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Multiplicity in proton-nucleus interactions at 200 GeV/c

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Abstract

Experimental results on multiparticle production in proton-nucleus collisions based on an exposure of emulsion stack to 200 GeV/c proton beam at NAL are presented. It is found that the average charged particle multiplicity $\langle n_s \rangle = 13.0 \pm 0.4$, $D \equiv (\langle n_s^2 \rangle - \langle n_s \rangle^2)^{1/2} = 8.5 \pm 0.4$ and $\langle N_h \rangle = 7.3 \pm 0.2$. Energy dependence of $R_{em} = \langle n_s \rangle / \langle n_{ch} \rangle$, where $\langle n_{ch} \rangle$ is the charged particle multiplicity in pp collisions, is investigated in the range 7.1 to 8000 GeV/c. R_{em} attains a constant value of 1.71 ± 0.04 beyond 200 GeV/c and furthermore this value of R_{em} implies an A dependence $R_A = A^{0.13}$. Predictions of R_{em} on various models are discussed and compared with the emulsion data. It is found that even for p-nucleus collisions D increases linearly with $\langle n_s \rangle$ with about the same slope as for pp collisions.

Last few years have seen a considerable spurt of interest in the study of hadron-nucleus interactions. There are a variety of reasons for this revival. First and foremost is the phenomenal success of Glauber theory [1] which has made it possible to correctly incorporate the nuclear effects. The second reason is the realisation that it may be possible to test the different models of multiparticle production in hadron-nucleon collisions by confronting their predictions for multiparticle production in hadron-nucleus collisions with the experimental data [2-6]. The third reason is the unique possibility that hadron-nucleus collisions offer for studying the space-time development of the particle production process [6].

Because of its unique spatial and ionisation resolution, an excellent producer-detector for studying hadron-nucleus collisions is the nuclear emulsion ($\langle A \rangle = 73$), which is mainly composed of Ag, Br, C, N, O, and H. Approximately 71% (39% in Ag and 32% in Br) of the collisions occur in the heavy nuclei, Ag and Br, 25% in the light nuclei, C, N and O, and only 4% in hydrogen.

In this note we present our results on multiplicity in proton-emulsion collisions at 200 GeV/c and analyse the available data in the range of 7.1 GeV/c to 8000 GeV/c to draw conclusions on multiplicity distribution and models of multiparticle production.

A stack of Ilford G5 emulsion pellicles was exposed to the 200 GeV/c proton beam at the NAL in September 1972. The emulsion plates were scanned carefully by the method of area scanning and events were classified according to the associated N_h, the black and grey prongs having $\beta < 0.7$, and n_s, the shower particle multiplicity (minium ionising) corresponding to $\beta \geq 0.7$. In this way a total of 1530 events with $N_h \geq 2$ were recorded.

The scanning efficiency for events with $N_h \geq 2$ is found to be 0.98. The ratio of the number of events with $2 \leq N_h \leq 5$ to that of $N_h \geq 2$ is 0.35 ± 0.02, compared to 0.38 ± 0.03 obtained by Cuer et al. [7] who obtained their events by the 'along the track' scanning method. This indicates that our sample of events with $N_h \geq 2$ has neglegible loss, if any, of events with low N_h. The detection efficiency for shower particles, from a track by track comparison of two independent assignments carried out for a sample of events, was found to be $\epsilon = 0.90 \pm 0.01$. In what follows, unless otherwise mentioned, necessary correction has been applied for the detection efficiency ϵ .

It is interesting to note that the highest multiplicity observed in an event is $n_s = 54$ (uncorrected for ϵ) and this event has $N_h = 18$. The highest observed n_s for $N_h = 2$ events is 23. These numbers may be compared to $n_{ch} = 22$, the highest multiplicity observed in pp interactions at 205 GeV/c [9]. The highest observed N_h is 39 (having $n_s = 22$).

Fig.1 shows the dependence of $\langle n_s \rangle$ on N_h at 200 GeV/c. The data for $N_h \geq 2$ is our own while the two points at $N_h = 0$ and 1 have been obtained from Babecki et al. [8]. As can be seen, $\langle n_s \rangle$ seems to increase linearly with N_h upto atleast $N_h = 22$. The line drawn is the best fit of the form

$$\langle n_s \rangle = a\, N_h + b \qquad .. \quad (1)$$

to our data with $N_h \geq 2$. The best fit values are $a = 0.63 \pm 0.04$, $b = 8.5 \pm 0.2$ with $\chi^2 = 18.0$ for 18 points (16 DOF). The values of $\langle n_s \rangle = 8.5 \pm 0.2$ and 9.1 ± 0.2 for $N_h = 0$ and 1 respectively, obtained from our fit, may be compared with the corresponding values of 8.51 ± 0.38 and 9.03 ± 0.59 obtained by Babecki et al. These values may also be compared with $\langle n_{ch} \rangle = 7.64 \pm 0.17$ observed in pp interactions at 205 GeV/c [9]. This indicates that nuclear effects are not neglegible even for $N_h = 0$ and 1 events. Fig.1 also shows the dependence of $\langle n_s \rangle$ on N_h at 7.1 GeV/c and 23.4 GeV/c [10] for comparison.

