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Remarks on e^+e^- Annihilation into Hadrons in Quantum Field Theory

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ABSTRACT

We show that the absorptive effect of particle production on e^+e^- annihilation reactions is to reduce the cross sections and give a constant value to $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$. In the approximation of summing leading terms, this effect simply gives a factor s^{-a} to all cross sections.

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In field theoretical studies of high-energy reactions, one finds that the production of slow particles in the CM system often plays an important role. Application to hadronic processes leads to the prediction of rising total cross sections.

Recently, the role of such slow particles (pionization products) is analyzed for electron-positron annihilation into hadrons.[1] The physical picture is that the photon with a large virtual mass creates a pair of elementary particles (or partons or quarks) which interact to produce many slow particles in a very short time. The cross section for producing n pairs of such particles is proportional to $(\ln s)^n/n!$, which leads, after summation, to the result

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \propto s^a \quad , \tag{1}$$

where a is a positive number.

Here we raise the question whether the creation of many slow particles can act back, in a way reminicent of unitarity, to cut down this factor $(\ln s)^n/n!$ and hence R. Physically, the question is this: Does the strong tendency for the original pair of elementary particles to create slow particles mean that it is unlikely for them to come out by themselves?

As an illustration, let us consider, for simplicity, an example in ϕ^3-theory. In the diagram of Fig. 1a, a scalar particle of large virtual mass creates a pair of scalar particles, which interact to produce another scalar particle, a pionization product. At large energies, the cross section for this production process is proportional to ln s, which arises from integration over the longitudinal momentum of the pionization product. This cross section is related to the imaginary part of the self-energy amplitude for the diagram illustrated in Fig. 1b. However, as one calculates this self-energy amplitude, one finds that its imaginary part has no ln s terms. Since this imaginary part is due to several cuts, the contribution of the three-particle cut represented by diagram 1a must be canceled by that of other cuts. It is easy to convince oneself that the only possible candidate must be the two-particle cut illustrated in Fig. 1c. This means that the form factor of a scalar meson must have a ln s term coming from the diagram at the left half of Fig. 1c, with a coefficient which is related to the cross section for the three particle creation process in Fig. 1a. We have thus seen that the probability of particle production modifies the form factor.

The situation remains exactly the same if we consider the case of fermions coupled to neutral gluons. In the same way, the probability of the pair creation process illustrated in Fig. 2a modifies the form factor of the fermion. More precisely, in order to take into account the absorptive effects of pair creation to the form factor, one must include, in addition to the diagram of Fig. 2b, the diagram of Fig. 2c. This latter diagram gives a ln s term to the form factor and is opposite in sign to the term corresponding to Fig. 2b.

The above considerations are readily generalized to the absorptive effects of n-pair creation. It becomes immediately clear that, for the purpose of taking into account the absorptive effects of n-pair creation illustrated in Fig. 3a, one must include the diagrams in Fig. 3b for the form factor.

The absorptive effects enter not only the form factor, but also all other reactions as well. For example, as the result of particle creation, the amplitude for producing 2 pairs of elementary particles is given by that of diagram 2a plus that of the diagrams such as the one illustrated in Fig. 4. The logarithmic factors of s coming from these absorptive effects are related to the logarithmic factors of the production cross section in such a way that the imaginary part of the photon propagator has no logarithmic factor of s . Thus the absorptive effects do act back to cut down the energy dependence and give a constant value for R at high energies.

It is now tempting to calculate the form factor as well as the amplitudes for various annihilation reactions, with the inclusion of the absorptive effects. Before we do so, we must reiterate the dangers of such calculations. First of all, the terms corresponding to successive chains of towers alternate in sign, and the sum of these terms is much smaller than each individual term. It is not at all clear that the procedure of summing leading terms is correct under this circumstance. Second, there are other diagrams which give logarithmic factors of s and have not been taken into consideration. In particular, the diagram in Fig. 5 for the form factor is well-known to be of ultra-violet divergence, and, after renormalization, gives a factor $\ln s$ to the form factor in the limit $s \to \infty$. This logarithmic factor of s is not due to longitudinal momentum integration, and we have no understanding of the role it plays in the physical picture.

If the method of summing the leading terms is nevertheless applied in spite of the above difficulties, we consider the multi-tower diagrams for each reaction, where a tower consists of one fermion loop and all permutations of the gluon vertices are included. The result is that the form factor or the amplitude for an annihilation reaction are equal to their lowest-order amplitudes (with no absorptive corrections) times a factor $s^{-\frac{1}{2}a}$. Thus, in the approximation of summing leading terms, the overall effect of absorption is simply reducing each amplitude by the same factor $s^{-a/2}$.

