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On the Separation of $\psi \rightarrow \pi^+\pi^-\gamma$ from $\psi \rightarrow \pi^+\pi^-\pi^0$ *

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ABSTRACT

Angular distributions provide a means for determining the frequency of the decay $\psi \rightarrow \pi^+\pi^-\gamma$ which is generally indistinguishable from $\psi \rightarrow \pi^+\pi^-\pi^0$ on the basis of the neutral missing mass alone. Radiative decays such as $\psi \rightarrow \pi^+\pi^-\gamma$ might be expected to be significant on the basis of vector dominance or as a consequence of charm or color models for the ψ.

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The new resonances[1-4] at 3.1 GeV and 3.7 GeV are so narrow that their second order electromagnetic decays are appreciable. A fortiori we expect their first order electromagnetic decays, $\psi \to \gamma$ + hadrons, to be important as well.

One such decay width may be estimated by conventional vector dominance arguments. The dominant decay of the $\psi'(=\psi(3.7))$ is[4] $\psi' \to \psi\pi^+\pi^-$. Using the phenomenological Lagrangian density[6,7]

$$\mathscr{L} = g\psi'_\mu \psi^\mu \pi^+\pi^- , \tag{1}$$

Jackson finds $\Gamma(\psi' \to \psi\pi^+\pi^-) = 13.6\,(g^2/4\pi)$ keV. This width may be[5] roughly 100 keV, indicating $g^2/4\pi \approx 10$. This effective Lagrangian does an inadequate job of describing the spectrum for $\psi' \to \psi\pi\pi$.[6,7] It should, however, suffice for the purpose of making the order of magnitude estimates we are interested in. In the same spirit we ignore final state π-π interactions which may have some effect for the spectrum near $m_{\pi\pi} = m_\epsilon$. Dominating $\psi \to \pi^+\pi^-\gamma$ with $\psi \to \psi'\pi^+\pi^-$, one finds

$$\frac{\Gamma(\psi \to \pi^+\pi^-\gamma)}{\Gamma(\psi' \to \mu^+\mu^-)} = \left(\frac{M_\psi}{M_{\psi}'}\right)\left(\frac{g^2}{4\pi}\right)\frac{1}{128\,\pi^2\alpha} , \tag{2}$$

where phase space has been calculated with $m_\pi = 0$. Using[5] $\Gamma(\psi' \to \mu^+\mu^-) = 2.2$ keV, we find $\Gamma(\psi \to \pi^+\pi^-\gamma) = 0.2\left(\frac{g^2}{4\pi}\right)$ keV. This estimate cannot be taken very seriously in view of the large extrapolation from the ψ mass to the photon mass. Moreover, we have ignored the interfering intermediate decay $\psi \to \psi\pi^+\pi^-$. Furthermore, the use of a field strength coupling of the $\psi'\psi\pi\pi$, $G^{\mu\nu}G'_{\mu\nu}\pi\pi$ restores gauge invariance for the process and leads to a very substantial reduction of the rate.[8] Contributions from $\psi \to \omega\pi^+\pi^-$ should be much smaller since[5] $\Gamma(\psi \to \omega\pi^+\pi^-) \approx 1$ keV, and the corresponding $\frac{g^2}{4\pi}$ is of the order of 10^{-3}. The computation, Eq. (2), however crude, does suggest this mode may be important. In charm and color schemes, radiative decays are again expected to be significant.

Of the observed 3π final states, many[5] are in fact $\rho\pi$. These particular events cannot have a misidentified γ since $\rho^0\gamma$ has $C = +1$. Therefore, only events without ρ mesons are candidates for $\pi^+\pi^-\gamma$. The final state $K^+K^-\gamma$ may also be interesting and can be analyzed in a fashion identical to that for $\pi^+\pi^-\gamma$.

Since it is difficult to distinguish the decay mode $\psi \to \pi^+\pi^-\pi^0$ from $\psi \to \pi^+\pi^-\gamma$ on the basis of the neutral missing mass, it is desirable to find an indirect means of separation. A hint of how this might be done is found by noting that in the $\pi^+\pi^-\pi^0$ final state, the $\pi^+\pi^-$ system must have $C_{\pi\pi} = -1$ so its angular momentum (in its rest frame), $\ell_{\pi\pi}$, is odd whereas for $\pi^+\pi^-\gamma$, $C_{\pi\pi} = +1$ and $\ell_{\pi\pi}$ is even. Armed with this observation, we proceed to a fuller investigation of the decay kinematics.

Let us define

$P = \psi$ momentum, $\epsilon = \psi$ polarization vector,

$k = \gamma$ momentum, $\eta = \gamma$ polarization vector,

$k^0 = \pi^0$ momentum,

$k^\pm = \pi^\pm$ momentum.

We shall also need

$q = k^+ - k^-$,

$\hat{n}$ = beam direction,

$\nu = P \cdot q = k \cdot q = (k^0 \cdot q)$,

$\omega = P \cdot k = (P \cdot k^0)$.

The general form of the $\psi \to \pi^+\pi^-\pi^0$ amplitude, assuming parity conservation, is[9]

$$\mathcal{M} = \epsilon^{\alpha\beta\gamma\delta} P_\alpha \epsilon_\beta k^+_\gamma k^-_\delta \bar{A}(\nu, \omega) \quad . \tag{3}$$

In the ψ rest frame (and absorbing a factor M_ψ) we may write

$$\mathcal{M} = \underset{\sim}{\epsilon} \cdot (\underset{\sim}{k}^+ \times \underset{\sim}{k}^-)\, A. \tag{4}$$

Now if the ψ is produced by e^+e^- annihilation, it is polarized transversely to the beam. Then the initial polarization average gives

$$|\mathcal{M}|^2_{\text{ave}} = \frac{1}{2} \left| \hat{n} \times (\underset{\sim}{k}^+ \times \underset{\sim}{k}^-) \right|^2 |A|^2 . \tag{5}$$

This implies the decay probability vanishes when

1) The normal to the decay plane coincides with the beam direction, or

2) The charged pi momenta are collinear.