Before we proceed further it should be mentioned that in order to obtain quantities for $N_h \geq 0$, i.e. for an average p-emulsion collision, we have used the data* of Babecki et al. [8] for $N_h = 0$ and 1 and our own for $N_h \geq 2$. For this purpose we have used 27.4 ± 1.2 and 72.6 ± 2.2 as percentages of events with $N_h \leq 1$ and $N_h \geq 2$ respectively; these are weighted averages of the values obtained by Cuer et al. and Babecki et al.

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* We are grateful to J. Gierula for providing us with n_s distribution for $N_h = 0$ and 1 events obtained by Babecki et al. [8].

Following the procedure outlined above, we obtain the average charged shower particle multiplicity

$$\langle n_s \rangle = 13.0 \pm 0.4, \qquad .. \quad (2a)$$

the dispersion

$$D = (\langle n_s^2 \rangle - \langle n_s \rangle^2)^{1/2} = 8.5 \pm 0.4 \qquad .. \quad (2b)$$

and the average N_h

$$\langle N_h \rangle = 7.3 \pm 0.2 \qquad .. \quad (3)$$

for $N_h \geq 0$, i.e. for an average p-emulsion collision, at 200 GeV/c. All possible sources of errors have been taken into account in arriving at the above errors. These values may be compared with $\langle n_s \rangle = 12.9 \pm 0.4$, $\langle N_h \rangle = 7.4 \pm 0.2$ [7] and $\langle n_s \rangle = 13.1 \pm 0.3$ [8] obtained in the other 200 GeV/c p-emulsion experiments. The weighted means of the three investigations at 200 GeV/c are $\langle n_s \rangle = 13.0 \pm 0.2$ and $\langle N_h \rangle = 7.35 \pm 0.1$.

An important quantity of interest is the ratio of average multiplicity in emulsion to that in pp collisions, $R_{em} = \langle n_s \rangle / \langle n_{ch} \rangle$, which may be used as a test to distinguish between different models. Using $\langle n_{ch} \rangle = 7.64 \pm 0.17$ at 205 GeV/c [9], we obtain

$$R_{em} = 1.71 \pm 0.06 \text{ for our data and}$$

$$R_{em} = 1.71 \pm 0.04 \qquad .. \quad (4)$$

for the weighted mean $\langle n_s \rangle$ at 200 GeV/c.

Fig.2 shows a plot of R_{em} as a function of p_{Lab} from 7.1 GeV/c to 8000 GeV/c. The data used is listed in Table 1. The

values of $\langle n_{ch} \rangle$ for $p_{Lab} \leq 27.9$ GeV/c have been obtained from the best fit described in Ref. [15] and for 69 GeV/c and 205 GeV/c from Refs. [16] and [9] respectively. The values of $\langle n_{ch} \rangle$ for $p_{Lab} = 1000$ to 8000 GeV/c have been calculated from $\langle n_{ch} \rangle = -3.02 + 1.81 (\ln s)$, which is the best fit to the accelerator and ISR data* in the range $p_{Lab} = 69.0$ GeV/c to 1500 GeV/c. Thus, one finds that the value of R_{em} shows a slow increase in the region of 10 to 68 GeV/c and attains an essentially <u>constant value of 1.71 ± 0.04 in the region of 200 to 8000 GeV/c</u>.

In order to abstract the A dependence of $\langle n_s \rangle$ at a given energy, we may express it as

$$\langle n_s(A) \rangle = \langle n_{ch} \rangle A^{\alpha}$$

$$R_A \equiv \langle n_s(A) \rangle / \langle n_{ch} \rangle = A^{\alpha} \qquad .. \quad (5a)$$

where $\langle n_{ch} \rangle$ is the average charged particle multiplicity in pp collisions at the same energy. Using the data in Fig.2 and averaging over the emulsion composition, we have evaluated the exponent α at various energies. We find that α increases from ~ 0 at 10 GeV/c to 0.12 at 68 GeV/c, and beyond 200 GeV/c it attains essentially a constant value of

$$\alpha = 0.131 \pm 0.005 \qquad .. \quad (5b)$$

Thus, not only is R_{em} nearly constant at $p_{Lab} > 100$ GeV/c, its absolute value of 1.71 ± 0.04 implies an A dependence

