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# Vector Meson Mixing, Lepton Pair Decays and Mass Formulas in the SU(4)/Z(2) Sextet Quark Model

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ABSTRACT

In the SU(4)/Z(2) sextet quark model the lepton pair decay rates and the quark wave functions are obtained for the neutral vector mesons $\rho$, $\omega$, $\phi$, $\psi$, $\psi'$, and $\psi''$ from Weinberg's first spectral function sum rule. Nine vector meson mass formulas are also given, three of which agree with existing data, while the remaining six predict the strange and charmed meson masses.

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We have recently proposed a sextet quark model[1] in which the quarks belong to the six dimensional representation of $SU(4)/Z(2) \sim O(6)$. Three of the quarks $p, n, \lambda$ have charges 2/3, -1/3, and -1/3 and belong to a 3, as usual, and the three new quarks $x, y, z$ have charges -1/3, -4/3, and -1/3 and belong to a $\bar{3}$ with charm[2] $C = -1$. In this model, the six neutral vector mesons $\rho(770)$, $\omega(783)$, $\phi(1020)$, $\psi(3095)$, $\psi'(3684)$ and $\psi''(4150)$ are all $^3S_1$ quark-antiquark ground states.

Here we use Weinberg's first spectral function sum rule[3] to obtain relations among the lepton pair decay rates of the neutral vector mesons, which agree well with present data,[4] and to determine their quark wave functions. In addition, we obtain nine nontrivial relations among the masses of all $6 \times 6$ vector mesons in this model from naive quark model arguments. Three of these mass formulas are found to be in remarkable agreement with existing data while the other six predict the masses of the, as yet unobserved, strange and charmed vector mesons.

The quark wave functions of the neutral vector mesons have components in the representations contained in the product $6 \times 6 = 1 + 15 + 20$ and can be written

$$|\rho\rangle = [|15,8,3\rangle - |20,8,3\rangle]/\sqrt{2} \tag{1a}$$

$$|\omega\rangle = \cos\theta[|1,1,1\rangle - |15,1,1\rangle]/\sqrt{2} + \sin\theta[|15,8,1\rangle - |20,8,1\rangle]/\sqrt{2} \tag{1b}$$

$$|\phi\rangle = \sin\theta[|1,1,1\rangle - |15,1,1\rangle]/\sqrt{2} - \cos\theta[|15,8,1\rangle - |20,8,1\rangle]/\sqrt{2} \tag{1c}$$

$$|\psi\rangle = \cos\Theta[|1,1,1\rangle + |15,1,1\rangle]/\sqrt{2} + \sin\Theta[|15,8,1\rangle + |20,8,1\rangle]/\sqrt{2} \tag{1d}$$

$$|\psi'\rangle = \sin\Theta[|1,1,1\rangle + |15,1,1\rangle]/\sqrt{2} - \cos\Theta[|15,8,1\rangle + |20,8,1\rangle]/\sqrt{2} \tag{1e}$$

$$|\psi''\rangle = [|15,8,3\rangle + |20,8,3\rangle]/\sqrt{2} \tag{1f}$$

where $|D_4, D_3, D_2>$ are the $I_3 = 0$ $q\bar{q}$ wave functions belonging to the SU(4), SU(3) and SU(2) representations with dimensions $D_4$, $D_3$ and $D_2$, respectively. In Eqs. (1) we have assumed that the constituent quarks of $\rho$, $\omega$ and $\phi$ are only p, n and $\lambda$ while those of $\psi$, $\psi'$ and $\psi''$ are only x, y and z; i.e., we have taken the mixing angles between the two SU(3) singlets as well as between the two SU(3) octets to both be $\pi/4$. In addition, we have assumed that both $\omega$-$\phi$ and $\psi$-$\psi'$ mixing are of the singlet-octet mass-mixing type so that $<\omega|\phi> = <\psi|\psi'> = 0$.

