national accelerator laboratory

FERMILAB-Pub-74/78-THY
August 1974

Some Applications of f/P Universality to Inclusive Processes in the Triple-Regge Region*

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ABSTRACT

The notion of f/P universality is shown to hold in a broad class of models even when an elementary version of the f-coupled Pomeron does not. It is then applied to triple-Regge vertices (ijk). At the same time we invoke exchange degeneracy, but only when $k \neq P$ (the case $k = P$ was recently shown to lead to inconsistencies, at least if applied exactly). Given the f and P Regge residues for a two-body process like $pp \rightarrow pp$, we can then relate PPR, RRP, RRR and RPP to PPP (where $R = f, \omega$) and $\pi\pi R$ to $\pi\pi P$, which can be calculated from the πp asymptotic cross section; the cross terms RPR and PRR drop out, but the RPP and PRP must be kept. This leaves only PPP to be actually fitted to the data. It serves mainly to fix the overall scale of the cross sections at a given t. The shapes of the missing-mass distributions and the deviations from scaling are all predicted in our scheme. The results are in fairly good agreement with both high and low missing mass data (the latter via a finite-mass sum rule). Finally, if we combine our approach with factorization, we can predict the cross sections for processes like $p\pi^- \rightarrow pX$ even in a region where non-scaling terms are important.

I. INTRODUCTION

There has recently been a considerable amount of interest in triple-Regge couplings, which play an important role in many high-energy situations. They can be most directly extracted from inclusive cross sections near the edge of phase space. Even here, however, this is often difficult because of the large number of terms which can come in. in $pp \rightarrow pX$, for example, we can have the PPP, PPR, RRP, RRR, $\pi\pi P$ and $\pi\pi R$ terms, even after we impose exchange degeneracy, where R is the leading meson trajectory and P is the Pomeron. Recently, it has been argued that an exact application of exchange degeneracy is inconsistent with G-parity so that we must also include at least the RPP and PRP terms.[1] It would therefore be desirable to have some symmetry or universality scheme which would relate the R and P to each other.

Several years ago, Chew and Snider[2] and Carlitz, Green and Zee (CGZ)[3] argued that in certain classes of models the f-P Regge-residue ratio is a universal quantity, independent of the process involved. The scheme can be generalized to include the f', and has been found to give reasonable agreement with experiment, both for two-body[3] and inclusive processes.[4] By applying it to processes where the "external lines" are Regge exchanges or complex systems we shall find that we can relate many of our triple-Regge vertices to each other, given only two-body scattering data. Some of our results have been reported elsewhere.[5,6] Here we include the RPP cross term, consider addition data, including low missing mass data through finite-mass sum rules, give a more general derivation of our universality scheme and combine it with factorization, which permits us to relate different cross sections to each other, even when scaling does not hold.

In Sec. II, we consider a derivation of f/P universality based on a generalized two-component Pomeron, factorization and a weak form of average duality. It is a generalization of the derivation of CGZ, but does not in general lead to the elementary CGZ version of the f-coupled Pomeron, even though the predictions are essentially the same. In Sec. III we consider the triple-Regge expansion and how f-ω exchange degeneracy can be applied, particularly in view of the remarks of Ref. 1. In Sec. IV we apply f/P universality to relate the different triple-Regge vertices to each other. We also write down simple models for π exchange. In Sec. V, we compare our results with the data, including low missing-mass data through finite mass sum rules. Finally in Sec. VI, we illustrate how factorization can be used to relate $pp \rightarrow pX$ to a process like $p\pi^- \rightarrow pX$, even when non-scaling terms are important.

II. A GENERALIZED DERIVATION OF f/P UNIVERSALITY

As in Ref. 3, we will combine duality with a general picture of the Pomeron. We will assume a much weaker form of (average) duality, however, and base ourselves on a broader class of models. We shall see that this no longer leads to an f-coupled P, at least in the original elementary sense of CGZ. But we find that we still get f/P universality as well as an "effective" f-coupled P, so that we essentially get the same kind of predictions as in Ref. 3.

The two-component picture of the Pomeron has been quite successful in accounting for many of the observed properties of multiparticle production.[7] It asserts that the production amplitude is made up of a "multiperipheral" component, with relatively small rapidity gaps, and a "diffractive" component with one large rapidity gap. We assume that these components can be represented by Figs. 1(a) and 1(b). The circles represent low subenergy(s') clusters (i.e., amplitudes cut off at $s' = s_1$), which may include background as well as resonance contributions. To avoid double-counting we would then have to insert a threshold term at $s = s_1$ on the Regge exchanges R and P of Fig. 1, where $s = s_1$ is the separation point between the low-energy resonance and the high-energy Regge regions. The rectangle in Fig. 1(b) would normally be something like a multiperipheral chain but could also be some more complicated production mechanism. The ingoing lines of Fig. 1 could be either ordinary particles or could themselves be Regge exchanges.

If we insert Fig. 1 into a unitarity relation we get the absorptive part A. Ignoring the cross terms, which can be argued to be small,[7] this is then made up out of the diagrams of Figs. 2(a) and 2(b). These require a knowledge of Fig. 3(a). Actually as long as one only needs A, one only has to know the absorptive part A_0 of Fig. 3(a).[8] If one assumes average duality this is then equal to Fig. 3(b) on the average. More precisely we have generalized finite-energy sum rules

$$\int_0^{s_1} A_0(s',t)\,\rho_n(s')\,ds' = \sum_m b_m(t) \int_0^{s_1} s'^{\,\alpha_m(t)}\,\rho_n(s')\,ds' \qquad (2.1)$$

where $\Sigma b_m s^{\alpha_m}$ is the high-s behavior of Fig. 3(b) and the $\rho_n(s)$ are weight functions. For ordinary particle scattering, $\rho_n(s) = s^n$ where n = integer, but if the "external" lines of Fig. 3 are Regge exchanges, ρ_n may be more complicated (see, e.g., Sec. III). In general A_0, ρ_n and b_m could also depend on the momentum-transfer variables or virtual "masses" of the "external" Regge lines of Fig. 3.

If we assume that Eq. (2.1) is sufficient to determine the overall normalization of A_0, then it must lead to the form

$$A_0(s,t) = \sum_m b_m(t)\,H[s,\alpha_m(t)]\,. \qquad (2.2)$$

This would be the case, for example, if we were to parametrize A_0 by a sum of delta-functions in s (or Breit-Wigner forms) and then used the same number of sum rules (2.1) to solve for the coefficients. We may do this either by considering a given s_1 and several ρ_n, or, if we assume semi-local duality, by taking the lowest ρ_n and several values of s_1, placed midway between resonances. Of course, in the ultimate limit of completely local duality, we simply have

$$H(s, \alpha) \approx s^{\alpha} . \tag{2.3}$$

If we assume factorization we can write

$$b_m(t) = \gamma_m(t)\bar{\gamma}_m(t) \tag{2.4}$$

where γ_m and $\bar{\gamma}_m$ are the left and right-hand couplings of Fig. 3(b). If we now use Eqs. (2.2) and (2.4) for Fig. 3(a)--in other words if we assume that Fig. 3(b) is a good approximation to Fig. 3(a) in the above average sense--we see that Figs. 2(a) and 2(b) reduce to Figs. 4(a) and 4(b). Both of these have the form of Fig. 5 so

$$A(s,t) = \sum_{mm'} \gamma_{acm}(t) B_{mm'}(s,t)\gamma_{bdm'}(t) \tag{2.5}$$

where B is independent of what a, c, b, d are. We must be careful in interpreting diagrams like Figs. 4 and 5, however. Strictly speaking the Regge "propagators" s^{α} for the m, m' lines must be replaced by the functions $H(s, \alpha)$, as we have seen.

For certain purposes it is desirable to make a partial-wave projection of Eq. (2.5). We then have

$$A(t,j) = \sum_{mm'} \gamma_{acm}(t) B_{mm'}(j,t)\gamma_{bdm'}(t) \tag{2.6}$$

If we make the additional assumption that $H(s, \alpha)$ in Eq. (2.2) does not depend on the momentum transfer variables or virtual "masses" of the R, P Regge lines of Fig. 4, we can further decompose

$$B_{mm'}(t,j) = H[j, \alpha_m(t)]\, V_{mm'}(j,t) H[j, \alpha_{m'}(t)] \tag{2.7}$$

where V no longer depends on α_m or $\alpha_{m'}$, but only on the internal couplings of Fig. 4. This would be true in the local duality limit (2.3), for example, in which case

$$H(j, \alpha) \approx (j - \alpha)^{-1} \tag{2.8}$$

at least if s_1 is sufficiently large and j is not too close to α.

So far we have assumed a two-component Pomeron. Clearly our derivation continues to apply in the much more general case where the Regge exchanges R of Figs. 1(a), 2(a) and 4(a) are allowed to be Pomerons as well as lower-lying Reggeons. The inclusion of cross terms likewise does not affect our conclusions.

Let us now turn to the t-channel isospin $I_t = 0$ state. Here, in general $m, m' = P, f, f'$. The CGZ scheme corresponds to dropping the P contribution, but we shall keep it, so we no longer have an f-coupled P. Suppose for a moment we consider a process like πp or pp scattering, where the f' decouples. Now the Pomeron, which corresponds to $A \approx b_P s^{\alpha_P}$, arises from the leading s behavior of B in Eq. (2.5), or equivalently from the leading j-singularity at $j = \alpha_P$ in Eq. (2.6) If, in addition, we assume that the Pomeron factorizes, as seems to be borne out by the data, we obtain

$$b_P = \gamma_{acP}\,\gamma_{bd\,P} = \sum_{m,m'=P,f} \gamma_{ac\,m}\Gamma_{mm'}\gamma_{bd\,m'} \tag{2.9}$$

where $\Gamma_{mm'}$ is independent of a, c, b, d.

Suppose we now consider the special case a = b, c = d. Then Eq. (2.9) is equivalent to a quadratic equation for the ratio $\gamma_{acf}/\gamma_{acP}$. This has two solutions

$$\gamma_{acf}/\gamma_{acP} = \{-\Gamma_{fP} \pm [\Gamma_{fP}^2 - \Gamma_{ff}(\Gamma_{PP}-1)]^{\frac{1}{2}}\}/\Gamma_{ff}\ . \tag{2.10}$$

However, the physically relevant solution must correspond universally to either the + or the - . If it corresponded to + for ac and - for bd, for example, Eq. (2.9) would no longer be satisfied. Since the $\Gamma_{mm'}$ are independent of ac, it then follows that $\gamma_{ac\,f}/\gamma_{ac\,P}$ is likewise independent of ac. This in turn means that

$$b_f/b_P = \gamma_{ac\,f}\,\gamma_{bd\,f}/\gamma_{acP}\,\gamma_{bd\,P} = \text{universal quantity}. \tag{2.11}$$

Although Eqs. (2.5) - (2.7) include m, m′ = P and so do not correspond to an f-coupled P in the elementary CGZ sense we can still obtain a kind of effective f-coupled Pomeron by rewriting Eq. (2.9) as

$$b_P = \gamma_{ac\,f}\,\bar{\Gamma}_{ff}\,\gamma_{bd\,f} \tag{2.12}$$

where

$$\bar{\Gamma}_{ff} = \Gamma_{ff} + \frac{\gamma_{bdP}}{\gamma_{bdf}} \Gamma_{fP} + \frac{\gamma_{acP}}{\gamma_{acf}} \Gamma_{Pf} + \frac{\gamma_{acP}}{\gamma_{acf}} \frac{\gamma_{bdP}}{\gamma_{bdf}} \Gamma_{PP} \tag{2.13}$$

Since all the quantities on the right-hand side are independent of a, b, c, d it follows that we can reproduce all the residue relations we would obtain by keeping only Γ_{ff}, i.e., if we had a true f-coupled Pomeron.