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* The data used is for momenta (in GeV/c), 69 [16], 102 [17], 205 [9], 303 [18], 405 [17], 259 [19], 510 [19,20], 1080 [19,20], 1498 [19,20]. The ISR data of Refs. [19,20] have been appropriately corrected for the fact that they used a constant value of $\sigma_{inel}(pp) = 32$ mb; we have scaled $\langle n_{ch} \rangle$ by $32.0/\sigma_{inel}(s)$, where $\sigma_{inel}(s) = 23.9\ s^{0.049}$ as given by Morrison [21].

of the type $A^{0.13}$, which is very slow indeed.

Before we compare the predictions of the different models for R_{em}, we shall discuss the experimental results on multiplicity distributions in pp and p-nucleus collisions. There has recently been a great deal of interest in the dependence of $D = (\langle n_{ch}^2 \rangle - \langle n_{ch} \rangle^2)^{1/2}$ on $\langle n_{ch} \rangle$ in p-p collisions. The first observation of an important regularity between $\langle n_{ch} \rangle$ and D in proton-nucleon collisions was made over ten years ago by Malhotra [22] who noted that $\langle n_{ch} \rangle / D \approx 2$ right from 9 GeV/c to 10^4 GeV/c. This regularity was explored in more detail by Czyzewski and Rybicki [23], Malhotra and Ganguli [24] and finally Wroblewski [25] who gave a more exact linear relationship in the range 4 to 70 GeV/c. Using the data compiled in Ref. [15] and the more recent NAL data [9,16-18] we have carried out a fit in the range 12.9 to 405 GeV/c and find that

$$D = a \langle n_{ch} \rangle + b \qquad .. \text{(6a)}$$

$$a = 0.581 \pm 0.004, \quad b = -0.558 \pm 0.007 \qquad .. \text{(6b)}$$

with $\chi^2 = 12.6$ for 13 data points.

In order to test the validity of a linear relation of type (6a) for p-nucleus collisions we have plotted in Fig. 3, D vs. $\langle n_s \rangle$ for $N_h \geq 0$ (curve A), $N_h \geq 9$ (curve B) and pp collisions (curve C - based on Eq. 6). The data used are compiled in Table 1. The results of a fit of the type (6a) are summarised in Table 2. The following observation can be made:

(i) the linear relation (6a) is well satisfied by $N_h \geq 0$ as well as $N_h \geq 9$ data, (ii) the slope 'a' for $N_h \geq 0$ is 0.63 ± 0.02, which is only slightly greater than that for pp collisions but the intercept 'b' is appreciably greater, resulting in significantly greater $D/\langle n_s \rangle$ for p-emulsion collisions compared to pp collisions and (iii) the slope for $N_h \geq 9$ is significantly less than that for $N_h \geq 0$ events. Similar observations have also been made by Babecki et al. [8].

While interpreting observation (ii) it should be noted that emulsion being a composite material, we are dealing with a sum of four (Ag, Br, CNO, H) different n_s distributions. Even if the relation between D and $\langle n_s(A) \rangle$ for a p-nucleus collision was the same as that for a pp collision, the presence of four components in p-emulsion collisions would lead to an increase in the ratio $D/\langle n_s \rangle$. To estimate this effect we calculate $\langle n_s(A) \rangle$ and D_A for each component using Eqs.(5) and (6) and adding them appropriately we obtain $\langle n_s \rangle = 12.9$ and $D = 7.2$ for p-emulsion collisions at 200 GeV/c. This yields the calculated value $D/\langle n_s \rangle = 0.56$ for emulsion, which is greater than 0.51 for pp data but much lower than the observed value of 0.65 for p-emulsion collisions at 200 GeV/c (see Table 1). Thus, we find that the composite nature of emulsion cannot entirely explain the observed high value of $D/\langle n_s \rangle$ for emulsion.

In order to understand (iii) we first note that the selection $N_h \geq 9$ implies that the data is restricted to collisions in Ag and Br only and furthermore to relatively central ones, i.e. those in which the number of collisions, suffered by the incident particle, is relatively large. Clearly, such a truncation would lead to a high value of $\langle n_s \rangle$ but not so high D, thus qualitatively explaining the observation (iii).

Before we consider the predictions of the different models and confront the same with the experimental data presented above, it is necessary to calculate $\langle \nu_{em} \rangle$, the average number of proton-nucleon inelastic collisions in emulsion nuclei.