The electromagnetic current in the SU(4)/Z(2) sextet model[1] is

$$J_\mu^{em} = V_\mu^3 + \frac{1}{\sqrt{3}} V_\mu^8 - \sqrt{\frac{2}{3}}\, V_\mu^{15} - \sqrt{\frac{2}{3}}\, V_\mu^0 \tag{2}$$

and the lepton pair partial width of the vector meson V is

$$\Gamma_V = \frac{4\pi\alpha}{3m_V^3} \left| <0|\epsilon_\mu J_\mu^{em}|V> \right|^2 \tag{3}$$

Evidently the amplitude $<0|\epsilon_\mu J_\mu^{em}|V>$ is a linear combination of the matrix elements $<0|\epsilon_\mu V_\mu^\alpha|D_4, D_3, D_2>$. The only such nonvanishing matrix elements can be written in terms of a single form factor

$$\begin{aligned} g(k^2) &= <0|\epsilon_\mu V_\mu^0|1,1,1> = <0|\epsilon_\mu V_\mu^{15}|15,1,1> \\ &= <0|\epsilon_\mu V_\mu^8|15,8,1> = <0|\epsilon_\mu V_\mu^3|15,8,3> \end{aligned} \tag{4}$$

assuming the hadron symmetry is U(4) in the equal mass limit. Although the physical masses are not equal, the different coupling constants that result when the form factor $g(k^2)$ is evaluated at $k^2 = m_V^2$ for each of the six vector mesons are related by the spectral function sum rule.

Weinberg's first spectral function sum rule,[3] extended to the 16 currents of U(4), is

$$\int dm^2 \frac{\rho_{\alpha\beta}(m^2)}{m^2} = S\,\delta_{\alpha\beta}\,,\ (\alpha, \beta = 0, 3, 8, 15)\quad , \tag{5}$$

where $\rho_{\alpha\beta}(m^2)$ is the spin 1 spectral function occurring in the Kallen-Lehmann representation of the current propagator $<0\,|\,T\,V_\mu^\alpha(x)V_\nu^\beta(0)\,|\,0>$. Saturating Eq. (5) with the six vector mesons $\rho, \omega, \phi, \psi, \psi'$ and $\psi''$ requires that the ratio $g^2(m_V^2)/m_V^2$ be independent of the vector meson mass. Then independent of the $\omega$-$\phi$ and $\psi$-$\psi'$ mixing angles there are three relations among the lepton-pair widths:[5]

$$m_\rho\Gamma_\rho = 3(m_\omega\Gamma_\omega + m_\phi\Gamma_\phi) = \frac{1}{3}(m_\psi\Gamma_\psi + m_{\psi'}\Gamma_{\psi'}) = m_{\psi''}\Gamma_{\psi''} \tag{6}$$

In addition, there are two relations which depend on the mixing angles:

$$\frac{m_\omega\Gamma_\omega}{m_\phi\Gamma_\phi} = \tan^2\theta \tag{7}$$

$$\frac{m_\psi\Gamma_\psi}{m_{\psi'}\Gamma_{\psi'}} = \left(\frac{2\sqrt{2} - \tan\Theta}{1 + 2\sqrt{2}\tan\Theta}\right)^2 . \tag{8}$$

From the experimental widths,[6] we find

$$\begin{aligned} &m_\rho\Gamma_\rho : 3(m_\omega\Gamma_\omega + m_\phi\Gamma_\phi) : \frac{1}{3}(m_\psi\Gamma_\psi + m_{\psi'}\Gamma_{\psi'}) : m_{\psi''}\Gamma_{\psi''} \\ &= 4.97 \pm 0.67 : 5.90 \pm 0.73 : 7.65 \pm 1.36 : 10 \text{ to } 20\ \text{MeV}^2 \end{aligned} \tag{9}$$

in rather good agreement with Eq. (6) except for the $\psi''$ whose width is poorly known. Using Eq. (6) to determine the $\psi''$ leptonic pair width yields