So far we have only considered cases where the f′ decouples. In general the situation is more complicated. If we assume that Regge vertices continue to satisfy SU(3), however, and if we assume that Eq. (2.7) applies, we can still get a relatively simple result by considering the SU(3) singlet (1) state instead of just the $I_t = 0$ state.[3] In order to guarantee that the f′ decouples from pions and nucleons, this state must correspond to the case of "ideal" mixing

$$1 = 3^{-1/2} [2^{1/2} f + f'] \tag{2.14}$$

as far as f and f′ is concerned. In this state then we have

$$V_{ff} = \sqrt{2}\, V_{f'f} = 2\, V_{f'f'} \,. \tag{2.15}$$

Since the Pomeron itself is assumed to be a pure singlet state we also have

$$V_{fP} = \sqrt{2}\, V_{f'P} \,. \tag{2.16}$$

We will now make the usual assumption that symmetry breaking only affects masses (and trajectories) but leaves couplings SU(3) symmetric.[3] Since B depends on α_m and $\alpha_{m'}$ only through the H factors in Eq. (2.7), as we have seen, Eqs. (2.15) and (2.16) then continue to hold even with symmetry breaking. If we now repeat the arguments leading up to Eq. (2.9) from Eqs. (2.6) and (2.7) we find that we recover exactly the same equation (2.9) but with the replacement

$$\gamma_{acf} \rightarrow \tilde{\gamma}_{acf} = \gamma_{acf} + \frac{r}{\sqrt{2}} \gamma_{acf'}, \tag{2.17}$$

$$r = H(\alpha_{f'}, \alpha_P) / H(\alpha_f, \alpha_P) \tag{2.18}$$

and similarly for γ_{bdf}. The argument for f/P universality proceeds exactly as before except that now it is quantities like $\tilde{\gamma}_{acf}/\gamma_{acP}$ which are universal, rather than $\gamma_{acf}/\gamma_{acP}$. We also recover the effective f-coupled Pomeron expression (2.12), but again with the replacement (2.17).

Perhaps the simplest example of f/P universality is in the case of forward πp and pp scattering where the f′ decouples. With $\alpha_f(0) = 0.5$,

$$b_f(0)/b_P(0) \approx 0.9, \ \pi p \text{ scattering} \tag{2.19}$$

$$\approx 1.2, \ pp \text{ scattering}. \tag{2.20}$$

The precise value actually depends to some extent on the intercept $\alpha_f(0)$.[3] Thus b_f/b_P is universal to the same sort of accuracy as the assumptions, such as duality and factorization, which went into the derivation of Eq. (2.11).

III. TRIPLE-REGGE FORMALISM AND EXCHANGE DEGENERACY

Let us now turn to an inclusive process $ab \to cX$. When the square of the missing mass M^2 is $<< s$, we have the Regge behavior $(s/M^2)^{\alpha_i(t)}$ (see Fig. 6). The cross section, which involves the squares of such graphs is then given by

$$\frac{s d\sigma}{dt\, dM^2} = \frac{1}{s} \sum_{ij} \left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)} \widetilde{A}_{ij}(M^2, t) \qquad (3.1)$$

where $\widetilde{A}_{ij}$ is proportional to the forward "absorptive part"[9] for the Regge-particle "scattering" process $\alpha_i(t) + b \to \alpha_j(t) + b$ (see Fig. 7). Thus, when $M^2 >> 1\ \mathrm{GeV}^2$, we expect $\widetilde{A}_{ij}$ itself to have the Regge behavior

$$\widetilde{A}_{ij}(M^2, t) = \sum_k G_{ijk}(t)(M^2)^{\alpha_k(0)} \qquad (3.2)$$

which means that Fig. 7 reduces to a sum of triple-Regge graphs[10] (Fig. 8); we are taking M^2, s and t in units of GeV^2 and the cross section of (3.1) in units of mb-GeV^2. Factorization then gives

$$G_{ijk}(t) = \gamma_{ac\,i}(t)\gamma_{ac\,j}(t)X_i^*(t)X_j(t)g_{ijk}(t)$$

$$\times \operatorname{Im} X_k(0)\ \gamma_{bbk}(0) \qquad (3.3)$$

where the γ are Regge-particle couplings, the X_i are the usual signature factors

$$X_i(t) = \frac{-1 \pm e^{-i\pi\alpha}}{-\sin \pi\alpha(t)},$$

and g_{ijk} is the actual triple-Regge coupling.

When M^2 is not large the cross section is more complicated. Since it proportional to the absorptive part of Fig. 7, however, it is still related to the triple-Regge couplings of Fig. 8 through Finite-Mass sum rules.[11] In the case of a process like $pp \to pX$ or $p\pi^- \to pX$, where $a = c$, the lowest-moment sum rule reduces to the form

$$\int_0^{\bar{M}_0^2} d\bar{M}^2 \, \bar{M}^2 \, \frac{sd\sigma}{dtdM^2} = \sum_{ijk} \frac{s^{-1}G_{ijk}(t)}{2+\alpha_k(0) - \alpha_i(t) - \alpha_j(t)} \left(\frac{s}{\bar{M}_0^2}\right)^{\alpha_i(t)+\alpha_j(t)} (\bar{M}_0^2)^{\alpha_k(0)+2} \tag{3.4}$$

where $\bar{M}^2 = M^2 - t - m_b^2$ is a crossing-symmetric variable which is a generalization of the familiar two-body scattering variable $\nu = \frac{1}{2}(s-u)$. The upper limit $\bar{M}_0^2$ is a point at which the asymptotic form (3.2) is already valid. If we assume semi-local duality, however, it could be taken to have a much lower value, provided this is taken half-way between resonances.

In the specific process $pp \to pX$ we have the exchanges $i, j, k = P, f, \omega, \pi$; the ρ and A_2 are known to couple only weakly to protons. If we assume exchange degeneracy the situation simplifies even further, since we can consider the f and ω as a single exchange R. Moreover, since P is approximately pure imaginary and R is real, the off-diagonal terms RPP, PRP, RPR and PRR must be absent. This argument has

has been used in the past to simplify the description of the triple-Regge region.[6, 12, 13]

Recently it has been pointed out that the above result is inconsistent with G-parity.[1] This is because the only way we can guarantee that the RPP + PRP term drops out for all s and M^2 is for the fPP + PfP to be cancelled by ωPP + PωP. However this must vanish by G-parity, whereas, as we shall see in the next section, the fPP + PfP term is nonzero. Thus there is no way in which we can maintain exchange degeneracy, except perhaps in a very limited range where the fPP + PfP and fPf + Pff could be roughly cancelled by the ωPω + P$\omega\omega$.[14]

In what follows we shall therefore give up exchange degeneracy for all triple-Regge graphs (Fig. 8) with k = P, but continue using it whenever k $\neq$ P. The justification for such an assumption is that the latter graphs should be dual to quasi-two-body scattering graphs (at least when we do not have i = j = P),[15] for which we know that exchange degeneracy has some validity. We shall therefore retain PfP + fPP but assume that Pff + fPf is cancelled by P$\omega\omega$ + ωPω so that there is no PRR + RPR term. However, for comparison, we also try fits where we deliberately drop PfP + fPP. We shall see that our predictions for the data are not too different in the two cases although the required PPP coupling is larger in the latter case.

In addition to PRR and RPR various other cross terms also drop out. In particular the fωP + ωfP drops out by G parity whereas the $f\omega k + \omega f k$ with $k \neq P$ drops out by exchange degeneracy. The term $i = P$, $j = \pi$ can only involve $k = \pi$, which, however, vanishes because the πpp vertex goes to zero in the forward direction. The $i = f$, $j = \pi$ term vanishes for the same reason. The $i = \omega$, $j = \pi$ term only permits $k = \rho$, which couples weakly to protons and so can also be neglected.

Since all the cross terms except the fPP + PfP drop out we can only have $k = P, f$. We are therefore left with the terms PPP, PfP, fPP, RRP, PPf, RRf, $\pi\pi P$ and $\pi\pi f$. Here $R = f, \omega$ so $RRf = fff + \omega\omega f$ and $RRP = ffP + \omega\omega P$. Now $g_{fff} = -g_{\omega\omega f}$ by exchange degeneracy. We cannot use the same argument to relate ffP to $\omega\omega P$, since we are not assuming it when $k = P$. However, exchange degeneracy does give $g_{fPf} = -g_{\omega P \omega}$ since the corresponding two diagrams must cancel, as we have seen. If we assume vertex symmetry for these triple-Regge couplings we can conclude that $g_{ffP} = -g_{\omega\omega P}$ also.

IV. RELATIONS BETWEEN TRIPLE-REGGE COUPLINGS

The simplest set of relations arises when we apply f/P universality to the exchanges k in the "absorptive part" of Eq. (3.2) for the Regge-particle "scattering" process $\alpha_i(t) + b \to \alpha_j(t) + b$. Here the functions $G_{ijk}(t)$ are proportional to the Regge residue $b_k(0)$ for this "process". As long as the f' decouples, f/P universality (2.11) then gives

$$G_{ijf}(t)/G_{ijP}(t) = b_f^p(0)/b_P^p(0) \tag{4.1}$$

where the right-hand side is the f/P Regge residue ratio for pp or πp scattering.

In our case, Eq. (4.1) gives the correct relation in all cases except when i = j = P. This is because f′ can couple to PP, whereas it cannot couple to any other ij combination (with i, j = P, f, ω, π), at least in the case of ideal mixing, since f′ is made up only of strange quarks and f, ω, π of non-strange quarks so that we cannot draw any quark-duality diagrams involving f′. We must therefore make the replacement (2.17) for the g_{PPf} coupling which means that Eq. (4.1) is replaced by

$$\frac{g_{PPf}(t) + \frac{r(0)}{\sqrt{2}}\, g_{PPf'}(t)}{g_{PPP}(t)}\ \frac{\gamma_{ppf}(0)}{\gamma_{ppP}(0)} = \frac{b_f^p(0)}{b_P^p(0)}. \tag{4.2}$$

Now we are assuming the P to be an SU(3) singlet so that the PP state is likewise a singlet. With the ideal mixing (2.14) we then have

$$g_{PPf'} = g_{PPf}/\sqrt{2} \tag{4.3}$$

which, when combined with Eq. (4.2), gives

$$[1 + \frac{1}{2}r(0)]\ G_{PPf}(t)/G_{PPP}(t) = b_f^p(0)/b_P^p(0)\,. \tag{4.4}$$

Equation (4.1) therefore has an extra correction factor when i = j = P. We can estimate it from Eqs. (2.18) and (2.8). Taking $\alpha_P(0) = 1$,

$\alpha_f(0) = 0.5$ and $\alpha_{f'}(0) \simeq 0$, we have $\frac{1}{2}r \simeq 0.25$, which is fairly small. We shall see in the next section that its inclusion only has a small effect on our predictions.

A second set of relations can be obtained by applying f/P universality directly to the exchanges i in the inclusive process (Fig. 6) itself, treating it as a quasi-two body process with Regge behavior $b_i X_i (s/M^2)^{\alpha_i}$. Since the inclusive cross section involves the square of sums of such graphs we obtain then

$$\frac{G_{ifk}(t)}{G_{iPk}(t)} = \frac{G^*_{fik}(t)}{G^*_{Pik}(t)} = \frac{b_f^p(t)X_f(t)}{b_P^p(t)\,X_P(t)}\,, \tag{4.5}$$

at least in situations where the f′ decouples. Once again the right-hand side involves the f/P ratio for pp or πp scattering.

As in the case of (4.1), Eq. (4.5) is always correct unless $i = k = P$. If we repeat the sort of procedure which led to Eq. (4.4) we obtain for the latter case

$$\frac{G_{PfP}(t)}{G_{PPP}(t)} = \frac{G^*_{fPP}(t)}{G^*_{PPP}(t)} = \frac{b_f^p(t)X_f(t)}{b_P^p(t)X_P(t)}\left[1 + \frac{1}{2}r(t)\right]^{-1}. \tag{4.6}$$

Once again, we shall see in the following section that the extra correction factor has only a small effect on our predictions.

Equations (4.1), (4.4) and (4.6) relate the ππf to ππP and PfP, fPP, PPf to PPP. In addition, if we use the exchange degeneracy and vertex symmetry results $g_{\omega\omega k} = - g_{ffk}$ obtained in the last paragraph of the preceding section, we obtain

$$G_{RRk} = G_{\omega\omega k} + G_{ffk} \tag{4.7}$$

with

$$G_{\omega\omega k}(t)\left|X_{\omega}(t)\right|^{-2} = G_{ffk}(t)\left|X_f(t)\right|^{-2} \tag{4.8}$$

for k = P, f. Together with Eqs. (4.1), (4.4), (4.5) and (4.6) these relations now permit us to relate RRP and RRf to PPP. We can then reduce everything to PPP and $\pi\pi P$, given $b_f^p(t)/b_P^p(t)$.