If P_ν is the probability of ν collisions in a nucleus with atomic weight A, then

$$\langle \nu_A \rangle = \frac{\sum_\nu \nu P_\nu}{\sum_\nu P_\nu} = \frac{2\pi \int \sigma_{in}\, T(b)\, db}{\sigma_{in}(A)} = \frac{A\, \sigma_{in}}{\sigma_{in}(A)} \quad .. \quad (7)$$

and $\sigma_{in}(A) = 2\pi \int \left[1 - e^{-\sigma_{in} T(b)}\right] b\, db \quad .. \quad (8)$

where σ_{in} and $\sigma_{in}(A)$ are the p-nucleon and p-nucleus inelastic cross sections, respectively, and T(b) is the number of nucleons per unit area in the path of the incident proton at impact parameter b. For the density distribution of the nucleus, we have used Wood-Saxon form (with $C=1.07\ A^{1/3}$ fm), which is known to be a good representation [1]. The A dependence of $\langle \nu_A \rangle$ so obtained can be expressed as $\langle \nu_A \rangle = 0.716\ A^{0.326}$ for $\sigma_{in} = 32.0$ mb, i.e. at 200 GeV/c. Note that $\langle \nu_A \rangle$ has an energy dependence since σ_{in} depends on s; we have used $\sigma_{in}(s)$ as given

by Morrison [21]. In this way, we obtain $\langle \nu_{em} \rangle = 2.72$, at 200 GeV/c ($\sigma_{in} = 32.0$ mb), averaged over the emulsion composition. Note that this value is significantly lower than 3.2 obtained [6] using a uniform density distribution for the nucleus.

We now consider the predictions of the different models of multiparticle production in proton-nucleus collisions. In the simple cascade model (see e.g. Refs. 3, 5a) one assumes that the incident hadron collides successively with a number of nucleons inside the nucleus producing secondary particles at each collision. Each of the secondaries may in turn suffer further collisions leading to a build-up of an intranuclear cascade. It is rather well known that such a model grossly overestimates multiplicities and $\langle N_h \rangle$ in proton-nucleus collisions, particularly at high energies. Curve A in Fig.2 shows the results of a calculation by Fishbane et al. [5a] based on the multiperipheral model and Glauber theory for the propagation of the secondary particles.

Attempts have been made to refine the cascade calculation by taking account of the fact that the secondary particles at high energies are highly collimated. Different recipes have been used. Curve B in Fig.2 presents the results of Artykov et al. [26] who have assumed that because of high collimation 'a large number of secondary particles interact with one and the same intranuclear nucleon'. As can be seen their predictions

agree rather well with the data upto about 1000 GeV/c, beyond which there are indications of disagreement. However, even this refined cascade calculation leads to far too high values of $\langle N_h \rangle$ for $p_{Lab} > 50$ GeV/c, e.g., at 200 GeV/c the predicted value is 13.0 ± 0.6 whereas our experimental value is 7.3 ± 0.2.

In the diffractive excitation model the particle production takes place through the diffractive excitation of the nucleons (fireballs, novas or isobars). Since each nova decay leads to $\langle n_{ch} \rangle/2$ particles, one can easily show that

$$R_A = \frac{1}{2} (\langle \nu_A \rangle + 1) - 0.5 \frac{\langle \nu_A \rangle}{\langle n_{ch} \rangle} \qquad .. \quad (9)$$

where the second term is a correction to take account of the fact that in a nucleus nearly half of the nucleons are neutrons. Using the value $\langle \nu_{em} \rangle = 2.72$ obtained above for emulsion, one finds that $R_{em} = 1.68$ at 200 GeV/c. Curve C in Fig.2 represents the prediction of the Eq.(9). As can be seen there is a very good agreement between the predictions and the data. This model also predicts that inelasticity for p-nucleus collisions should be nearly same as for p-p collisions which is in agreement with observations [27]. However, it should be pointed out that the simple diffractive excitation model fails [24] to explain the charged particle multiplicity distribution and the observed energy dependence of $\langle n_{ch} \rangle/D$ in pp collisions.

We next consider the hydrodynamical model due to Landau [28]. This model predicts energy dependence of multiplicity in

pp collisions as $\langle n_{ch} \rangle = a\, s^{1/4}$ which is in good agreement with the data [15]. An extension of this model to a hadron-nucleus collision implies that at high enough energies the hadron will essentially collide with $\langle \nu_A \rangle$ nucleons at rest, where $\langle \nu_A \rangle$ is the mean number of nucleons contained in the 'tube' traversed by the incident hadron. After carrying out a rather complex computation, Belenkij and Landau [28] give following prediction

$$R_A = A^{0.19} \qquad .. (10)$$

However, since in this model multiplicity grows as $s^{1/4}$ and for a hadron-nucleus collision $s_A = s \langle \nu_A \rangle$, ignoring transverse motion one expects

$$R_A = \langle \nu_A \rangle^{1/4} = 0.92\ A^{0.08} \qquad .. (11)$$

which may be compared with the experimental result $R_A = A^{0.131 \pm 0.005}$, given by Eq.(5). It is not clear why (10) and (11) differ so much even at high energies. It seems to us that there is a need for a better computation of A dependence of R_A on Landau's model. Finally, we note that Landau's model predicts essentially the same linear relation between D and n_s for p-nucleus collisions as for pp collisions.

