Ref.TH.2066-CERN

Fermilab Pub-75/68-THY
September 1975

COMMENT ON CALCULATIONS OF THE $K_L \to \mu\bar{\mu}$ DECAY RATE AND THE $K_L K_S$ MASS DIFFERENCE IN GAUGE THEORIES

M. K. Gaillard

CERN -- Geneva
and
Laboratoire de Physique Théorique
et Hautes Energies, Orsay *)

and

Benjamin W. Lee and R.E. Schrock

Fermi National Accelerator Laboratory, Batavia

A B S T R A C T

We reexamine the calculations of the $K_L \to \mu\bar{\mu}$ decay rate and the $K_L K_S$ mass difference in the Weinberg-Salam model with the GIM mechanism incorporated. We consider both the free quark model and corrections due to strong interactions in an asymptotically free theory, and compare our results with those of other recent calculations. Our conclusions are basically identical to those drawn from our original free quark calculation : the decay amplitude for $K_L \to \mu^+\mu^-$ is dominated by the conventional two-photon exchange, and the decay rate places no useful limit on the charmed quark mass, whereas the K_L, K_S mass difference constrains this mass by $m_c \lesssim$ a few GeV.

---

*) Laboratoire associé au C.N.R.S.

Ref.TH.2066-CERN
15 August 1975

1. - INTRODUCTION

A number of authors have computed the decay rate $K_L \to \mu\bar{\mu}$ and the $K_L K_S$ mass difference in the Weinberg-Salam model of electromagnetic and weak interactions which incorporates the so-called GIM mechanism, and commented on the mass scale of the fourth quark, i.e., the charmed quark, which appears in this scheme. More recently, several authors discussed the effects of strong interactions on these amplitudes assuming that strong interactions of hadrons are described by an asymptotically free gauge theory.

One of the purposes of this note is to clarify the discrepancy between Ref. 1) and those of Russian workers [2)], especially of Flambaum, on $K_L \to \mu\bar{\mu}$. We find that the result of I is in error, and our correct result agrees with Ref. 2). However, we disagree with Flambaum regarding the Ward-Takahashi identity for the Zds vertex. Under the circumstances, we feel it necessary to describe our calculation in some detail. The second subject we wish to discuss here has to do with estimating the size of strong interaction effects in the proceeses. We obtain results which are not completely in accord with previous authors [3)], including Nanopoulos and Ross, and Vainshtein, Zakharov, Novikov and Shifman.

2. - WARD-TAKAHASHI IDENTITY

The effective Zsd vertex discussed in Appendix B of (I) is a sum of diagrams depicted in Figs. 1 where black circles represent one-loop corrections. These diagrams are separately divergent, but the sum is not. To extract the finite sum, we make use of the Ward-Takahashi identity described below. We shall also outline the direct calculation of Figs. 1.

It was shown elsewhere [4)] that in a gauge specified by

$$F^a[\phi] = f_i^a \phi_i \tag{1}$$

the Ward-Takahashi identity for the generating functional for proper vertices may be written as

$$\Gamma_i^a[\phi] \frac{\delta}{\delta\phi_i} \Gamma_0[\phi] = 0 \tag{2}$$

where $\Gamma = \Gamma_0 - \frac{1}{2}F_a^2$ is the generating functional of proper vertices with external ϕ lines, and $\bar{c}^a f_i^a \Gamma_i^b[\phi] c^b$ is the generating functional of proper vertices with two external ghost lines and an arbitrary number of external ϕ lines. The index a refers to the adjoint representation of the gauge group in question.

We consider a special case in which the index a in Eq. (2) refers to the transformation which leaves the photon field invariant, but changes the Z_μ field by a translation :

$$\delta A_\mu = 0$$

$$\delta Z_\mu = \partial_\mu \lambda(x) \tag{3a}$$

Under this transformation, the d and s fields and the Higgs field ϕ_1 change by

$$\delta \begin{pmatrix} d \\ s \end{pmatrix} = i T \begin{pmatrix} d \\ s \end{pmatrix} \tag{3b}$$

where

$$T = \Big\{ -\frac{1}{2} - (Q-1)\sin^2\theta_W \; L + \left[-(Q-1)\sin^2\theta_W \right] R \Big\} (g^2 + g'^2)^{1/2}, \tag{3c}$$

$$L = \frac{1}{2}(1-\gamma_5), \qquad R = \frac{1}{2}(1+\gamma_5),$$

and

$$\delta\phi_1 = i\left(-\frac{1}{2}\right)\sqrt{g^2+g'^2}\,(i\,\phi_2). \tag{3d}$$

In one-loop approximation, we consider

$$\frac{\delta}{\delta \bar{d}} \frac{\delta}{\delta s}\left[\Gamma_i^a[\phi] \frac{\delta \Gamma_0[\phi]}{\delta \phi_i} \right]\Bigg|_{\phi = v} = 0. \tag{4}$$

We need to concentrate on the case in which $\Gamma_i^a[\phi]$ takes its bare form, and Γ_0 is given by the one-loop approximation. To this order, Eq. (4) is a statement that Γ_0 is invariant under the gauge transformations (3a-d). Taking into account Eqs. (3a-d), we obtain the expression (B.2) in I :

$$(q-p)^\mu \Gamma_\mu^{(Z)}(q,p) - [\Sigma(q) T - T^* \Sigma(p)]$$
$$= i \tfrac{1}{2}(g^2+g'^2)^{\frac{1}{2}} v \Gamma_2(q,p) \ . \tag{5}$$

As explained in I, to lowest order in $(m_c/m_W)^2$, the effective vertex $E_\mu^{(Z)}$ can be written as

$$E_\mu^{(Z)} = \gamma_\mu \left(\frac{1-\gamma_5}{2}\right) E \ , \tag{6}$$

where the on-shell value of E is related to Γ_2 through Eq. (4) :

$$\tfrac{1}{2} i (g^2+g'^2)^{\frac{1}{2}} v \, \bar{d}(q) \Gamma_2(q,p) S(p) \Big|_{\not p = m_s, \not q = m_d} = \bar{d}(q)(m_d L - m_s R) S(p) E. \tag{7}$$

