Fermi National Accelerator Laboratory

FERMILAB-Lecture-75/02-THY
September 1975

# Gauge Fields and Strong Interactions*

THOMAS APPELQUIST†
Fermi National Accelerator Laboratory,** Batavia, Illinois 60510
and
Department of Physics, Yale University‡
New Haven, Connecticut 06520

---

*Lectures given at the 1975 Cargèse Summer Institute on Weak and Electromagnetic Interactions at High Energies. Institut D'Études Scientifiques de Cargèse, Cargèse, France. June 30-July 21, 1975.

†Alfred P. Sloan Foundation Fellow.

‡ Permanent Address after November 1, 1975.

** Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration

## ABSTRACT

These three lectures deal with several aspects of the Yang-Mills quark gluon theory of strong interactions. An overview is followed by a plunge into the physics of electron positron annihilation into hadrons. Looming large in this discussion are the newly discovered, long-lived mesons. The role of these new heavy hadrons in the quark gluon model is examined. It is suggested that they are heavy quark antiquark bound states and that their properties could provide a rather clear and simple experimental handle on the underlying field theory.


## NOTE

At the time the lectures were being given and written up (July through September 1975), the experimental situation with respect to the new particles was developing and changing rapidly. At the risk of giving the published version of the lectures a somewhat acausal character, I have incorporated a discussion of some of these developments.

## LECTURE 1

### 1. INTRODUCTION

There now exists an attractive and viable candidate (really a class of candidates) for a local quantum field theory of the strong interactions. The elementary fields in the strong interaction Lagrangian are quarks and vector gluons. Very little is known about most features of this theory. For example, it is not at all clear how to calculate the ground state properties of hadrons or even to ennumerate the spectrum of physical states. Nevertheless, its short distance behavior is well understood and its long range structure is very tantalizing. This structure suggests the possibility that the theory contains long range forces that might permanently confine quarks and gluons to the interior of physical hadrons.

The discovery[1,2] of heavy, long lived $J^P = 1^-$ hadrons within the last year is an important development for the quark gluon theory of strong interactions. It means that new, heavy quarks must be included in the model.[3] Furthermore, the new particles should provide us with an important new experimental handle on the dynamics of the model. By this I mean that their features should reflect the properties of the underlying field theory more directly than the other hadrons. This is due to the large mass of the new quarks and their subsequent nonrelativistic bound state motion.

These three lectures are intended to be a survey of the Yang-Mills quark gluon theory of strong interactions with special emphasis on the role of the new hadrons. I will occasionally digress into related matters such as weak and electromagnetic interactions but the main thrust will be strong interaction dynamics. In the first lecture, I will describe the model and discuss renormalization, the renormalization group and asymptotic freedom. Lecture II will begin with a discussion of quark mass renormalization. The remainder of this lecture will be devoted largely to electron positron annihilation into hadrons. The computation of the total cross section behavior in terms of the underlying quark gluon field theory is discussed and sources of possible perturbation theory breakdown identified. In Lecture III, particular attention is paid to the breakdown in the vicinity of a heavy quark-antiquark threshold. The possible role of asymptotic freedom in explaining the narrowness of the new hadrons is examined. I will review the status of the heavy quark-antiquark bound state as a nonrelativistic system and look at the spin dependent forces and the role of the Bethe-Salpeter equation.

## 2. A DESCRIPTION OF THE MODEL

I will assume that the strong interactions are described by a local, renormalizable quantum field theory of quarks and gluons. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{\psi} (i\not{D} - m_0) \psi \qquad (1)$$

where $\psi$ is a set of quark fields coming in several flavors, u, d, s along with one or more heavy quarks. Each flavor comes in three colors[4,5] and color is taken to be an exact SU(3) gauge symmetry. Thus each quark color multiplet has a single mass and the colored vector mesons remain massless. $F^a_{\mu\nu}$ is the gauge covariant curl and $D_\mu$ is the covariant derivative

$$(D_\mu \psi)_n = \partial_\mu \psi_n - \frac{1}{2} g A^a_\mu (\lambda^a)_{nm} \psi_m \qquad (2)$$

where $\psi_m$ is one of the color triplets. The symmetries of the theory are determined by the bare mass matrix $m_0$. Each flavor has its own electrical charge and the colored gluons are electrically neutral. I will always assume the existence of at least one heavy quark c. There may be others, but in the discussion of $e^+e^-$ annihilation, I will espouse the view that only the c quark is operative at present energies.

Many of the properties of the colored quark model have been discussed by Fritzsch, Gell-Mann and Leutwyler.[5] The emphasis in these lectures will be on the short distance structure of the model and I will devote most of the remainder of this lecture to introducing the essential ideas. Reference 5 is a good introductory overview of the model for those of you unacquainted with it.

## 3. RENORMALIZATION

The quantization of Yang-Mills theories has been discussed by many people. A recent lucid treatment is that of Lee and Zinn-Justin[6] who develop the Feynman rules and discuss regularization and Ward-Slavnov identities. The Feynman rules are shown here in Fig. 1. Higher order computations (one or more closed loops) are best done using the dimensional continuation scheme[6] as a regulator. I will occasionally be looking at higher order diagrams but only for the purpose of examining general features. It will not be necessary to do explicit computations requiring the explicit use of dimensional continuation.

By power counting, the theory is renormalizable so that a finite number of counterterms is sufficient to define ultraviolet finite Green's functions to all orders in perturbation theory. We thus have a perfectly satisfactory theory of Green's functions, all of which are calculable and finite as long as the external momenta are kept away from mass shell. In that limit, which unfortunately is the relevant limit for exploring particle structure and constructing the S-matrix, the theory is plagued by infrared divergences. It is easy to see, for example, that the quark gluon vertex diagram of Fig. 2a behaves like $\log(p^2 - m^2)$ in the limit $p^2 = p'^2 \to m^2$. Similarly, the gluon self-energy graph of Fig. 2b behaves like $(q_{\mu\nu}q^2 - q_\mu q_\nu)\log q^2 + q_\mu q_\nu$ terms in the limit $q^2 \to 0$. On the $n^{th}$ loop level, there will be $n$ log factors and unlike

QED, the true infrared structure of the theory is unknown. In the next lecture, I will take a quick look at what little is known about the infrared structure and its speculated connection to quark confinement and the physical particle spectrum.

Because of the infrared divergences, the renormalized Green's functions must be defined by subtracting away from mass shell. A convenient way of defining the renormalized coupling constant $\bar{g}$ (M) is in terms of the gluon propagator and the 1PI three gluon vertex at a symmetric Euclidean point. This is shown in Fig. 3. Each of the external line factors symbolizes the object $\sqrt{d(k^2)}$ where $d(k^2)$ appears in the transverse part of the complete gluon propagator $-i(g_{\mu\nu} - k_\mu k_\nu/k^2)\frac{1}{k^2}\, d(k^2) - i\alpha\; k_\mu k_\nu/k^4$. The Ward-Slavnov identities[5] assure us that $\bar{g}(M)$ can equivalenty be defined using say the quark-quark-gluon vertex.

I have so far considered only wave function and coupling constant renormalization. Mass renormalization and the question of how to define the renormalized quark masses is equally important but it is best to return to that after a discussion of the renormalization group and asymptotic freedom.

## 4. THE RENORMALIZATION GROUP AND ASYMPTOTIC FREEDOM

The ideas of the renormalization group[7] and the property of asymptotic freedom[8] underlie much of what I will say about $e^+e^-$

annihilation and the physics of $c\bar{c}$ bound states. The renormalization group is really the subject of Dr. Crewther's lectures and all I intend to do is to outline the notions that will be essential in lectures 2 and 3.

A renormalization mass M must be introduced to define the theory perturbatively. On the other hand, the physical content of the theory cannot depend on the choice of M and it is both possible and convenient to move it about. The existence of the renormalization group is the statement that a shift in M can be reabsorbed completely into multiplicative rescalings of field strengths and the coupling strength $\bar{g}(M)$. This feature of any renormalizable theory is most usefully expressed in terms of the partial differential equations of the renormalization group[7,9].