For the f/P ratio we will take the intermediate value $b_f^p/b_P^p = 1.1$ at t = 0; see Eqs. (2.19) and (2.20). For t $\neq$ 0 we could use the results of detailed Regge fits to, say, pp scattering. A simpler procedure is simply to require that we reproduce σ_{tot} and σ_{el}, given a spin-averaged amplitude

$$T = \sum_{i = P,\, \omega,\, f} b_i^p(t) X_i(t) s^{\alpha_i(t)} \tag{4.9}$$

with

$$\alpha_P(t) = 1 + 0.2t, \tag{4.10}$$

$$\alpha_f = \alpha_\omega = 0.5 + t, \tag{4.11}$$

$$b_P^p(t)\left|X_P(t)\right| = b_P^p(0)e^{a_P t} \tag{4.12}$$

and, assuming f-ω exchange degeneracy,

$$b_\omega^p(t) = b_f^p(t) = b_f^p(0)e^{a_f t}\sqrt{\pi}/\Gamma\left[\alpha(t)\right] \tag{4.13}$$

where the $\Gamma(\alpha)$ is put in to guarantee that T does not blow up at $\alpha = 0$. These forms are approximately consistent with what is required to reproduce the correct $d\sigma/dt \simeq 16\pi |T|^2/s^2$. We then adjusted $b_P(0)$ to the experimental $\sigma_{tot} \simeq 16\pi s^{-1}$ Im T(t = 0) at each energy; since σ_{tot} actually varies slightly with energy, $b_P(0)$ will vary slightly also--this is then an approximate way of taking into account the fact that the P is not a pure pole at j = 1. Given $b_f^p(0)/b_P^p(0)$, we then adjusted the constants a_p and a_f so as to reproduce the correct experimental $\sigma_{el} \simeq \int_{-\infty}^{0} dt(d\sigma/dt)$ in the range $10 < s < 200$ GeV^2, using the curve drawn by Morrison.[16] We obtained $a_P - a_f \simeq 3.8$ with $b_f^p(0)/b_P^p(0) = 1.1$ and $a_P - a_f \simeq 3.0$ with $b_f^p(0)/b_P^p(0) = 1.2$, the value most appropriate for pp scattering.[17] In most of our fits we took the intermediate value $a_P - a_f \simeq 3.2$, although the values $a_P - a_f = 3.7$ and 2.7 were also tried, mainly for purposes of comparison.

Although $G_{\pi\pi P}$ could be treated as a parameter in our scheme it is also possible to write down a simple model for it. From Fig. 6 with $i = \pi$ we see that this contribution must be proportional to the total πp asymptotic cross section $\sigma_P(\pi p) \approx 21$ mb,[17] as well as the πN coupling $g^2/4\pi = 14.4$. Specifically we have[18]

$$G_{\pi\pi P}(t) = \frac{1}{4\pi}\left(\frac{g^2}{4\pi}\right)\frac{-tF(t)}{(t - m_\pi^2)^2}\,\sigma_P(\pi p), \tag{4.14}$$

where the denominator comes from the pion propagator and F(t) is a "form factor" to take into account the fact that the pion is off-shell. We took two different models:

(a) A Regge form with

$$\alpha_\pi(t) \simeq t, \qquad F(t) = 1 \tag{4.15}$$

(b) An elementary-pion form with

$$\alpha_\pi(t) \simeq 0, \quad F(t) \simeq (1-t)^{-2[\alpha_P(0)+1]} \tag{4.16}$$

This F(t) is the off-shell extrapolation suggested by a simplified solution to the Amati, Bertocchi, Fubini, Stanghellini, Tonin (ABFST)[19] multiperipheral model (see, e.g., Ref. 20).

One way of checking the above models for π exchange is to look at the process $np \to pX$, which can be extracted from $dp \to pX$. Here $\pi\pi P$ and $\pi\pi f$ are essentially the only surviving terms, with couplings which are exactly twice as large as for $pp \to pX$. Figure 9 shows the resulting cross sections for both case (a) and (b). The results are in approximate agreement with preliminary Fermilab data.[21]

V. COMPARISON WITH EXPERIMENT

In the preceeding section we saw that we can relate all the triple-Regge terms for $pp \to pX$ to the PPP and $\pi\pi P$ couplings. If we assume Eq. (4.15) or (4.16) we moreover have an explicit expression for $\pi\pi P$. This means that $G_{PPP}(t)$ is now the only quantity which actually has to be fitted to the data. Its main function then is to fix the overall magnitude of the cross sections at a given t value. The shapes of the missing-mass distributions and the deviations from scaling are all predicted in our scheme, and are not sensitive to the precise value of the overall magnitude.

Figure 10 shows the results if we keep all the couplings, including fPP + PfP, and use the Regge-π of Eq. (4.15). The solid lines correspond to the approximation of neglecting the f' correction, so $\frac{1}{2} r \approx 0$. The overall magnitudes correspond to taking $G_{PPP}(t) \simeq 2.1\, e^{3.9t}$, although there is no particular significance to the particular exponential parametrization we have chosen. The other parameters, $a_P - a_f = 3.2$ and $b_f^p(0)/b_P^p(0) = 1.1$, came from two-body data, as we have seen, and did not have to be fitted to the inclusive cross sections. They lead to the couplings listed in Table I. The dashed lines at $t = -0.33$ correspond to $\frac{1}{2} r(t) \simeq 0.25$. This is the correct value at $t = 0$ with $\alpha_{f'}(0) = 0$ and continues to be approximately true for $t \neq 0$. We see that the curve is not too different from the corresponding (solid) curve with $\frac{1}{2} r = 0$. Of course, the G_{PPP} for $\frac{1}{2} r = 0.25$ has to be taken somewhat larger but it turns out to be within 15% of the one for $\frac{1}{2} r = 0$. For this reason we will take $\frac{1}{2} r = 0$ in all subsequent fits.

The data in Fig. 10 is that of Ref. 22. We see that the agreement is fairly good and that we can reproduce the shallow dip in $x (= 1 - M^2/s)$. This is non-trivial if we remember that some of the individual triple-

Regge terms which make up the cross section themselves vary quite rapidly in the region of interest. In fact, a comparison of the solid and dot-dash-dot lines, for which we deliberately took the somewhat bigger value of 1.6 for $b^p_f(0)/b^p_P(0)$, shows that the position of the dip would not be given correctly if we allowed the value to deviate too far from that predicted by f/P universality. We can also approximately reproduce the deviation from scaling. Our deviation is somewhat smaller than the one given by the data. However, the latter itself seems to vary from one experiment to another, as can be seen e.g., by comparing Refs. 22 and 23.

Figure 11 shows the results if we drop the cross terms PfP and fPP, as would be the case if exchange degeneracy could be applied to this process (see Sec. III). We considered both the elementary-π of Eq. (4.16) (solid lines) and the Regge-π of (4.15) (dot-dash-dot lines). There is no major difference between the two cases although the latter is somewhat better. There is also no major difference from the solid lines of Fig. 10, which included the fPP + PfP term, although we now do have a somewhat bigger deviation from scaling. Of course, the G_{PPP} needed in the two cases is different.

The dashed line of Fig. 11 corresponds to the case $a_P - a_f = 3.7$ and is presented mainly for purposes of comparison. We see that the position of the dip has shifted to the right. This indicates that we will not get the correct position if we allow $a_P - a_f$ to deviate too much from the value predicted by f/P universality.

In Fig. 12 we again consider situations [without the PfP + fPP term and with the elementary π of (4.16)] where $b_f^p(0)/b_P^p(0)$ was deliberately chosen to have the somewhat larger value of 1.6, again mainly for comparison. In part this was done because the deviation from scaling is somewhat larger for the data of Ref. 23, with which we compare our results. We also tried to vary $a_P - a_f$. The results, especially when compared with those of Fig. 11 again suggest that we cannot allow our parameters to vary too much from the values required by f/P universality. This statement may not be valid, of course, if we took a completely different parametrization from the one adopted here.

We next used the parametrization of the solid lines of Fig. 11 and extended it to much smaller values of M^2. The results are shown as the solid lines of Fig. 13 and compared with the ISR data of Ref. 24 at $s = 930$ GeV^2, which essentially overlaps with that of Ref. 22, at $t = -0.25$. We continue to get agreement in spite of the seeming tendency of the larger x data in Fig. 11 to lie above the triple-Regge curves. The dashed line uses the parametrization of the solid lines of Fig. 12.

In comparing our predictions with the data it must be kept in mind that the triple-Regge expansion of (3.1) and (3.2) is itself expected to break down at $x = 0.80$.[12] The expansion cannot then be considered more than an approximate formula in the first place, particularly for smaller x. Besides, the situation for two-body scattering suggests that f/P universality and exchange degeneracy are not expected to be better than about 20% either. As far as the data is concerned,it must also be remembered that different experiments often give quite different results in overlapping regions. This can be seen for example by comparing Ref. (22) and (23) which have both different normalizations and different deviations from scaling.

So far we have only been considering high-M^2 data. However, the finite-mass sum rule (3.4) also permits us to check if our scheme is at the same time consistent with low-M^2 data. (Conversely, since we only have one unknown parameter G_{PPP}, we could have used the latter to predict what the cross sections should be for high M^2.) For the left-hand side (LHS) we will simply use the integrals evaluated by Ellis and Sanda[25] with $\bar{M}_0^2 \approx M_0^2 = 8\ GeV^2$. At $s = 56$, $t \simeq -0.16$, for example, we have LHS = 18.4 ± 0.9 mb. On the other hand, if we use the parametrization of the solid lines of Fig. 11 we have a right-hand side RHS = 16.9 mb., so that the sum rule is approximately satisfied. On the other hand, if we increase the RHS by a factor of 1.16, so as to bring it into agreement with the normalization of Ref. 23, we obtain a RHS = 19.2 mb.

VI. RELATION BETWEEN $pp \to pX$ AND $p\pi^- \to pX$

Factorization of Regge couplings has led to numerous relations between various inclusive cross sections. Most of these are only applicable in the scaling region, where the $k = P$ exchange dominates. Since f/P universality relates the P to lower-lying exchanges, however, it can often enable us to relate cross sections even when non-scaling terms are important.

Let us now consider $p\pi^- \to pX$. We again have the exchanges $i, j = P, f, \omega, \pi$ in Fig. 8, since the ρ and A_2 only couple weakly to protons. These will certainly couple to $k = P, f$ in exactly the same way as they did in $pp \to pX$. However, whereas in the latter case the fPf + Pff term was cancelled by an $\omega P\omega + P\omega\omega$ which was related to it by exchange degeneracy, in the $p\pi^-$ case, the $\omega P\omega + P\omega\omega$ term vanishes because the $\omega\pi\pi$ coupling is zero by G-parity. We therefore have an extra fPf + Pff term which, however, can be related to the PPP by f/P universality. The $f\omega k + \omega f k$ terms drop out but this time not by exchange degeneracy (which does not apply in this case), but because the only such possibility is $f\omega\omega + \omega f\omega$ which again vanishes because the $\omega\pi\pi$ coupling is zero. The $i = P$, $j = \pi$ and $i = f$, $j = \pi$ vertices can only involve $k = \pi$, which vanishes because the $\pi\pi\pi$ vertex is zero by G-parity.