It is important to observe here that the Ward-Takahashi identity (5) is correct in all linear gauges of the form (1), and to all orders in strong interactions if they are invariant under the gauge symmetry of electromagnetic and weak interactions. We note that this Ward-Takahashi identity disagrees with that of Flambaum [2]. It will be useful for the calculation of $E^{(2)}$ to recall the following facts from (I). Again to lowest order in $(m_q/m_W)^2$, and assuming $m_0 \simeq m_d \simeq m_s \ll m_c$, Σ and $\Gamma_\mu^{(z)}$ have the form

$$\Gamma_\mu^{(Z)}(q,p) = \gamma_\mu L x \ ,$$
$$\Sigma(p) = \not p L a + b L + c R \ . \tag{8}$$

$E_\mu^{(z)}$ can thus be written

$$i\,\bar{d}(q)\,E_\mu^{(z)}\,s(p) = \bar{d}(q)\left[i\Gamma_\mu^{(z)} + i\gamma_\mu T \frac{i}{\not p - m_d}\, i\,\Sigma(p) + i\,\Sigma(q)\frac{i}{\not q - m_s}\, i\gamma_\mu T \right] s(p)$$

$$= i\,\bar{d}(q)\,\gamma_\mu L\, s(p)\left[x + \left\{ \tfrac{1}{2} + (Q-1)\sin^2\theta_W \right\} (g^2 + g'^2)^{\frac{1}{2}}\, a \right] \tag{9}$$

3. - $K_L \to \mu\bar{\mu}$

We proceed to the computation of Γ_2 and of $E_\mu^{(z)}$. To ascertain gauge independence of our result, we carry out the computation in the R_ξ gauge, wherein the $W^\pm$ and $\phi^\pm$ propagators are, respectively,

$$i\Delta_{\mu\nu}(k) = \left(-i g_{\mu\nu} + i\,\frac{k_\mu k_\nu (1 - 1/\xi)}{k^2 - m_W^2/\xi} \right) \frac{1}{k^2 - m_W^2}$$

and

$$i\Delta(k) = i\,(k^2 - m_W^2/\xi)^{-1}.$$

Dimensional regularization is used for the treatment of divergent integrals.

There are altogether five diagrams which contribute to Γ_2 to order $G_F\alpha(m_c/m_W)^2$, shown in Fig. 2. [Henceforth we neglect m_u, but we cannot neglect the external masses as the Ward identity, Eq. (7), is valid only on the mass shell.] The graph of Fig. 2a is convergent, but the graphs 2d-e have a logarithmically divergent term proportional to the external mass. This divergence cancels when the u and c contributions are added. Explicit evaluation of the diagrams 2a,d,e yields :

$$i\Gamma_2^{(2a)} = \left[\ln \frac{m_W^2}{m_c^2} - 1 - \frac{1}{4} \ln \xi \right] C \;, \tag{10}$$

$$i\Gamma_2^{(2d+2c)} = -\left[\frac{3}{2} \frac{\xi \ln \xi}{\xi - 1} + \frac{1}{2} \right] C \;, \tag{11}$$

where the common factor

$$C = g^2 \frac{\sin\theta_c \cos\theta_c}{v(16\pi^2)} \frac{m_c^2}{m_W^2} \bar{d}(\not{q} - \not{p}) L s \tag{12}$$

has been extracted. The contributions of graph 2b is exactly cancelled by the counter term 2c [see Eqs. (24) and (25) below] :

$$i\Gamma_2^{(2b)} = -i\Gamma_2^{(2c)} = \frac{1}{2}\left[\frac{2}{4-n} + 1 + \ln \xi \right] C \;, \tag{13}$$

in the limit $n \to 4$, where n is the dimension of space-time. Combining Eqs. (10)-(13), we find

$$i\Gamma_2(q,p) = g^2 \frac{\sin\theta_c \cos\theta_c}{v(16\pi^2)} \frac{m_c^2}{m_W^2} \left[\ln \frac{m_W^2}{m_c^2} - \frac{3}{2} - \frac{(7\xi - 1)}{4(\xi - 1)} \ln \xi \right] \times \bar{d}(q)(\not{q} - \not{p}) L s(p) \,. \tag{14}$$

The zds vertex may be extracted from Eq. (14) using the Ward identity of Eq. (7). However, as we wish to demonstrate explicitly the validity of Eq. (6) we shall proceed to the direct evaluation of $E_\mu^{(z)}$. The relevant diagrams are those of Fig. 1, which are made explicit in Figs. 3 and 4.

Graph 3a gives the following contribution to the (non-diagonal) self-energy :

$$\Sigma(p)_{(3a)} = \frac{g^2 \sin\theta_c \cos\theta_c}{2(16\pi^2)} \frac{m_c^2}{m_W^2} \left(1 - \frac{3}{2} \ln \xi\right) \not{p} L \,. \tag{15}$$

Graphs 4a and 4c-e yield

$$\Gamma_\mu^{(4a)} = \Big[\Big(1 - 2\ln\frac{m_W^2}{m_c^2} + 2Q\sin^2\theta_W\Big) + (2 - 3Q\sin^2\theta_W)\ln\zeta\Big] K_\mu \ , \qquad (16)$$

$$\Gamma_\mu^{(4c)} = 3\cos^2\theta_W \Big[1 + \frac{\ln\zeta}{\zeta - 1}\Big] K_\mu \ , \qquad (17)$$

$$\Gamma_\mu^{(4d+4e)} = \sin^2\theta_W \Big[1 + \frac{3\zeta\ln\zeta}{\zeta - 1}\Big] K_\mu \ , \qquad (18)$$

where, for brevity of notation we have extracted the common factor

$$K_\mu = -\frac{g^2(g^2+g'^2)^{1/2}}{4(16\pi^2)} \sin\theta_c\cos\theta_c \frac{m_c^2}{m_W^2} \gamma_\mu L \qquad (19)$$

Substituting these contributions to Σ and $\Gamma_\mu^{(z)}$ into Eq. (9), we obtain

$$E_\mu^{(z)} = \frac{g^2(g^2+g'^2)^{1/2}}{2(16\pi^2)} \sin\theta_c\cos\theta_c \frac{m_c^2}{m_W^2} \Big[\ln\frac{m_W^2}{m_c^2} - \frac{3}{2} - \frac{(7\zeta-1)}{4(\zeta-1)}\ln\zeta\Big]\gamma_\mu L. \qquad (20)$$

This is in fact the final result for $E_\mu^{(z)}$, as will now be shown.