Recently Gottfried has proposed an energy flux cascade model [6] for hadron-nucleus collisions. In common with Landau's model, this model assumes that the energy flux of hadronic matter is the essential variable that governs the early evolution of the system, and it is a cascade of this flux, and not of conventional hadrons, that occurs in a nucleon-nucleus collision. The

essential difference between the two models lies in the temporal structure of the developing state, e.g., whereas in Landau's model the expansion phase is relatively slow, in Gottfried's model the expansion occurs with a rapidity close to that of the incident particle. An important prediction of Gottfried's model [6] is that

$$R_A = \frac{1}{3} (\langle \nu_A \rangle + 2) + O (\ln^{-1} s) \qquad .. (12)$$

If we ignore the $O(\ln^{-1} s)$ term which is quite small even at 200 GeV/c, then it implies that $R_{em} = 1.57$ at 200 GeV/c. Curve D in Fig.2 represents the prediction of Eq.(12). We consider the agreement to be rather good; the slight deviation at 200 GeV/c may imply that transverse motion, which is neglected in arriving at Eq.(12), cannot be altogether ignored at energies as low as 200 GeV/c. This model also seems to explain, atleast qualitatively, features such as low $\langle N_h \rangle$ at $p_{Lab} > 200$ GeV/c, near independence of inelasticity of A and the observed nuclear ln (tan θ) distribution in emulsion at 200 GeV/c [6,7].

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Table 1. Compilation of $\langle n_s \rangle$, D and R_{em} for an average emulsion collision, and $\langle n_s \rangle$ and D for a sub-class of collisions with $N_h \geq 9$.

p_{Lab} GeV/c	$\langle n_{ch} \rangle$	$N_h \geq 0$			$N_h \geq 9$		Ref
		$\langle n_s \rangle$	D	R_{em}	$\langle n_s \rangle$	D	
7.1	2.94 ± 0.03	2.80 ± 0.04	1.60 ± 0.03	0.95 ± 0.02	3.24 ± 0.06	1.66 ± 0.05	10
9.9	3.24 ± 0.03	3.2 ± 0.2	-	0.99 ± 0.06	-	-	11
23.4	4.22 ± 0.04	5.61 ± 0.11	3.31 ± 0.08	1.31 ± 0.03	7.28 ± 0.19	3.49 ± 0.12	10
27.9	4.46 ± 0.04	6.23 ± 0.2	3.59 ± 0.12	1.40 ± 0.05	8.51 ± 0.27	3.70 ± 0.21	12
67.9	5.89 ± 0.07	9.57 ± 0.23 9.73 + 0.23[a]	- 5.90 + 0.22[a]	1.62 ± 0.04	13.75 ± 0.41	6.26 ± 0.37	8
200	7.64 ± 0.17	13.04 ± 0.4	8.51 ± 0.4	1.71 ± 0.06	18.2 ± 0.4	9.7 ± 0.4	This work
200	7.64 ± 0.17	13.08 ± 0.3 13.31 ± 0.3[a]	- 8.34 ± 0.3[a]	1.71 ± 0.06	19.3 ± 0.6	9.4 ± 0.4	8
200	7.64 ± 0.17	12.9 ± 0.4	-	1.69 ± 0.06	-	-	7
1000	10.6 ± 0.6	19.2 ± 1.9	13.7 ± 1.5	1.81 ± 0.20	30.8 ± 3.7	15.1 ± 2.7	8
3000	12.6 ± 0.7	21.7 ± 1.6[b]	-	1.72 ± 0.16	-	-	13
8000	14.4 ± 0.8	23.3 ± 2.0	17.0 ± 2.3	1.62 ± 0.16	-	-	14

a : These numbers pertain to data from which coherent events have been excluded.

b : A small correction has been applied to the data of Ref. (13) for the fact that events with $n_s \leq 5$ had been excluded.

