



Fermi National Accelerator Laboratory

FERMILAB-Pub-75/20-EXP
7100.063

(Submitted to Phys. Rev. Letters)

EVIDENCE FOR RADIAL SCALING
IN A NEW KINEMATIC RANGE

D. C. Carey, J. R. Johnson, R. Kammerud, D. J. Ritchie,
A. Roberts, J. R. Sauer, R. Shafer, D. Theriot,
and J. K. Walker

Fermi National Accelerator Laboratory
Batavia, Illinois 60510

and

F. E. Taylor
Physics Department, Northern Illinois University
DeKalb, Illinois 60115

February 1975

EVIDENCE FOR RADIAL SCALING IN A NEW
KINEMATIC RANGE

D. C. Carey, J. R. Johnson,* R. Kammerud, D. J. Ritchie,
A. Roberts, J. R. Sauer, R. Shafer, D. Theriot, and
J. K. Walker
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

and

F. E. Taylor
Physics Department, Northern Illinois University
DeKalb, Illinois 60115

Abstract

The cross section for the reaction $p + p \rightarrow \pi^0 + \text{anything}$ has been measured for incident momenta between 50 and 400 GeV/c, $0.3 \leq p_{\perp} < 3.5$ (GeV/c) and $20^{\circ} < \theta_{\pi^0}^*$ (center of mass angle) $\leq 150^{\circ}$. We find that this cross section can be expressed as a function of x_R and p_{\perp} where $x_R = 2E^*/\sqrt{s}$ and E^* is the pion center of mass energy. Scaling in the variable x_R is therefore demonstrated in a new kinematic range.

The study of single pion production in hadron-hadron collisions has been a subject of great interest in the last few years. The theoretical predictions of Feynman¹ and Yang and collaborators² suggested that the invariant cross section $E d^3\sigma/dp^3$ for the process $p + p \rightarrow \pi^0 + X$ should approach a limiting function of $x_{||}$ and p_{\perp} at high energies i.e.

$$\lim_{s \rightarrow \infty} E \frac{d^3\sigma}{dp^3} \rightarrow f(x_{||}, p_{\perp})$$

where $x_{||} = \frac{2p_{||}^*}{\sqrt{s}}$ is the usual Feynman variable and $p_{||}^*$ and p_{\perp}

are respectively the longitudinal and transverse momentum of the π^0 in the center of mass system. The data³ on pion production at low p_{\perp} (< 1 GeV/c) support these theoretical conjectures. However, at large transverse momenta ($p_{\perp} \gg 1$ GeV/c) experimental results⁴ indicate that $E d^3\sigma/dp^3$ at $x_{||} = 0$ is a rapidly increasing function of s even up to the highest energies available at the ISR. The purpose of this paper is to present experimental data which show that scaling sets in very early even for large p_{\perp} when a different choice of variables is made, namely

$$E \frac{d^3\sigma}{dp^3} = F(x_R, p_{\perp})$$

where $x_R = \frac{2E^*}{\sqrt{s}}$ and E^* is the center of mass energy of the detected pion.⁵ We have termed this behavior radial scaling.⁶

The range over which this scaling has been obtained is $50 \leq E_{\text{inc}} < 400$ GeV/c and $0.3 < p_{\perp} < 3.5$ GeV/c.

We have previously reported⁶ preliminary results of measurements of the reaction $p + p \rightarrow \pi^0 + \text{anything}$ for the laboratory angles 80, 100 and 120 mrad. In this letter we give the preliminary results of our completed set of measurements at eight angles, namely 30, 40, 65, 80, 100, 120, 200, 275 mrad. These data demonstrate radial scaling over a new and enlarged kinematic range. Details of the experimental method have been given previously⁶ and will not be repeated here.