The cancellation of logarithmic divergences which occurs for the above graphs does not occur for graphs 3b, 4b or 4f, since the divergent parts of these diagrams are proportional to m_c^2. The counter terms required to render Σ and $\Gamma_\mu^{(z)}$ finite are given by

$$\bar{d}\Big[i\gamma\cdot\partial - (g^2+g'^2)^{1/2}\Big\{\tfrac{1}{2} + (Q-1)\sin^2\theta_W\Big\}\gamma\cdot Z\Big] L s \sin\theta_c\cos\theta_c (z_L - z_L') + \text{h.c.} \qquad (21)$$

where, as in (I), z_L and z_L' are the wave function renormalization constants for the left-handed quark doublets. The same renormalization constants render Σ and $\Gamma_\mu^{(z)}$ finite, and can be chosen to cancel the sd transition on mass shell. z_L and z_L' can be determined from graph 3b :

$$\Sigma(p)_{(3b)} = \frac{g^2 \sin\theta_c \cos\theta_c}{2(16\pi^2)} \frac{m_c^2}{m_W^2} \left[\frac{1}{2} \not{p} L \left\{ \frac{2}{4-n} + \frac{1}{2} + \ln \right\} \right.$$

$$\left. - (m_d L + m_s R) \left\{ \frac{2}{4-n} + 1 + \ln \right\} \right], \tag{22}$$

where thr expression represents the limit as $n \to 4$, with n the dimension of space-time. Thus,

$$(Z_L - Z'_L) = -\frac{g^2 \sin\theta_c \cos\theta_c}{4(16\pi^2)} \frac{m_c^2}{m_W^2} \left[\frac{2}{4-n} + \frac{1}{2} + \ln \right\}]. \tag{23}$$

It is readily checked that the corresponding Zds counter term cancels the sum of the contributions of graphs 4b and 4f to $\Gamma_\mu^{(z)}$. As is evident from Eqs. (9) and (21) the sd and Zsd counter terms give equal and opposite contributions to $E_\mu^{(z)}$. Thus, alternatively, one could simply add all of the graphs in Figs. 3 and 4 together directly, without adding the counter terms in graphs 3c and 4g, and the result would be that the divergences arising from Σ would exactly cancel those from $\Gamma_\mu^{(z)}$. Both procedures are equivalent as regards the final answer for $E_\mu^{(z)}$.

The second term in Eq. (22) represents a direct sd transition, as does the first, and must be cancelled by another counter term in the Lagrangian. This term is of the form

$$A \bar{d} L s (\phi_1 + v + i\phi_2) + B \bar{d} R s (\phi_1 + v - i\phi_2) + h.c. \tag{24}$$

where φ_1 is the physical Higgs scalar and $\langle \varphi \rangle_0 = \binom{0}{v/\sqrt{2}}$. The renormalization constants A and B are given by

$$A = \frac{m_d}{m_s} B = \frac{g^3 \sin\theta_c \cos\theta_c}{4(16\pi^2)} m_d \frac{m_c^2}{m_W^3} \left[\frac{2}{4-n} + 1 + \ln \right\}]. \tag{25}$$

This is just the counter term for the φ_2 sd vertex as shown in Fig. 2e.

Now we may compare the expressions obtained for $E_\mu^{(z)}$, Eq. (20), and for Γ_2, Eq. (14), and we see that the Ward identity of Eq. (7) is indeed satisfied. Our correct result agrees with that of Ref. 2).

To conclude our discussion, the Z exchange contribution to the process $s+\bar{d}\to\mu+\bar{\mu}$, relevant to K_L decay, is given by

$$-i\frac{G_F}{\sqrt{2}}\frac{\alpha}{\pi}\cos\theta_c\sin\theta_c\left(\frac{m_c}{38\,\mathrm{GeV}}\right)^2\frac{1}{4}\left(\bar{\mu}\gamma_\alpha\gamma_5\mu\right)\left(\bar{d}\gamma^\alpha L s\right)$$
$$\times\left[\ln\left(\frac{m_W}{m_c}\right)^2-\frac{3}{2}-\frac{7\xi-1}{4(\xi-1)}\ln\xi\right], \tag{26}$$

whereas the W^+W^- contribution to the process is

$$-i\frac{G_F}{\sqrt{2}}\frac{\alpha}{\pi}\cos\theta_c\sin\theta_c\left(\frac{m_c}{38\,\mathrm{GeV}}\right)^2\frac{1}{4}\left(\bar{\mu}\gamma_\alpha\gamma_5\mu\right)\left(\bar{d}\gamma^\alpha L s\right)$$
$$\times\left[-\ln\left(\frac{m_W^2}{m_c^2}\right)-\frac{1}{2}+\frac{7\xi-1}{4(\xi-1)}\ln\xi\right]. \tag{27}$$

The sum of Eqs. (26) and (27) is gauge independent ; while the sum is not equal to zero, as asserted in I, it is nevertheless small, being of order $G_F\alpha(m_c/38\ \mathrm{GeV})^2$ without any logarithmic factor. If $m_c \lesssim$ a few GeV as the K_LK_S mass difference implies, then the dominant mechanism for $K_L\to\mu\bar{\mu}$ would be the conventional one of $K_L\to 2\gamma(\mathrm{virtual})\to\mu\bar{\mu}$.