Table 2. Results of fits to Eq. (~~12a~~ 6a) together with χ^2 over number of degrees of freedom.

	p_L (GeV/c)	a	b	χ^2/DOF
pp	12.9 - 400	0.581 ± 0.004	-0.558 ± 0.007	12.6/11
$N_h \geq 0$	7.1 - 8×10^3	0.63 ± 0.02	-0.17 ± 0.06	5.8/6
$N_h \geq 9$	7.1 - ~~10^4~~ 8×10^3	0.49 ± 0.02	0.05 ± 0.08	12.8/5

Figure Captions

1. Dependence of $\langle n_s \rangle$ on N_h at 200 GeV/c, 23.4 GeV/c [10] and 7.1 GeV/c [10]. Line shown is the best fit representing Eq. (1) to our data with $N_h \geq 2$.

2. Dependence of R_{em} on p_{Lab}. The 200 GeV/c point represents the value obtained by using $\langle n_s \rangle = 13.0 \pm 0.2$ which is the weighted mean of the value obtained in this, Cuer et al. [7] and Babecki et al., [8] investigations. See text for explanations of the curves A, B, C and D.

3. Plot of the dispersion D vs. $\langle n_s \rangle$ for $N_h \geq 0$, $N_h \geq 9$ and pp data. The lines A, B and C are the best fits represented by Eq. (~~12a~~ 6a) to the data.

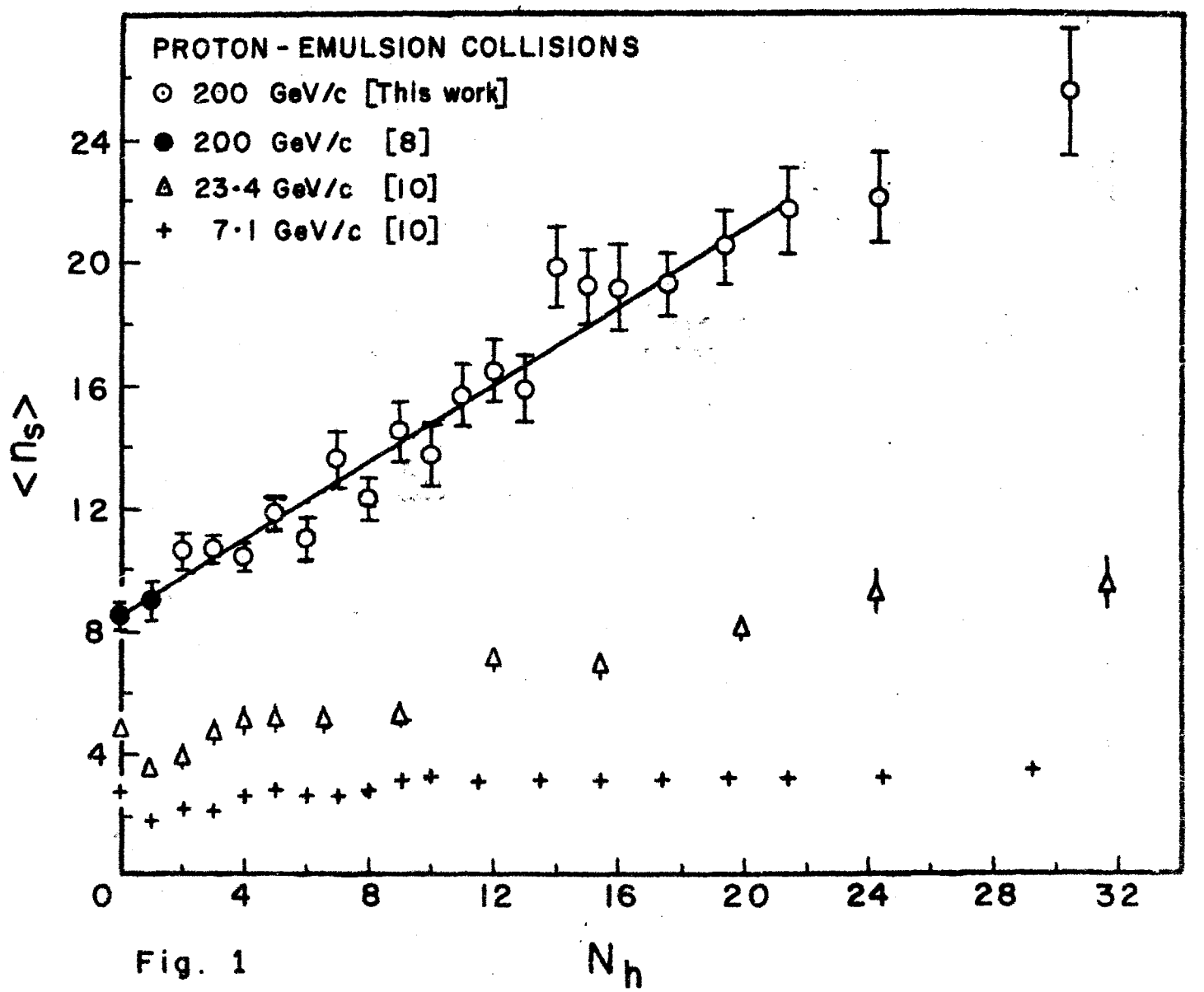
PROTON - EMULSION COLLISIONS
200 GeV/c [This work]
200 GeV/c [8]
23·4 GeV/c [10]
7·1 GeV/c [10]
<ns>
Nh
0
4
8
12
16
20
24
28
32