These provide a framework for discussing both infrared and ultraviolet asymptotic behavior of the theory. This is in general a nontrivial problem precisely <u>because</u> of the essential presence of M. Even when the quark masses can be neglected the naive use of scale invariance is impossible. Perturbation theory is plagued by arbitrarily high powers of logarithms involving M which can sum up to modify the naive dimensional predictions. The differential equations of the renormalization group provide a framework for discussing these modifications, which depend on the properties of a given field theory. One such modification is asymptotic freedom.

A field theory is asymptotically free if $\bar{g}(M) \to 0$ as $M \to \infty$.

The deciding one loop calculation reveals that the color SU(3) gauge theory is asymptotically free providing that $N \leq 16$. Recall that N is the number of quark flavors. As $M \rightarrow \infty$, $\bar{g}^2(M) \sim 1/\log M$, indicating that the short distance structure of the theory can be calculated perturbatively in terms of a small coupling constant. Since this is only an asymptotic theory, the question of when the small coupling regime is reached can only be answered experimentally. The experimental applications of asymptotic freedom are very limited since very few experimental quantities depend <u>only</u> on the short distance structure of the theory. The most direct experimental application is inelastic lepton scattering. The momentum transfer dependence of the structure function moments can be shown, by the use of the Wilson operator product expansion,[10] to depend only on short distance structure. Approximate Bjorken scaling of the structure functions can then be explained by asymptotic freedom providing that for $M > 1$ or 2 GeV, $\alpha_s(M) \equiv \bar{g}^2(M)/4\pi << 1$. This establishes the scale for the onset of the weak coupling region.

Some of the dynamical considerations in the next two lectures will involve some unconventional uses of asymptotic freedom. For this reason, it is important to understand in some detail the limitations on its use. Why is it difficult to experimentally isolate short distance behavior and thereby allow the use of perturbation theory? The coupling constant $\bar{g}(M)$ has been defined in the deep Euclidean region ($M > 1$-2 GeV) so that it is small. The Green's functions of the theory can be calculated as a perturbation expansion in $\bar{g}$ (M) providing that this small coupling constant really is the appropriate expansion parameter. This will not be the case if large dynamical factors accompany each power of $\bar{g}$ (M). This can happen only if the object being calculated is sensitive to dimensional factors (such as a momentum or a mass) which are much smaller than M. The dynamical factors are typically powers of $\log(p/M)$ where p is the small momentum and if they accompany each power a $\bar{g}(M)$, then the true expansion parameter is not $\bar{g}(M)$ but instead of $\bar{g}$ appropriate to the smaller momentum scale. If this scale is less than a few hundred MeV or, say one GeV to be conservative, $\bar{g}$ must become strong and the use of perturbation theory is impossible.

This then is the restriction on the use of asymptotic freedom. With M taken greater than one or two GeV, $\bar{g}$ (M) is small but perturbation theory can be used only if small ($< 1$ GeV) dimensional parameters do not crucially enter the calculation. This is the case for example with deep Euclidean Green's functions and the leptoproduction structure function moments. In the later, the Wilson expansions must be used to disentangle the large dimensional parameters (momentum transfer and M) from the small ones (mass and binding energy of the target nucleon). The disentangling or elimination of small dimensional parameters will be of great concern in the next two lectures.

## 5. THE CHOICE OF FLAVORS

The fourth quark flavor was given a raison d'etre long ago in the classic paper of Glashow, Iliopoulos and Maiani.[11] This is the celebrated GIM mechanism for the suppression of $\Delta S = 1$ weak neutral currents.[12] The fourth quark whose role in the strong interactions I will be discussing, could well be the GIM quark. That is, it could enter the weak currents as prescribed by GIM and have an electrical charge of 2/3. It is to be emphasized, however, that this is not necessary. No commitment to a particular theory of weak interactions need be made for the discussion of the color gauge theory of strong interactions.

There are by now many reasons for considering the possibility of even more than four quark flavors. In my opinion, none of these are yet as compelling as the theoretical (GIM) and experimental (new particles) evidence in favor of the fourth quark. Nevertheless, when one begins to contemplate the behavior of $\sigma_{TOT}(e^+e^- \to \text{hadrons})$, the strange and wonderful things being discovered in the high energy neutrino experiments and the arcane problems of triangle anomalies in weak and electromagnetic interactions, a natural if not profound question emerges: Why not? Indeed, a variety of models incorporating additional heavy quarks and/or heavy leptons have already been constructed. A lucid review of this work has recently been given by R. M. Barnett.[13] For the remainder of these lectures, I will assume that only the fourth quark c is playing a role in the current $e^+e^-$ experiments. As we shall see, $\sigma_{TOT}$ is too large at high energies to be explained by only four colored quarks. A possible explanation for the increment is the production of heavy leptons. This mechanism might also be the origin of the $\mu^+e^-$ events, recently discovered at SPEAR.[14]

## LECTURE II

### 6. MASS RENORMALIZATION

The strong interaction Lagrangian (1) has a bare quark mass matrix $m_0$. From the point of view of strong interaction phenomenology, it is reasonable to assume that it is a God given parameter in the strong interaction Lagrangian. The question of where it comes from is very interesting but very likely a deeper problem than strong interaction physics. Its origin is surely connected with the ultimate unification of the fundamental interactions and the breakdown of weak and electromagnetic symmetries.[15] Our strong interaction Lagrangian is an effective Lagrangian. It can be used in isolation up to energies $\Lambda$ above which the weak and electromagnetic interactions become comparable to the strong.[16] If the effective weak and electromagnetic Lagrangian is renormalizable (a Higgs-gauge theory for example), $\Lambda$ will be well above attainable laboratory energies.[17] I will assume this to be the case.

It is useful to think of $m_0$ as a renormalized mass matrix defined at the Euclidean momentum scale $\Lambda$. The effective mass matrix at laboratory energies will be related to $m_0(\Lambda)$ by renormalization effects due mainly to strong interactions since the weak and electromagnetic interactions stay small below $\Lambda$. The question of how to define these laboratory renormalized masses is a matter requiring some thought, especially since the quarks may be permanently confined inside color singlet hadrons. One can imagine defining a mass matrix $m(M)$ at a sliding Euclidean scale $M$,[18] which approaches $m_0$ as $M \to \Lambda$, which stays nearly equal to $m_0$ through the weak coupling region down to one or two GeV, and which finally becomes some appropriate constituent quark model mass below one GeV. In the weak coupling regime, $m_0(M)$ may be experimentally accessible[19] but its connection to the constituent masses is obscured by strong coupling effects.

In the case of the charmed quark, it is possible to define a useful constituent mass in a precise field theoretic way. As we shall see, this relies on the fact that this mass sits in the weak coupling region and that binding energies are small. A convenient, but by no means unique, scheme is the following. A renormalized mass matrix m is obtained by the common rescaling $m_i = Zm_{0i}$. Then Z is adjusted so that for the c quark, $m_c$ is the threshold of a cut in the propagator to any finite order of perturbation theory. Wave function renormalization must be done off shell as discussed in Lecture 1. Then as $p^2 \to m_c^2$, the c quarks propagator will behave like

$$\frac{1}{p^2 - m_c^2} \left[\log\left(p^2 - m_c^2\right)\right]^n$$

where n is the order of perturbation theory. This completely imitates quantum electrodynamics but, unlike that theory, the threshold behavior to all orders is unknown due to infrared instability. Nevertheless, we shall see that $m_c$, defined in this way, is experimentally accessible in a way that light quark masses are not. The experimental significance of the light quark masses as defined here is obscure but that isn't important for the heavy quark computations I will describe.

A crucial ingredient in some of our applications of asymptotic freedom is the rapid onset of the weak coupling regime. In particular, perturbation theory becomes possible well before the heavy c quark can be excited. For Euclidean momenta on the order of $m_c$, perturbation theory can be used although $m_c$ will play a prominent role and the result will be far from scale invariant. This takes some getting used to since the familiar short distance applications of the renormalization group are to an energy region well above all the mass parameters. The onset of the weak coupling region at about 1 GeV is determined by the light quark and gluon sector of the theory.