One term, which could be excluded for $pp \to pX$, but is not so easy to exclude for $p\pi^- \to pX$ is the $\omega\pi\rho + \pi\omega\rho$ term. We shall nevertheless assume that its contribution is small. This is suggested by the relative smallness of the decay into $\pi\omega$ of the g resonance, which lies on the ρ trajectory and tends to decay primarily into $\pi\pi$ and $\rho\rho$.[26]

We are now left with the PPP, PfP, fPP, RRP, PPf, RRf, $\pi\pi P$

and $\pi\pi f$ terms, which were also present for $pp \to pX$, and the Pff and fPf terms, which only occur for $p\pi^- \to pX$. From the factorization relation (3.3), the former terms are related to the corresponding terms for $pp \to pX$ via the relations

$$G_{ijk}^{\pi p}(t)/G_{ijk}^{pp}(t) = \gamma_{\pi\pi k}(0)/\gamma_{ppk}(0) \tag{6.1}$$

with

$$\frac{\gamma_{\pi\pi f}(0)}{\gamma_{ppf}(0)} = \frac{\gamma_{\pi\pi P}(0)}{\gamma_{ppP}(0)} = \frac{\sigma_\infty(\pi p)}{\sigma_\infty(pp)} \simeq \frac{2}{3} \tag{6.2}$$

where we have used Eq. (2.11), factorization and the quark model relation between the asymptotic total cross sections σ_∞. The Pff and fPf terms can be calculated from the PfP and fPP terms by using Eq. (4.1). We then have a complete description of $p\pi^- \to pX$ in terms of quantities which were extracted from $pp \to pX$.

If the extra Pff + fPf term were zero, the $\pi^- p$ cross section would be just $\frac{2}{3}$ of the pp cross section. Since it is not, the relation is more complicated. Figure 14 shows the result of our predicted $p\pi^- \to pX$ cross sections at $t = -0.25$, and $s = 48, 76$. We have used the parametrization of the solid lines of Fig. 11, which included the PfP + fPP term and used a Regge-π. The resulting cross sections (solid lines) are slightly too low compared with the $0.17 < -t < 0.35$ data of Ref. 27. This may, however, be due to a normalization discrepancy. If, for example, we increase everything by a factor of 1.16, to bring the $pp \to pX$ cross section into agreement with the normalization of Ref. 23, we obtain the dashed lines of Fig. 11. These curves are closer to experiment and we can conclude that we have agreement with the data within normalization uncertainties.

VII. CONCLUSION

We have considered a generalized derivation of f/P universality which holds even when an elementary version of the f-coupled Pomeron does not. We then applied it to the Regge exchange in inclusive processes (mainly $pp \to pX$) in the triple-Regge region. Since ω as well as f exchange are involved this was combined with exchange degeneracy, which was, however, applied only when $k \neq P$ in Fig. 8, a graph which is dual to quasi-two-body scattering and avoids the inconsistencies which arise when $k = P$. Using only data which can be extracted from $pp \to pp$ we can then relate the PRP, RPP, RRP, PPR and RRR terms to PPP and $\pi\pi R$ to $\pi\pi P$. Since we can write down a simple model for $\pi\pi P$, only PPP actually had to be fitted to the data.

Our scheme gives a fairly good description of the data. In particular, we can predict the shapes of the distributions in x, with a dip at about the right position, as well as an approximately correct deviation from scaling. The former is particularly nontrivial because it depends crucially on the relative magnitudesof the different couplings which come in (for example, the region near $x = 1$ is determined mainly by $i = P$ exchange while that near $x = 0.8$ by $i = \pi$). In fact the position of the dip deviates from its correct value if we vary some of our parameters away from the values predicted from two-body scattering data via f/P universality.

Our $G_{PPP}(t)$ [$\simeq 2.1\, e^{3.9t}$] is not too different from the one obtained in some previous triple-Regge fits, at least in the t-range we have been considering.[28] The other couplings are more difficult to compare because we have a somewhat different parametrization from the one assumed in other fits. We keep more terms, and , besides, in a more general fit, where the terms are unconstrained, we can usually vary the relative magnitudes by quite large amounts without affecting the fit in any serious way (in our own case the terms are very strongly constrained). Even so, it is interesting to note that, at least for $-t > 0.16$, the fits of Roy and Roberts[13] for example, are consistent with $G_{PPP} \approx G_{PPM}$ and $G_{MMP} \approx G_{MMM}$, as would be required by the f/P relation (4.1) with the ratio of Eq. (2.20). The experimental situation for smaller $|t|$ is in any case more confusing at the present time (see the Appendix).

Although we have only considered $pp \to pX$ and $p\pi^- \to pX$ in the present paper, our methods should be applicable in many other cases, both for reducing the number of independent triple-Regge couplings and for relating different cross sections to each other via factorization as was done in Sec. VI. Additional results can also be obtained by using something like SU(3) or the quark model. In many cases, of course, data does not yet exist in the triple-Regge region. However, in such cases one may still be able to check the results by using low missing mass data and finite-mass sum rules.

ACKNOWLEDGMENTS

The author would like to express his gratitude to Dr. M. Bishari, Prof. A. Pagnamenta, Prof. G. F. Fox, and Prof. K. Kang for their valuable help, and to Professors G. F. Chew and B. W. Lee for their hospitality at the Lawrence Berkeley Laboratory and the Fermi National Accelerator Laboratory, respectively.

APPENDIX: A FIT AT SMALL $|t|$

The small - $|t|$ region is particularly interesting because different theoretical models give different predictions about the PPP coupling as $|t| \to 0$. On the other hand, the data for $|t| < 0.16$ is still rather confusing. The most accurate is at rather low energies and much of it is not strictly in the triple-Regge region.[29,30] The data of Ref. 31 is in the triple-Regge region but seems to be inconsistent with other data, as has been stressed, e.g., by Roy and Roberts.[13]

We will only consider a somewhat simplified description in which we drop the interference terms PfP and fPP and use the elementary-π expression (4.16). We saw in Sec. V that this should not affect the shapes of the distributions in x too much. It does lead to a somewhat bigger $G_{PPP}(t)$ but should not change its general t dependence.

Suppose we simply take the parametrization of the solid lines of Fig. 12 and try to fit the data of Refs. 29 and 30. It turns out that the resulting cross section at $t = -0.20$, for example, is too low. This is presumably just a normalization discrepancy between the data of Refs. 29 and 22., so we simply increase G_{PPP} by 20%. In other words, we now have $G_{PPP}(t) = 1.2 \times 3.1\, e^{4.8t}$ with $a_P - a_f = 3.2$. The resulting curves (dashed line) are shown in Fig. 15, where they are compared with the data of Ref. 29 at $s \simeq 50$, Ref. 30 at $s \simeq 40$ and Ref. 32 at $s \simeq 400$. The small $\bar{M}^2/s$ data is not strictly in the triple-Regge region in the first two cases but is nevertheless described quite well by our model on the average. We see that our $G_{PPP}(t)$, which is the dashed line in Fig. 16 is consistent with this data. Figure 16 also shows the corresponding $\bar{g}_{PPP}(t) = g_{PPP}(t)G_{PPP}(t)/g_{PPP}(0)$, for which we used Eq. (3.3).

On the other hand, if we attempt to fit the data of Ref. 31, we find that we do need a more complicated G_{PPP}. Figure 17 shows the result of a fit using the form shown as the solid lines in Fig. 16. We see that $g_{PPP}(t)$ in particular shows a rapid decrease as $t \to 0$. However, if we now try to see what the resulting model gives for the M^2 dependence, we find that it is only fair. In particular we cannot reproduce the dip in the data at $M^2 \approx 20$, a feature which does not seem to be present in other experiments.

It is difficult to account for the discrepancy between the dashed and solid lines of Fig. 16. One possibility is that the data of Refs. 30 and 32 involves values of M^2 which are too low for the triple-Regge description to be valid, even in an average sense; there is, after all, no reason in principle for $\bar{M}_0^2$ in the finite mass sum rule (3.4) to be small. The data of Ref. 32 does involve much larger M^2, but is also much less accurate. A more likely explanation, however, is simply that the data are inconsistent at this time. Hopefully additional experiments will enable us to resolve this inconsistency.

REFERENCES

[1] R. Shankar, Criticism of the P′-ω Exchange Degeneracy Arguments in the pp→pX Triple-Regge Region, Lawrence Berkeley Laboratory Preprint, LBL-2678 (March 28, 1974).

[2] G. F. Chew and D. R. Snider, Phys. Rev. D3, 420 (1971).

[3] R. Carlitz, M. B. Green and A. Zee, Phys. Rev. D4, 3439 (1971).

[4] R. C. Brower, Phys. Lett. 37B, 121 (1971).

[5] L. A. P. Balazs, Phys. Lett. 48B, 232 (1974).

[6] L. A. P. Balazs, A Description of pp → pX in the Triple-Regge Region Incorporating f/P Universality, to be published in Phys. Lett.

[7] W. Frazer, R. Peccei, S. Pinsky, and C. -I. Tan, Phys. Rev. D7, 2647 (1973); K. Fialkowski and H. Miettinen, Phys. Lett. 43B, 49 (1973); H. Harari and E. Rabinovici, Phys. Lett. 43B, 49 (1973).

[8] L. Bertocchi, S. Fubini and M. Tonin, Nuovo Cimento 25, 626 (1962); D. Amati, A. Stanghellini and S. Fubini, ibid. 26, 896 (1962).

[9] A. M. Mueller, Phys. Rev. D2, 2963 (1970).

[10] C. E. DeTar et al., Phys. Rev. Lett. 26, 675 (1971).

[11] A. I. Sanda, Phys. Rev. D6, 280 (1972); M. B. Einhorn, J. Ellis and J. Finkelstein, Phys. Rev. D5, 2063 (1972).

[12] K. Abe et al., Phys. Rev. Lett. 31, 1530 (1973).

[13] D. P. Roy and R. G. Roberts, Triple-Regge Analysis of pp → pX and Some Related Phenomena--A Detailed Study, Rutherford Laboratory Preprint RL-74-022 (1974).

[14] J. W. Dash, Phys. Rev. D9, 200 (1974).

[15] G. C. Fox, private communication.

[16] D. R. O. Morrison, Vth Hawaii Topical Conference Proceedings (the University Press of Hawaii, 1974).

[17] V. D. Barger and D. B. Cline, Phenomenological Theories of High Energy Scattering (Benjamin, N. Y. 1969).

[18] M. Bishari, Phys. Lett. 38B, 510 (1972).

[19] D. Amati, A. Stanghellini and S. Fubini, Nuovo Cimento 26, 896 (1962); L. Bertocchi, S. Fubini and M. Tonin, Nuovo Cimento 25, 626 (1962).

[20] C. Sorenson, Phys. Rev. D6, 2554 (1972).

[21] A. Pagnamenta, private communication.

[22] K. Abe et al., Phys. Rev. Lett. 31, 1527 (1973).

[23] F. Sannes et al., Phys. Rev. Lett. 30, 766 (1973).

[24] M. G. Albrow et al., Nucl. Phys. B54, 6 (1973).

[25] S. D. Ellis and A. I. Sanda, Phys. Rev. D6, 1347 (1972).

[26] N. M. Cason et al., Phys. Rev. D7, 1971 (1973).

[27] Y. M. Antipov et al., Phys. Lett. 40B, 147 (1972).

[28] A. Capella, Phys. Rev. D8, 2047 (1973)

[29] J. V. Allaby et al., Nucl. Physics B52, 316 (1973).

[30] R. M. Edelstein, Phys. Rev. D5, 1073 (1972).

[31] S. Childress et al., Phys. Rev. Lett. 32, 389 (1974).

[32] S. J. Barish et al., Phys. Rev. Lett. 31, 1080 (1973).

Table I

Triple Regge Couplings Corresponding to the

Solid-Line Parametrization of Figure 10

$-t$	0.16	0.20	0.25	0.33
G_{PPP}	1.14	0.97	0.80	0.58
G_{RRP}	9.1	9.1	9.1	9.5
$G_{\pi\pi P}$	116.0	97.0	80.0	63.0
$G_{PfP} + G_{fPP}$	2.82	2.40	1.91	1.19
G_{PPf}	1.25	1.07	0.88	0.64
G_{RRf}	10.0	10.0	10.0	10.4
$G_{\pi\pi f}$	127.0	107.0	88.0	69.0

FIGURE CAPTIONS

Fig. 1 (a) Multiperipheral component of a Multiparticle production amplitude

(b) Diffractive component of a multiparticle production amplitude.

Fig. 2 Contributions to the absorptive part coming from

(a) the multiperipheral and

(b) the diffractive component.

Fig. 3 Diagrammatic representation of the duality relation (2.1) relating the low-subenergy absorptive part (a) to Regge exchange (b).