$$\Gamma_{\psi''} = 1.5 \text{ KeV} \quad . \tag{10}$$

From the data[6] Eqs. (7) and (8) imply $\tan\theta = 0.66 \pm 0.08$ and $\tan\Theta = 0.31 \pm 0.08$. These empirical angles suggest the magic mixing hypothesis for which

$$\tan\theta = 1/\sqrt{2} \simeq 0.71 \tag{11}$$

$$\tan\Theta = 1/(2\sqrt{2}) \simeq 0.35. \tag{12}$$

With magic mixing $\omega$ and $\phi$ are composed of purely nonstrange and strange quarks, respectively, while $\psi$ and $\psi'$ transform purely as octet and singlet components, respectively, under the SU(3) subgroup whose multiplets lie in the planes normal to the direction of strangeness plus charm.[7] Under the usual SU(3), whose multiplets are normal to the direction of charm, $\psi$ is predominantly a singlet and $\psi'$ is predominantly an octet component.

For the magic mixing angles in Eqs. (11) and (12), the vector meson quark wave functions are

$$|\rho> = (p\bar{p} - n\bar{n})/\sqrt{2} \tag{13a}$$

$$|\omega> = (p\bar{p} + n\bar{n})/\sqrt{2} \tag{13b}$$

$$|\phi\rangle = \lambda\bar{\lambda} \tag{13c}$$

$$|\psi\rangle = (x\bar{x} + y\bar{y} + 2z\bar{z})/\sqrt{6} \tag{13d}$$

$$|\psi'\rangle = (x\bar{x} + y\bar{y} - z\bar{z})/\sqrt{3} \tag{13e}$$

$$|\psi''\rangle = (x\bar{x} - y\bar{y})/\sqrt{2} \tag{13f}$$

and the lepton pair widths are in the ratios

$$m_\rho\Gamma_\rho : m_\omega\Gamma_\omega : m_\phi\Gamma_\phi : m_\psi\Gamma_\psi : m_{\psi'}\Gamma_{\psi'} : m_{\psi''}\Gamma_{\psi''} :: 9 : 1 : 2 : 16\tfrac{1}{3} : 10\tfrac{2}{3} : 9 \tag{14}$$

Experimentally[6]

$$\frac{m_\rho\Gamma_\rho}{9} : \frac{m_\omega\Gamma_\omega}{1} : \frac{m_\phi\Gamma_\phi}{2} : \frac{m_\psi\Gamma_\psi}{16\ 1/3} : \frac{m_{\psi'}\Gamma_{\psi'}}{10\ 2/3} : \frac{m_{\psi''}\Gamma_{\psi''}}{9}$$

$$= 0.55 \pm 0.07 : 0.60 \pm 0.14 : 0.69 \pm 0.05 : 0.91 \pm 0.11 : 0.76 \pm 0.21 : 1 \text{ to } 2\ \text{MeV}^2 \; . \tag{15}$$

In the naive quark model the neutral vector meson wave functions, Eqs. (13), lead to two mass formulas: the well-known relation[8]

$$m_\rho^2 = m_\omega^2 \simeq (0.783\ \text{GeV})^2 \tag{16}$$

and the new relation

$$m_{\psi''}^2 = 2\, m_{\psi'}^2 - m_\psi^2 \simeq (4.19\ \text{GeV})^2 \; . \tag{17}$$

These agree very well with the observed values[4] $m_\rho = 0.770$ GeV and $m_{\psi''} = 4.15$ GeV. By extending these naive quark model arguments to the strange and charmed $^3S_1$ $q\bar{q}$ ground states that occur in the product $6 \times 6$ one can calculate the masses of all the 36 vector mesons in the SU(4)/Z(2) sextet quark model.[1]