The π^0 invariant cross section reveals a strikingly simple behavior when it is expressed as a function of x_R and p_{\perp} ; in particular, any additional s -dependence is eliminated. Figure 1 shows this s -dependence for small p_{\perp} (< 1 GeV/c) in the so called central region at $x_R = 0.05$. Only statistical errors are shown in Figure 1. In addition, there are systematic relative normalization errors for the measurements made at different angles ($\pm 30\%$) and uncertainties in the energy calibration ($\pm 7\%$) of the lead glass counter. These are not included in the figures but result in uncertainties of the overall slopes of $E d^3\sigma/dp^3$ versus \sqrt{s} of $\pm 15\%$ for $p_{\perp} \leq 1$ GeV/c which increase to $\pm 30\%$ for $p_{\perp} \geq 2$ GeV/c. The data have been plotted in bins of $\Delta p_{\perp} = 0.25$ GeV/c and $\Delta x_R = 0.1$. A small correction factor has therefore been applied to each measured cross section to center the data in each bin. These correction factors are typically $\leq 30\%$ and errors due to this procedure are estimated to be $\leq 5\%$.

The data for $0.3 < p_{\perp} \text{ (GeV/c)} < 1.0$ in Fig. 1 are seen to be s -independent, even starting from their respective thresholds. On the other hand, when we plot our data for $p_{\perp} = 1.0 \text{ GeV/c}$ for fixed Feynman $x_{\parallel} = 0$, the π^0 invariant cross section rises over this energy range by approximately a factor of two. For fixed x_{\parallel} this energy dependence of the cross section becomes larger as p_{\perp} is increased.

The invariant cross sections $E \frac{d^3\sigma}{dp^3}$ are plotted in Figure 2 for $p_{\perp} = 1.5, 2.25, 2.75$ and 3.25 GeV/c and for various x_R . All of the data are consistent with a hypothesis of s -independence. The data at the three lower values of p_{\perp} have significantly greater statistical accuracy and show this consistency most conclusively. The dependence of the cross section on only p_{\perp} and x_R demonstrates that, in general, x_R is a good scaling variable to use in analyzing single particle production. In addition, we have data at other values of x_R and the above values of p_{\perp} which are also consistent with scaling. These data will be reported at a later time.⁷

It is instructive to examine the methods by which single particle inclusive measurements are made to contrast radial scaling with Feynman scaling. In both cases, consider the invariant cross section at a fixed p_{\perp} and compare $x_{\text{Feynman}} = 0.3$ to $x_{\text{Radial}} = 0.3$. Figure 3 shows the curves along which these

measurements are made as a function of s in both cases. This plot is made in the $x_{\perp} = 2p_{\perp}/\sqrt{s}$ and $x_{\parallel} = 2p_{\parallel}^*/\sqrt{s}$ plane. In the limit of $s \rightarrow \infty$

$$F(x_R, p_{\perp}) = F(x_{\parallel}, p_{\perp}).$$

However, the approach to this limit is quite different in the two cases. In the case of Feynman scaling the point at which the measurement is made moves away from the kinematic boundary. A large s -dependent increase in the cross section due to increasing phase space is thereby introduced. On the other hand measurements at fixed x_R require that the fractional distance to the kinematic boundary remain constant. In this way, it appears that the s -dependence of the dynamics may be more directly probed.

Finally, we have investigated whether the invariant cross section could be factorized in the form

$$E \frac{d^3\sigma}{dp^3} = F(x_R, p_{\perp}) = f(x_R) g(p_{\perp}).$$

At each laboratory angle the cross section data were analyzed to extract in an iterative and self consistent manner⁶ the functions $g(p_{\perp})$ and $f(x_R)$. The resulting functions $g(p_{\perp})$ and $f(x_R)$ are shown in Figures 4 and 5 for each angle. A unique curve is drawn through all sets of data for $g(p_{\perp})$. Similarly a different but unique curve is drawn through all sets of data

for $f(x_R)$.⁸ It can be seen that to a good approximation $g(p_{\perp})$ is a universal function for all

a) center of mass angles 20°

$\leq \theta^* \leq 150^{\circ}$

b) x_R ($0.05 \leq x_R \leq .6$)

c) center of mass energies

$9 \leq \sqrt{s} \leq 27$ GeV.

