of the language of the solid-state physicists so that I could be a nuclear expert in explaining the Mössbauer effect to them because I knew a little nuclear physics necessary for the understanding of the effect and spoke their language. I soon found that momentum transfer to bound systems and the general form factor Eqs. (17) appear in all areas of physics. But in this era of specialization each group has its own language for describing these phenomena and is comparatively unaware that the same phenomena exists in other areas.

I therefore think that the Mössbauer effect and all the phenomena of momentum transfer to bound systems is ideal for a student studying elementary quantum mechanics. I have applied it to as many examples as possible whenever I teach the quantum mechanics course. This impresses the students with the unity of physics and the power of an interdisciplinary approach in obtaining results in many areas with a little fundamental input.

Let me conclude with the hope that the next 50 years of the development of quantum mechanics will continue to be productive and exciting and that perhaps quantum mechanics can provide the "interdisciplinary attractive force" to bring all of physics together again.