How does this change the results in Ref. 1? First of all, R approaches a constant as $s \to \infty$ instead of rising like s^a. Similarly, the expressions for cross sections in Ref. 1 should be divided by a factor s^a. Thus all the results, when scaled by σ, remain the same. For the purpose of clarity, we reiterate these results below: The average multiplicity is given by

$$\langle n \rangle \sim a \ln s \quad . \tag{2}$$

The single particle distribution $d\sigma/dp$ scaled by the cross section is given by

$$\frac{d\sigma}{dp}\Big/\sigma \rightarrow E^{-1}F(p) \quad , \tag{3}$$

valid when p is finite. In the limit when p is large compared to the mass scale but still much less than $\frac{1}{2}\sqrt{s}$, we have

$$F(p) \rightarrow \text{constant} \quad . \tag{4}$$

In the limit $p \ll 1$, we have

$$F(p) \propto p^2 \quad . \tag{5}$$

The inclusive distribution for two particles of momentum $\vec{p}_1$ and $\vec{p}_2$ respectively is given by

$$E_1 E_2 \frac{d\sigma}{dp_1 dp_2}\Big/\sigma \sim h(\vec{p}_1, \vec{p}_2) \quad . \tag{6}$$

When $\vec{p}_1$ and $\vec{p}_2$ are far apart in the momentum space, the function $h(\vec{p}_1, \vec{p}_2)$ factorizes into

$$h(\vec{p}_1, \vec{p}_2) \sim f(p_1)f(p_2) \quad . \tag{7}$$

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REFERENCE

[1]H. Cheng and J. Mandula, "e^+e^- Annihilation into Hadrons in Quantum Field Theory" (to be published).

FIGURE CAPTIONS

Fig. 1 Examples of diagrams in ϕ^3 theory. The leading contribution of diagram (a), used twice, to diagram (b) is cancelled by that due to the two-particle cuts shown in diagram (c).

Fig. 2 Similar examples in neutral vector gluon theory. The leading contribution of diagram (a), used twice, to the photon self-energy is cancelled by that due to diagrams (b) and (c).

Fig. 3 Effect of producing two pairs of pionization product.

Fig. 4 Absorptive effect in a production process.

Fig. 5 Example of a diagram that is unrelated to pionization but gives $\ln s$ at high energies.