It appears however, that for the radial scaling function $f(x_R)$ there are significant deviations. Thus, factorization of the invariant cross section is a good approximation within a large, but not the entire, range of the kinematic variables.

In conclusion, we have shown that in the reaction $p + p \rightarrow \pi^0 + \text{anything}$, a simple scaling law holds for the invariant production cross section when it is expressed as a function of $x_R = 2 E^*/\sqrt{s}$. It is a striking experimental fact that this scaling works for all p_{\perp} out to about 3 GeV/c and over the energy range 50 to 400 GeV.⁹

We gratefully acknowledge the enthusiastic support of D. Jovanovic, P. Mantsch and the staff of the Internal Target Area at Fermi Laboratory. D. Burandt and R. Olsen provided invaluable technical assistance. Finally, we would like to recognize the encouragement given to us by R. R. Wilson, E. L. Goldwasser and J. Sanford in the course of this work.

References

- *Northeastern University, Physics Department, Boston, Massachusetts.
- ¹R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969).
- ²J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. 188, 2159 (1969).
- ³A. M. Rossi, G. Vannini, A. Bussiere, E. Albin, D. D'Alessandra and G. Giacomelli, CERN Preprint 26 July 1974, submitted to Nuclear Physics.
- ⁴F. W. Busser, L. Camilleri, L. Di Lella, G. Gladding, A. Placci, B. G. Pope, A. M. Smith, J. K. Yoh, E. Zavattini, B. J. Blumenfeld, L. M. Lederman, R. L. Cool, L. Litt and S. L. Sedler, Phys. Letters 46B, 471 (1973).
- J. W. Cronin, M. J. Frisch, M. J. Shochet, J. P. Boymond, P. A. Piroue and R. L. Sumner, Phys. Rev. Lett. 31, 1426 (1973).
- ⁵As far as we are aware the variable x_R was first suggested by Feynman in 1969 (private communication from N. Byers) and later by K. Kinoshita and H. Noda, Prog. Theor. Phys. 6, 1639 (1971).
- ⁶D. C. Carey, J. R. Johnson, R. Kammerud, M. Peters, D. J. Ritchie, A. Roberts, J. R. Sauer, R. Shafer, F. E. Taylor, D. Theriot and J. K. Walker, Phys. Rev. Lett. 33, 327 (1974).
- ⁷D. C. Carey, J. R. Johnson, R. Kammerud, D. J. Ritchie, A. Roberts, J. R. Sauer, R. Shafer, F. E. Taylor, D. Theriot and J. K. Walker, to be published in Phys. Rev.

⁸The function used for the $f(x_R)$ curve was:

$$f(x_R) = (1 - x_R)^4 \left(1.0 + 1.22 x_R - 0.203 x_R^2 + \frac{9.47 \times 10^{-5}}{x_R^2} \right)$$

and for the $g(p_{\perp})$ curve was:

$$g(p_{\perp}) = 4.5 / (p_{\perp}^2 + 0.86)^{4.5}$$

where p_{\perp} is in units of GeV/c. It should be emphasized

that these curves were drawn to show angle to angle comparisons of the data and thereby to check the factorization hypothesis. The curves are not fits to the data. At the smallest laboratory angles differences between the curves and the data become large at large x_R .

⁹It should be remarked that the present energy range where radial scaling is found to be valid is also the energy range where the total pp cross section is constant. It is therefore of interest to investigate the validity of radial scaling for single particle production in the energy range where the total cross section begins to rise significantly. If radial scaling begins to break down at these higher energies, then the precise range of x_R and p_{\perp} where this breakdown occurs may give insight into the origin of the phenomenon of rising total cross sections.

