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Pomerons and Fermions In Infrared Free and Almost Free Reggeon Field Theories

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ABSTRACT

A class of Reggeon field theories with pomerons and fermions is described and its infrared behavior deduced using the renormalization group. Unlike previous theories with fermions, the ones considered here possess fixed points which are completely infrared stable. Moreover, one of these theories has infrared free pomeron and fermion Green's functions. The mechanism required to remove the fermion parity partners is present in all these theories, including the one which is infrared free. In order to calculate the critical exponents, a generalization of the ϵ-expansion, based on analytic regularization, is introduced since the usual ϵ-expansion cannot be applied to these theories.

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I. INTRODUCTION

In constructing a complete Reggeon field theory for the description of hadronic processes, one must ultimately face the problem of the renormalization of ordinary Regge trajectories by the pomeron. In renormalizing the pomeron in the infrared region, t near zero and α_p near one, the effect of lower lying trajectories should be negligible and so field theories which concern themselves only with the renormalizatio of the pomeron do not include them. Nevertheless, such lower lying singularities do impose restrictions on an acceptable form for the pomeron, since the same pomeron which accounts for high energy diffraction scattering couples to ordinary Reggeons and must also renormalize them in an acceptable way.

What constitutes an acceptable Reggeon field theory? Aside from the obvious constraint that it should agree with experiment, we can identify at least three theoretically motivated criteria. First, as emphasized by White, ,[1] it would be nice if the bare propagators of the theory were poles. Since perturbation theory is expected to be valid for j and t positive and large (i.e., outside of the infrared region), this would ensure that the leading singularities in this region are poles, and we would be guaranteed to have constructed a solution to the t-channel discontinuity formulae. A second criterion, suggested by Abarbanel, is that the Reggeon field theory should be infrared free in the sense that the effective coupling constant(s) go to zero in the infrared limit. In this way, the machinery of the renormalization group can be thought of as a filter for selecting those field theories which almost reproduce themselves in the infrared limit.[4] A third criterion may be stated as follows: if there are fixed points in the renormalized parameter space of the complete field theory, at least some of these should be infrared stable. Clearly, the most desirable situation from this point of view would be one in which we would always approach one particular fixed point in the infrared limit, regardless of where in the parameter space we started. (one could perhaps present a fourth criterion here; namely, that the theory force the fermion parity partners to disappear,[5] but perhaps this falls more properly under the heading of confrontation with experiment.)

We emphasize that none of these criteria are rigorously necessary. The failure of a theory to satisfy any or all of them does not mean that the theory must be discarded - only confrontation with experiment can do that - but a theory possessing some of the above features does have an edge in theoretical attractiveness.

The Reggeon field theory which has been studies most extensively has a pomeron whose bare propagator is

$$\frac{i}{E+\alpha_0' q^2+i\epsilon}$$

where $E = 1 -$ (angular momentum), and $q^2 = t$. The pomerons interact via a bare three-point coupling. The infrared behavior of this pomeron theory has been described,[3] and the effect of this pomeron on linear boson trajectories[4] and on square-root type fermion trajectories[5] has also been examined. In most respects this theory is acceptable. It satisfies criterion numer one - all the bare propagators and poles. Criterion two is not satisfied, it is not infrared free, but the most unsatisfactory feature of the theory is that there appear to be no completely infrared stable fixed points. The instability appears when one couples fermions into the theory and can be traced to the fact that the slope parameters of the pomeron and fermion do not have the same dimensions. A modification of the fermion and/or pomeron propagators is evidently necessary to remove this instability, and that is the subject of the present paper.

There are a number of ways in which one may try to cure this instability. We have chosen to modify the pomeron propagator. For our bare pomeron propagator we take the form

$$i\left[E^2+a_0^2q^2\right]^{-s} ,$$

and we consider a theory with both this pomeron and a bare fermion pole. We have not examined the effect of this pomeron on boson trajectories. (The pure pomeron theory with $s = \frac{1}{2}$ has been discussed by Abarbanel, Bartels, and Dash.[6]) This theory has several attractive features. First, it is completely infrared stable for any $s \geq \frac{1}{2}$. For $s = \frac{1}{2}$, both the pomeron (as discussed in Ref. 6) and the fermion are infrared free. However, there is an important difference: there is apparently no fixed j-plane cut in the renormalized pomeron propagator, but there is a soft (logarithmic) cut in the renormalized fermion propagator. The presence of this cut is crucial in the fermion, since it provides the mechanism by which the fermion parity partners are forced to disappear. This difference can be traced to the fact that the bare fermion is a pole, while the bare pomeron is not.

An important technical comment should be made here. In the theories we are considering, the standard technique of the ϵ-expansion for calculating the critical exponents cannot be used; that is, there is no dimension of space-time in which all the coupling constants of the theory are dimensionless. To overcome this problem we have developed a generalization of the ϵ-expansion which uses dimensional and analytic regularization simultaneously, and lets us find and expand about a fully scale invariant theory. The technique will be described more fully later.