4. - STRONG INTERACTION EFFECTS ON $K_L \to \mu\bar{\mu}$

We treat the strong interactions by an asymptotically free gauge theory - specifically colour SU(3). We quantize the gluons in the Lorentz gauge : $\alpha = 1/\xi = 0$. In this gauge, there is no renormalization of the gauge parameter α, and no quark wave function renormalization to lowest order.

The box diagram $s + \bar{d} \to W^+ + W^- \to \mu + \bar{\mu}$ is cut off at $p \approx m_W$ where p is the internal loop momentum, for removal of both W propagators renders the integral logarithmically divergent. For this diagram, therefore, the analysis of Nanopoulos and Ross is correct.

The operator product expansion relevant to this amplitude is

$$j_\mu(x) j_\nu^\dagger(0) \simeq \cdots + C_{\mu\nu\lambda}(x) m_c^2 J^\lambda(0) + \cdots \tag{28}$$

where j_μ is the standard charged weak current :

$$j_\mu = \bar{u}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\left(d\cos\theta_c + s\sin\theta_c\right)$$

$$+ \bar{c}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\left(-d\sin\theta_c + s\cos\theta_c\right),$$

and

$$J_\lambda = \bar{d}\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)s.$$

Since both j_μ and J_λ have zero anomalous dimensions,

$$m_c^2 C_{\mu\nu\lambda}(\xi^{-1}x, g) = m_c^2(\xi) C_{\mu\nu\lambda}(x, g(\xi))\,\xi$$

$$\simeq m_c^2 C_{\mu\nu\lambda}(x,0)\,\xi^{-1} e^{-2\int_{m_c/\mu}^{\xi}\gamma_\Theta[g(\xi')]\frac{d\xi'}{\xi'}},$$

(29)

where $g(\xi)$ is the running gluon coupling constant, $g(1) = g$. μ is the momentum subtraction point, and the anomalous dimension associated with the quark mass operator, γ_Θ is given by

$$\gamma_\Theta = \frac{g^2}{8\pi^2} \cdot 4 \tag{30}$$

Thus, the effect of strong interactions is to multiply the amplitude of Eq. (27) by the factor

$$e^{-2\int_{m_c/\mu}^{m_W/\mu} \gamma_\Theta [g(\xi')] \frac{d\xi'}{\xi'}}$$

$$= \left[\frac{\alpha_4(m_W)}{\alpha_4(m_c)}\right]^{-24/25} \tag{31}$$

where

$$\alpha_n(m_W) = 1 + \frac{g^2}{8\pi^2} b_n \ln \frac{m_W}{\mu} ,$$

$$b_n = \frac{33 - 2n}{3} ,$$

and n is the number of quark degrees of freedom in ordinary symmetry space $SU(n)$ (i.e., flavours).

Equation (31) takes into account the fact that in the momentum range $m_c \lesssim |p| \lesssim m_W$, all four kinds of quark degrees of freedom are excited. Strictly speaking, we should add to Eq. (29) a contribution from the momentum region $\mu^2 \lesssim |p'|^2 \leq m_c^2$, but this gives a correction factor

$$e^{-2\int_1^{m_c/\mu} \frac{d\xi'}{\xi'} \gamma_\Theta [g(\xi')]} = \left[\frac{\alpha_3(m_c)}{\alpha_3(\mu)}\right]^{-24/27} \simeq 1.$$

Furthermore, Eq. (31) ignores any complications that might arise from the breakdown of perturbation theory near the charm particle threshold.

For the Z exchange diagrams, we note again that the relation (7) is valid in the presence of strong interactions. The bare $\phi_2 cc$ vertex is of the form

$$\mathcal{L}_{\phi_2 cc} = i\,\frac{m_c}{v}\,\bar{c}\,\gamma_5\,c\,\phi_2$$

and we are led to consider the operator product expansion

$$j_\mu(x)\,j_\nu^+(y)\,p(0) \simeq \cdots + c'_{\mu\nu\lambda}(x,y)\,J_\lambda(0) + \cdots \tag{32}$$

where

$$p = \bar{c}\,\gamma_5\,c$$

is a pseudoscalar density. Since we have set the external masses to zero, the relevant operator on the right is necessarily a V-A current operator. One c mass insertion is necessary to obtain a non-vanishing result since p is a right-left transition operator and j_μ is a left-left operator.

The analysis of this contribution differs from the above in that the operator $p(0)$ on the left has anomalous dimensions. We find

$$\gamma_p = \gamma_\Theta = 4\,g^2/8\pi^2 .$$

Solution of the Callan-Symanzik equation then gives

$$m_c\,c'_{\mu\nu\lambda}(\xi^{-1}x, \xi^{-1}y, g) = m_c(\xi)\,\xi^5\,c_{\mu\nu\lambda}[x, y, g(\xi)]\,e^{-\int_{m_c/\mu}^{\xi}\gamma_\Theta[g(\xi')]\frac{d\xi'}{\xi'}} ,$$

with $m_c(\xi)$ scaling as before. Then the expression on the right-hand side is modified with respect to the free field case by a factor

$$e^{-2\int_{m_c/\mu}^{\xi}\gamma_\Theta[g(\xi')]\frac{d\xi'}{\xi'}} . \tag{33}$$

This is equivalent to the result of Nanopoulos and Ross who instead considered the expansion of the product of three currents with two mass insertions. This corresponds to the effective $\bar{d}sZ$ vertex which one is actually trying to determine. However, our analysis is more straightforward as it is free of the complications entailed by the sum over individually divergent diagrams and the wave function renormalization arising from the weak $s \to d$ transition.