## 7. $\sigma_{TOT}(e^+e^- \to$ HADRONS) AND ASYMPTOTIC FREEDOM

The most direct application of asymptotic freedom to $e^+e^-$ annihilation is a dispersive constraint on the total cross section. The hadronic vacuum polarization tensor $\Pi_{\mu\nu}(q) = (g_{\mu\nu}q^2 - q_\mu q_\nu) \times \Pi(q^2)$ can be calculated[20] perturbatively for space like $q^2$ less than $-1$ GeV$^2$. Since this is a deep Euclidean Green's function, the light quark masses can be scaled to zero and $\Pi(q^2)$ becomes a function of dimensionless ratios involving $q^2$, $m_c$ and the renormalization mass M:

$\Pi[q^2/M^2, m_c^2/M^2, \bar{g}(M)]$. The absence of sensitivity to the light quark masses (the small dimensional parameters) is insured by the mass singularity analysis of Kinoshita.[21] In effect, the external momentum provides an infrared shield insuring the existence of the limit $m_q \to 0$. $\Pi(q^2)$ is related to the total cross section by the dispersion relation

$$\Pi(q^2) = \frac{q^2}{\pi} \int_0^\infty \frac{ds \; R(s)}{s(s - q^2)} \tag{3}$$

where

$$R(s) \equiv \frac{3s}{4\pi\alpha^2} \sigma_{TOT}(e^+e^- \to \text{hadrons}) \tag{4}$$

and $s = E_{CM}^2$ .

This can be used to put a bound on the total cross section behavior.[22] This is an important and useful fact but I want to go on to a more speculative use of asymptotic freedom.

$R(s)$ appears to be flat from $E_{CM} \approx 1$ GeV to $E_{CM} \approx 3$ GeV and may again be flattening out above 5 to 6 GeV.[23] In the lower region, $R(s) \approx 2$ so that it is given quite nicely by the parton model with the nine light quarks. The parton model calculation is zeroth order perturbation theory directly in the time like region as indicated in Fig. 4. Since quarks and gluons have not been seen and may never be seen as physical particles, can this be justified? In the language of the parton model, one simply says that once the probability for the creation of the $q\bar{q}$ pair has been correctly computed, the final state interactions that produce the physical hadrons occur with probability one. They do not affect the total cross section. In the remainder of this section and the next, I will use the property of asymptotic freedom to give some quantitative justification to this direct use of perturbation theory in the time like region. There are sources of perturbation theory breakdown however, and I will discuss them in Sec. 9.

There are three energy regions to consider.

I. Low energies. Light quark masses negligible but the heavy c quark cannot be excited.

II. High energies. All quark masses negligible.

III. A transitional region.

I will first consider the use of asymptotic freedom in regions I and II and consider how the perturbation expansion could break down. Then I will look at the transitional region III. Here the perturbation expansion surely breaks down and we shall look in some detail at the way this happens.

To examine regions I or II in low orders of perturbation theory, we can set the masses of the operative quarks equal to zero. The zeroth order (parton) contribution is shown in Fig. 4 and we have $R^{(0)} = \sum_i Q_i^2$ where $Q_i$ is the charge of the $i^{th}$ quark. This gives $R^{(0)} = 2$ below charm threshold and $R^{(0)} = 10/3$ in the 4 quark GIM model. To see whether this result is reliable, we examine the higher orders. The second order contribution is shown in Fig. 5. An explicit calculation for zero quark mass gives R(s) through second order.

$$R^{(2)}(s) = \Sigma\ Q_i^2 \left\{ 1 + \frac{4}{3}\ \frac{3\ \alpha_s(M)}{4\pi} \right\} \tag{5}$$

This result is taken directly from electrodynamics.[24,25] The only modification is the factor of 4/3 in second order which comes from an SU(3) color sum. Since $\alpha_s(M) << 1$ for $M > 1$ GeV, the second order term is a small correction.

## 8. MASS SINGULARITIES

It is important to understand the simplicity of the second order result before going on. In doing so, we shall encounter the essential property of the theory which underlies the use of perturbation theory at timelike as well as spacelike momenta. R(s) is dimensionless and could therefore be a function of dimensionless ratios of $E_{CM}$, quark masses $m_q$ and the renormalization mass M. There is no M dependence through second order since no renormalization subtractions need be done. The overall divergence is absent since an imaginary part is being taken and the subgraph divergences in graphs (5a) and (5b) cancel through the electromagnetic Ward identity. Explicit M dependence will enter in the next order.

The absence of explicit quark mass dependence (the existence of the limit $m_q \to 0$) is insured by the same mass singularity theorem of Kinoshita[21, 26] that underlies perturbation theory in the Euclidean region. Thus no small dimensional parameters enter the calculation to this order and R(s) remains constant. Perturbation theory seems to be converging.

The application of the Kinoshita theorem to $\sigma_{TOT}(e^+e^- \to \text{hadrons})$ is particularly straightforward. The idea is that any graph contributing to $\Pi(q)$ is finite when all internal masses are taken to zero since the external momentum provides an infrared cutoff. This is true for timelike as well as spacelike $q^2$ and in the time-like case, it applies to the absorptive as well as dispersive parts. The sum of the contributions to $\sigma_{TOT}(e^+e^- \to \text{hadrons})$ corresponding to the different Cutkosky cuts of a single Feynman diagram will be free of mass singularities. This is a generalization of the Block-Nordseick analysis in quantum electrodynamics to a situation with self-coupled massless fields. Individual contributions will contain mass singularities leading typically to terms like $\log E_{CM}/m_q$ or logarithmic singularities due to the masslessness of the gluons. However, they will cancel in the total cross section.

Let me make a list of a few general remarks about the mass singulary theorem before going on.

1. It applies only to renormalizable theories. It has been proven[21] for all except Yang-Mills theories and I have checked in low orders that the theorem is true. I am assuming here that it is true to all orders,[27] as in other renormalizable theories.

2. It is applicable to any properly defined total transition probability. In general, however, there are mass singularities associated with the incoming lines and these must be treated carefully.[21] For $e^+e^-$ annihilation the initial line is a single off shell photon and there are no mass singularities.

3. It says nothing about partial transition probabilities. In the case of $e^+e^-$ annihilation into hadrons, it can be applied only to the total

cross section. In particular, it completely leaves open the question of quark confinement. To answer this question, one would have to look at more than $\sigma_{TOT}$. A constraint would have to be put on the final state to force the macroscopic separation of the $q\bar{q}$ pair. A confining theory would presumably lead to zero in this case. However, such an investigation would inevitably lead to the presence of small dimensional parameters such as the inverse $q\bar{q}$ separation length or the energy resolution of the quark detector or perhaps mass shell singularities of the type discussed in Sec. 6. Perturbation theory is not likely to be useful.

In fourth order and beyond, renormalization subtractions become necessary and R(s) picks up s dependence in the form of powers of $\log s/M^2$. They remain inocuous until s becomes much larger than M. It is then sensible to express the perturbation expansion in terms of a $\bar{g}(s)$ appropriate to the larger energy scale. As $s \to \infty$, $\bar{g}^2(s) \sim 1/\log s$ and the parton model result is approached from above. I will imagine keeping M fixed at around 3 GeV. Then for a sizable energy range, $\log s/M^2$ is of order one and it isn't necessary to introduce a sliding component constant $\bar{g}(s)$. The perturbation expansion will converge as long as there are no mass singularities and this is insured to this order and beyond by the Kinoshita analysis. It is important to be able to scale all the light quark masses to zero. It is also possible to scale $m_c$ to zero in region III although this isn't always appropriate and certainly not necessary since $m_c > 1$ GeV.

The mass singularity theorem has apparently justified the use of perturbation theory for $\sigma_{TOT}(e^+e^- \to \text{hadrons})$. The parton model is the dominant, zeroth order contribution and higher order corrections can be calculated. The second order result (5) is in fact a good fit to the total cross section from one to three GeV since with $\alpha_s(M) \approx 0.25$,[28] $R^{(2)}(s) \approx 2.2$. Above the transitional region ($E_{CM} > 5.5$ GeV), R(s) is again flat and equal to about 5.[23] This clearly indicates there is something going on in addition to, or in place of, the excitation of 12 GIM quarks. An attractive possibility from several points of view is the production of heavy leptons.[14, 29] As far as the quark sector is concerned however, it seems both experimentally and theoretically that perturbation theory could be a reliable tool for the total cross section.