The outline of this paper is as follows: In Section II, we discuss the field theory, present the Feynman rules, and set up the renormalization conditions. Section III is devoted to a derivation of the renormalization group equations, and a discussion of the fixed points and their stability properties. The infrared behavior of the renormalized Green's functions is the subject of Section IV. The mechanism for the removal of fermion parity doublets in our theories is also elucidated in this section. Finally, a general discussion and conclusions are presented in Section V.

II. THE FIELD THEORY AND RENORMALIZATION PROGRAM

We now want to set up the field theory and renormalization procedure for the problems we shall consider. The usual approach in this regard, is to write down a Lagrangian which expresses the interactions and energy-momentum relations of the particles of the theory. For our purposes, however, this formal device is rather awkward. The reason is that the "particles" of the theory are represented by momentum space propagators whose singularities are not poles. The bare pomeron propagator, for example, is

$$i\left[E^2+a_0^{\prime 2}q^2\right]^{-s} \tag{1}$$

where s is, in general, non-integer. To derive such a propagator from an action principle probably requires a non-local Lagrangian and generalized Euler-Lagrange equations. Rather than pursuing these problems here, we shall assume that a theory with such a propagator exists, and that the momentum space Feynman rules of the theory are like those which one could derive for $s = 1$. We also need to assume that renormalization is multiplicative in the usual way, as we shall discuss below.[7]

With these remarks in mind, let us proceed to describe the theory.

In addition to the pomeron whose propagator is given by Eq. (1), we also introduce a fermion trajectory. Its momentum space propagator is

$$i\left[E-\Delta_0+b_0'\hat{q}\right]^{-\sigma} \tag{2a}$$

which may be written as

$$i\left(E-\Delta_0+b_0'q\right)^{-\sigma}\Lambda^{+}+i\left(E-\Delta_0-b_0'q\right)^{-\sigma}\Lambda^{-} \quad . \tag{2b}$$

$\Lambda^{\pm}=\frac{1}{2}\left(\mp\frac{\hat{q}}{\sqrt{q^2}}\right)$ are the positive and negative parity fermion projection operators, $\hat{q}=-\gamma_{\perp}\cdot q_{\perp}$, and the two-dimensional vector q satisfies $q^2=q_{\perp}^2=u$. Notice that for $u<0$ q is purely imaginary. As usual in the Reggeon calculus, $E=1-$ (angular momentum) . For a detailed discussion of the meaning of this propagator when $t=1$, see Ref. 5 and references therein. Notice, finally, that the bare propagators (1) and (2) imply that the bare trajectories near $u=0$ are proportional to $\sqrt{u}$.

We choose to couple the pomeron and fermion together in the simplest way possible, namely, by introducing bare triple pomeron and fermion-fermion-pomeron couplings. Let ϕ be the pomeron field, and ψ the fermion field. ϕ contains both creation and destruction operators, while ψ has only destruction operators. The interaction Lagrangian then has the form

$$\mathscr{L}_{\pm}=\frac{-i\lambda_0\phi^3}{6}\quad\frac{-ir_0}{2}\psi^{+}\phi\psi \quad . \tag{3}$$

Notice that this Lagrangian will not generate any diagrams with closed fermion loops. This is quite reasonable since the fermion intercept is far below that of the pomeron, and such closed loops will not be an important contribution to the pomeron self-energy in the infrared region. As pointed out elsewhere,[4, 5] this has the consequence that the renormalization of the pomeron decouples from the lower lying trajectories.

The objects of the theory with which we shall be concerned are the connected, amputated, one particle irreducible Green's functions, $\Gamma^{(k,n)}$. k is the number of incoming plus outgoing fermions (= twice the number of incoming fermions, since fermion number is conserved), and n is the number of incoming plus outgoing pomerons. Because of the relativistic nature of the pomeron propagator, no distinction need be made between incoming and outgoing pomerons. The amputated Green's functions, $\Gamma^{(k,n)}$, in momentum space are related to the unamputated connected Green's functions, $G_c^{(k,n)}$, by

$$\Gamma^{(k,n)}\left(E_1, q_1 \ldots E_k, q_k, E_{k+1}, q_{k+1} \ldots E_{k+n}, q_{k+n}\right)$$

$$= \prod_{j=1}^{k} G^{(2,0)}\left(E_j, q_j\right) \prod_{\ell=k+1}^{n+1} G^{(0,2)}\left(E_\ell, q_\ell\right) G_c^{(k,n)}\left(E_1, q_1 \ldots E_{k+n}, q_{k+n}\right). \quad (4)$$

The rules for calculating in unrenormalized perturbation theory the momentum space Green's functions, $G^{(k,n)}$, in D space and one time dimension are

1. Draw all topologically distinct graphs with $k/2$ incoming fermions and n pomerons. There are no arrows on the pomeron lines.