Our result therefore reduces to that of Nanopoulos and Ross if the momentum cut-off is effectively m_W, i.e., $\xi \to m_W/\mu$ in Eq. (33). This is precisely the point which is contested by Vainshtein et al. [3)] and which is crucial if the cancellation of the leading contributions $[\sim \ln(m_W^2/m_c^2)]$ is to be maintained in the presence of strong interactions. Vainshtein et al. argue that while the effective distance $x - y$ is determined by the W propagator (see Fig. 5a) :

$$|x-y|^2 \sim 1/m_W^2$$

the effective distances x, y are determined by the charmed quark propagator :

$$|x|^2, \; |y|^2 \sim 1/m_c^2$$

In such a case the short distance behaviour of the coefficient function $C_{\mu\nu\lambda}(x,y)$ must be treated more carefully, as will be illustrated below in the discussion of the K_L, K_S mass difference.

In the free quark case, to lowest order in $1/m_W^2$, the hadronic weak vertex reduces to a local current-current interaction by the replacement

$$(m_W^2 - k^2)^{-1} \longrightarrow m_W^{-2}$$

This replacement is legitimate in a loop diagram (Fig. 5b) if the remaining integral is convergent ; then the effective cut-off of the integral is clearly independent of the W mass. However, for the effective $\bar{d}s\phi_\mu$ vertex considered here, removal of the W propagator [which is equivalent to taking first the limit $x - y \to 0$ in Eq. (32)] leads to a divergent integral. Thus the momentum cut-off must be provided by the W mass, and we conclude that the result of Nanopoulos and Ross is the correct one.

A further argument supporting this conclusion is the following. We saw in Section 3 that the WW and Z contributions to $K_L \to \mu\mu$ are not separately gauge invariant with respect to the gauge group of the weak interactions. Since the hadronic operators appearing in the Wilson expansion are the same for the gauge dependent and gauge independent pieces of each contribution, gauge invariance cannot be maintained unless both contributions have the same scaling behaviour. Thus we conclude that the cancellation of $\ell n(m_W^2/m_c^2)$ terms must be maintained in the presence of strong interactions.

5. - THE K_L, K_S MASS DIFFERENCE

To evaluate the K_L, K_S mass difference in an asymptotically free theory, Nanopoulos and Ross consider the operator product expansion (see Fig. 6)

$$j^\mu(x) j_\mu^\dagger(y) j^\nu(z) j_\nu^\dagger(0) \simeq \cdots + C(x,y,z)\, m_c^2\, J^\mu(0) J_\mu(0) + \cdots$$

In addition to the anomalous dimension associated with the mass insertions, the local current-current operator

$$J^\mu(0) J_\mu(0) = \bar{d}\,\gamma^\mu\left(\frac{1-\gamma_5}{2}\right) s\, \bar{d}\,\gamma_\mu\left(\frac{1-\gamma_5}{2}\right) s \equiv \frac{1}{4} O_s'(0) \qquad (34)$$

has an anomalous dimension. This has been calculated in Refs. 5) where it was found that the anomalous dimension associated with a V-A current-current operator which is symmetric under the exchange of colour indices of two quark or anti-quark fields is negative :

$$\gamma_s = -2\frac{g^2}{8\pi^2} .$$

If the momentum cut-off is effectively m_W, the over-all effect of strong interactions is to modify the free quark amplitude by a factor

$$\left[\frac{\alpha_n(m_W)}{\alpha_n(\mu)} \right]^{(-2d_{\oplus} + d_s)/b_n} \tag{35}$$

where d_i is defined by

$$\gamma_i = d_i \frac{g^2}{8\pi^2} , \qquad i = s, \oplus : \; d_{\oplus} = 4, \; d_s = -2.$$

This is the result of Nanopoulos and Ross who evaluated Eq. (35) for $n = 4$ and subtraction point $\mu \simeq 1$ GeV.

However, in this case the GIM mechanism has the effect of subtracting both fermion propagators, so the loop integral remains convergent when both W propagators are removed (Fig. 7). Therefore, the correct procedure is first to evaluate the corrections to the effectively local current-current interaction obtained by removing the W propagator (Fig. 7), then to consider the time-ordered product of two local current-current operators thus obtained. In this instance we agree with the approach of Vainshtein et al., but differ with their numerical result. Our evaluation is described in what follows.

First, we consider the operator product expansion of two currents ; it was shown in Ref. 5) that the leading contribution is :

$$j^{\mu}(x) j^{+}_{\mu}(0) = \frac{1}{\sqrt{2}} \frac{1}{4} \left\{ C_s(x) O_s(0) + C_a(x) O_a(0) \right\} \cos\theta_c \sin\theta_c$$

where $O_{s,a}$ is a local V-A current operator with the internal symmetry structure :

$$O_{s,a} = \frac{1}{\sqrt{2}} \left[(\bar{d}u)(\bar{u}s) \pm (\bar{d}s)(\bar{u}s) \right] - (u \rightarrow c) ,$$

and

$$C_{s,a}(\xi^{-1}x, g) = e^{-\int_1^{\xi} \gamma_{s,a}[g(\xi')] \frac{d\xi'}{\xi'}} .$$

The anomalous dimension γ_a is given above and

$$\gamma_s = 4\frac{g^2}{8\pi^2} .$$

The effectively local current-current operator is obtained by inserting the intermediate boson propagator and integrating over x :

$$\mathcal{H}_{eff.}(0) = g^2 \int d^4x \, \Delta_F(x, m_W^2) \, T(j^\lambda(x) j_\lambda^\dagger(0))$$

$$= \frac{G_F}{\sqrt{2}} \cos\theta_c \sin\theta_c \frac{1}{\sqrt{2}} \left\{ \left[\frac{\alpha_n(m_W)}{\alpha_n(\mu)}\right]^{d_s/b_n} O_s \right.$$

$$\left. + \left[\frac{\alpha_n(m_W)}{\alpha_n(\mu)}\right]^{d_a/b_n} O_a \right\} . \qquad (36)$$

