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The KNO Scaling Hypothesis and the Total Multiplicity Distribution for Produced Hadrons in High Energy pp Collisions*

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The observed high energy scaling behavior of the charged particle multiplicity distribution in pp collisions is shown to be consistent with the hypothesis that the total multiplicity distribution for all the produced hadrons satisfies the KNO scaling prediction at present energies. This hypothesis, plus simple, experimentally valid, assumptions concerning leading particle effects and neutral to charged particle correlations is found to yield the measured multiplicity characteristics of particle production in high energy pp collisions.

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The theoretically motivated suggestion by Z. Koba, H. B. Nielsen, and P. Olesen (KNO) that secondary particle multiplicity distributions arising in high energy inelastic hadron-hadron collisions should asymptotically follow scaling laws of the form[1]:

$$P(n) = \frac{\sigma_n}{\sigma_{inel}} \xrightarrow[s \to \infty]{} \frac{1}{\langle n \rangle} \psi\left(\frac{n}{\langle n \rangle}\right) \tag{1}$$

with functions ψ which are energy independent, has been shown to be in unexpectedly good agreement with charged particle multiplicity data in the NAL energy range[2]. Moreover, a simple empirical modification of Eq. (1) has been found to be sufficient to extend this type of scaling to the description of low energy data as well. Explicitly, scaling laws of the form:

$$P(n) = \frac{1}{\langle n \rangle - \alpha} \psi\left(\frac{n - \alpha}{\langle n \rangle - \alpha}\right) \tag{2}$$

with energy independent parameters α, have been successful in describing charged particle multiplicity data from pp and $\pi^{\pm}$p collisions over a range of incident momenta spanning approximately two orders of magnitude[3]. (The optimum values of α are found to depend on the type of collision being fit, but are of order unity for each of the above reactions; in particular $\alpha \approx .9$ for pp collisions.)

One consequence of Eq. (2) is that the successive moments of the multiplicity distributions should satisfy equations of the form[4]:

$$\langle (n-\alpha)^q \rangle = C_q \langle n-\alpha \rangle^q \quad \text{for } q = 2,3,4,\ldots \tag{3}$$

with the C_q being energy independent constants. Consequently, Eq. (2) may be regarded as a generalization and a refinement of an earlier observation by Wróblewski that the dispersion of the charged particle multiplicity distribution could be written as a linear function of the mean of this distribution:[5]

$$D_{ch} = \sqrt{\langle n_{ch}^2 \rangle - \langle n_{ch} \rangle^2} = A(\langle n_{ch} \rangle - B) \tag{4}$$

For pp collisions, Wróblewski found the parameter values A = 0.58 and B = 1.

Since Eq. (2) asymptotically approaches Eq. (1), the overall conclusion which emerges is that the KNO scaling hypothesis is impressively well satisfied by all available charged particle production data. Moreover, studies of neutral particle production have also shown these data to be consistent with the KNO hypothesis, although, at present, the statistical accuracy of the available data on neutral particle production is much less than that of the corresponding charged particle data.[6]

Nevertheless, a very troublesome question arises as soon as one attempts to interpret the significance of this level of agreement between prediction and theory. This difficulty arises because the central assumption of the KNO derivation, namely exact Feynman scaling at asymptotic energies, is very badly violated at the energies for which Eq. (2) is found to apply.[7] Consequently, there

is no obvious explanation why such a simple modification of exact KNO scaling should suffice to fit low energy data.

In this paper we wish to investigate whether the observed multiplicity data for the high energy production of hadrons is consistent with an hypothesis which avoids this problem--namely, that the overall multiplicity distribution for all the produced hadrons (neutral as well as charged) satisfies the KNO hypothesis at present energies. Naturally, this type of scaling behavior would require an explanation quite different from that presented by KNO, but the empirical success of Eq. (2) in itself strongly suggests the need for a different theoretical basis for the KNO hypothesis than the particular arguments which led these authors originally to suggest it. Our procedure will be to investigate whether the above hypothesis plus reasonable assumptions concerning leading particle effects and for the production of neutrals are sufficient to reproduce the observed behavior of the charged particle multiplicity distribution, and also the experimental neutral to charged particle correlation data.[8]

We begin with the assumption that, over the energy range spanned by present experiments, a general inelastic proton-proton collision may be adequately described by the equation:

$$p + p \longrightarrow 2\ \text{Baryons} + n\ \text{Mesons} \tag{5}$$

We will regard the final state baryons as leading particles, while

the mesons will be assumed to be produced particles. We will not distinguish between mesons which are fragmentation products, and those which are created approximately at rest in the collision center-of-mass. Consequently, we will be assuming that the combined effect of all the dynamical processes which lead to the high energy production of hadrons is to produce a smooth overall hadron multiplicity distribution to which we can attempt to apply the KNO scaling hypothesis. Finally, while in this note we will confine our explicit discussion to pp collisions only, for which the available experimental data is the most extensive, an analogous analysis could clearly also be applied to the other measured types of hadron-hadron collisions.