2. Integrate around each closed loop $\int d^D q dE$.

3. For each three pomeron vertex multiply by $\dfrac{-i\lambda_0}{(2\pi)^{\frac{D+1}{2}}}$.

4. For each fermion-fermion-pomeron vertex multiply by $\dfrac{-ir_0}{(2\pi)^{\frac{D+1}{2}}}$.

5. For each pomeron line insert a factor $i\left[E^2+a_0'^2 q^2+ie\right]^{-s}$.

6. For each fermion line insert a factor $i\left[E-\Delta_0+b_0'\hat{q}+ie\right]^{-\sigma}$.

7. For each two pomeron loop multiply by $\frac{1}{2}$.

8. Conserve energy and momentum at each vertex.

Before proceeding to a discussion of the renormalization program, it is useful to perform some naive dimensional analysis on the bare quantities which appear in our theory. When these quantities are renormalized, their dimensions will not, of course, change. Using the fact that the action

$$A = \int d^D x dt$$

is dimensionless, and the observation that the momentum space propagator is given by a space-time integral over a product of two fields, we can easily derive the following results. ([ ] means "dimension of")

$$[\phi] = E^{\frac{1}{2}-s} q^{\frac{D}{2}} ,$$

$$[\psi] = E^{\frac{1-t}{2}} q^{\frac{D}{2}} ,$$

$$\left[\lambda_0\right] = E^{3s-\frac{1}{2}} q^{-\frac{D}{2}} \quad ,$$

$$\left[r_0\right] = E^{2s+\frac{t}{2}-\frac{1}{2}} q^{-\frac{D}{2}} \quad ,$$

$$\left[a_0'\right] = \left[b_0'\right] = E k^{-1} \quad . \tag{5}$$

We now want to renormalize our theory. To define the renormalized parameters of the theory, we adopt the following renormalization conditions

$$i\,\Gamma_R^{(0,2)}\Big|_{\substack{E=0 \\ q=0}} = 0 \quad , \tag{6a}$$

$$\frac{\partial}{\partial E^2}\, i\,\Gamma_R^{(0,2)}\Big|_{\substack{E^2=-E_N \\ \underset{\sim}{q}=0}} = (-E_N)^{2s-2} \quad , \tag{6b}$$

$$\frac{\partial}{\partial q^2}\, i\,\Gamma_R^{(0,2)}\Big|_{\substack{E^2=-E_N \\ \underset{\sim}{q}=0}} = s^2(-E_N)^{2s-2} \quad , \tag{6c}$$

$$i\,\Gamma_R^{(2,0)}\Big|_{\substack{F=0 \\ \underset{\sim}{q}=0}} = 0 \quad , \tag{7a}$$

$$\frac{\partial}{\partial E}\, i\,\Gamma_R^{(2,0)}\Big|_{\substack{F=-E_N \\ \underset{\sim}{q}=0}} = t(-E_N)^{t-1} \quad , \tag{7b}$$

$$\frac{\partial}{\partial \hat{q}} \, i\, \Gamma_R^{(2,0)} \Bigg|_{\substack{E=-E_N \\ \underset{\sim}{q}=0}} = t(-E_N)^{t-1} \, b \quad , \tag{7c}$$

$$i\, \Gamma^{(0,3)} \Bigg|_{\substack{E=-E_N \\ E_2=-RE_N \\ E_3=-(1-R)E_N}} = \frac{\lambda}{(2\pi)^{\frac{D+1}{2}}} \quad , \tag{8}$$

$$i\, \Gamma_R^{(2,1)} \Bigg|_{\substack{F_1=-E_N \\ F_2=-(1-R)E_N \\ E_3=-RE_N}} = \frac{r}{(2\pi)^{\frac{D+1}{2}}} \quad . \tag{9}$$

In Green's functions with fermion lines, we have shifted the bare mass away by defining $F=E-\Delta_0$. This can be done in the calculation of all Feynman diagrams because of the absence of fermion loops, as described above. The operator in (7c) is define as

$$\frac{\partial}{\partial \hat{q}} \equiv \frac{1}{D^2} \, T_R(\gamma \cdot \frac{\partial}{\partial q}) \quad . \tag{10}$$

Conditions (6a) and (7a) assure that the pomeron and fermion trajectories have their required intercepts. Conditions (6b) and (7b) determine the pomeron and fermion wave function renormalization constants, Z and W , since

$$\Gamma_R^{(k,n)} = Z^{n/2} \, W^{k/2} \, \Gamma_u^{(k,n)} \tag{11}$$

Equations (6c) and (7c) define the renormalized pomeron and fermion slopes respectively, and finally, Eqs. (8) and (9) define the renormalized triple pomeron and fermion-fermion-pomeron coupling constants.[8]