## 9. SOURCES OF BREAKDOWN

The theoretical situation isn't nearly that good. There is a rather obvious source of perturbation theory breakdown that appears in higher orders. In the Euclidean region, the Kinoshita analysis assures us that the internal masses any diagram can be scaled to zero. Since the light quark masses are assumed to be small ( $<< 1$ GeV), this is an appropriate thing to do for momenta well above 1 GeV. In the physical region, however, the perturbation expansion contains branch points corresponding to nominal multiple quark thresholds. If quarks are confined, these thresholds disappear in favor of physical particle thresholds when the theory is saloed to all orders. Nevertheless they are there in finite orders and it is only appropriate to scale $m_q \to 0$ in a given diagram if $E_{CM}$ is well above <u>all</u> the nominal quark thresholds of that diagram.

For any value of $E_{CM}$, diagrams exist (perhaps in very high order) which contain thresholds at or above $E_{CM}$. This is the source of a new small dimensional parameter--the distance to nearby nominal quark thresholds. This will no doubt cause perturbation theory breakdown in high orders and it is probably connected to the formation of the physical particles. In Ref. 3, some plausibility arguments are given that the high order effects average out to be a small contribution to R(s) but this has not been proven. In the next lecture, I will assume that because of the convergence of the perturbation expansion in low orders, R(s) can indeed be computed perturbatively in regions I and II.

There is a way to avoid the multiple threshold singularities and yet to improve upon the dispersive constraint (4). $\Pi(q^2)$ can be calculated in the complex $q^2$ plane with Re $q^2 > 0$ provided that Im $q^2$ is taken non-zero and large enough to shield from the singularities of the physical region. With Im $q^2 > 1$ GeV, there will be no sensitivity to small (< 1 GeV) dimensional parameters and perturbation theory can be applied. The dispersion relation for $(q^2)$ then leads to a prediction for R(s) averaged over a region of order 1 GeV.[30] It must be given by the parton model computation.

It remains a problem to understand why the parton model works on a much more local levels. The experiments average over energy intervals on the order of 1 MeV and the parton model works well from 1 to 3 GeV. The use of perturbation theory locally for R(s) is not fully justified but I will assume that it is possible unless an obvious breakdown appears in low orders.

# LECTURE III

## 10. $c\bar{c}$ THRESHOLD BEHAVIOR

Recall from the last lecture that away from the transitional region, perturbation theory converges through low orders. This consequence of the Kinoshita analysis was used to give some justification to the use of perturbation for $\sigma_{TOT}$. In the transitional region, on the other hand, the breakdown of perturbation theory is immediate, coming already at second order.

The second order computation of R(s), (Eq. 5) is easily generalized to include the heavy quark mass. The result is[24, 25]

$$R^{(2)}(s) = \sum_{\substack{\text{light} \\ \text{quarks}}} Q_i^2 \left\{ 1 + \frac{4}{3} \frac{3\alpha_s(M)}{4\pi} \right\}$$
$$+ \sum_{\substack{\text{heavy} \\ \text{quarks}}} Q_i^2 \, \theta(s - 4m_c^2) \, v \, \frac{3 - v^2}{2} \left\{ 1 + \frac{4}{3} \alpha_s(M) f(v) \right\} , \tag{6}$$

where v is the velocity of the heavy quark or antiquark, $v = \sqrt{1 - 4m_c^2/s}$. The overall heavy quark factor $v \frac{3-v^2}{2}$ is just s-wave phase space and it approaches 1 as $m_c \to 0$. The function f(v) is complicated, involving combinations of Spence functions, but it is well represented (to ± 1 percent) by an interpolating formula due to Schwinger:[25]

$$f(v) = \frac{\pi}{2v} - \frac{3+v}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right) \tag{7}$$

This formula is exact in the limits $v \to 0$ and $v \to 1$ and as $v \to 1$, $f(v) \to \frac{3}{4\pi}$ in agreement with the zero mass calculation.

As $v \to 0$, $f(v) \to \frac{\pi}{2v}$. This behavior comes from graph (5a) and is a consequence of a Coulomb-like final state interaction. In $n^{th}$ order, n uncrossed gluon exchanges give n factors of $\frac{1}{v}$ and the perturbation expansion breaks down. In the limit $v \to 0$, these leading terms are just the expansion of the nonrelativistic Coulomb enhancement factor[25]

$$\frac{\frac{4}{3}\alpha_s \frac{\pi}{v}}{1-\exp\left(-\frac{4}{3}\alpha_s \frac{\pi}{v}\right)} = \frac{|\psi_{COUL}(0)|^2}{|\psi_{COUL}(\infty)|^2}, \tag{8}$$

representing the increased probability for $c\bar{c}$ production in an attractive Coulomb field. This breakdown of the perturbation expansion for small v is connected to a breakdown below $E_{CM} = 2\,m_c$. There, the sum of uncrossed gluon exchanges is responsible for the formation of positronium-like bound states. A typical contribution below threshold is shown in Fig. 6.

If this were the only breakdown of perturbation theory, the uncrossed ladders could be summed as in QED and one could perturb about that result. There is, however, a more serious perturbation theory breakdown due to the Yang-Mills infrared structure. As threshold is approached from above or below, the typical momentum transfer flowing through exchanged gluons decreases. This means that the higher order corrections to these lines become more and more important, increasing the effective coupling strength. When the typical exchanged momentum becomes less than about one GeV, the theory becomes strongly coupled and the perturbation expansion breaks down.

I can estimate the width of the transitional region inside of which the Yang-Mills infrared structure leads to a breakdown of the perturbation expansion. For $E_{CM}$ above $2\ m_c$, a measure of the typical momentum transfer $\langle k \rangle$ is $\langle k^2 \rangle \approx \frac{1}{4}(E_{CM}^2 - 4\ m_c^2)$. If $\langle k \rangle$ is to be less than one GeV, then $E_{CM}^2 < 4\ (m_c^2 + 1\ \mathrm{GeV}^2)$. Below $E_{CM} = 2\ m_c$, the typical momentum transfer in a graph like the one in Fig. 6 is $\langle k \rangle \approx \sqrt{m_c^2 - E_{CM}^2/4}$. In a Coulombic theory, this is just the Bohr momentum at each bound state. Thus if we take $E_{CM} = 2\ m_c - \frac{1}{4}(\frac{3}{3}\ \alpha_s)^2\ m_c \frac{1}{n^2}$, then $\langle k \rangle = \frac{1}{2}(\frac{4}{3}\alpha_s) m_c \frac{1}{n}$. The entire range in $E_{CM}^2$ inside of which the typical momentum transfer is less than one GeV is thus

$$\Delta(E_{CM}^2) \approx 2E_{CM}\Delta E_{CM} \approx 8\ \mathrm{GeV}^2 . \tag{9}$$

With $E_{CM} \approx 3$-$4$ GeV, $\Delta E_{CM} \gtrsim 1$ GeV. In the absence of some careful higher order calculations, this can be taken only as an order of magnitude estimate. The experimental curve for R(s)[23] suggests the transitional region to be perhaps a factor of two larger than this. It extends from $E_{CM} \approx 3$ GeV to $E_{CM} \approx 5$ GeV.

Beyond $E_{CM} \approx 5$ GeV, perturbation theory should again be applicable.

R(s) will be given, through second order, by Eq. 6 with $\alpha_s$ replaced by the running coupling constant $\bar{\alpha}(s)$:

$$\bar{\alpha}(s) = \frac{\alpha_s}{1 + \frac{25}{12\pi}\,\alpha_s \log s/M^2} \quad . \tag{10}$$

If the GIM model were correct, the approach to 10/3 would be quite rapid. At $E_{CM} = 6$ GeV for example, we find $R(s) = 3.5 \pm 0.2$ where the estimated error comes from the uncertainty in the value of $\alpha_s$ and from the uncalculated higher order terms. Beyond $E_{CM} = 6$ GeV, the rate of decrease is very slow and in fact R(s) should remain nearly constant through $E_{CM} \approx 9$ GeV. This results from an interplay between the slowly rising two particle phase space factor and the slowly decreasing $\bar{\alpha}(s)$. The experiments clearly indicate that the GIM model is not the entire description of $e^+e^-$ physics above $E_{CM} = 3$ GeV.