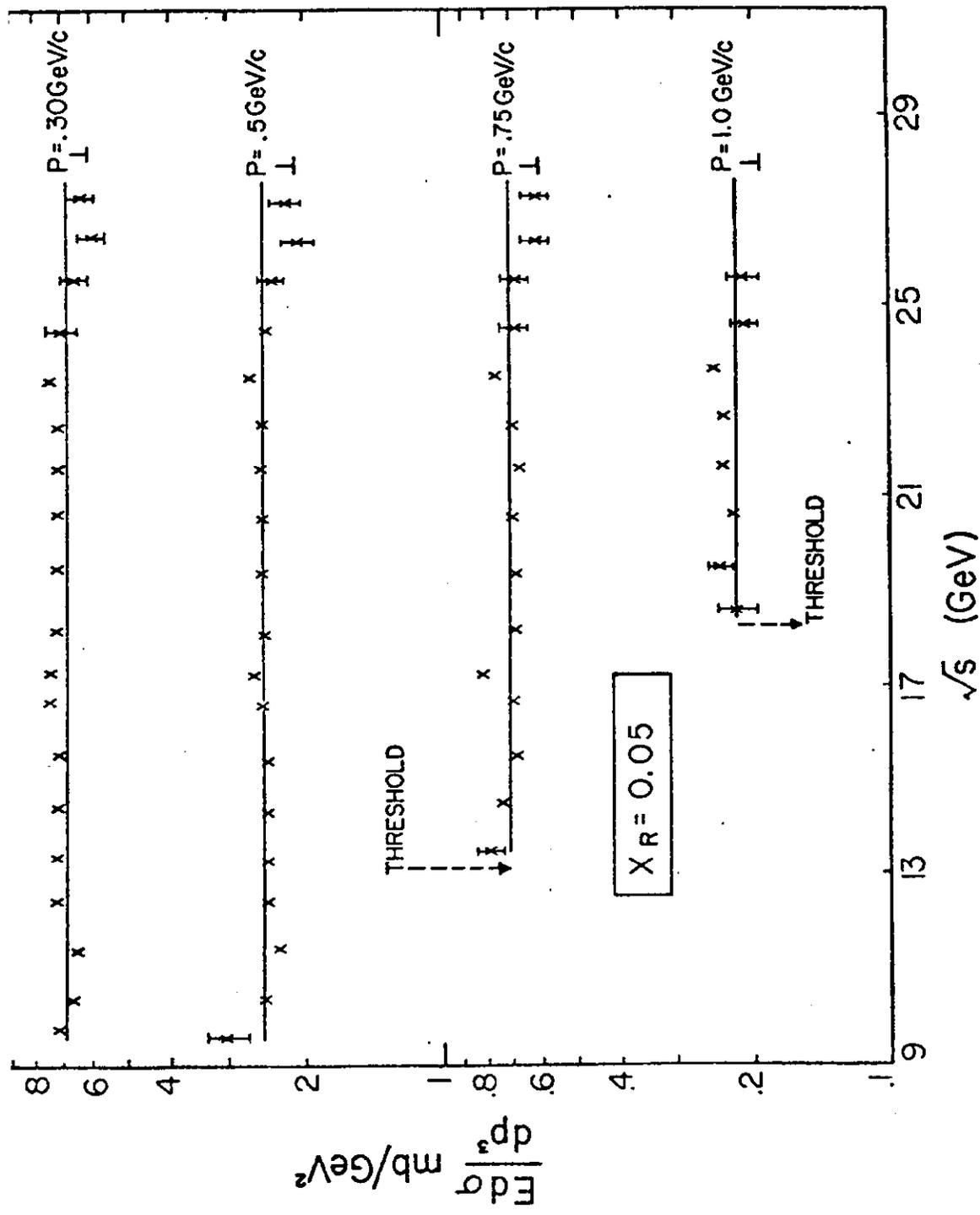


Fig. 1. The invariant π^0 production cross section for fixed $x_R = 0.05$ and $P_{\perp} = 0.3, 0.5, 0.75,$ and 1.0 GeV/c is plotted versus center-of-mass energy (\sqrt{s}) in GeV. The straight lines are drawn parallel to the horizontal axis.