We want to digress here for a moment and anticipate a development which we shall discuss in more detail later. The reader familiar with the renormalization group and the ϵ-expansion will recognize from (5) that when $D = 6s-1$ the triple pomeron coupling constant is dimensionless, while the fermion-fermion-pomeron coupling constant is dimensionless when $D = 4s+\sigma-1$. In general, there is no value of D which satisfies both these equations, and so it is not clear how to use the ϵ-expansion to calculate critical exponents in this case. However, there is a line in the three dimensional s, σ, D Space along which both these conditions are satisfied, and the whole theory scale invariant. One can solve the theory at a point on this line and expand in a small parameter, which plays the role of ϵ, to the theory of physical interest. In particular, if we take $\sigma = 1$, as we shall, for simplicity, then the theory is scale invariant for $D = 2$, $s = \frac{1}{2}$. To get to a general s, we make a δ-expansion: $s = \frac{1}{2} + \delta$. The fact that for $\sigma = 1$ scale invariance obtains at $D = 2$, which happens to be the physical number of dimensions, is a rather fortuitous feature of problem we are considering and means that by varying δ we sample a range of physical Reggeon field theories. While this particular feature may not occur in all cases, such a generalized ϵ-expansion, which incorporates both dimensional and analytic regularization has a wide range of application to critical phenomena, and is one of the major results of this paper. We shall discuss it again, below.

III. SCALING ARGUMENTS AND THE RENORMALIZATION GROUP

In this section we will set up and solve the renormalization group equations for our theory. For brevity, we shall omit some steps in the derivation. The reader interested in filling in the details may refer to references 3, 4 and 5, where similar arguments are presented more fully for other theories.

Consider the relation (11) between the renormalized and unrenormalized Green's functions. The renormalization group equations is based on the observation that the unrenormalized theory, having no knowledge of the renormalization point cannot depend on E_N:

$$E_N \frac{d}{dE_N}\bigg|_{\text{Bare}} \Gamma_u^{(k,n)} = 0 \quad , \tag{12}$$

where the bare unrenormalized parameters of the theory are held fixed. To use this equation in a meaningful way, we proceed in three steps. First, we use Eq. (11) to write (12) in terms of Z, W, and Γ_R, expressing Γ_R as a function of the renormalized parameters. Next, we replace the dimensional renormalized parameters of the theory with as many independent dimensionless parameters as we can. Finally, remembering that these renormalized parameters are functions of E_N, we carry out the differentiation indicated in (12). The result of all this is:

$$\left\{E_N \frac{\partial}{\partial E_N} + \zeta \frac{\partial}{\partial a} + \Theta \frac{\partial}{\partial \rho} + \beta_p \frac{\partial}{\partial x} \; \beta_f \frac{\partial}{\partial y} - \frac{n}{2}\gamma_p - \frac{k}{2}\gamma_f\right\}$$

$$\Gamma_R^{(k,n)}(E_i, q_i, a, \rho, x, y, E_N) = 0 \quad , \qquad (13)$$

where

$$\zeta \equiv E_N \frac{\partial a}{\partial E_N}\bigg|_{\text{Bare}} \quad ,$$

$$0 = E_N \frac{\partial \rho}{\partial E_N}\bigg|_{\text{Bare}} \quad ,$$

$$\beta_p = E_N \frac{\partial x}{\partial E_N}\bigg|_{\text{Bare}} \quad ,$$

$$\beta_p = E_N \frac{\partial y}{\partial E_N}\bigg|_{\text{Bare}} \quad ,$$

$$\gamma_p = E_N \frac{\partial}{\partial E_N} \ln Z\bigg|_{\text{Bare}} \quad ,$$

$$\gamma_f = E_N \frac{\partial}{\partial E_N} \ln W\bigg|_{\text{Bare}} \quad , \qquad (14)$$

and the dimensionless parameters, ρ, x, and y are defined as

$$x = \frac{\lambda}{a^{D/2}} E_N^{\frac{D+1}{2} - 3s} ,$$

$$y = \frac{r}{a^{D/2}} E_N^{\frac{D+1}{2} - \sigma - s} ,$$

$$\rho = \frac{b}{a} . \tag{15}$$

Notice that since pomeron renormalization decouples from the fermion, β_p is a function only of x, while Θ and β_f are, a priori functions of x, y, and ρ.

Now, we are interested in the behavior of the Γ_R when the energies, E_i, are scaled to zero. To learn about this limit, we note first that

$$\left[\Gamma_R^{(k,n)}\right] = E^{n\left(\frac{1}{2}+s\right)+\frac{k}{2}\left(1+\sigma\right)-1} q^{(n+k-2)\frac{D}{2}} , \tag{16}$$

so that we can write $\Gamma_R^{(k,n)}$ as

$$\Gamma_R^{(k,n)}(E_i, q_i, b, \rho, x, y, E_N) =$$

$$= E_N^{n\left(\frac{1}{2}+s+\frac{D}{2}\right)+\frac{k}{2}(1+\sigma+D)-D-1} a^{(2-n-k)\frac{D}{2}} \Psi\left(\frac{E_i}{E_N}, \frac{aq_i}{E_N}, \rho, x, y\right), \tag{17}$$

where Ψ is a dimensionless function of dimensionless variables.