## 11. $c\bar{c}$ BOUND STATES

If one or more of the $c\bar{c}$ bound states lies outside the strong coupling transitional region, then it will be essentially Coulombic. A simple way to see if this is the case for the J(3.1) is to estimate the Bohr momentum $k_B = \frac{1}{2}(\frac{4}{3}\alpha_s)m_c$. For $\alpha_s \leq 0.25$ and $m_c \leq 2$ GeV, $k_B \leq 300$ MeV and with momentum transfers this small, the effective coupling strength associated with the binding will be large. Thus the J(3.1) cannot be Coulombic. With distances as large as about one Fermi being important, the long-range confining forces are already playing a role. Computations in terms of the underlying field theory must be replaced by more phenomenological considerations.

It is important to point out that a heavy quark-antiquark system becomes more Coulombic as the quark mass increases. The Bohr momentum is determined by the equation

$$k_B = \frac{1}{2}\left(\frac{4}{3}\alpha_s(k_B)\right)m_Q \tag{11}$$

and if $m_Q$ is large enough so that $k_B > 1$ GeV, $\alpha_s(k_B)$ will be small ($\lesssim 0.25$) and the $Q\bar{Q}$ ground state will be Coulombic. For $m_Q \gtrsim$ 6-7 GeV, this is the case and all the ground state properties (in particular the hyperfine splitting) are computable. What better reason could there be for building the next generation $e^+e^-$ machines than to look for a truely Coulombic hadron?

A great deal of phenomenological work has been done on the $c\bar{c}$ system.[31,32,33] The essential ingredient of this work is non-relativistic motion. Despite the fact that the $c\bar{c}$ bound state is non-Coulombic, there is good evidence that it remains non-relativistic and loosely bound. The first generation of this work[32,33] used a spin independent Schroedinger equation formalism with an effective long range $c\bar{c}$ potential. A popular choice for this effective potential includes a Coulombic short distance piece, a linearly rising long range piece and a constant $V_0$ to represent intermediate range and spin-dependent forces.

$$V(r) = V_0 + ar - \frac{4/3\, \alpha_s}{r} \quad . \tag{12}$$

The linear long range growth has been suggested by several theoretical considerations[34] but it has in no way been derived from the underlying Yang-Mills field theory.

Two important things have emerged from this work. Firstly there is the consistency check that the system is indeed nonrelativistic. From fits to the leptonic width of the J(3.1) (the wave function at the origin) and the J(3.1) - $\psi$(3.7) splitting, all groups agree that $a/m_c^2 << 1$. This qualitative feature is almost certainly independent of the specific form of the confining potential.

Secondly, there are quantitative predictions. In addition to the J(3.1) and $\psi$(3.7), other levels are predicted:

1. Additional radial recurrances, the recurrance at 4.2 GeV being the first. The 4.2 is already quite broad, sitting well above the threshold for the production of a pair of "charmed" mesons, $(c\bar{q})$ and $(\bar{c}q)$.

2. Pseudoscalar partners of the $J^{PC} = 1^{--}$ vector mesons, the $\eta_c$, $\eta_c'$, ----.[12] They can be reached by magnetic dipole transition from the vector states.

3. Four intermediate P-wave states centered at around 3.5 GeV. Their quantum numbers are $J^{PC} = 0^{++}, 1^{++}, 2^{++}$ and $1^{+-}$ with the even C states reachable by electric dipole transitions from the $\psi(3.7)$.

4. D-wave states in the neighborhood of 4.1 - 4.2 GeV. These can mix with the $\psi(4.2)$ and this might offer an explanation for the structure emerging in this region.[23] Fine structure involves the spin dependent forces and is more difficult to predict. Of particular interest are the $J(3.1) - \eta_c$ hyperfine splitting and the splittings among the P-states. I will return to a discussion of spin dependent forces in Section 13.

The recent discoveries of intermediate levels around 3.4 and 3.5 GeV and a possible level at around 2.8 GeV offer a great deal of support for this general picture.[35,36] The level at 2.8 GeV could well be the $\eta_c$ and the intermediate levels could be one or more of the P-states and/or the $\eta_c'$. A great deal of work will be necessary to to complete the correspondence between the predicted and experimentally discovered levels.

Electric and magnetic dipole transitions have been estimated[31] or computed[32,37] by several groups. Most of the predicted transitions are quite sensitive to the fine structure splittings. For example, the transition $J(3.1) \to \eta_c \gamma$ varies as $(\Delta M)^3$ where $\Delta M$ is the $J(3.1) - \eta_c$ hyperfine splitting. A rather simple line of reasoning (see Section 13) suggested that this splitting might be on the order of 100 MeV.[37,38] In this case, the magnetic dipole width is on the order of 1 KeV. If the newly discovered level at around 2.8 GeV[35] is indeed the $\eta_c$, then the width will be substantially larger. Scaling up by $(\Delta M)^3$ will probably be an overestimate because of corrections to the dipole approximation and spin dependence in the overlap intergral. It can be no more than about 10 KeV.[39]

The physics of the $\psi(3.7)$ is likely to be much more complicated than a $c\bar{c}$ pair moving in some effective potential. It sits very close to the threshold for the production of a $(c\bar{q})$ and $(\bar{c}q)$ pair. Coupling to these decay channels can affect the position and decays of the $\psi(3.7)$. The radiative widths to the P-states seem to be about one order of magnitude smaller than the original estimates which neglected this effect as well as fine structure. Some recent work has begun to take this into account.[40,41]

## 12. TOTAL HADRONIC WIDTHS

Surely the outstanding problem for the $c\bar{c}$ model of the new hadrons is the narrow width of the J(3.1). The Okubo-Iizuka-Zweig rule[42] which describes the large enhancement of $\phi \to K\bar{K}$ over $\phi \to \pi\pi\pi$ seems to be much more strongly operative here. The hadronic width of the J(3.1) is about 50 KeV which is perhaps a factor $10^{-4}$ below a "typical" width for a hadron of this mass. A mechanism was suggested by Politzer and myself[3] to explain this fact making use of the asymptotic freedom of the model and I want to review the argument behind this rather unusual use of asymptotic freedom. It is far from air tight but I would at least like to convince you that some analysis has gone into it and that it remains a viable possibility. It makes one rather striking experimental prediction which I will discuss at the end of this section.

I suggested in the last lecture that if the perturbation expansion converges in low orders, perturbation theory can be applied locally to the computation of R(s). This is manifestly not the case at the position of a resonance but it may still be possible to understand the widths of the J(3.1) and $\eta_c$ by the use of perturbation theory. The widths involve both a large mass scale ($E_{CM} \approx 3$ GeV) and a small one (momentum transfer $\approx 300$ MeV), and if perturbation theory is to be useful, then these two dependences must be disentangled. The part depending only on the large energy scale can then be computed in perturbation theory.

First of all, recall that the leptonic width of the J is given by the expression

$$\Gamma(J \rightarrow \bar{\ell}\,\ell) = |\psi(0)|^2 \; 16\,(Q_c\alpha)^2 \; \frac{1}{M_J^2} \; . \qquad (13)$$

$\psi(0)$ is the non-relativistic wave function at the origin and $Q_c$ is the charge of the c quark in units of e. $|\psi(0)|^2$ if of course not calculable in perturbation theory but the leptonic width is a measure of this probability factor.

What we suggested[3] is that the hadronic width could also be expressed as the product of $|\psi(0)|^2$ and a calculable matrix element. The annihilation of the $c\bar{c}$ state into some final state consisting of light quarks and gluons is shown in Fig. 7. The $B_n$ amplitude is defined to be two particle irreducible in the $c\bar{c}$ channel. The A amplitude contains the forces that produce the bound state and it contains mass singularities since the c and $\bar{c}$ lines sit very close to $p^2 = m_c^2$. (Recall from the phenomonological work that the $c\bar{c}$ system is non-relativistic and weakly bound.) It cannot be calculated perturbatively and it is responsible for the factor $|\psi(0)|^2$ when the subsequent annihilation is local.