Effect (1) is well-known theoretically[9] and experimentally.[10] Any estimation of the rate $\Gamma(\psi \to \pi^+\pi^-\pi^0)$ should incorporate Eq. (5) into the detector efficiency computation.

The general form of the amplitude for $\psi \rightarrow \pi^+\pi^-\gamma$ is

$$\mathcal{M} = \eta^{\mu*} T_{\mu\nu} \epsilon^{\nu} \quad . \tag{6}$$

By gauge invariance for electromagnetic interactions,

$$k^{\mu} T_{\mu\nu} = 0. \tag{7}$$

By the gauge condition for the vector particles, $\partial_{\nu}\psi^{\nu} = 0$,

$$T_{\mu\nu} P^{\nu} = 0 \quad . \tag{8}$$

There are three independent amplitudes as can be verified by counting helicity amplitudes: $\langle 1|1\rangle$, $\langle -1|1\rangle$, $\langle 1|0\rangle$.

Amplitudes free of kinematic singularities may be chosen so that

$$T_{\mu\nu} = T_1(P_{\mu}k_{\nu} - P\cdot k\, g_{\mu\nu}) + T_2(\nu q_{\mu}k_{\nu} - \nu^2 g_{\mu\nu} + \nu P_{\mu}q_{\nu} - q_{\mu}q_{\nu}k\cdot P)$$
$$+ T_3(\nu^2 P_{\mu}P_{\nu} - \nu P^2 P_{\mu}q_{\nu} - k\cdot P\nu q_{\mu}P_{\nu} + k\cdot PP^2 q_{\mu}q_{\nu}). \tag{9}$$

Since the photon polarization is not observed, we may take η to be real. In addition, we are free to choose the gauge $\eta\cdot P = 0$ (i.e., η spatial only, in the e^+e^- c.m.) so that

$$\eta^{\mu}T_{\mu\nu}\epsilon^{\nu} = \eta\cdot\epsilon(-k\cdot PT_1 - \nu^2 T_2) + \eta\cdot q\,k\cdot\epsilon\,\nu T_2$$
$$+ \eta\cdot q\; q\cdot\epsilon\,(-k\cdot PT_2 + k\cdot PP^2 T_3) \quad . \tag{10}$$

For convenience, we define three new amplitudes (with $\underset{\sim}{q}, \underset{\sim}{k}$ measured in the ψ rest frame)

$$
\begin{aligned}
B_1 &= k \cdot P T_1 + \nu^2 T_2 , \\
B_2 &= |\underset{\sim}{q}|\ |\underset{\sim}{k}|\ \nu T_2 , \\
B_3 &= |\underset{\sim}{q}|^2 k \cdot P(-T_2 + P^2 T_3) .
\end{aligned}
\tag{11}
$$

Then in the ψ rest frame,

$$
|\mathscr{M}|^2_{\text{ave}} = \frac{1}{2} \left| \hat{n} \times (B_1 \hat{\eta} + B_2 \hat{\eta} \cdot \hat{q} \hat{k} + B_3 \hat{\eta} \cdot \hat{q}\ \hat{q}) \right|^2 . \tag{12}
$$

We select two orthogonal photon polarizations,

$$
\begin{aligned}
\hat{\eta}_1 &= \frac{\hat{k} \times \hat{q}}{|\hat{k} \times \hat{q}|} = \hat{c} , \\
\hat{\eta}_2 &= \hat{k} \times \hat{c},
\end{aligned}
\tag{13}
$$

which are convenient for doing the sum over outgoing photon polarizations.

The full angular distribution is given by inserting these polarizations into Eq. (12)

$$
\sum_{\text{pol}} |\mathscr{M}|^2_{\text{ave}} = \frac{1}{2} |\hat{n} \times \hat{c}|^2 |B_1|^2 + \frac{1}{2} \left| \hat{n} \times (B_1 \hat{k} \times \hat{c} - |\hat{k} \times \hat{q}| \hat{k} B_2 - |\hat{k} \times \hat{q}| \hat{q} B_3) \right|^2 . \tag{14}
$$

This equation shows that while the contribution from photons polarized normal to the decay plane vanishes when the beam direction coincides with the normal to the decay plane, the contribution from photons polarized in the decay plane does not vanish. If θ is the angle between the normal to the decay plane, $\hat{c}$, and the beam direction, $\hat{n}$, the 3π final state must have a distribution proportional to $\sin^2\theta$, but the $\pi^+\pi^-\gamma$ final state should not, and in particular there should not be a zero at $\theta = 0$ or π.

The process $e^+e^- \to \pi^+\pi^-\gamma$ has been discussed by Creutz and Einhorn[11] in a more general context. They consider interference between photons emitted by the electrons and those coming from the $\pi^+\pi^-$. This is not important for the case of the ψ since initial bremsstrahlung would move the event off resonance. Creutz and Einhorn[11] emphasize that there is no final state interaction theorem for $e^+e^- \to \pi^+\pi^-\gamma$ and thus no information about the $\pi\pi$ phase shift can be obtained. Such information is available in the decay $\psi' \to \psi\pi^+\pi^-$ if certain conditions are satisfied.[12]

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