From (17) it is a simple matter to see that

$$\Gamma_R^{(k,n)}(\xi E_i, q_i, a, \rho, x, y, E_N) = \xi^{n\left(\frac{1}{2}+s\right)+\frac{k}{2}(1+\sigma)-1}\,\Gamma_R^{(k,n)}\left(E_i, q_i, \frac{a}{\xi}, \rho, x, y, \frac{E_N}{\xi}\right), \tag{18}$$

which implies that

$$\left[\xi\frac{\partial}{\partial\xi} + a\frac{\partial}{\partial a} + E_N\frac{\partial}{\partial E_N} + 1 - n(\tfrac{1}{2}+s) - \frac{k}{2}(1+\sigma)\right]\Gamma_R^{(k,n)}(\xi E_i, q_i, a, \rho, x, y, E_N) = 0\,. \tag{19}$$

Finally, using (19) in (13), we have

$$\left\{-\xi\frac{\partial}{\partial\xi} + (\zeta - a)\frac{\partial}{\partial a} + \Theta\frac{\partial}{\partial\rho} + \beta_p\frac{\partial}{\partial x} + \beta_f\frac{\partial}{\partial y}\right.$$

$$\left. -\frac{n}{2}\gamma_p - \frac{k}{2}\gamma_f + n(\tfrac{1}{2}+s) + \frac{k}{2}(1+\sigma) - 1 \quad \Gamma_R^{(k,n)}(\xi E_i, q_i, a, \rho, x, y, E_N)\right\} = 0\,, \tag{20}$$

whose solution is

$$\Gamma_R^{(k,n)}(\xi E_i, q_i, a, \rho, x, y, E_N)$$

$$= \Gamma_R^{(k,n)}(E_i, q_i, \tilde{a}(-t), \tilde{\rho}(-t), \tilde{x}(-t), \tilde{y}(-t), E_N)$$

$$\times \exp\int_{-t}^{0} dt'\left[n(\tfrac{1}{2}+s)+\frac{k}{2}(1+\sigma)-1-\frac{n}{2}\gamma_p\left(\tilde{x}(t')\right) - \frac{k}{2}\gamma_f\left(\tilde{x}(t'), \tilde{y}(t'), \tilde{\rho}(t')\right)\right], \tag{21}$$

where

$$\frac{dx(t)}{dt} = -\beta_p\,,$$

$$\frac{dy(t)}{dt} = -\beta_f \quad ,$$

$$\frac{d\rho(t)}{dt} = -\Theta \quad ,$$

$$\frac{da(t)}{dt} = a - \zeta \quad ,$$

and

$$t = \ln \xi \quad . \tag{22}$$

As usual, we want to calculate the functions, β_p, β_f, γ_p, γ_f, Θ and ζ in perturbation theory, and look for the fixed points, hoping that the values of the effective coupling constants at the fixed point are small so that the approximation is self-consistent. Since we work to lowest non-trivial order in perturbation theory, we must calculate the graphs shown in Figs. 1 and 2. Only the graphs of Fig. 1 contribute to the functions β_p, γ_p, and ζ, while all the graphs of Figs. 1 and 2 contribute, in principle, to β_f, γ_f, and Θ. For simplicity, we will specialize our considerations to the case $\sigma = 1$. The class of theories we will be left with will be sufficiently general to make all the physical points we wish to make. (Notice however, that by setting $\sigma = 1$, we have reduced the three dimensional (D, s, σ) space to a plane, and have limited ourselves to expanding the theory about $D = 2$, $s = \frac{1}{2}$, as discussed above.)

Using the Feynman rules of the last section, and the relations (14), it is straightforward to calculate the auxilliary functions, (22).

They are presented in the Appendix for arbitrary D and s. Here we present them keeping only terms which are lowest order in $\delta = s - \frac{1}{2}$:

$$-\frac{d\tilde{g}}{dt} = \beta_p = \left(\frac{D+1}{2} - 3s\right)\tilde{g} + \frac{\tilde{g}^3}{2\pi^2} = -3\delta\,\tilde{g} + \frac{\tilde{g}^3}{2\pi^2} \quad , \tag{23}$$

$$-\frac{d\tilde{h}}{dt} = \beta_f = \left(\frac{D-1}{2} - s\right)\tilde{h} + c\tilde{h}^3 = -\delta\tilde{h} + c\tilde{h}^3 \quad , \tag{24}$$

$$-\frac{d\tilde{\rho}}{dt} = \Theta = \frac{2\tilde{\rho}\tilde{h}^2}{3\pi^2} \quad , \tag{25}$$

$$\zeta = 0 \quad , \tag{26}$$

$$\gamma_f = \frac{-\tilde{h}^2}{6\pi^2} \quad , \tag{27}$$

$$\gamma_p = -\tilde{g}^2\frac{3\delta}{\sqrt{\pi}} \quad . \tag{28}$$

According to the usual signature analysis, the two coupling constants should be purely imaginary, so we have for convenience defined $g = ix$ and $h = iy$. g and h are now real. c is a positive real constant, a representation of which is given in the appendix.