Fig. 4 Reduced form of Fig. 2(a) and (b) after applying the duality relation (2.1).

Fig. 5 A general form which includes both Figs. 4(a) and (b).

Fig. 6 Regge exchange for the inclusive process $ab \rightarrow cX$.

Fig. 7 Regge-particle "scattering" amplitude.

Fig. 8 Triple-Regge graph.

Fig. 9 Prediction of $np \rightarrow pX$ using the pion exchange model. The solid lines correspond to the elementary-π of Eq. (4.16) and the dashed lines to the Regge-π of Eq. (4.15).

Fig. 10 Triple Regge description of the experimental $pp \rightarrow pX$ data of Ref. 22 which includes the PfP + fPP term and uses the

Regge-π of Eq. (4.15). The solid lines at $-t = 0.33$, 0.25 0.20, and 0.16 (the average t values for each interval) use $\frac{1}{2} r = 0$, $b_f^p(0)/b_P^p(0) = 1.1$ and $a_P - a_f = 3.2$. The dashed line at $t = -0.33$ uses $\frac{1}{2} r = 0.25$. The dot-dash-dot line at $t = -0.16$ corresponds to $\frac{1}{2} r = 0$ but $b_f^p/b_P^p(0) = 1.6$, a value which was deliberately taken bigger than the f/P value for purposes of comparison.

Fig. 11 Fit with the PfP term. The solid lines, with $G_{PPP} = 3.1e^{4.8t}$ correspond to the elementary-π of Eq. (4.16) and the dot-dash-dot lines with $G_{PPP} = 3.2e^{5.2t}$ to the Regge-π of Eq. (4.15). Both use $\frac{1}{2} r = 0$, $b_f^p/b_P^p(0) = 1.1$ and $a_P - a_f = 3.2$. The dashed line, by contrast, corresponds to taking $a_P - a_f = 3.7$, with Eq. (4.16), at $-t = 0.33$ and 0.16, where it is indistinguishable from the solid line.

Fig. 12 Fits to the data of Ref. 23 without PfP + fPP and with $\frac{1}{2} r = 0$, and $b_f^p(0)/b_P^p(0) = 1.6$, a value which was deliberately taken to be larger than the f/P value. The dot-dash-dot line did not use any π exchange and corresponds to $a_P - a_f = 3.7$, $G_{PPP} = 3.1e^{5.4t}$. The other lines used the elementary-π of Eq. (4.15). The solid line corresponds to $a_P - a_f = 3.7$, $G_{PPP} = 2.0e^{4.3t}$ and the dashed line to $a_P - a_f = 2.7$, $G_{PPP} = 2.0e^{3.8t}$.

Fig. 13 The parametrization of the solid lines of Fig. 11 extended to smaller M^2 and compared with the data of Ref. 24 at $s = 930\ \mathrm{GeV}^2$ (solid line). The dashed line uses the parametrization of the solid lines of Fig. 12.

Fig. 14 The $p\pi^- \to pX$ cross sections (solid lines) predicted from $pp \to pX$ using Eqs. (6.1) and (6.2) and the parametrization of the solid lines of Fig. 11. The dashed lines are 16% higher to bring them into better agreement with the $pp \to pX$ normalization of Ref. 23. The difference between the solid and dashed lines is thus a measure of experimental normalization uncertainties. The upper curve corresponds to $s = 48$ and the lower to $s = 76$ in each case. The data is that of Ref. 27.

Fig. 15 The parametrization of the solid lines of Fig. 12 extended to smaller $|t|$ but with $G_{PPP}(t)$ increased by 20% (dashed lines). The solid lines represent the data of Ref. 29 at $-t = 0.10$, 0.20 and of Ref. 30 at $-t = 0.05$. The crosses represent the data of Ref. 32.

Fig. 16 Plots of $G_{PPP}(t)$ (lower curves) and $\bar{g}_{PPP}(t) = g_{PPP}(t) \times G_{PPP}(0)/g_{PPP}(0)$ (upper curves). The dashed lines correspond to the parametrization of Fig. 15 and the solid lines to that of Fig. 17.

Fig. 17 A description of the t dependence of the data of Ref. 31 using the parametrization of Fig. 15 but with the solid line of Fig. 16 for G_{PPP}.

Fig. 18 A description of the M^2 dependence of the cross section of Ref. 31 using the same parametrization as in Fig. 17.

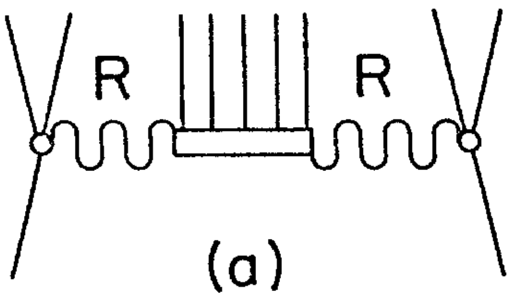


Fig. 1

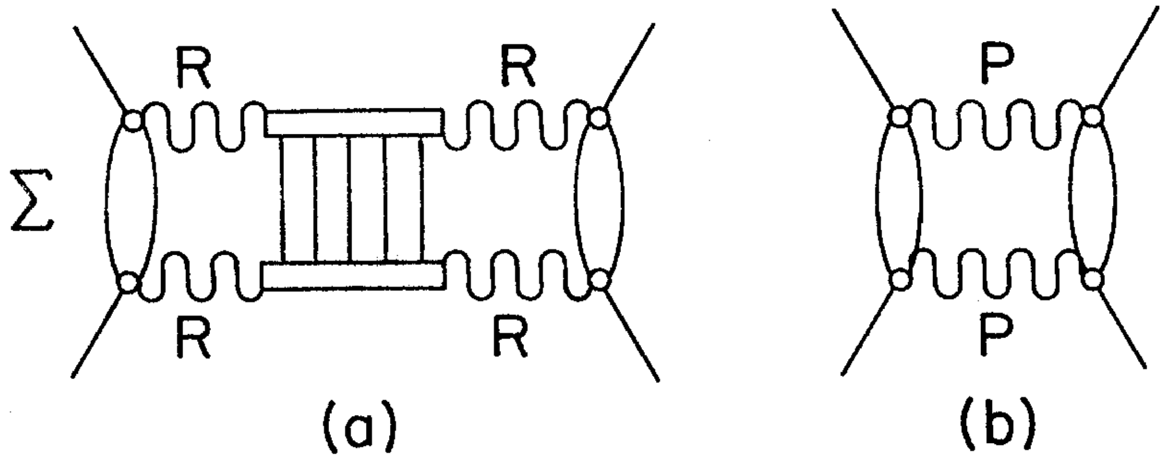


Fig. 2

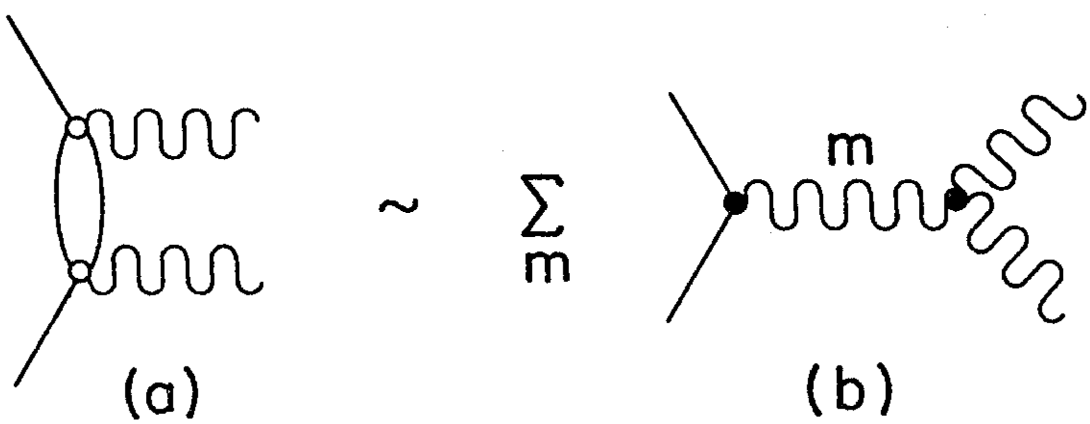
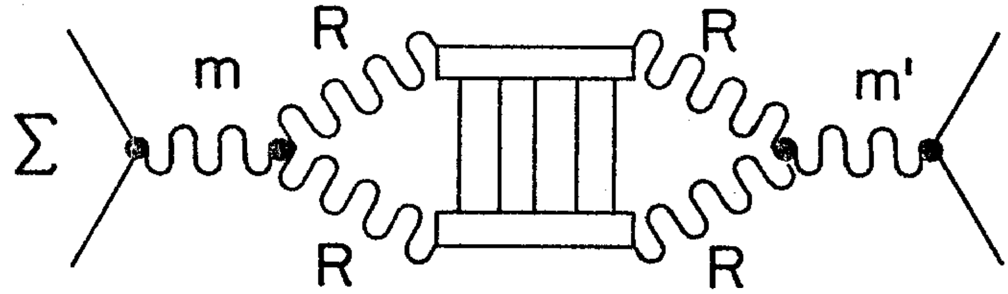
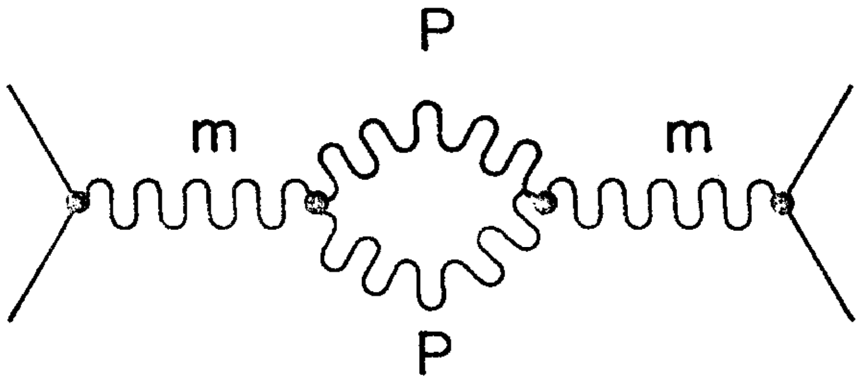


Fig. 3



(a)



(b)