We will calculate the net probability for any particular configuration of final state particles from the product of three individual probabilities which will be discussed separately.

1) P(n)--the probability that a total of n mesons (both charged and neutral) will be produced in a given inelastic pp collision. Our goal will be to investigate whether the assumption that the distribution P(n) obeys the KNO scaling hypothesis (Eq. (1)) at present energies is compatible with the observation that only in an empirically modified form (Eq. (2)) does such a scaling law describe charged particle data at these same energies. More precisely, we will examine whether the assumption:

$$\langle n^2 \rangle = C_2 \langle n \rangle^2 \tag{6}$$

with an energy independent constant C_2, leads naturally to Eq. (4), when coupled with the observed degree of correlation between neutral and charged particle production at these energies. We choose to confine our analysis to just the lowest moments of the multiplicity distribution because these are experimentally the most accurately determined, and as a result Eq. (4) represents an economical summary of the observed regularities in the energy variation of the charged particle multiplicity distribution arising in pp collisions.

The particular parameterization which we will here employ for P(n) is the following:[9]

$$P(n) \propto \int_{n-1}^{n} z^{\alpha} e^{-\beta z^2} dz \tag{7}$$

for n = 1,2,3,... (We note that P(0) = 0, since we are restricting our analysis to inelastic collisions only.) The overall normalization, plus the numerical values of the parameters α and β, are fixed at each energy by Eq. (6) plus the two constraints:

$$\sum_n P(n) = 1 \tag{8a}$$

$$\sum_n n P(n) = \langle n \rangle \tag{8b}$$

To avoid introducing a largely superfluous assumption concerning the precise energy dependence of the average total multiplicity $\langle n \rangle$, throughout this paper $\langle n \rangle$ will be employed as a measure of the collision energy. The constant C_2 in Eq. (6) was treated as an adjustable parameter; the curves to be presented were calculated using the fitted value $C_2 = 1.3$.

The known information concerning the correlation between neutral and charged meson production in pp collisions was incorporated via the following distributions.

(2) P(q)--the probability that the net charge of all the mesons produced in a given pp collision will equal q. Through overall charge conservation, we may estimate P(q) from measurements of the average number of leading protons in such collisions. This quantity, which is not at present experimentally very well determined, appears to decrease slowly with energy from a value of approximately 1.7 at 12 GeV/c to about 1.2 at the highest NAL energies.[10] If we define P_{cex} to be the probability that in a given pp collision one of the incident protons will become a neutral baryon, and if we assume that each of the incident protons fragments independently, then P(q) is Binomial in form, and given by:

$$P(q) = \frac{2}{q!(2-q)!} P_{cex}^{q} \left(1 - P_{cex}\right)^{2-q} \qquad (9)$$

for q = 0,1,2. The probability P_{cex} may be directly calculated using the equation:

$$P_{cex} = 1 - \frac{1}{2} \langle p_2 \rangle \tag{10}$$

where $\langle p_2 \rangle$ is the average number of leading protons at a given collision energy. Since the measured data are consistent with a linear decrease of $\langle p_2 \rangle$ versus log s over the range of energies of interest to this analysis, we will assume a relationship of the form:

$$P_{cex} = a \langle n \rangle + b \tag{11}$$

(This implies that the parameter $\langle n \rangle$ may also be regarded as a linear function of log s to within the accuracy of the above assumption concerning the energy variation of $\langle p_2 \rangle$.) Deviations from the simple linear relationship assumed in Eq. (11) must necessarily be present at both small and large $\langle n \rangle$ values by virtue of the bounded nature of the probability P_{cex}. (As $\langle n \rangle \to 0$, the value of P_{cex} must smoothly approach 0, while as $\langle n \rangle \to \infty$, it is not unreasonable to anticipate that P_{cex} may approach 1/2.) While it would be straight-forward to modify Eq. (11) in a phenomenological manner to include these effects, they are not of significance in the energy range of interest to us here; therefore, in the interest of overall simplicity, such refinements have not been introduced. The values

of the constants a and b in Eq. (11) are fixed by the measured $\langle p_\ell \rangle$ values; the values employed in our analysis were a = .025, and b = .1.