However, it is probably more accurate to consider two regions of integration ; one above the scaling threshold for ordinary hadrons, but below excitation of the charmed quark :

$$\mu \lesssim |x|^{-1} \lesssim m_c \quad , \qquad n = 3 ,$$

and one above charm threshold :

$$m_c \lesssim |x|^{-1} \lesssim m_W \quad , \qquad n = 4 .$$

Then the expression on the right in Eq. (36) is modified by :

$$\left[\frac{\alpha_n(m_W)}{\alpha_n(\mu)}\right]^{\gamma/b_n} \longrightarrow \left[\frac{\alpha_3(m_c)}{\alpha_3(\mu)}\right]^{3d/27} \left[\frac{\alpha_4(m_W)}{\alpha_4(m_c)}\right]^{3d/25} .$$

To obtain the effectively local $|\Delta S| = 2$ operator, one must next consider the time-ordered product :

$$\mathcal{H}_{\Delta S = 2} = \int d^4x \, T(\mathcal{H}_{eff}(x), \mathcal{H}_{eff}(0)) . \qquad (37)$$

There are, of course, contributions from low energy (i.e., below scaling threshold) intermediate states, which are by themselves of the correct order of magnitude :

$$\langle K | \mathcal{H}_{\Delta S=2} | \bar{K} \rangle \sim (m_S - m_L) + i \frac{(\Gamma_S - \Gamma_L)}{2} .$$

The K_S lifetime $\Gamma_S \simeq \Gamma_S - \Gamma_L$ measures the absorptive part of the low energy contributions and one expects the corresponding dispersive part to be of the same order of magnitude, which is the case for the measured mass difference : $m_L - m_S \simeq \Gamma_S/2$.

The rôle of the GIM mechanism is to ensure that the high energy contributions are cut off at the charmed quark mass so that they give a contribution which is not too large, i.e., is of the same order of magnitude. We shall evaluate that contribution assuming that the distance $|x| \sim 1/m_c$ is "short" on the scale of uncharmed hadrons, which is <u>a posteriori</u> justified by the observed scaling below charm threshold. Then we must consider the short distance expansion of the operator product in Eq. (37). Henceforth we consider only the enhanced part of $\mathcal{H}_{eff}$:

$$\mathcal{H}_{eff} \sim \frac{G_F}{\sqrt{2}} \cos\theta_c \sin\theta_c \frac{1}{\sqrt{2}} \left[\frac{\alpha_3(m_c)}{\alpha_3(\mu)}\right]^{12/27} \left[\frac{\alpha_4(m_W)}{\alpha_4(m_c)}\right]^{12/25} O_a . \tag{38}$$

The relevant part of the operator product expansion is now

$$O_a(x) O_a(0) \sim m_c^2 c'(x,g) J^\mu(0) J_\mu(0) = \frac{1}{4} m_c^2 c'(x,g) O_s'(0). \tag{39}$$

The scaling properties of the coefficient function c' are determined by the anomalous dimensions of both the operators O_a and O_s'. Solution of the Callan-Symanzik equation for $c'(x,g)$ gives

$$m_c^2 c'(\xi^{-1}x, g) \underset{\xi \gg 1}{\longrightarrow} m_c^2 c'(x,0)\, e^{\int_1^\xi \frac{d\xi'}{\xi'} [\gamma_s - 2\gamma_a - 2\gamma_\Theta]} . \tag{40}$$

The absolute normalization is obtained by a summation over colour indices in forming the product $O_a^2 \to O_s'$. Then inserting Eqs. (38)-(40) in Eq. (37), the effective $|\Delta S| = 2$ operator is modified relative to the free quark case by the factor :

$$\frac{1}{2}\left[\frac{\alpha_4(m_W)}{\alpha_4(m_c)}\right]^{24/25}\left[\frac{\alpha_3(m_c)}{\alpha_3(\mu)}\right]^{-10/9} \simeq 0.8 \quad , \tag{41}$$

where we have evaluated this expression taking

$$\frac{g^2}{4\pi} \simeq 1 \; , \; m_W \simeq 100 \, , \; m_c \simeq 2\,GeV, \; \mu \simeq 1\,GeV.$$

The explicit evaluation of the expression (41) is of course subject to caution, particularly for the contribution from the momentum region $\mu < |p| < m_c$. However, the qualitative result is that the effects of strong interactions make a negligible difference with respect to the free quark model calculation.

ACKNOWLEDGEMENTS

Two of us (B.W.L. and M.K.G.) thank the CERN Theoretical Study Division and the Fermilab Theoretical Physics Group, respectively, for hospitality extended during periods of this collaboration. We also thank Dr. D.V. Nanopoulos and Dr. G.G. Ross for informative discussions.

REFERENCES

1) M.K. Gaillard and B.W. Lee - Phys.Rev. D10, 897 (1974), referred to in the text as I.

2) A.I. Vainshtein and I.B. Khriplovich - JETP Letters 18, 141 (1973) ;

V.V. Flambaum - Novosibirsk Institute of Nuclear Physics Preprint (1975) : we single out this paper since it gives a fairly detailed account of the computation which allows comparison with our work.

3) D.V. Nanopoulos and G.G. Ross - Phys.Letters B56, 279 (1975) ;

E.B. Bogomolny, V.A. Novikov and M.A. Shifman - Moscow Preprint ITEP-42 (1975) ;

A.I. Vainshtein, V.I. Zakharov, V.A. Novikov and M.A. Shifman - Moscow Preprint ITEP-44 (1975).

4) B.W. Lee - Phys.Rev. D9, 933 (1974).

5) M.K. Gaillard and B.W. Lee - Phys.Rev.Letters 33, 108 (1974) ;

G. Altarelli and L. Maiani - Phys.Letters 52B, 531 (1974).

FIGURE CAPTIONS

Figure 1 Diagrams contributing to the effective $Z\bar{d}s$ vertex.

Figure 2 Diagrams contributing to the effective $\phi_2\bar{d}s$ vertex to order $G_1 = \alpha(m_c/m_W)^2$.