If the $B_n$ amplitude can be shown to be insensitive to the small dimensional factors, it can be calculated perturbatively. What we suggested is that $|B_n|^2$ summed over all final states $\sum_n |B_n|^2$ depends only on the large factors $E_{CM}$ and M (the renormalization mass) and can be computed in perturbation theory. An analysis through low orders of perturbation theory leads to the following conclusions:

1. $\sum_n |B_n|^2$ is free of mass singularities associated with the initial lines by virtue of its two particle irreducibility in the $c\bar{c}$ channel. This can be seen most easily for $\eta_c$ decay where the Born term is the annihilation into two gluons. It is straight forward to check that the two particle irreducibility does indeed shield out mass singularities and therefore eliminate dependence on small dimensional parameters like $p^2 - m_c^2$. I recommend verifying this for a few simple graphs.

2. $\sum_n |B_n|^2$ is free of final state mass singularities because of the inclusive sum. This application of the Kinoshita theorem is analogous to the computation of R(s) off resonance. The analysis has only been carried to order $\alpha_s^6$ and I am currently trying to generalize it.[43]

If this analysis is correct to all orders, then the dominant contribution to J(3.1) and $\eta_c$ decay is the annihilation into 3 gluons and 2 gluons respectively. As in the parton model, these minimal constituent states then evolve into ordinary hadrons with probability one. The expressions for the widths can be taken over from the corresponding expressions for ortho and para positronium decay.

$$\Gamma_{\mathrm{Had}}(J) = |\psi(0)|^2 \frac{16}{9\pi} (\pi^2 - 9) \frac{5}{18} \alpha_s^3 \frac{1}{m_c^2} \tag{14}$$

$$\Gamma_{\mathrm{Had}}(\eta_c) = |\psi(0)|^2 \frac{8}{3} \alpha_s^2 \frac{1}{m_c^2} \quad . \tag{15}$$

These are zero binding approximations which are subject to relativistic and binding corrections. We have estimated that these corrections could be as much as 20%.[3]

The $J(3.1)$ width can be fit by choosing $\alpha_s(3.1) \approx 0.2$. One is then led to the rather striking prediction that the hadronic width of the $\eta_c$ should be a <u>few MeV.</u> If the $\eta_c \rightarrow \gamma\gamma$ width is a few KeV,[44] then the $\gamma\gamma$ branching ratio will be on the order of $10^{-3}$. There is so far one experimental number relevant to this prediction. The DESY group has reported[37]

$$\frac{\Gamma(J \rightarrow \gamma\eta_c)}{\Gamma(J \rightarrow \mathrm{all})} \times \frac{\Gamma(\eta_c \rightarrow \gamma\gamma)}{\Gamma(\eta_c \rightarrow \mathrm{all})} \approx 2 \times 10^{-4} . \tag{16}$$

In the last section, it was suggested that the $J \rightarrow \gamma\eta_c$ branching ratio might well be on the order of 10%. If this is the case, then the $\gamma\gamma$ branching ratio of might well be $10^{-3}$. If this branching ratio is much larger, then the very small magnetic dipole transition becomes hard to understand.

If the minimal gluon mechanism is wrong, then it is hard to see why the $\eta_c$ width should be much larger than the $J$ width. Although no sign of breakdown appears in low orders, it could be that this is misleading and that the expansion breaks down in high orders.[45] If this is the case, then the decay is described by some other mechanism, for example charmed meson intermediate states, $\psi \to D\bar{D} \to$ non charmed hadrons.[46] As long as the minimal gluon mechanism is at least an important part of the whole story, the $\eta_c$ should be broad. A large width for this state would be an important piece of support for the quark gluon theory.

The application of weak coupling methods to the light hadrons is much more speculative.[47] The direct channel energies are small and the bound states are relativistic. It is nevertheless an interesting possibility that these ideas have something to do with the classic examples of the O-I-Z rule.[42]

## 13. CONCLUDING REMARKS

Since the $c\bar{c}$ system appears to be non relativistic, is should be easier to deal with than any other hadronic system. The motion of the $c\bar{c}$ pair is governed by an effective Hamiltonian corresponding to some effective Bethe-Salpeter kernel. If this kernel has some simple structure in terms of Greens functions, then a connection can be established between the bound state properties and the color gauge theory which is much more direct than for the old hadrons. Whether the dominant part of the kernel is simple enough to do this is not yet clear.

For a Coulombic system, the dominant kernel is single photon exchange $[\gamma^\mu]_1 \; 1/k^2 \; [\gamma_\mu]_2$ and this leads to the Breit-Fermi Hamiltonian.[48] The appropriate kernal for the $c\bar{c}$ system is not a priori evident. An analysis of diagrams indicates that any contribution to the kernel without internal $c\bar{c}$ loops is equally important. This is because every additional factor of $\alpha_s(M_J)$ is accompanied by a logarithm of momentum transfer or binding energy. This product must be taken to be $0(1)$.

In order to make some numerical estimates, several groups assumed a simple form for the $c\bar{c}$ kernel. Although surely over simplified, it seemed to me at least to be a sensible order of magnitude guess. The guess was $[\gamma_\mu]_1 \, D(k^2) \, [\gamma^\mu]_2$ with $D(k^2)$ chosen to give the spin independent potential discussed in Section II.[37,38]

This structure looks like single gluon exchange with a dressed gluon propagator. It then must be assumed, for example, that $D(k^2) \sim 1/k^4$ as $k \to 0$.

This assumption fixes the spin dependent forces and the analog of the Breit-Fermi Hamiltonian can be written down. Numerical computations are still underway and it looks as though sensible fine structure emerges. The spin-spin interaction goes like $1/r\ \bar{\sigma}_1 \cdot \bar{\sigma}_2$ at large distances and the hyperfine splitting comes out around 70-90 MeV. The experimental number may be larger by a factor of three or four.[35]

The true structure of the kernel is surely more complicated than this. Certainly there is no reason to assume a tensor structure of the form $[\gamma_\mu]_1 [\gamma^\mu]_2$. Even if a dressed single gluon exchange is dominant, the vertex corrections each have a Pauli momentum piece in addition to the Dirac piece. The truth is probably still more complicated[49] and $c\bar{c}$ fine structure measurements are extremely important to get a handle on this structure.

The fundamental problems remain. Are the colored quarks and gluons permanently confined? Can $c\bar{c}$ properties, much less the properties of ordinary hadrons be computed starting from the color gauge theory? Absolute confinement is an attractive and widely discussed idea[50]. The necessary strongly attractive long range forces could well be a consequence of the infrared instability of the asymptotically free guage theory. Some quantitative support for this idea has come from the lattice gauge work of Witson and Kogut and Susskind[51] but the way it comes about in a continuum theory is far from clear.

Finally, the existence of charmed hadrons made of a heavy quark and one or more light quarks is an inescapable consequence of the color guage theory. The experimental situation with respect to these states is confused at the moment.[52] They may or may not have been seen in several experiments. Since they decay weakly, the experimental signature is of course dependent on how the heavy quark enters the weak current. Nothing could be more welcome at the moment than some strong evidence for the existence of charmed hadrons.

## REFERENCES AND FOOTNOTES

[1] J. J. Aubert et al., Phys. Rev. Lett. 33, 1404 (1974).

[2] J.-E. Augustin et al., Phys. Rev. Lett. 33, 1406 (1974).

[3] Much of what I will say in these lectures about the physics of heavy quarks is based on work by myself and H. D. Politzer. Most of it is published in two papers. T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975); T. Appelquist and H. D. Politzer, Institute for Advanced Study preprint, February 1975, to be published in the Physical Review.

[4] O. W. Greenberg, Phys. Rev. Lett. 13, 598 (1964).

[5] For a modern treatment and a discussion of the many experimental advantages of the colored quark model, see H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. 47B, 365 (1973).

[6] B. W. Lee and J. Zinn Justin, Phys. Rev. D5, 3121 (1972).

[7] F. Low and M. Gell-Mann, Phys. Rev. 95, 1300 (1954); N. N. Bogoliubov and D. V. Shirkov, Introduction to the Theory of Quantized Fields, New York, Interscience Publ. 1959.

[8] H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); A recent review of asymptotic freedom and some of its applications has been given by H. D. Politzer, Phys. Reports 14C, 130 (1974).

[9] C. Callan, Phys. Rev. D2, 1541 (1970). K. Symanzik, Comm. Math. Phys. 18, 227 (1970).