3) $P(n_o | n,q)$--the probability that n_o neutral mesons will be produced in a pp collision in which a total of n mesons are emitted and have a net charge q. Given values for n_o, n, and q, we may directly calculate the number of positive and negative mesons which were also produced via the relationships:

$$n_+ = \frac{1}{2}\left(n - n_o + q\right) \tag{12a}$$

$$n_- = \frac{1}{2}\left(n - n_o - q\right) \tag{12b}$$

(Only n_o, n, and q combinations which yield non-negative, integral values of n_+ and n_- are permissible.)

The existence of hadron resonances, plus the available experimental data concerning π^0/π^- correlations, both clearly indicate the presence of positive correlations between the charged and neutral mesons produced in any hadron-hadron collision.[8] Moreover, the very large number of low mass, strongly decaying, resonant states which are known to exist strongly suggests that the net effect of all the possible production processes which

may physically occur in a high energy pp collision is to result in an overall neutral to charged meson ratio which can be adequately reproduced by a very simple model employing just the following two assumptions:

i) Each produced meson in a given event has an equal probability for being positive, negative, or neutral.

ii) The resulting probability for any particular configuration of charged mesons is, for fixed n and q, simply proportional to the number of distinguishable charge permutations, viz.

$$N(n_+, n_0, n_-) = \frac{(n_+ + n_0 + n_-)!}{n_+!\, n_0!\, n_-!} \tag{13}$$

where n_+ and n_- are related to n_0, n, and q by Eqs. (12a) and (12b).[11]

A parameter-free expression for $P(n_0; n,q)$ subsequently results:

$$P(n_0 | n, q) = \frac{N(n_+, n_0, n_-)}{\sum_{n_0} N(n_+, n_0, n_-)} \tag{14}$$

where $N(n_+, n_0, n_-)$ is given by Eq. (13), and the sum includes only those n_0 values for which Eqs. (12a) and (12b) apply.

The combined probability for any particular configuration of final state mesons is given by the product of the preceding three separate probabilities:

$$P(n_+, n_0, n_-) = P(n_0 | n, g)\, P(n)\, P(g) \tag{15}$$

The consequences of this expression for $P(n_+, n_o, n_-)$ have been investigated using Monte Carlo methods. The figures to be presented were obtained using samples of 10,000 generated events at each of 8 uniformly spaced values of $\langle n \rangle$ ranging from 2 through 16.

In Fig. 1, we present the resulting D_{ch} versus $\langle n_{ch} \rangle$ plot. The straight line represents the results of a least squares fit to the 8 data points, and has a χ^2 of 3.6. Clearly, the Wróblewski parameterization is well reproduced. (The value of .9 for the x-intercept is in agreement with the aforementioned recent fit to the pp data.[3]) We have therefore succeeded in our goal of obtaining Eq. (4) for n_{ch} from assuming Eq. (6) for n.

Our calculation also yields results which are consistent with the other known multiplicity characteristics of high energy pp collisions. This agreement will be discussed in an essentially qualitative manner, however, since experimentally these other features of the data are much less well determined than D_{ch} or $\langle n_{ch} \rangle$, and consequently place far weaker quantitative constraints on any empirical model.

In Fig. 2, we present the various mean multiplicities for secondary particles plotted versus the parameter $\langle n \rangle$. The following observations can be made:

1) $\langle n_{ch} \rangle$ is an effectively linear function of $\langle n \rangle$ over the

range presented. Therefore, since $\langle n_{ch} \rangle$ is measured to be an approximately linear function of log s, our assumption that $\langle n \rangle$ was also an approximately linear function of log s is internally consistent and experimentally justified.

2) $\langle p_\ell \rangle$ is 1.6 for $\langle n_{ch} \rangle = 4$ and 1.2 for $\langle n_{ch} \rangle = 9$; consequently, Eq. (11), with the indicated values of a and b, successfully reproduces the measured data.

3) $\langle n_+ \rangle$ is always greater than $\langle n_- \rangle$, in agreement with what is experimentally observed. The relative magnitudes of these averages are also consistent with the measured values except that for large $\langle n \rangle$, the curves in Fig. 2 continue to separate whereas the measured values appear to approach a constant separation. This discrepancy, however, is caused entirely by our neglect of the high energy flattening of the $\langle p_\ell \rangle$ curve which sets in at approximately $\langle n_{ch} \rangle = 9$. When this effect is included, the $\langle n_+ \rangle$ and $\langle n_- \rangle$ curves generated from Eq. (15) also rise in parallel.