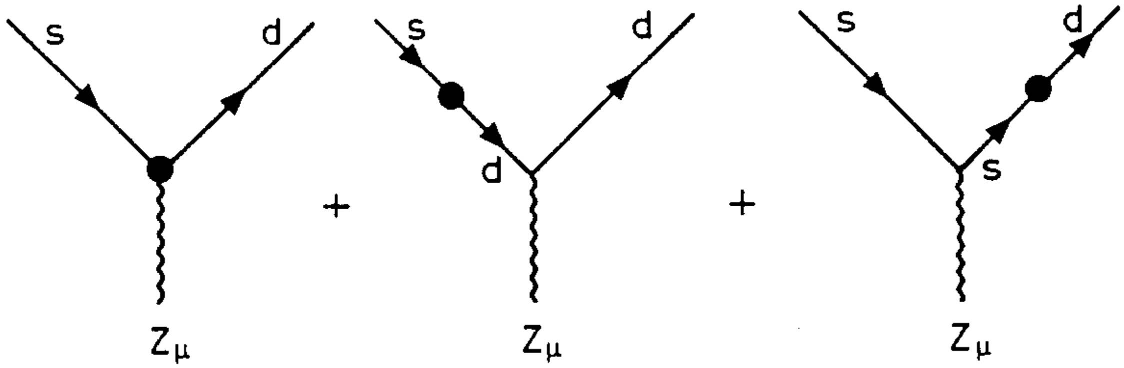
Figure 3 Lowest order diagrams contributing to Fig. 1a.

Figure 4 Lowest order diagrams contributing to the n-λ transition (Figs. 1a, 1b).

Figure 5 Diagram (a) contributing to the effective operator of Eq. (32) ; the same diagram (b) in the limit $|x-y| \to 0$.

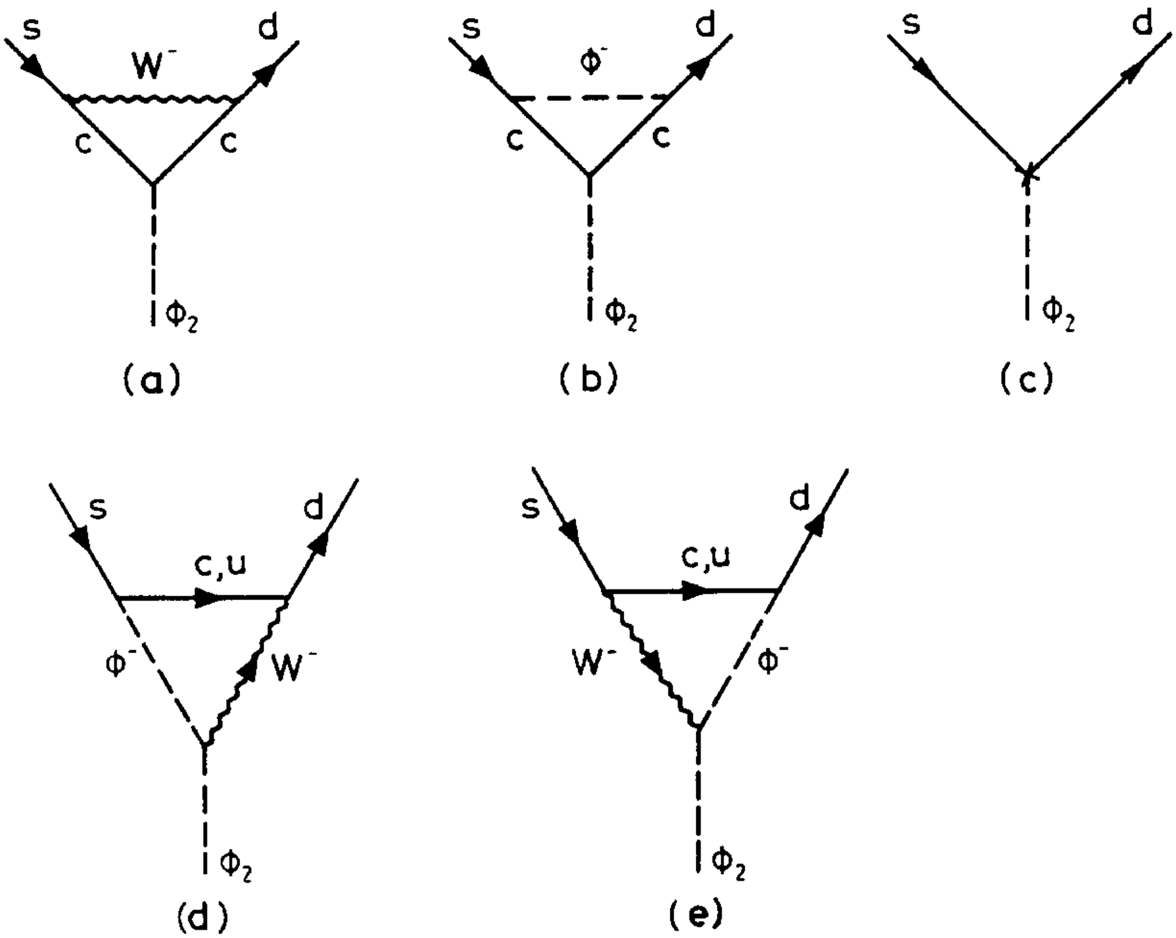
Figure 6 Diagrams contributing to the effective operator of Eq. (34).