Before describing the fixed points associated with the zeros of the three functions (23) - (25), we want to make some technical comments. From (23), we see that β_p depends only on g , the triple pomeron coupling and not on h or ρ . This was expected since pomeron renormalization decouples from the fermion. Notice, however, the rather surprising result (24) that β_f depends only on h , not on g . The reason is that in the scale invariant limit, $\delta = 0$, graphs which are usually thought to be (logarithmically) divergent in ordinary renormalizable field theories are convergent when propagators are raised to fractional power, as is our pomeron. In particular, graphs (1a) and (2b) are finite at $\delta = 0$, which accounts for the absence of terms proportional to g^2h and hg^2 to lowest order in δ in β_f.

This effect is also the source of the factor δ on the right hand side of (28). For a general s and D, Eq. (28), which in lowest order is derived from Fig. (1a) has the general structure

$$\gamma_p \sim g^2\left(3s - \frac{D+1}{2}\right)\,\Gamma\left(1+2s - \frac{D+1}{2}\right) \quad ,$$

so only if $s = 1, 2, 3 \ldots$ ($s = 1$ corresponds to ordinary ϕ^3 theory) will the coefficient of g^2 be finite when the parameter, $3s - \frac{D+1}{2}$, which measures the distance from the scale invariant theory goes to zero. This appears to be a fairly general property of theories whose propagators are not necessarily poles, and charges considerably the set of graphs which are summed by the renormalization group in lowest order.[9] For comparison, the reader's attention is drawn to Eq. (27). In this equation, which comes from Fig. 2a, the coefficient of h^2 is finite as $\delta \to 0$, because the fermion (although not the pomeron) is a pole. Finally, we notice that $\zeta = 0$, since, as pointed out in Ref. 6, the speed of light does not get renormalized in the theory we are discussing. (However, the effective speed of light does go to ∞ in the infrared limit, as we shall see.)

In alanyzing (23) - (25) for fixed points, it is convenient to consider the cases $\delta = 0$ and $\delta \neq 0$ separately. Suppose $\delta \neq 0$. Then it is easy to see that there are infrared $(t \to -\infty)$ stable zeros of β_p, β_f, and ρ at

$$g^2 = 6\pi^2\delta \ , \ h^2 = \frac{\delta}{c} \ , \ \rho = 0 \ . \tag{29}$$

For $\delta \neq 0$, the fixed points with h and/or $g = 0$ are not infrared stable from all directions in the (h, g, ρ) parameter space. Assuming that the values of the renormalized parameters are sufficiently close to the fixed point values (29), we can linearize (23) - (25) and solve for the functions $\tilde{g}$, $\tilde{h}$, and $\tilde{\rho}$. In the region of the infrared stable fixed point, we have

$$\tilde{g}(t) = g e^{-6\delta t} \pm \pi\sqrt{6\delta}\left(1 - e^{-6\delta t}\right) \ ,$$

$$\tilde{h}(t) = h e^{-2\delta t} \pm \sqrt{\delta / c}\left(1 - e^{-2\delta t}\right) \ ,$$

$$\tilde{\rho}(t) = \rho e^{-\frac{2\delta t}{3\pi^2 c}} \ . \tag{30}$$

Suppose now that $\delta = 0$. Then, the system of Eq. (23) - (25) posses a fixed point at

$$\tilde{g} = 0 \ , \ \tilde{h} = 0 \ , \ \tilde{\rho} = \rho_\infty \ . \tag{31}$$

It is an easy matter to solve for $\tilde{g}(t)$, $\tilde{h}(t)$, and $\tilde{\rho}(t)$, and we find

$$\tilde{g}^2(t) = \frac{\pi^2 g^2}{[illegible]} \xrightarrow[t \to \infty]{} \frac{g^2}{t} \ ,$$

$$\tilde{h}^2(t) = \frac{h^2}{1+2ct} \xrightarrow[t\to\infty]{} \frac{h^2}{t} \quad ,$$

$$\tilde{\rho}(t) = \frac{\rho}{(1+2ct)^{h^2/3\pi^2 c}} \xrightarrow[t\to\infty]{} \sim \frac{1}{t^{h^2/3\pi^2 c}} \quad . \quad (32)$$

So the fixed point in (31) has $\rho_\infty = 0$, and is infrared stable. The functions $\tilde{g}$, $\tilde{h}$, and $\tilde{\rho}$ given in (30) and (32) can be used via (21) to learn about the infrared behavior of the Γ_R. This is the task to which we now turn.