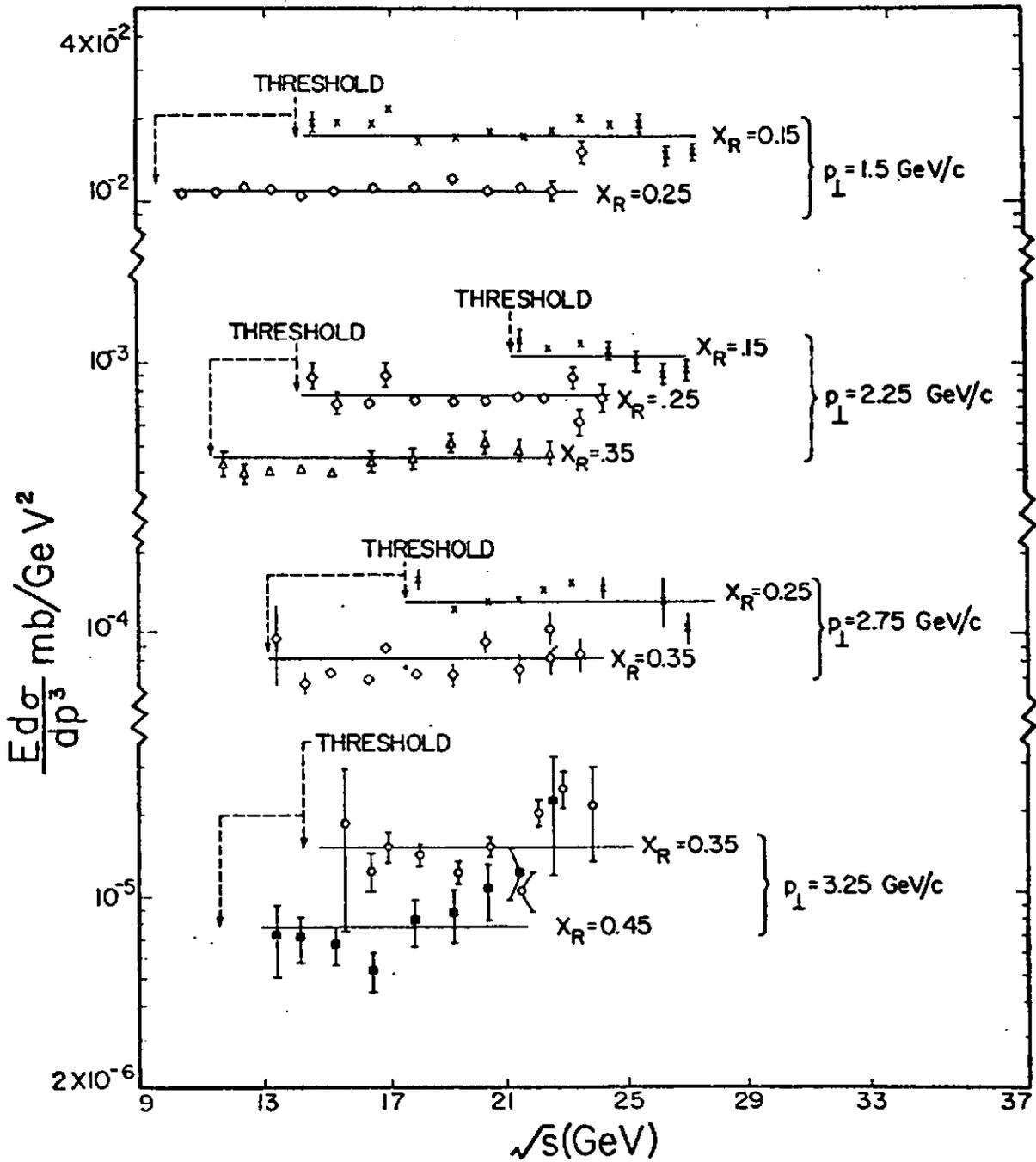


Fig. 2. The invariant π^0 production cross section for various combinations of x_R and p_{\perp} versus center-of-mass energy (\sqrt{s}) in GeV. The straight lines are drawn parallel to the horizontal axis.