Fig. 1

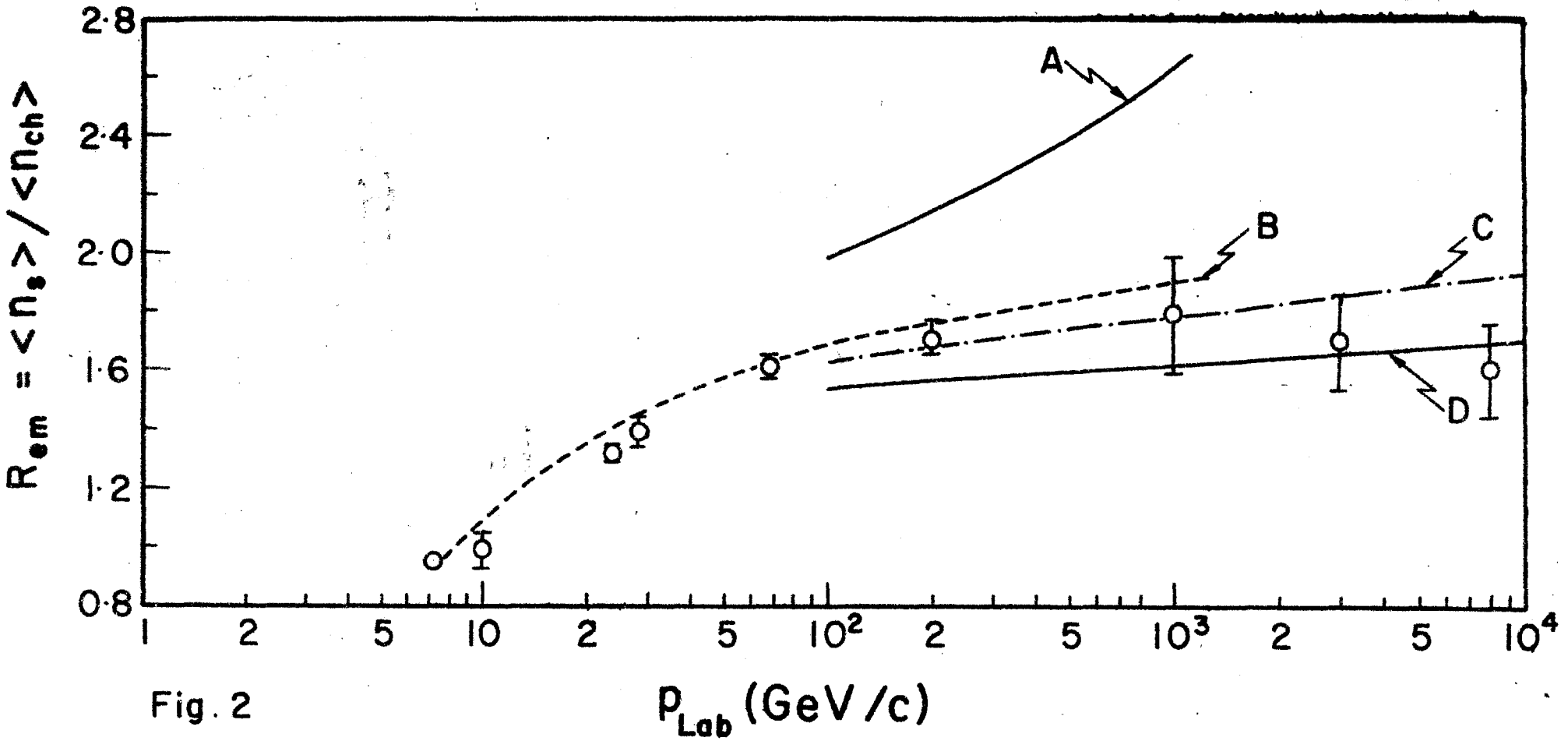
2·8
2·4
2·0
1·6
1·2
0·8
$R_{em} = \langle n_s \rangle / \langle n_{ch} \rangle$
A
B
C
D
1
2
5
10
2
5
10^2
2
5
10^3
2
5
10^4
P_{Lab} (GeV/c)