Figure 7 The diagrams of Fig. 6 in the limit $|x-y| \to 0$, $|z| \to 0$.



s
d
+
s
d
d
+
s
d
s
Z_μ
Z_μ
Z_μ

FIG.1



s
W^-
d
c
c
Φ_2
(a)
s
Φ^-
d
c
c
Φ_2
(b)
s
d
Φ_2
(c)
s
c,u
d
Φ^-
W^-
Φ_2
(d)
s
c,u
d
W^-
Φ^-
Φ_2
(e)

FIG. 2

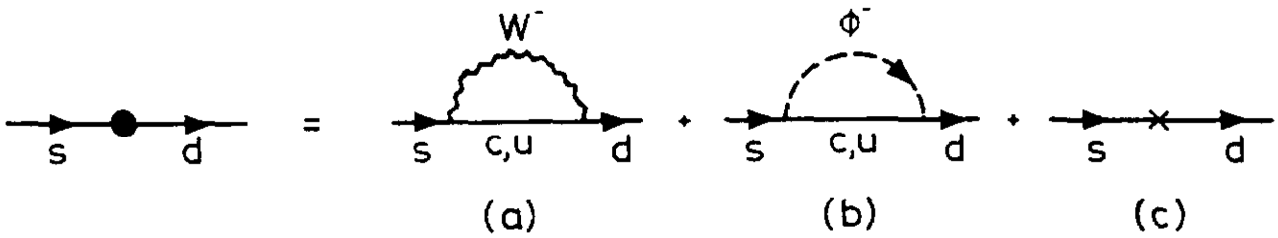

s
d
=
W⁻
s
c,u
d
+
Φ⁻
s
c,u
d
+
s
d
(a)
(b)
(c)

FIG. 3

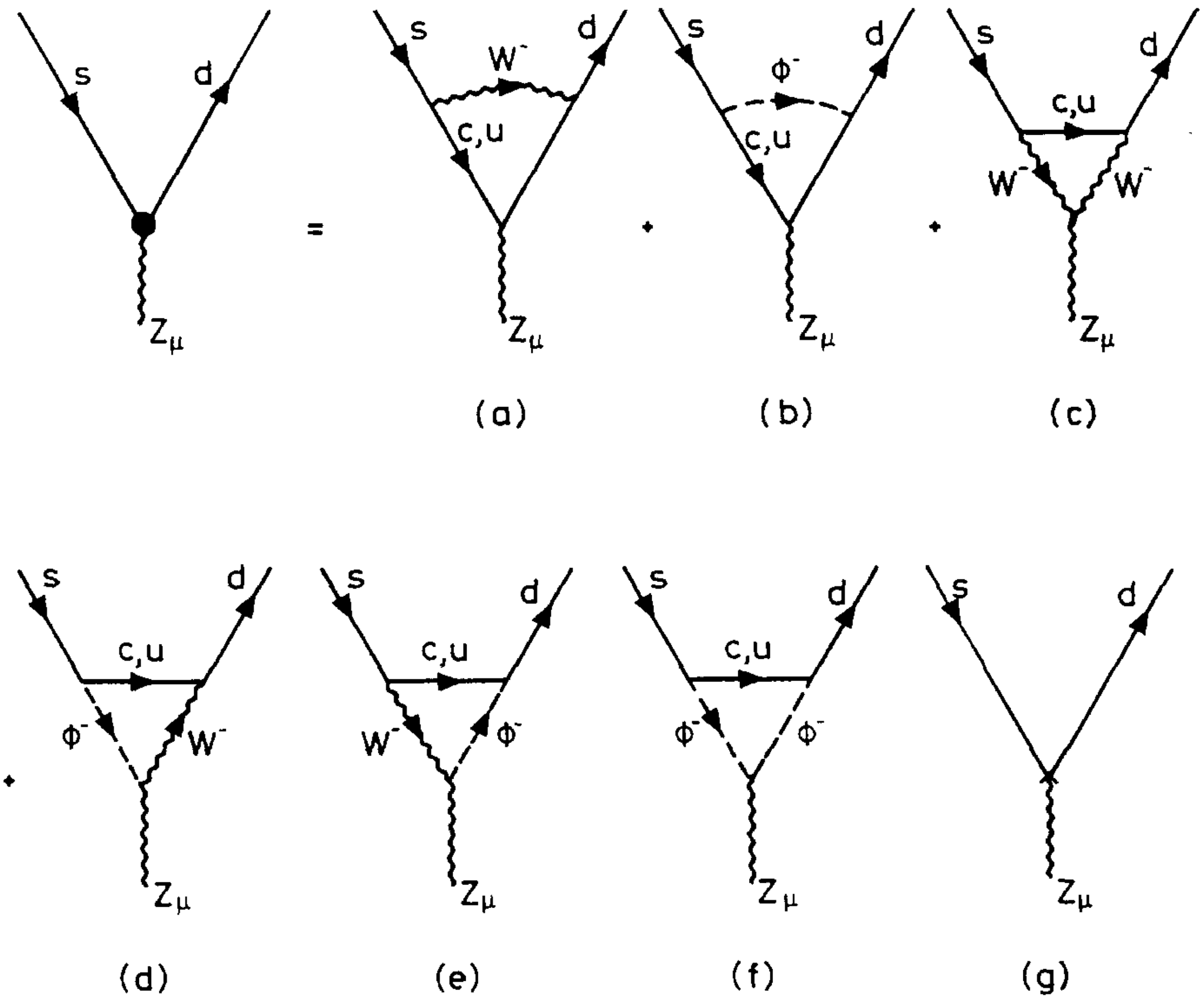

s
d
Z_μ
=
s
d
W⁻
c,u
Z_μ
+
s
d
Φ⁻
c,u
Z_μ
+
s
d
c,u
W⁻
W⁻
Z_μ
(a)
(b)
(c)
+
s
d
c,u
Φ⁻
W⁻
Z_μ
s
d
c,u
W⁻
Φ⁻
Z_μ
s
d
c,u
Φ⁻
Φ⁻
Z_μ
s
d
Z_μ
(d)
(e)
(f)
(g)

FIG. 4

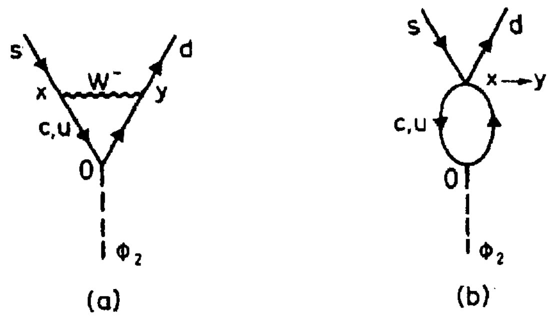


FIG. 5

Figures 6 & 7 not available.