Fig. 4

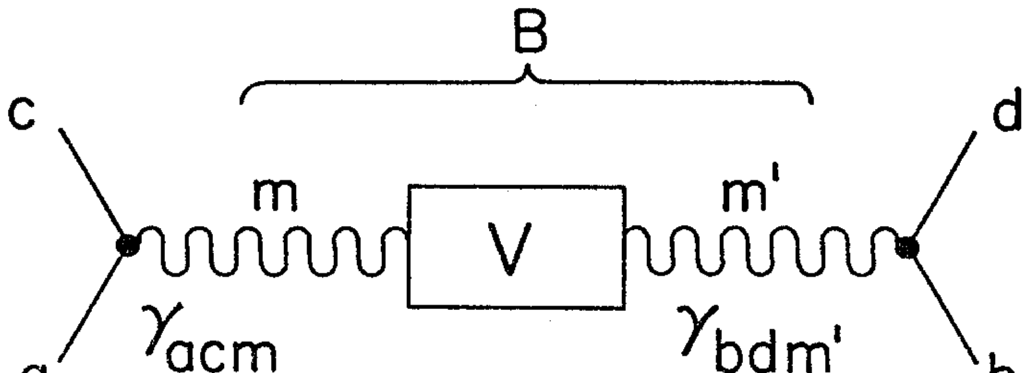


Fig. 5

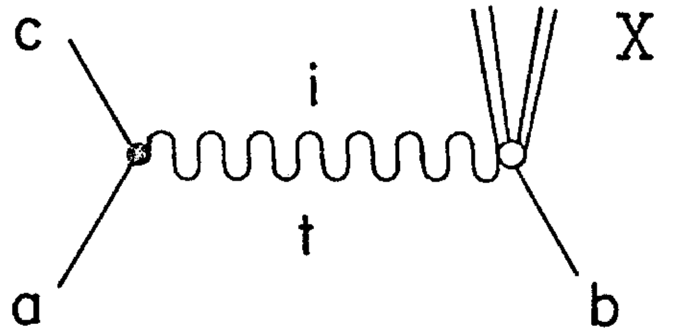


Fig. 6

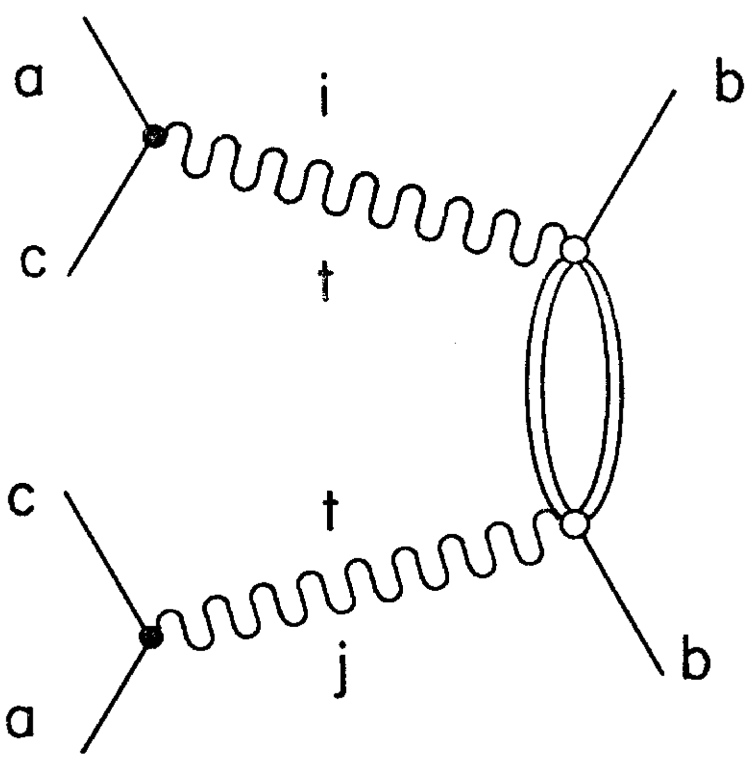


Fig. 7

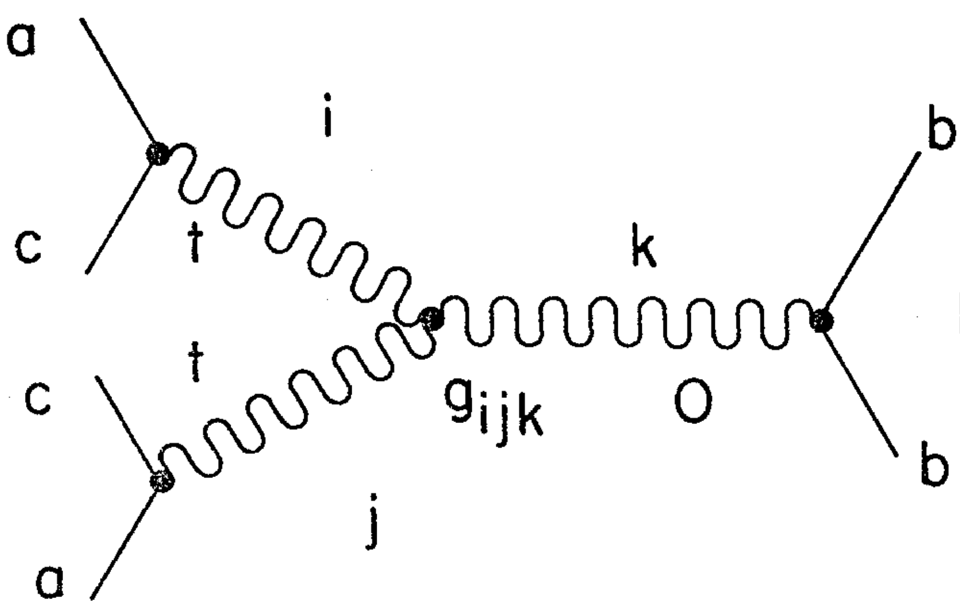
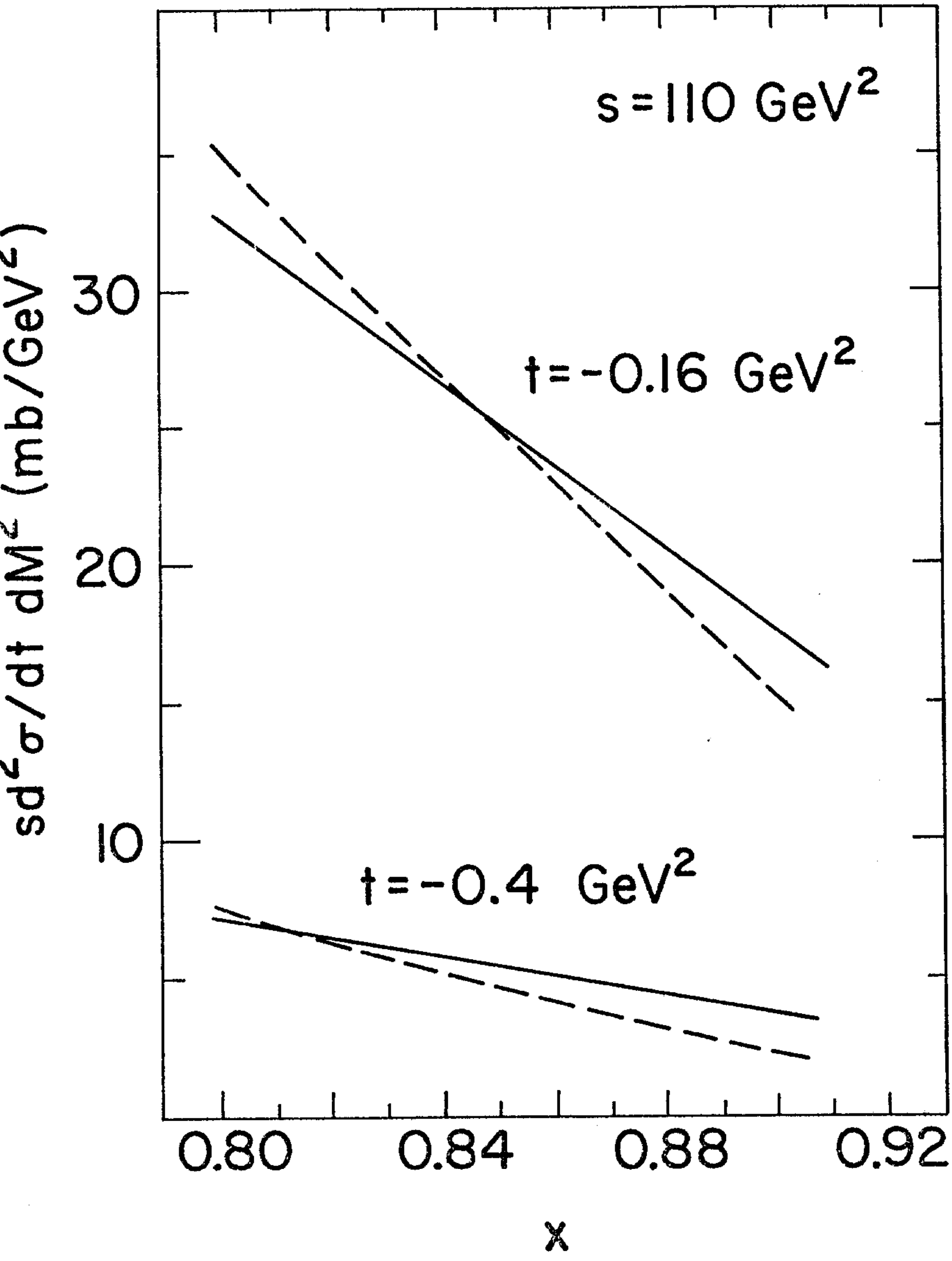


Fig. 8



s=110 GeV^2
t=-0.16 GeV^2
t=-0.4 GeV^2
30
20
10
sd$^2\sigma$/dt dM^2 (mb/GeV^2)
0.80
0.84
0.88
0.92
x