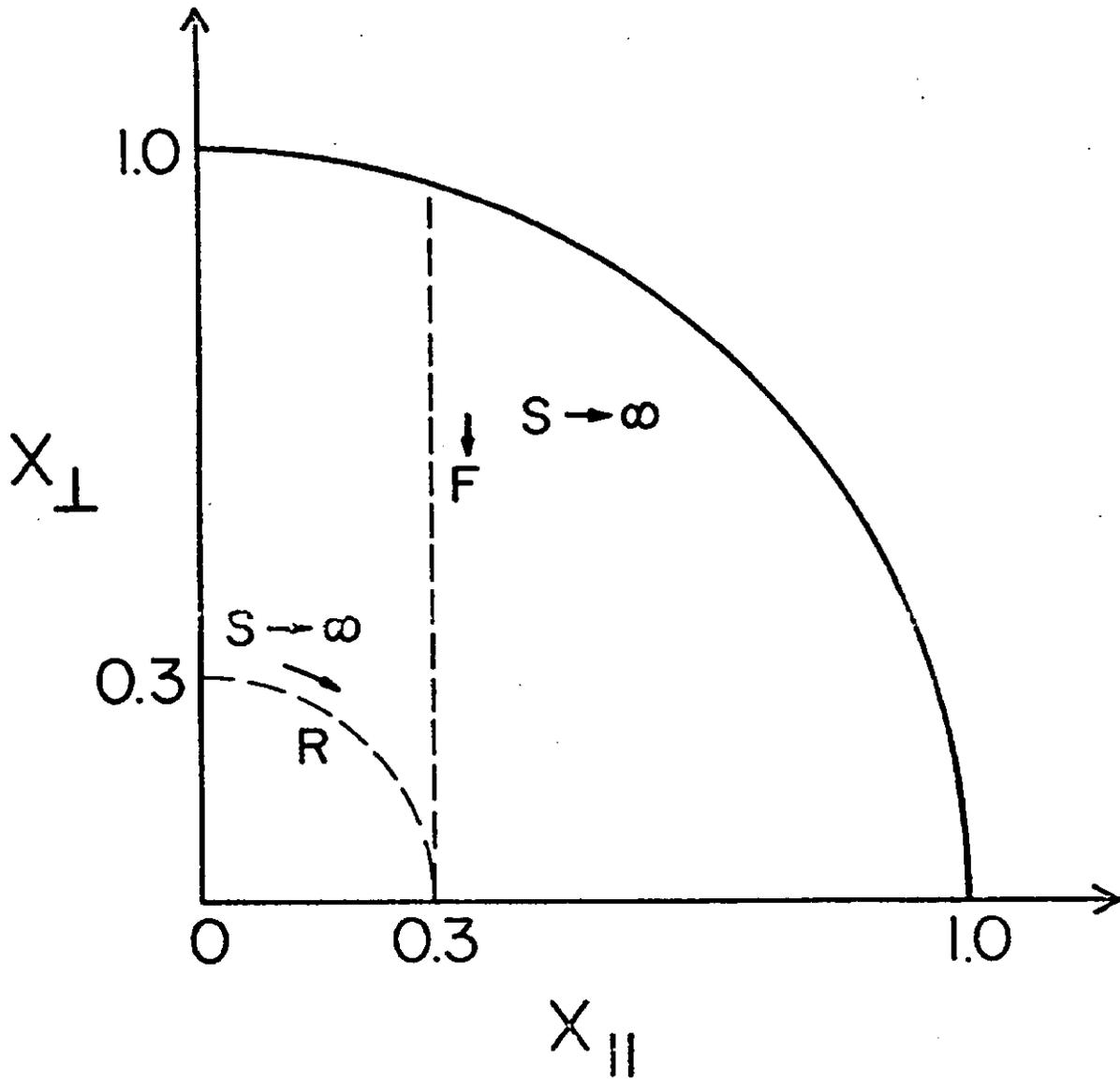


Fig. 3. The plot shows the lines along which measurements are made to study scaling in the case of fixed Feynman (F) variable x_{\parallel} , and fixed radial (R) variable x_R .