4) $\langle n_o \rangle$ is approximately equal to $\langle n_+ \rangle$ for $\langle n_{ch} \rangle$ values from 4 to 9. This is also in agreement with the experimental data. Above $\langle n_{ch} \rangle = 9$, the combined influences of probable λ production, plus the effect of the flattening out of $\langle p_\ell \rangle$, should combine to preserve this approximate equality between $\langle n_o \rangle$ and $\langle n_+ \rangle$. (In contrast to the behavior of the curves in Fig. 2.)

The remaining feature of the multiplicity data to be compared is the $\langle n_o \rangle$ to n_- correlation results. In Fig. 3 we plot this correlation for our generated data. The trend of these data is in

complete agreement with that observed experimentally. It should be recalled that no low multiplicity clustering was built into Eq. (14) even though the presence of such clusters in physical pp data is well established. The effect of introducing such clusters on Fig. 3 was investigated using alternate expressions for Eq. (14). The following assumptions were examined:

i) All produced particles are ρ mesons which subsequently decay into π mesons (an example of 2 particle clustering).

ii) All produced particles are A mesons which subsequently decay by the chain $A \rightarrow \rho\pi \rightarrow \pi\pi\pi$ (an example of 3 particle clustering)

In either case the same general trends were observed that are displayed in Fig. 3. The only significant difference is that the fewer the number of particles making up the clusters, the steeper is the rise of the correlation function. (The points displayed in Fig. 3 can be regarded as resulting from the assumption of a single n body cluster per event.) Consequently, the effect of adding the effects of low multiplicity clustering to our calculation would be to increase the slope of the points displayed in Fig. 3. (The slope of the measured data does appear steeper than the rate of rise in Fig. 3, although the experimental error bars are too large to solidly establish this point.)

In summary, therefore, we conclude that all available data concerning secondary hadron multiplicities in high energy pp collisions is consistent with the hypothesis that the overall multiplicity distribution for all the produced hadrons satisfies

the KNO scaling hypothesis at finite energies. Besides being a useful observation from the point of view of those attempting to construct simple empirical models of high energy production processes, this conclusion is also important because it would appear to require the presence of long range correlations between the secondary hadrons produced in high energy collisions. (We note that Eq. (6) implies that $f_2 = (C_2-1)\langle n\rangle^2 - \langle n\rangle$, and hence asymptotically this correlation moment will increase as $\langle n\rangle^2$, rather than as $\langle n\rangle$.)

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11. More rigorously, assumption (ii) is a reasonable consequence of assumption (i) only if just one type of meson exists. Consequently, a more accurate description of the experimental data would be expected from a model in which the above two assumptions were applied separately to pions and kaons. Furthermore, assumption (i) ignores those final state pions which result from the electromagnetically decaying η meson. Since, instead of being in the vicinity of 1/2, the neutral to charged pion ratio for the products of η decay is approximately 8/3, any appreciable amount of η production at high energies will have significant effects. Nevertheless, the present state of our knowledge of neutral particle production in high energy pp collisions does not really justify introducing the above refinements. Consequently, in the interests of simplicity, the influence of kaons and eta mesons was not explicitly included.