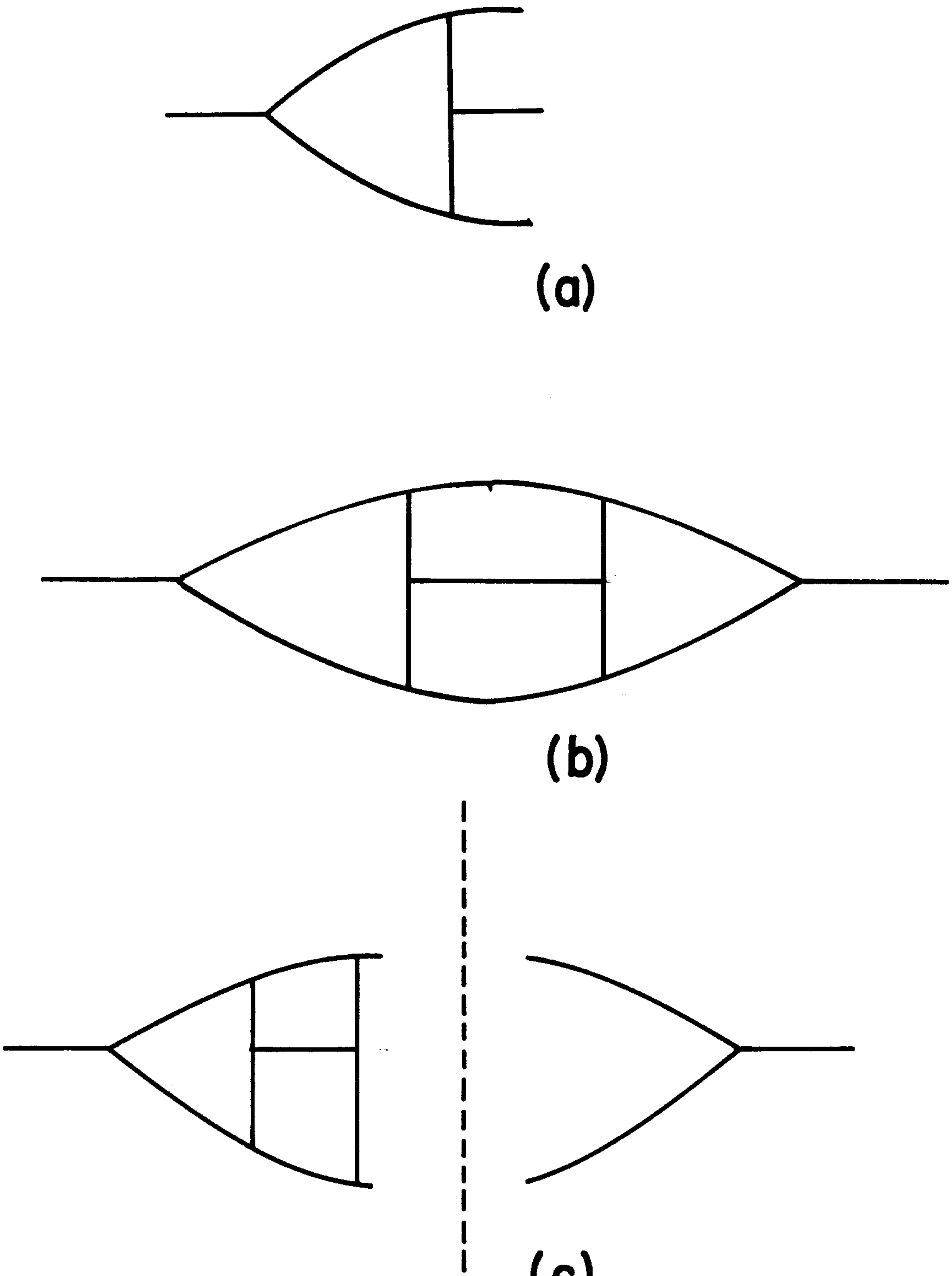


Fig. 1

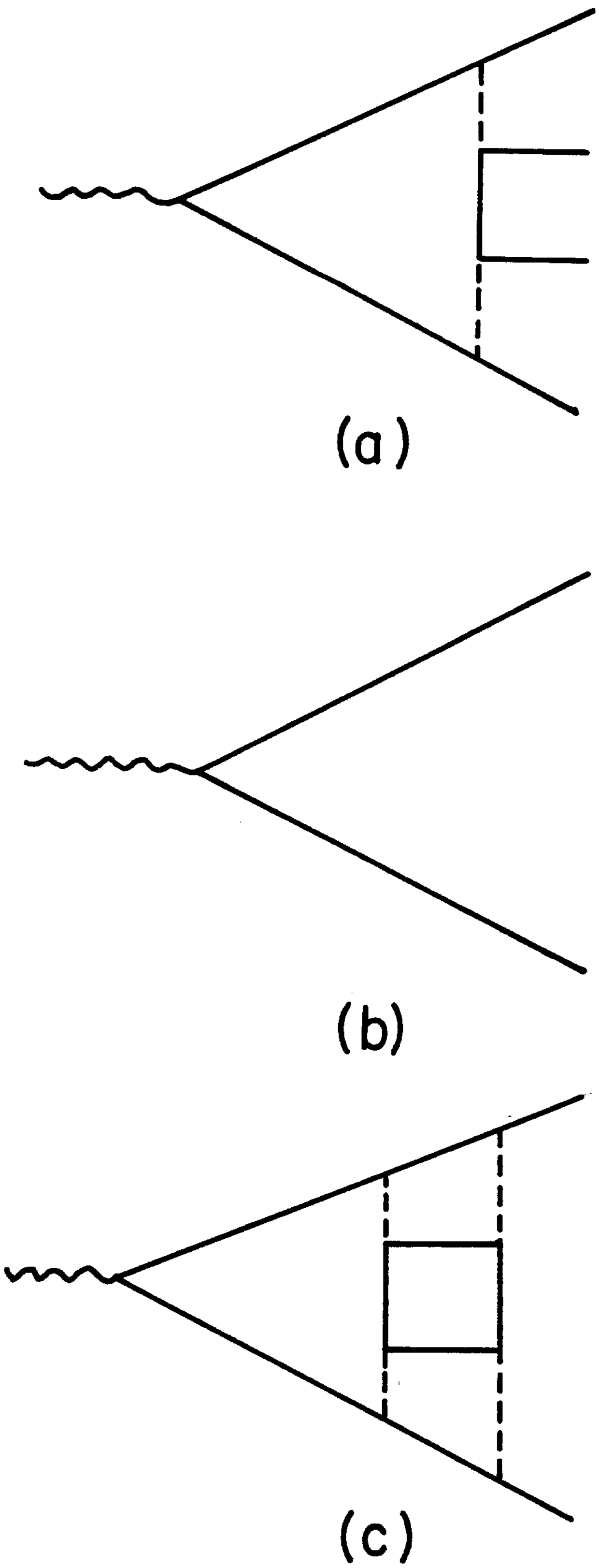
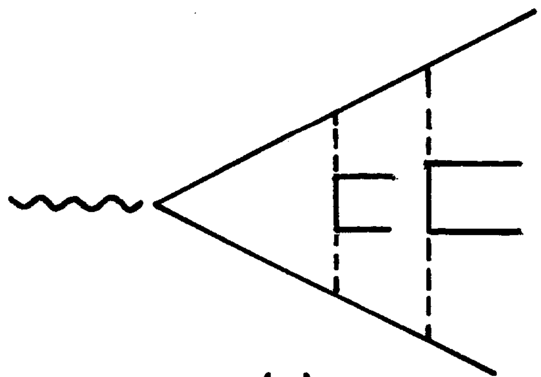
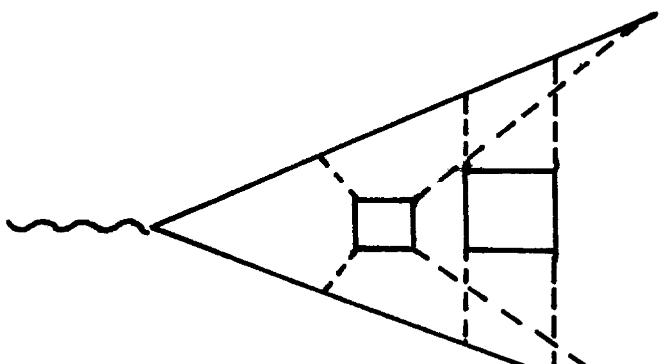