IV. INFRARED BEHAVIOR OF THE GREEN'S FUNCTIONS.

Because they are qualitatively so different, we will consider the two cases $\delta \neq 0$ and $\delta = 0$ separately. Since our main concern here is the behavior of the renormalized fermion propagator $\Gamma_R^{(2,0)}$, we will concentrate on that, and on the pomeron propagator $\Gamma_R^{(0,2)}$.

First consider the case $\delta \neq 0$. From (21) (27), (28) and (29), we find at the fixed point

$$\Gamma_R^{(k,n)}(\xi E_i, q_i, a, \rho, g, h, E_N)$$

$$= \xi^{n\left(1+\delta+a\pi^{3/2}\delta^2\right)+k\left(1+\frac{\delta}{12c\pi^2}\right)-1} \Gamma_R^{(k,n)}(E_i, q_i, \tilde{a}(-t), \tilde{\rho}(-t), \tilde{g}(-t), \tilde{h}(-t), E_N) \; . \quad (33)$$

Notice that, although, at the fixed point $\gamma_p \sim \delta^2$, the scaling dimension of pomeron Green's functions still contains a term proposition to δ coming from the term $h(\frac{1}{2}+s)$ in Eq. (21).[10] Of course, if the renormalized parameters g, h and ρ do not have fixed point values there will be other terms in (33), which are less important as $\xi \to 0$.

Recognizing that $\tilde{g}^2(\infty)$, $\tilde{h}^2(\infty) \sim \delta$, we can calculate the right hand side of (33) to lowest order in δ. For the pomeron and fermion propagators, this gives

$$i\Gamma_R^{(2,0)}(\xi E, \ldots) \propto \xi^{1+2\delta+18\pi^{3/2}\delta^2}\left[A+\frac{a^2}{\xi^2}q^2\right]^{\frac{1}{2}+\delta} , \qquad (34)$$

$$i\Gamma_R^{(2,0)}(\xi F, \ldots) \propto \xi^{1+\frac{\delta}{6c\pi^2}}\left[B+a\rho\xi^{\frac{2\delta}{3\pi^2 c}-1}\hat{q}\right]$$

$$= \xi^{1+\frac{\delta}{6c\pi^2}}\left\{\left[B+a\rho\xi^{\frac{2\delta}{3\pi^2 c}-1}q\right]\Lambda^+ + \left[B-a\rho\xi^{\frac{2\delta}{3\pi^2 c}-1}q\right]\Lambda^-\right\} , \qquad (35)$$

where A and B are constants, and where we have used $\tilde{b}(-t) = \tilde{\rho}(-t)\tilde{a}(-t)$. $\tilde{a}(-t)$ may be thought of as the effective speed of light, (as least for the pomeron), and from (22) and (26) is given by

$$a(1t) = ae^{-t} = \frac{a}{\xi} , \qquad (36)$$

which goes to ∞ , as provided. As we see from (34), this form is necessary to ensure that the q^2 dependence of the pomeron trajectory is still of the square root type.

Let us turn now to the renormalized fermion propagator (35). First we notice the presence of the j-plane cut which we have come to expect in non-infrared free theories. From the last form of the function given in (35), it is a simple matter to analyze the positions of the zeros of the positive and negative parity parts of the inverse propagator. Writing $q^2 = u = ye^{i\phi}$, we see that the positions of the zeros are given by

$$F_+ = Ny^{\frac{1}{2}} e^{i\frac{\phi}{2}\frac{1}{1-z}} ,$$

$$F_- = Ny^{\frac{1}{2}} e^{i\left(\frac{\phi}{2}-\pi\right)\frac{1}{1-z}} , \tag{37}$$

where N is a real constant, and $z = \frac{2\delta}{3\pi^2 c} > 0$. For small enough δ, $z < \frac{1}{2}$, and the positions of the singularities of $G_R^{(2,0)}$ in the F-plane are illustrated in Fig. 3. As in the theory described in Ref. 5, both parity poles are on the physical sheet (in complex conjugate positions) for $u < 0$, but for $u > 0$, one of them (in this case, the negative parity pole) moves under the cut and off the physical sheet. If $1 > z > \frac{1}{2}$, neither pole will be on the physical sheet for $u < 0$, but there will still be one pole on the physical sheet for $u > 0$. Of course, this latter situation must be taken <u>cum grano salis</u>, since for such a large δ higher order terms are undoubtedly quite important. Finally, we remark that there are other poles on other sheets of the cut F-plane, but they are quite far away, and do not change our argument.[11]

Let us now turn to the case $\delta = 0$. Using (21), (27), (28), and (32), we have

$$\Gamma_R^{(k,n)}(\xi E_i, q_i, a, \rho, g, h, E_N)$$

$$= \xi^{n+k-1}\left[1-2c\ \ln\xi\right]^{\frac{-kh^2}{24\pi^2 c}} \Gamma_R^{(k,n)}\left(E_i, q_i, \tilde{a}(1t), \tilde{\rho}(-t), \tilde{g}(-t), \tilde{h}(-t), E_N\right) . \tag{38}$$

Notice here the important difference between Green's functions with pomerons and with fermions: For each external fermion leg there is a logarithmic cut in the F-plane, but such cuts are not generated by pomeron fields. This difference is due to the presence of the factor δ in (28) which is absent in (27), as discussed above.