FIGURE 9

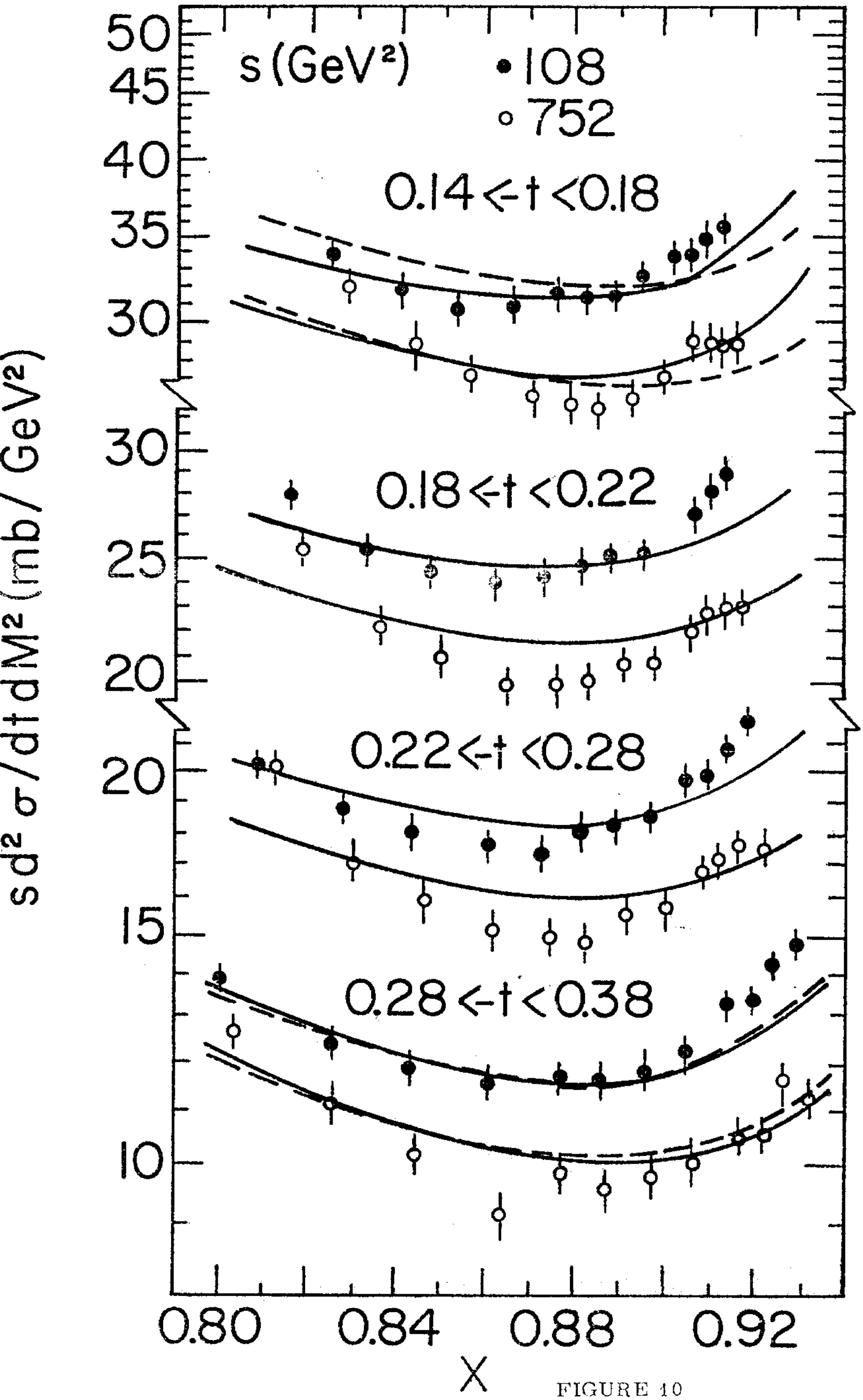
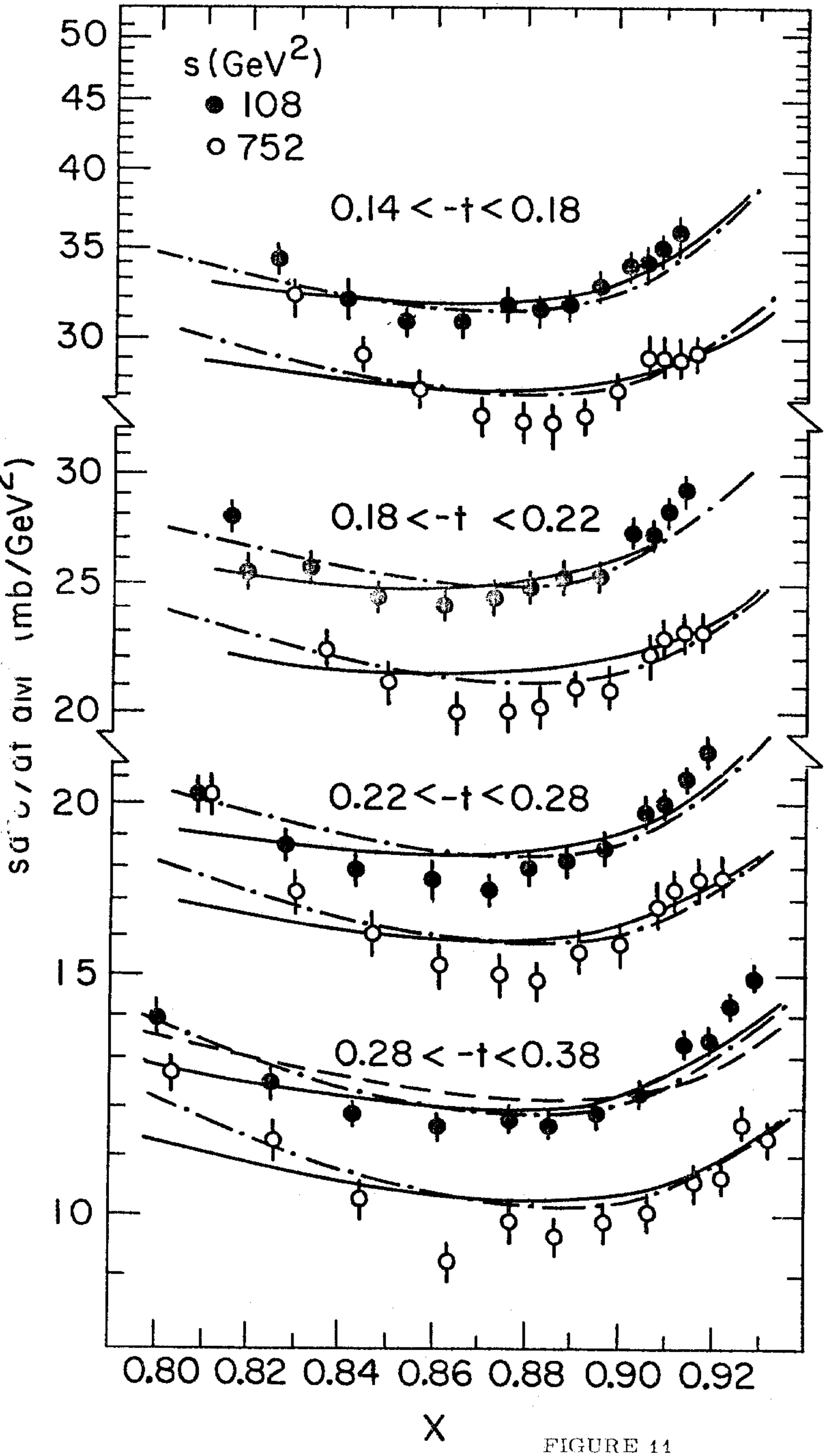
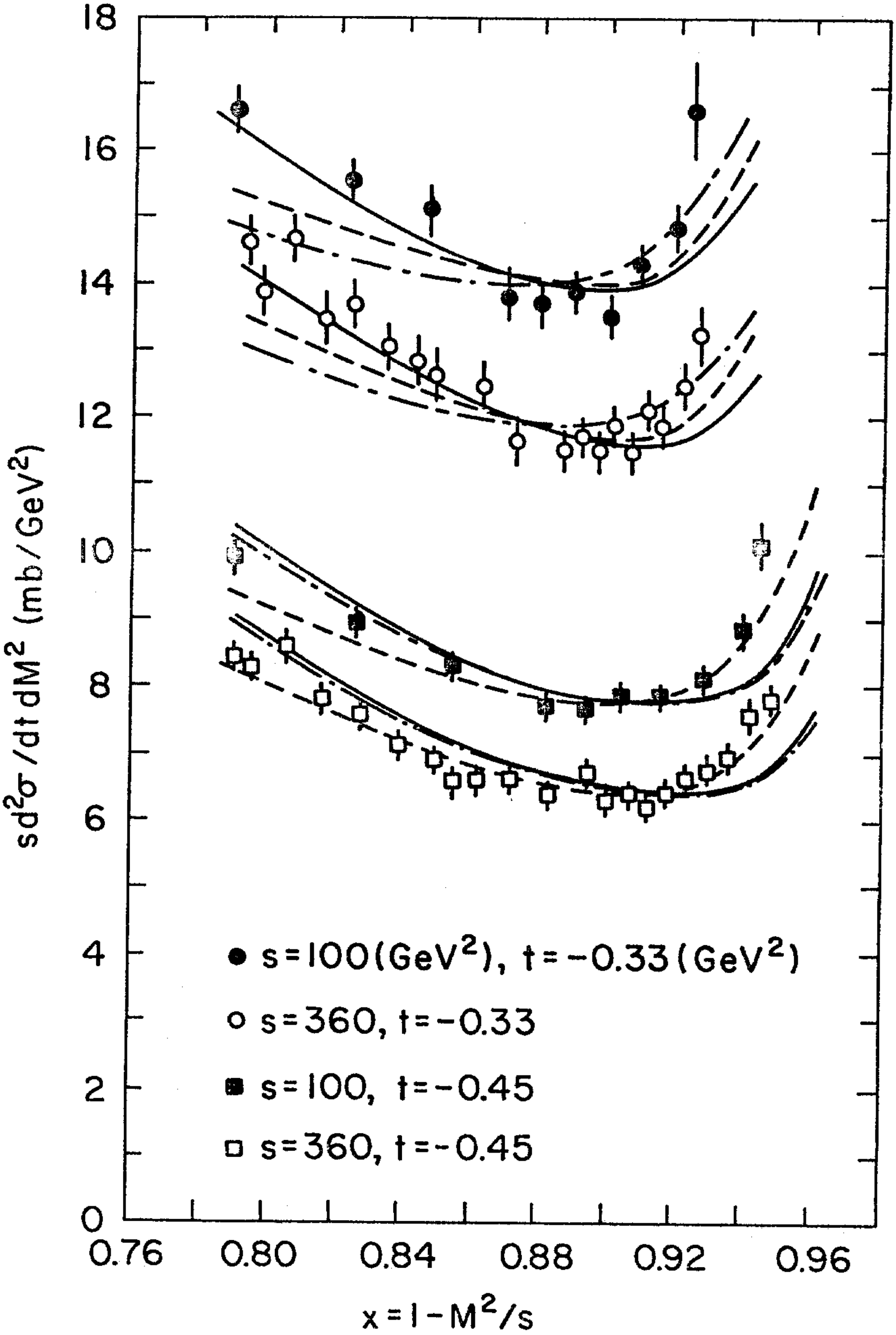

s (GeV²)
• 108
○ 752
0.14 < -t < 0.18
0.18 < -t < 0.22
0.22 < -t < 0.28
0.28 < -t < 0.38
s d² σ / dt dM² (mb / GeV²)
50
45
40
35
30
30
25
20
20
15
10
0.80
0.84
0.88
0.92
X

FIGURE 10



s(GeV^2)
● 108
○ 752
0.14 < -t < 0.18
0.18 < -t < 0.22
0.22 < -t < 0.28
0.28 < -t < 0.38
sd$^2\sigma$/dt dM (mb/GeV^2)
X

FIGURE 11



18
16
14
12
10
8
6
4
2
0
0.76
0.80
0.84
0.88
0.92
0.96
$sd^2\sigma/dt\,dM^2$ (mb/GeV^2)
$x = 1 - M^2/s$
● s=100(GeV^2), t=−0.33(GeV^2)
○ s=360, t=−0.33
■ s=100, t=−0.45
□ s=360, t=−0.45

FIGURE 12

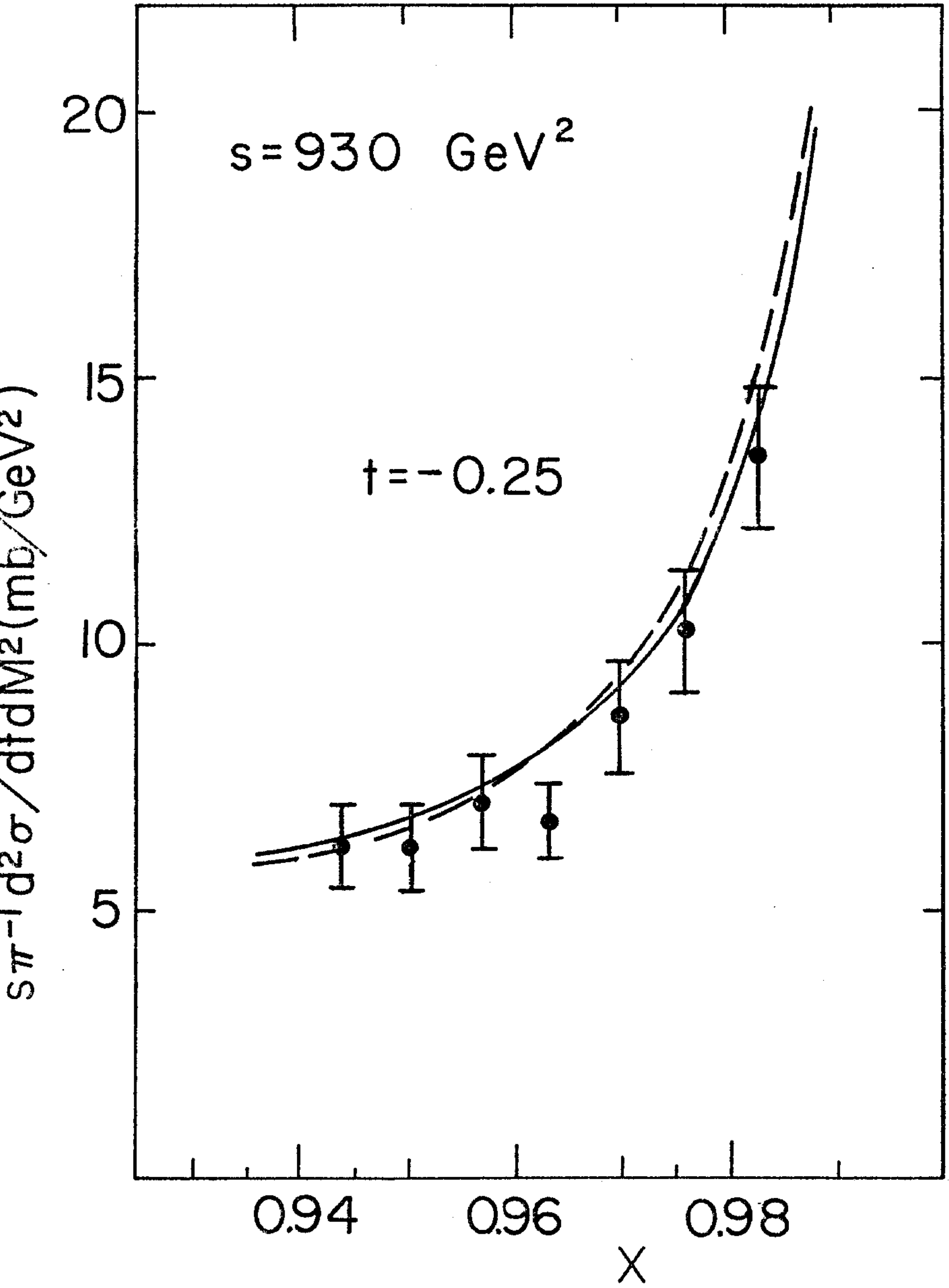

s=930 GeV²
t=-0.25
$s\pi^{-1} d^2\sigma / dt dM^2$ (mb/GeV²)
20
15
10
5
0.94
0.96
0.98
X