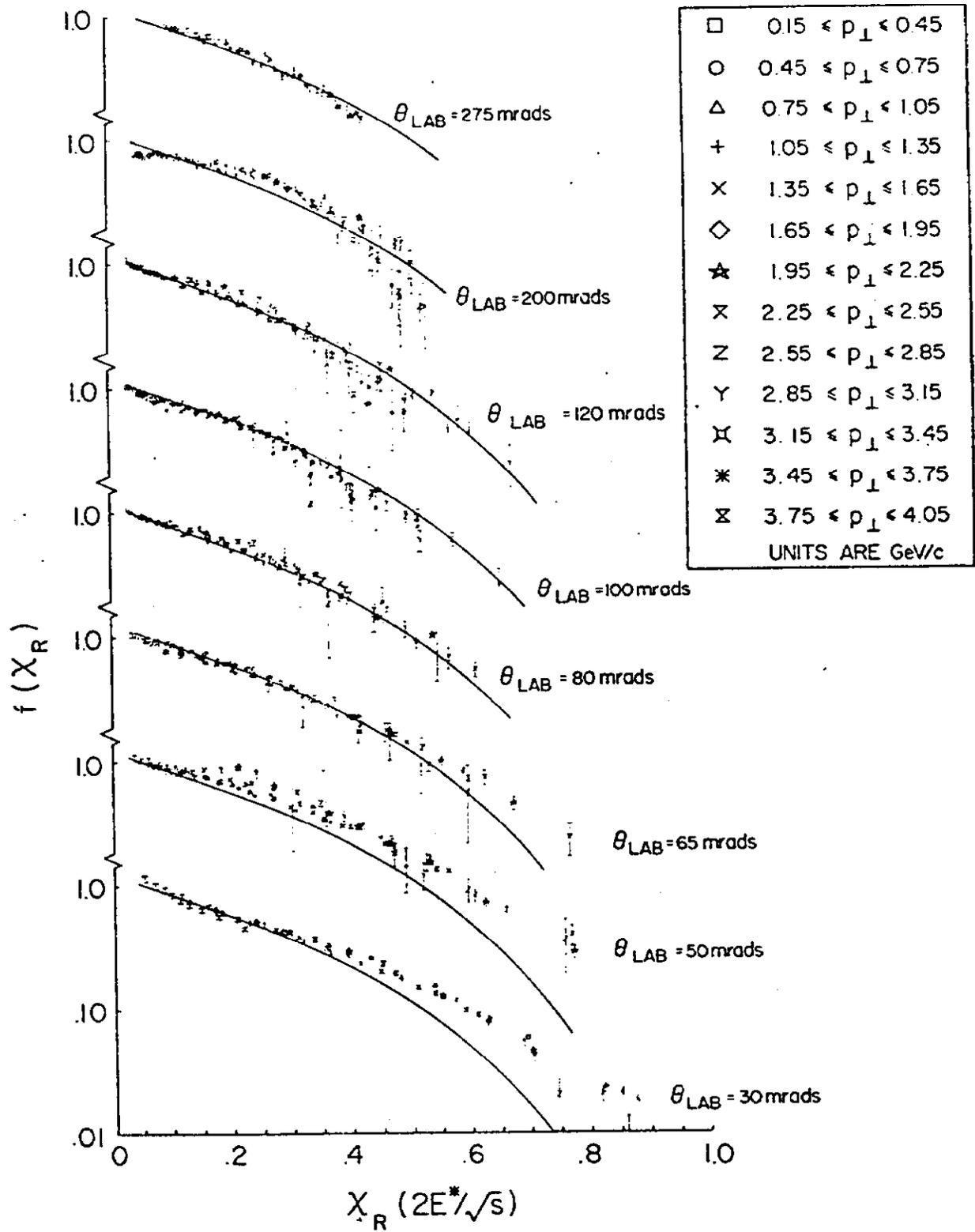


Fig. 4. The radial function $f(x_R)$, defined in the text, is plotted for various laboratory angles.