Evaluating Γ_R on the right hand side of (38) to lowest order in $\tilde{g}^2$ and $\tilde{h}^2$, we find, using (32), the infrared behavior of the renormalized fermion and pomeron propagators:

$$i\,\Gamma_R^{(0,2)}(\xi E, \ldots) \propto \xi\left[A' + \frac{a^2}{\xi^2} q^2\right] , \tag{39}$$

$$i\,\Gamma_R^{(2,0)}(\xi F, \ldots) \propto \xi(-\ln\xi)^{-v}\left\{\left[B' + \frac{a\rho}{\xi}(-\ln\xi)^{-4v} q\right]\Lambda^+ + \left[B' - \frac{a\rho}{\xi}(-\ln\xi)^{-4v} q\right]\Lambda^-\right\} . \tag{40}$$

where $v = \dfrac{h^2}{12\pi^2 c} > 0$, and A' and B' are constants. Notice that while both these Green's functions are infrared free there are logarithmic modifications to the fermion propagator and trajectory which are absent in the pomeron.

The implications of this form for the renormalized pomeron Green's functions has already been discussed in Ref. 6, so let us concentrate on the fermion propagator. We want to find the positions of the zeros of the coefficients of $\Lambda^{\pm}$ in the cut F-plane as a function of u . Writing

$F = fe^{i\theta}$, it is easy to see that for small f, the trajectories of the positive and negative parity pieces of the fermion propagator satisfy

$$f_+ R_+ = N' y^{\frac{1}{2}} \quad , \tag{41a}$$

$$\theta_+ + 4v \sin^{-1}\left[\frac{-\theta_+}{R_+}\right] = \frac{\phi}{2} \quad , \tag{42b}$$

and

$$f_- R_- = N' y^{\frac{1}{2}} \quad , \tag{42a}$$

$$\theta_- + 4v \sin^{-1}\left[\frac{-\theta_-}{R_-}\right] = \frac{\phi}{2} - \pi \quad , \tag{42b}$$

where

$$R_\pm = \left[\ln^2 f_\pm + \theta_\pm^2\right]^{\frac{1}{2}} \quad ,$$

$$u = ye^{i\phi} \quad ,$$

and N' is a constant.

The analytic structure of the fermion propagator, $G^{(2,0)}$, in the F-plane is shown in Fig. 4. The cut generated by the factor $(-\ln F)^v$ lies along the negative real axis. Now, for small $|u|$, (41) and (42) tell us that $f_\pm$ is small, and $R_\pm$, which is proportional to $|-\ln f_\pm|$, is large. With this in mind, we can deduce from small $|u|$ the values $\theta_\pm$ when $u > 0$ $(\phi=0)$ and when $u < 0$ $(\phi=\pi)$. We find

$$u > 0 : \qquad \theta_+ = 0$$

$$\theta_- = -\pi - \Delta$$

$$u < 0 : \qquad \theta_+ = \frac{\pi}{2} + \Delta'$$

$$\theta_- = -\frac{\pi}{2} - \Delta'' ,$$

where Δ, Δ', $\Delta'' > 0$, and to lowest order in $|\ln f_\pm|^{-1} \simeq |\frac{1}{2}\ln y|^{-1}$ have the values

$$\tfrac{1}{2}\Delta \cong \Delta' \cong \Delta'' \cong \frac{-4\pi v}{\ln y} . \tag{43}$$

From Fig. 4 we see the by now familiar F-plane structure of the fermion propagator. For $u > 0$ both parity poles are on the physical sheet in (nearly complex conjugate positions, but for $u < 0$ one of them moves under the cut and disappears from the physical sheet. Notice also (for instance, from (43)) that unlike the non-infrared free theories which have been considered before, the negative parity pole, which is under the cut for positive u, moves further away from the cut as u increases. So even without considering higher order terms in the trajectory (which, however, one must do) the influence of the fermion parity partner is felt less and less as the u-channel resonance region is approached. We also remind the reader that the present, theory differs from those previously discussed in that the F-plane cut in this theory is very soft - a power of a logarithm - rather than just a power. By the usual arguments this implies a quite small deviation from power behavior in backward π-N scattering, namely, a power of $\ln \ln s$.

There were no references or conclusion with the original paper.