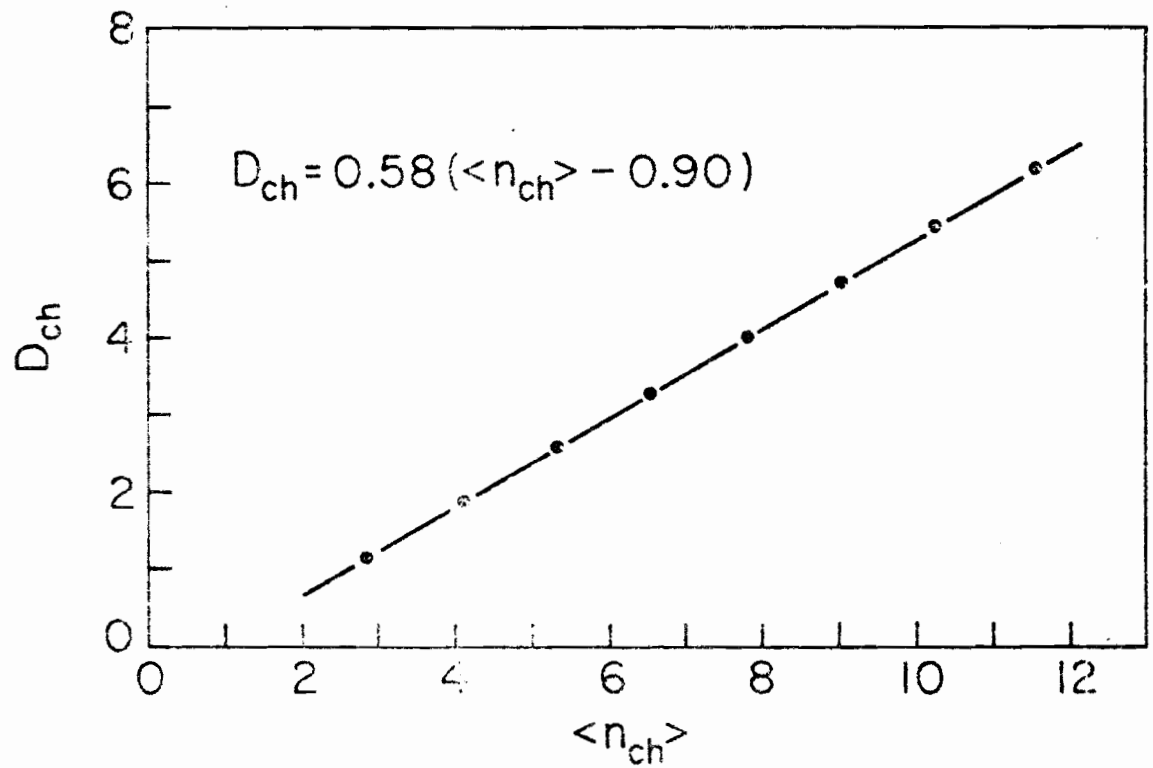
$D_{ch} = 0.58(\langle n_{ch}\rangle - 0.90)$
D_{ch}
$\langle n_{ch}\rangle$
0
2
4
6
8
10
12