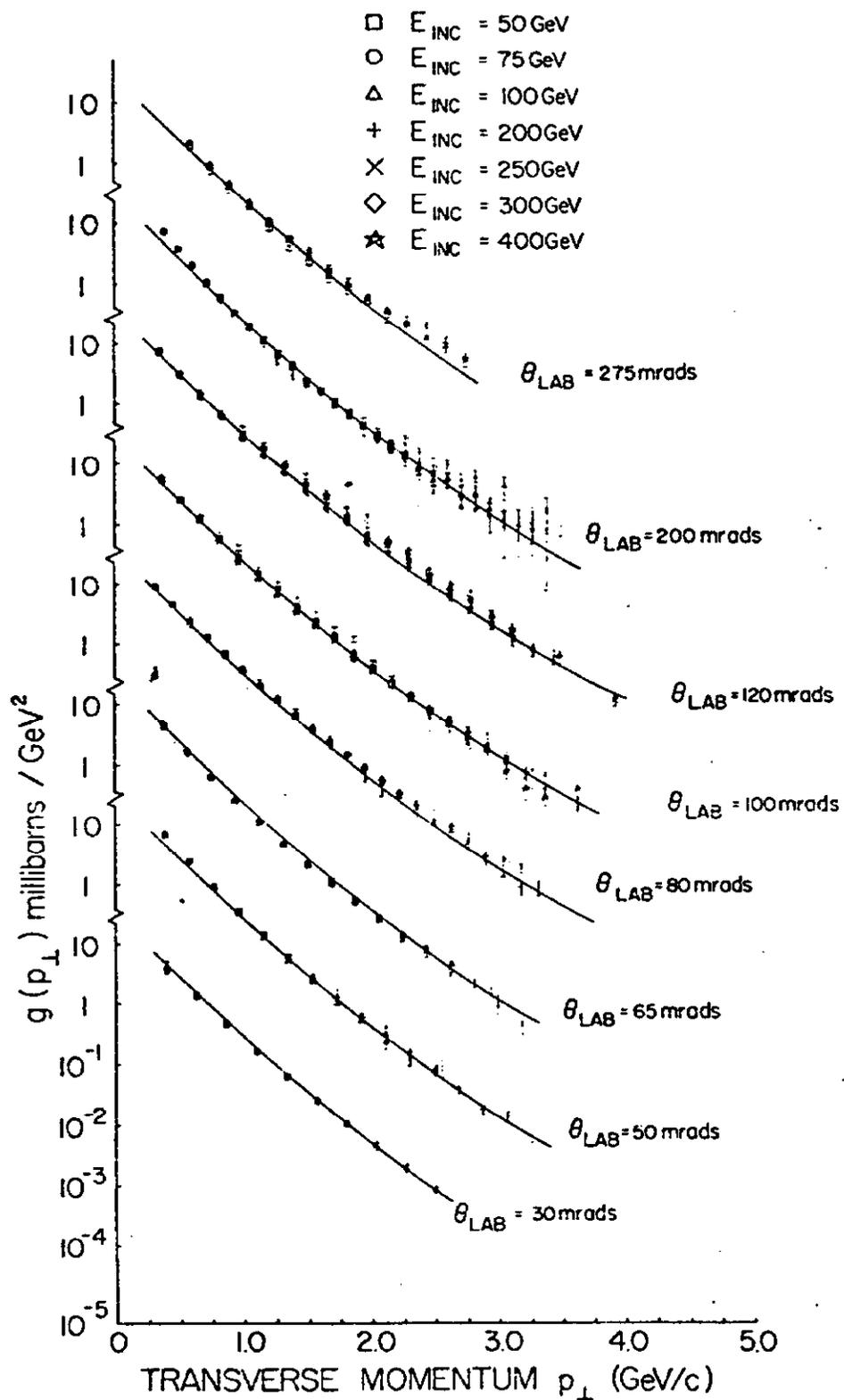


Fig. 5. The perpendicular momentum function, $g(p_{\perp})$, defined in the text, is plotted for various laboratory angles. In both 4 and 5 a common curve is drawn through the data at each angle.