[10] K. G. Wilson, Phys. Rev. 179, 1499 (1969), and unpublished work. See also W. Zimmerman in "Lectures on Elementary Particles and Quantum Field Theory," edited by S. Deser et al., MIT Press, Cambridge, Mass., 1970.

[11] S. L. Glashow, I. Iliopoulos and L. Maiani, Phys. Rev. D2, 1285 (1970).

[12] For a recent review of the GIM mechanism and its implications for weak phenomenology see M. K. Gaillard, B. W. Lee and J. Rosner, Rev. Mod. Phys. 47, 277 (1975).

[13] R. M. Barnett, Invited talk, 6th Hawaii Topical Conference in Particle Physics, University of Hawaii, Honolulu, August 1975 and FERMILAB-Conf-75/71-THY, September 1975.

[14] G. Feldman, Rapporteurs talk at the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, Calif. M. Perl, Invited talk at the 1975 meeting of the Division of Particles and Fields of the American Physical Society, Seattle.

[15] M. Weinstein, Phys. Rev. D1, 1854 (1973). J.C. Pati and A. Salam, Phys. Rev. D8, 1240 (1973) and Phys. Rev. Lett. 31, 661 (1973). H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[16]K. Wilson, Phys. Rev. D3, 1818 (1971). H. Georgi, H. Quinn and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).

[17]If the effective weak Lagrangian is nonrenormalizable (the Fermi theory for example), then $\Lambda$ might be large but ultimately attainable ($E_{CM} \approx 10^5$ GeV in the Fermi theory for example).

[18]This corresponds to performing the mass renormalization subtraction at the same large Euclidean momentum at which the wave function subtraction is carried out.

[19]R. L. Jaffe and C. H. Llewellyn Smith, Phys. Rev. D7, 2506 (1973). H. Leutwyler, Phys. Lett. 48B, 431 (1974). M. Testa, Physics Letters 56B, 53 (1975).

[20]T. Appelquist and H. Georgi, Phys. Rev. D8, 4000 (1973); A. Zee, Phys. Rev. D8, 4038 (1973).

[21]T. Kinoshita, I. Math. Phys. 3, 650 (1962).

[22]S. L. Adler, Phys. Rev. D10, 3714, (1974). This analysis for the 9 colored Gell-Mann Zweig quarks can easily be extended to include one or more heavy quarks.

[23]R. Schwitters, Rapporteurs talk at the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California.

[24]R. Jost and J. M. Luttinger, Helv. Phys. Acta. 23, 201 (1950). G. Kallen and A. Sabry, Dansk Vidensk. Selsk. 29, no. 17 (1955).

[25] J. Schwinger, "Particles, Sources and Fields, Vol. II", Chapter 5-4, Addison-Wesley, New York, 1973.

[26] T.D. Lee and M. Nauenberg, Phys. Rev. 133, B1544 (1954).

[27] The theorem has been checked to low orders (two loops) by myself and J. Carazzone. I have been informed by Professor T. Kinoshita that such a check has also been performed by Dr. A. Ukawa.

[28] This order of magnitude for $\alpha_s(M)$ (with M equal to a few GeV) has been suggested by analyses of Bjorken scaling in electroproduction. See the third paper of Ref. 8.

[29] H. Harari, Rapporteurs talk at the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California. The heavy lepton should decay semi-leptonically at least 60 percent or 70 percent of the time so that it makes a sizable contribution to R(s).

[30] H.R. Quinn, Private communication and E. Poggio, H.R. Quinn and S. Weinberg, manuscript in preparation.

[31] T. Appelquist, A. DeRujula, H.D. Politzer and S.L. Glashow, Phys. Rev. Lett. 34, 365 (1974).

[32] E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K.D. Lane and T.M. Yan, Phys. Rev. Lett. 34, 369 (1975).

[33] B. Harrington, S.Y. Park and A. Yildiz, Phys. Rev. Lett. 34, 168 (1975). H. Schnitzer and J.S. Kang, Brandeis University, preprint, 1975. K. Jhung, K. Chung, and R.S. Willey, University of Pittsburgh preprint (1975).

[34] The Coulomb potential in one spatial dimension is proportion to r so that this long range growth is a natural feature of any scheme in which hadronic physics is essentially one dimensional at large distances.

[35] Bjorn Wiik, Rapporteurs talk at the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California.

[36] G. J. Feldman et al., Phys. Rev. Lett. 35, 821 (1975).

[37] J. Borenstein and R. Shankar, Phys. Rev. Lett. 34, 619 (1975). J. Borenstein, Harvard Univ. Ph.D. thesis, May 1975. Unpublished. J. Pumplin, W. Repko and A. Sato, Michigan State Preprint, July 1975.

[38] H. J. Schnitzer, Brandeis University Preprints, June 1975 and July 1975.

[39] The total width of the J(3.1) is about 70 KeV. By adding up the observed partial widths [Ref. 23] and estimating isospin related neutral channels, no more than 10 KeV remains unaccounted for.

[40] J. Kogut and L. Susskind, Phys. Rev. Lett. 34, 767 (1975) and Cornell preprint CLNS-303, (1975). R. Barbieri et al. CERN preprint TH2026 (1975).

[41] E. Eichten et al., Cornell preprint CLNS-316, (1975)

[42] S. Okubo, Phys. Letters 5, 165 (1963); J. Iizuka, K. Okada and O. Shito, Prog. Theo. Phys. 35, 1061 (1966); G. Zweig, 1964, unpublished.

[43]Details of this analysis will be presented in a future publication.

[44]This value is predicted if the decay is described by the Born graph for $c\bar{c}$ into $\gamma\gamma$ [Ref. 3]. This is on even less firm ground than the other predictions since two real photons are being emitted. The perturbation expansion can break down due to vector dominance-like corrections on each photon line.

[45]A small momentum dependence could enter from a sharing of the momentum with multiple gluon emission. See Ref. 3 for a discussion of this.

[46]G. Cohen-Tannoudji, C. Gilain, G. Girardi, U. Maor and A. Morel, I would like to thank Uri Maor for several informative discussions.

[47]A. DeRujula and S. L. Glashow, Phys. Rev. Lett. <u>34</u>, 46 (1975).

[48]J. Schwinger, <u>Particles, Sources and Fields</u> (Addison-Wesley, Reading, 1973), Vol. 2, 326-346, 420; A. Akhiezer and V. Beretski, <u>QED</u> (Interscience, New York, 1965), 514-530; G. Kallen, <u>QED</u> (Springer-Verlay, New York, 1972), 182-188; H. Bethe and E. Salpeter, <u>Quantum Mechanics of One and Two Electron Atoms</u> (Academic Press, New York 1957), 178-193.

[49]The structure of the $c\bar{c}$ kernel has been looked at in some detail by Ivan Muzinich and myself. If it is assumed that the potential (12) is due to dressed single gluon exchange, then the fact that $a/m_c^2 << 1$ can be used to show that this is a consistent assumption.

The higher order terms in the kernel skeleton expansion are supressed by powers of $a/m_c^2$. The main problem with dressed ladder dominance is gauge invariance. It has not been demonstrated.

[50] See for example R. Dashen, Rapporteurs talk at the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California.

[51] K. Wilson, Phys. Rev. D10, 2445 (1974); J. Kogut and L. Susskind, Phys. Rev. D11, 395 (1975).

[52] See Rapporteurs talks by G. Feldman, C. Rubbia, B. Barish and N. Samios at the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California.

## FIGURE CAPTIONS

Fig. 1 Feynman rules for the pure Yang-Mills theory. The conventions are those of Bjorken and Drell. The inclusion of the quark propagator and the quark-quark-gluon vertex is straightforward.

Fig. 2 One loop diagrams with infrared divergences on mass shell.

Fig. 3 A definition of the renormalized (running) coupling constant in a Yang-Mills theory.

Fig. 4 The zeroth order, parton model contribution to R(s). It gives $R^{(0)}=2$ in the colored triplet model.

Fig. 5 The second order (order $\alpha_s$) contributions to R(s).

Fig. 6 A contribution to $\sigma_{TOT}$ below threshold. Positronium-like resonances arise from uncrossed ladder exchanges in the c quark loops.

Fig. 7 The transition of a $c\bar{c}$ pair into a state consisting of gluons and light quarks.