There are two charm 0, strangeness + 1 isodoublets $K^*(892)$ and $K_c^*$ and their masses are[8]

$$m_{K^*}^2 = \frac{1}{2}(m_\omega^2 + m_\phi^2) \simeq (0.909\ \text{GeV})^2 \qquad (18)$$

and

$$m_{K_c^*}^2 = \frac{1}{2}(m_\psi^2 + m_{\psi'}^2) \simeq (3.40\ \text{GeV})^2 \quad . \qquad (19)$$

The $K^*$ mass agrees well with the observed value[4] $m_{K^*} = 0.892$ GeV and Eq. (19) is a prediction. The singlet $\sigma^{*+}$ and triplet $\tau^{*++}$, $\tau^{*+}$, $\tau^{*0}$ vector mesons have C = 1, S = 0 and their masses are

$$m_{\sigma^*}^2 = m_{\tau^*}^2 = \frac{1}{2}(m_\rho^2 + m_{\psi''}^2) \simeq (2.99\ \text{GeV})^2 \quad . \qquad (20)$$

There are two C = +1, S = -1 vector meson doublets $\xi^{*+}$, $\xi^{*0}$ and $\zeta^{*+}$, $\zeta^{*0}$ with masses[9]

$$m_{\xi^*}^2 = m_{\zeta^*}^2 = \frac{1}{4}(m_\omega^2 + m_\phi^2 + M_\psi^2 + m_{\psi'}^2) \simeq (2.49\ \text{GeV})^2 . \qquad (21)$$

Lastly, the mass of the C = +1, S = -2 singlet vector meson $\theta^{*0}$ is

$$m_{\theta^*}^2 = \frac{1}{2}(m_\omega^2 + m_\phi^2 + m_\psi^2 + m_{\psi'}^2 - m_\rho^2 - m_{\psi''}^2) \simeq (1.87\ \text{GeV})^2 \qquad (22)$$

Observe that $\tau^*$, $\zeta^*$ and $\theta^*$ which belong to an SU(3) 6 are equally spaced and their splitting is equal to that of $\sigma^*$ and $\xi^*$ which belong to an SU(3) $\bar{3}$ .

Altogether, nine mass formulas have been given above, three of which can be compared with existing data and the agreement is excellent (2 %). The other six relations predict the masses of the strange and charmed vector mesons which remain to be discovered.

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[5]If the vector mesons are arbitrary mixtures of all six quarks, then, independent of all the mixing angles, the first sum rule only requires that $m_\rho \Gamma_\rho + m_{\psi''} \Gamma_{\psi''} = \frac{5}{3}(m_\omega \Gamma_\omega + m_\phi \Gamma_\phi + m_\psi \Gamma_\psi + m_{\psi'} \Gamma_{\psi'})$.

[6]We use the $e^+e^-$ partial widths $\Gamma_\rho = 6.45 \pm 0.86$ KeV, $\Gamma_\omega = 0.76 \pm 0.17$ KeV, $\Gamma_\phi = 1.34 \pm 0.11$ KeV, $\Gamma_\psi = 4.8 \pm 0.6$ KeV, $\Gamma_{\psi'} = 2.2 \pm 0.6$ KeV and $\Gamma_{\psi''} = 2.4$ to 5 KeV taken from reference 4.

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[9]If $\xi^*$ and $\zeta^*$ are mixed, then $m_{\xi^*}^2$ and $m_{\zeta^*}^2$ lie between $(2m_\psi^2 - m_{\psi'}^2 + m_\omega^2)/2 \simeq (1.76\ \mathrm{GeV})^2$ and $(2m_{\psi'}^2 - m_\psi^2 + m_\phi^2)/2 \simeq (3.05\ \mathrm{GeV})^2$ with $m_{\xi^*}^2 + m_{\zeta^*}^2 = (m_\omega^2 + m_\phi^2 + m_\psi^2 + m_{\psi'}^2)/2 \simeq (3.52\ \mathrm{GeV})^2$.