Fig. 2

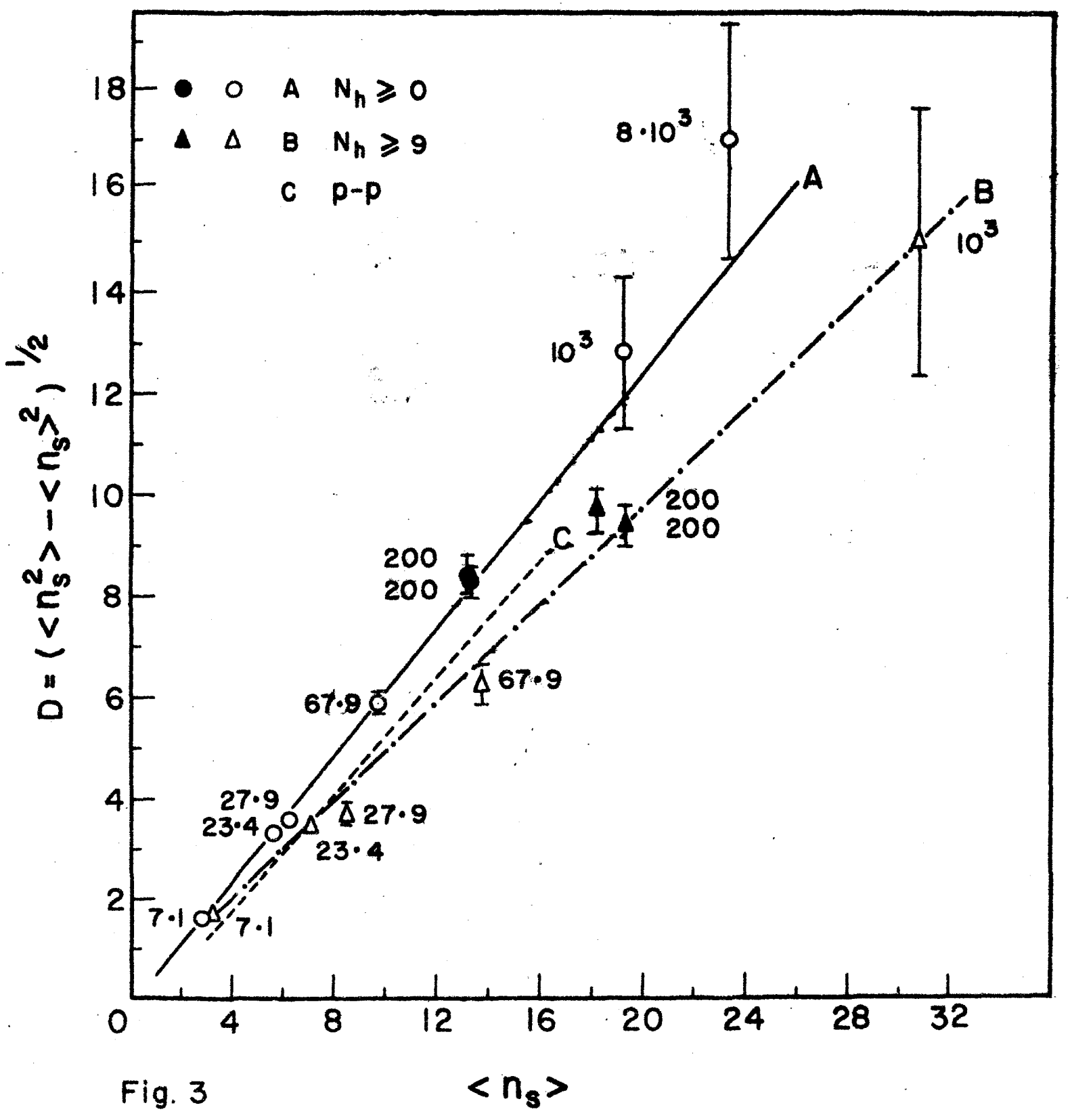
A $N_h \geq 0$
B $N_h \geq 9$
C p-p
$D = (\langle n_s^2 \rangle - \langle n_s \rangle^2)^{1/2}$
$\langle n_s \rangle$
A
B
C
$8 \cdot 10^3$
10^3
10^3
200
200
200
200
200
200
67·9
67·9
67·9
27·9
27·9
27·9
23·4
23·4
23·4
7·1
7·1
7·1
0
2
4
6
8
10
12
14
16
18
4
8
12
16
20
24
28
32

Fig. 3