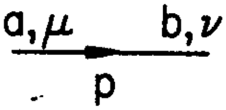
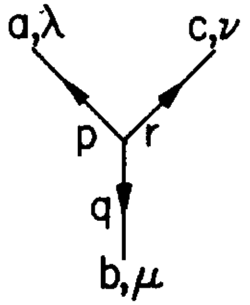
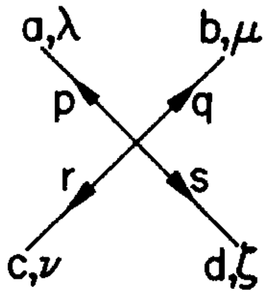
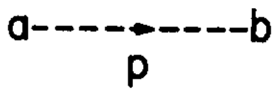
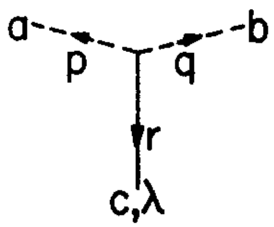
| | VERTICES | BARE VERTICES |
|---|---|---|
|  | $-i\delta^{ab}\Delta_{\mu\nu}(p)$ | $-i\delta^{ab}\left[(g_{\mu\nu}-p_\mu p_\nu/p^2)1/p^2+ap_\mu p_\nu(p^2)^{-2}\right]$ |
|  | $i\Gamma^{abc}_{\lambda\mu\nu}(p,q,r)$ $p+q+r=0$ | $f^{abc}\left[(p-q)_\nu g_{\lambda\mu}+(q-r)_\lambda g_{\mu\nu}+(r-p)_\mu g_{\nu\lambda}\right]$ |
|  | $i\Gamma^{abcd}_{\lambda\mu\nu\zeta}(p,q,r,s)$ $p+q+r+s=0$ | $-if^{abf}f^{cdf}(g_{\lambda\nu}g_{\mu\zeta}-g_{\lambda\zeta}g_{\mu\nu})$ $-if^{acf}f^{bdf}(g_{\lambda\mu}g_{\nu\zeta}-g_{\lambda\zeta}g_{\mu\nu})$ $f^{adf}f^{cbf}(g_{\lambda\nu}g_{\mu\zeta}-g_{\lambda\mu}g_{\zeta\nu})$ |
|  | $\delta^{ab}iG(p)$ | $i\delta^{ab}\,1/p^2$ F-P Propagator |
|  | $i\gamma^{abc}_{\lambda}(p,q;r)$ | $f^{abc}p_\lambda$ |

Fig. 1

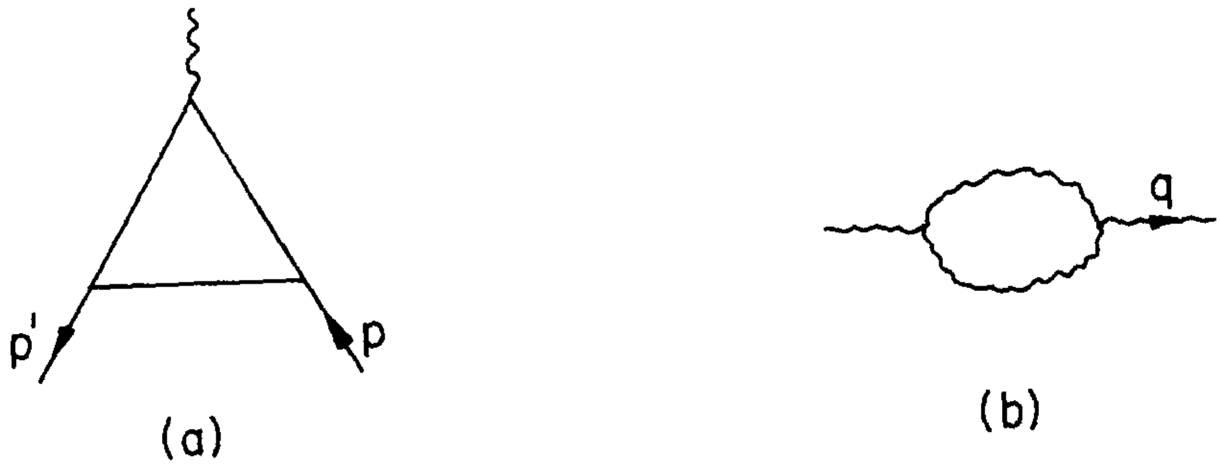



Fig. 2

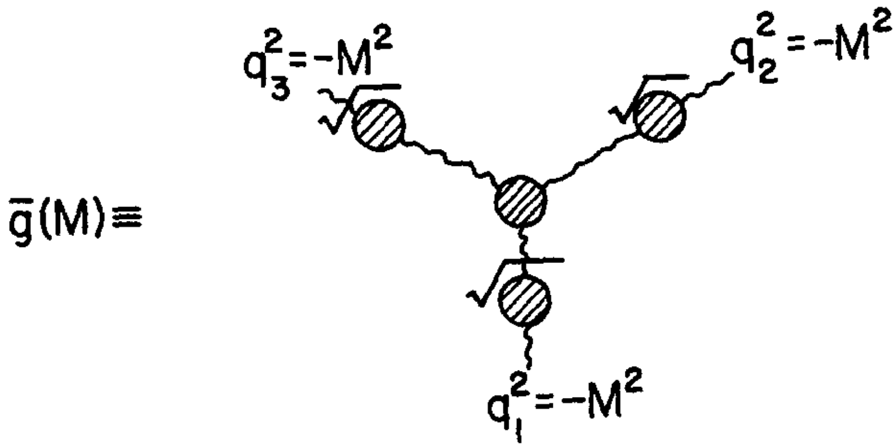



Fig. 3

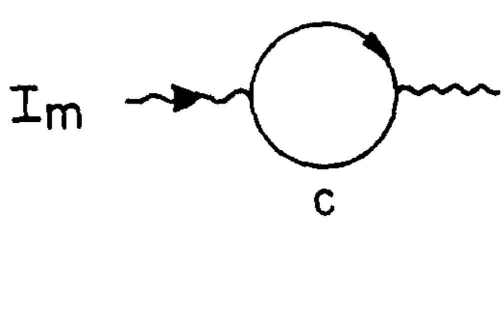



Fig. 4

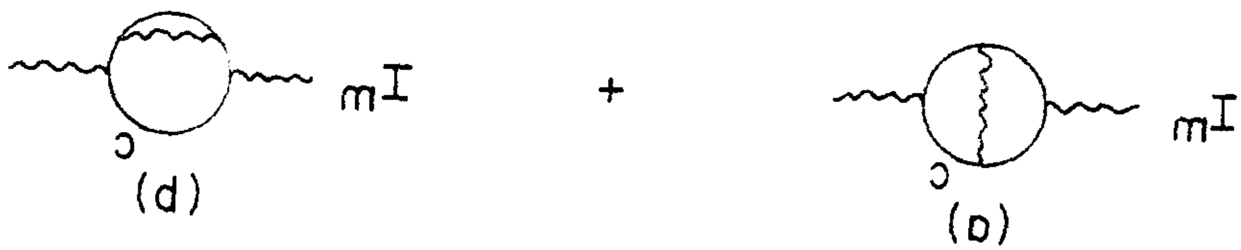



Fig.5

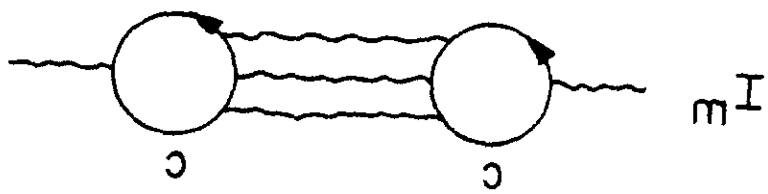



Fig.6

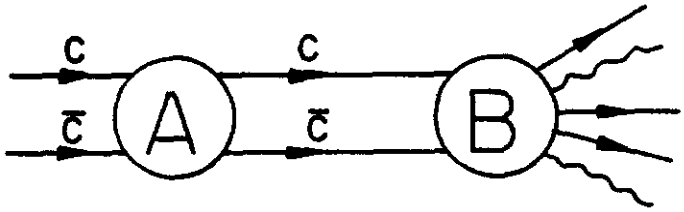



Fig. 7