Fig. 2



(a)



(b)

Fig. 3

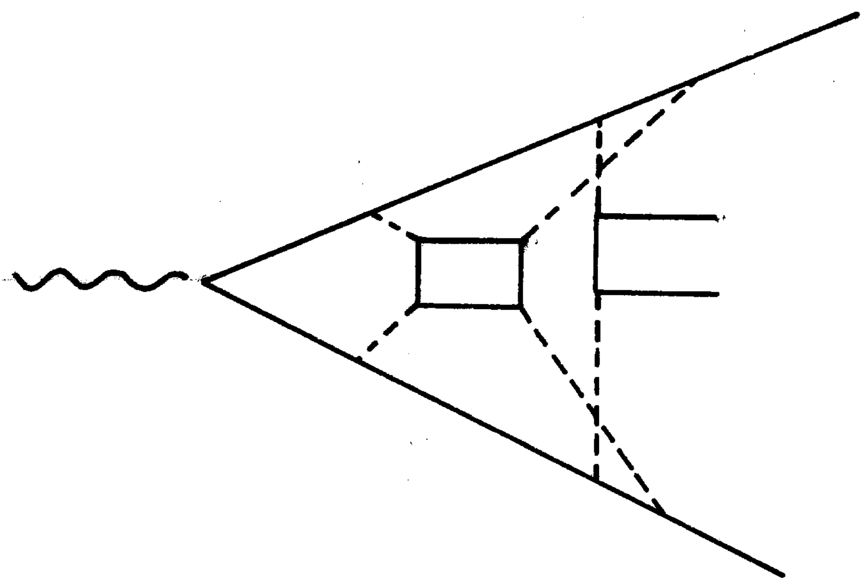


Fig. 4

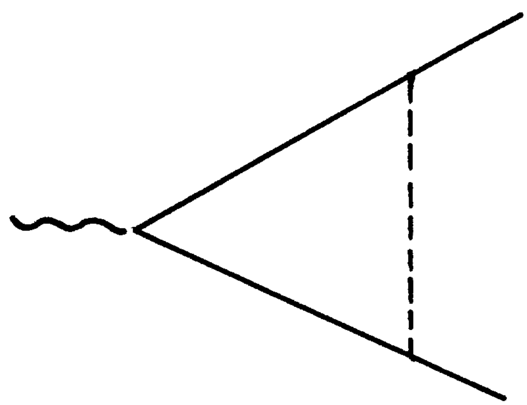


Fig. 5