Fig. 1

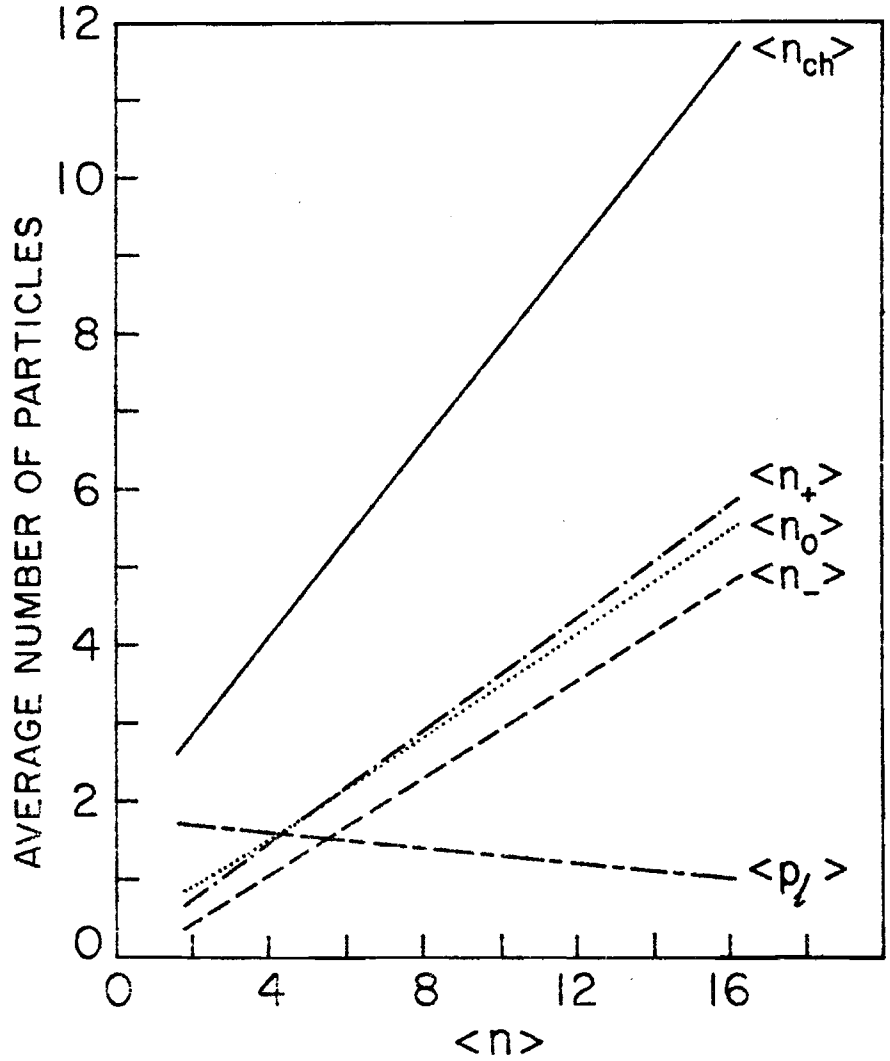

Fig. 2

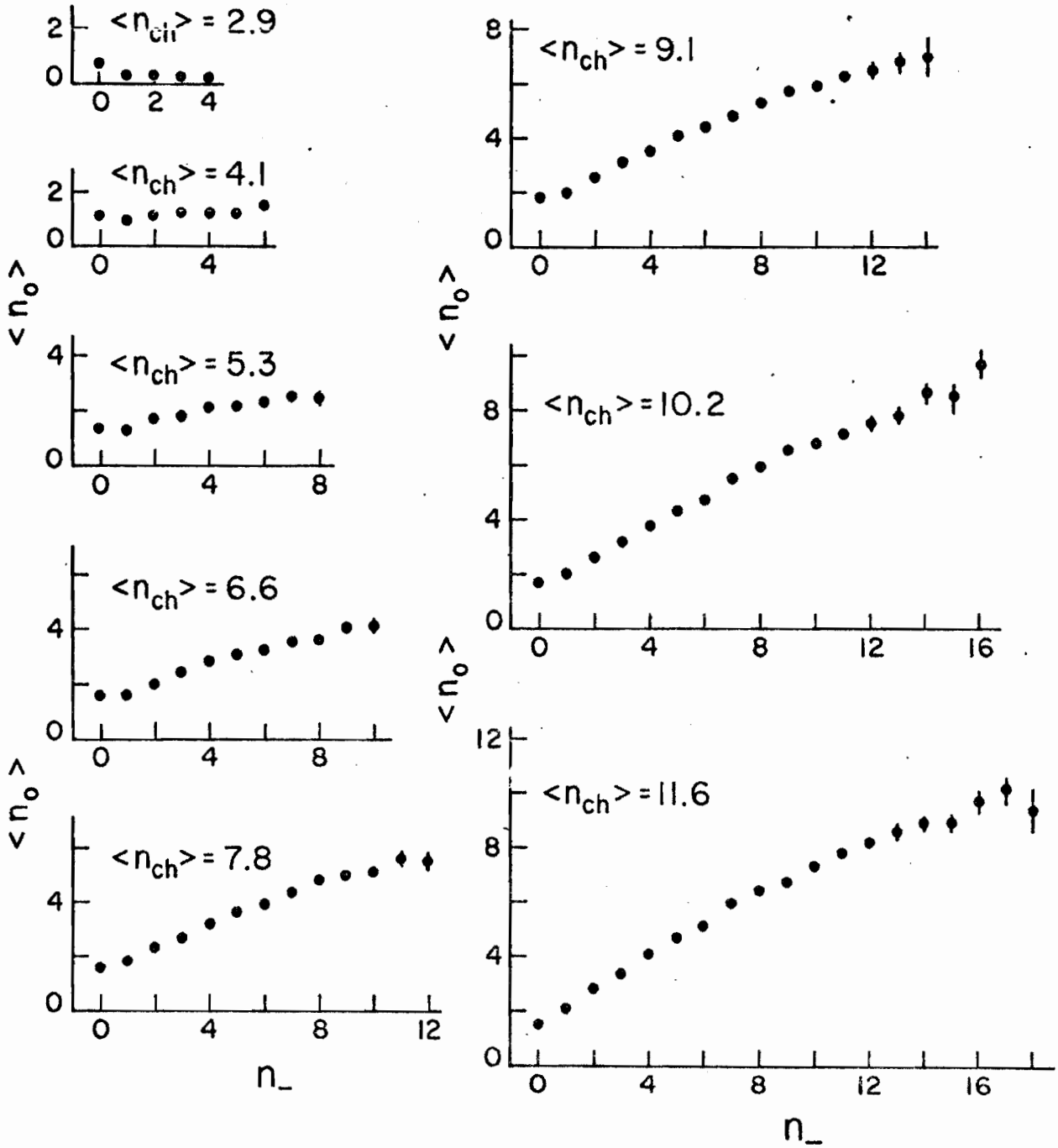

Fig 3