FIGURE 13

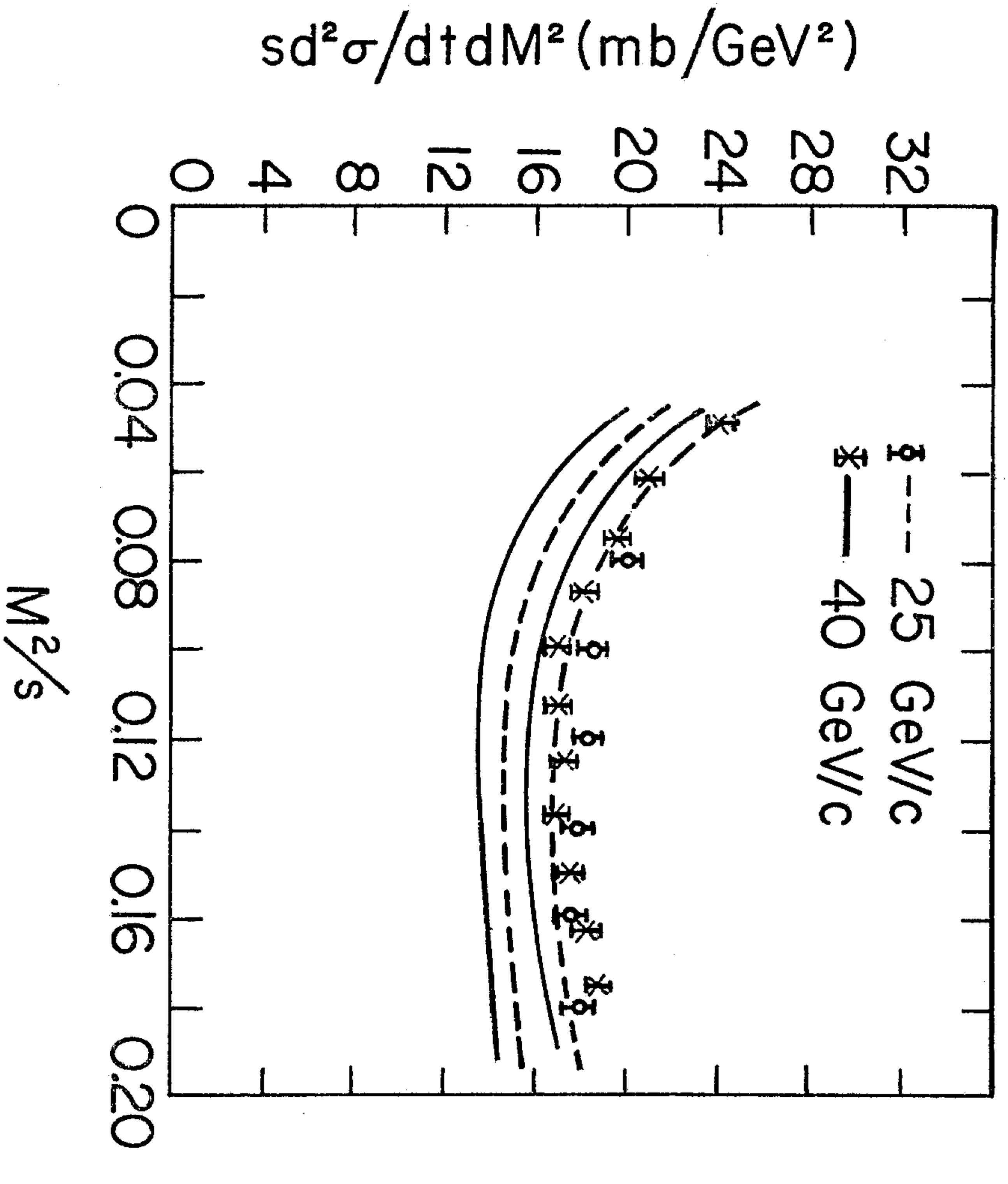
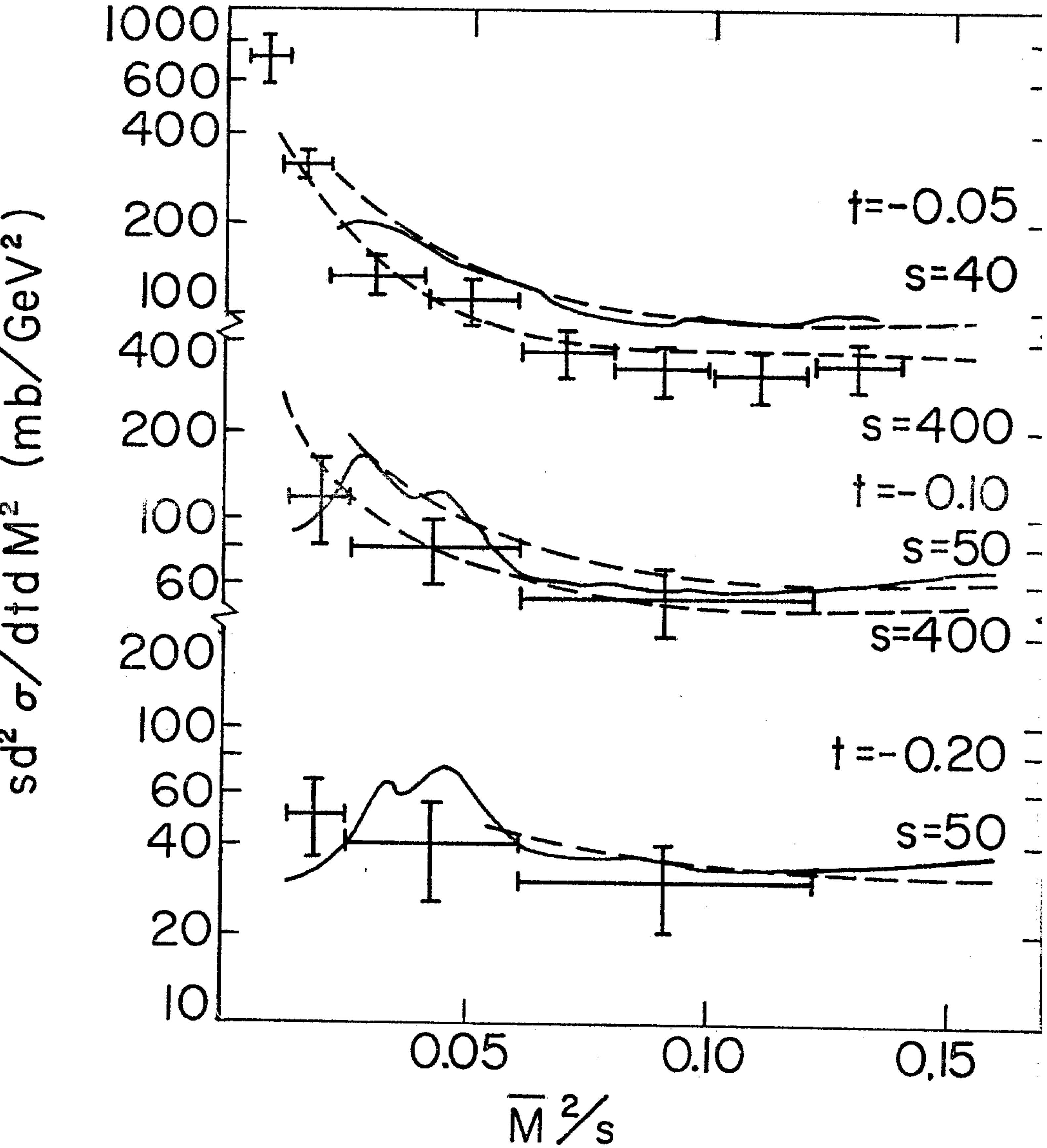

sd²σ/dtdM² (mb/GeV²)
M²/s
25 GeV/c
40 GeV/c
0
4
8
12
16
20
24
28
32
0.04
0.08
0.12
0.16
0.20

FIGURE 14



1000
600
400
200
100
400
200
100
60
200
100
60
40
20
10
$sd^2\sigma/dtdM^2$ (mb/GeV^2)
t=-0.05
s=40
s=400
t=-0.10
s=50
s=400
t=-0.20
s=50
0.05
0.10
0.15
$\overline{M}^2/s$

FIGURE 15

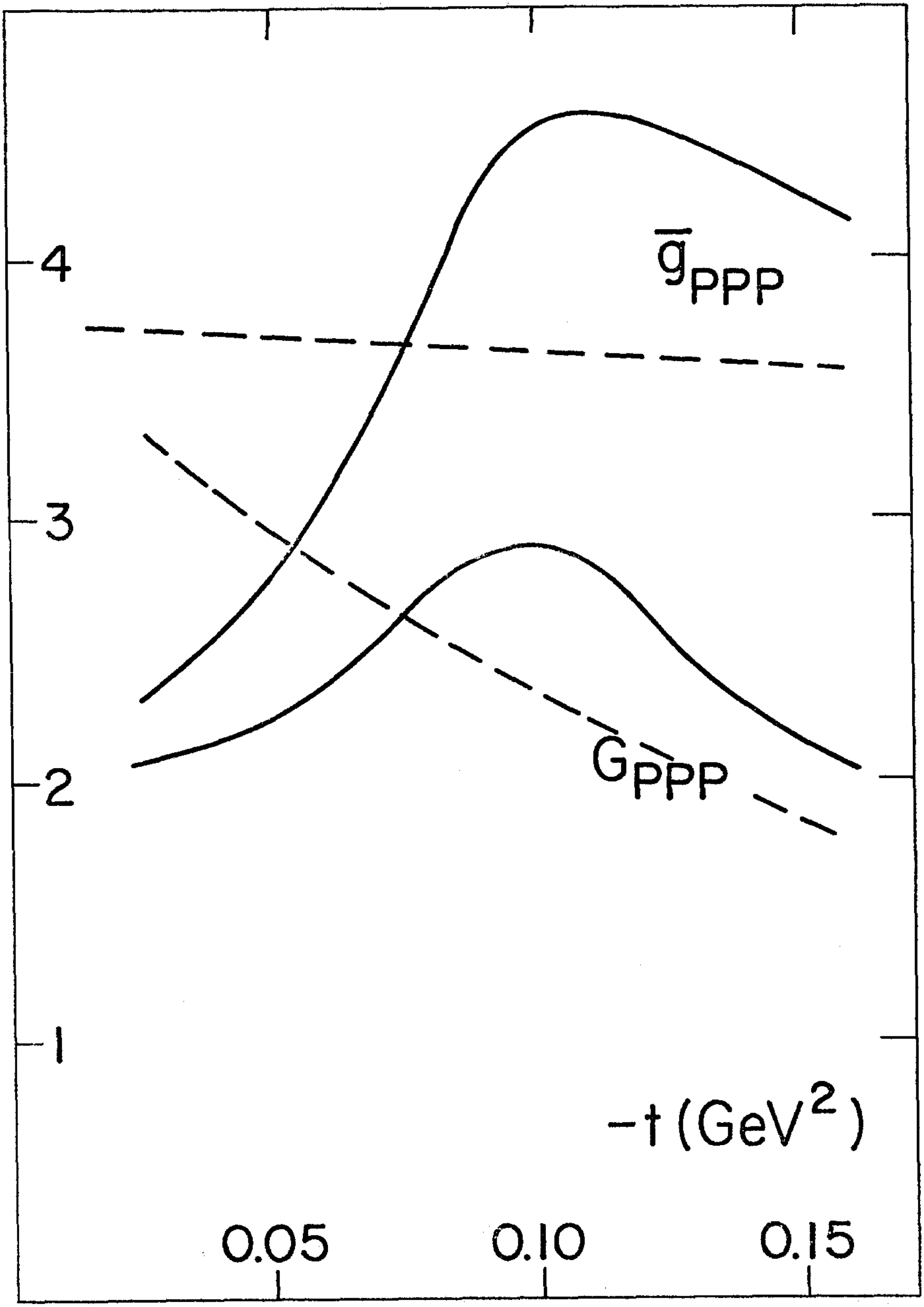
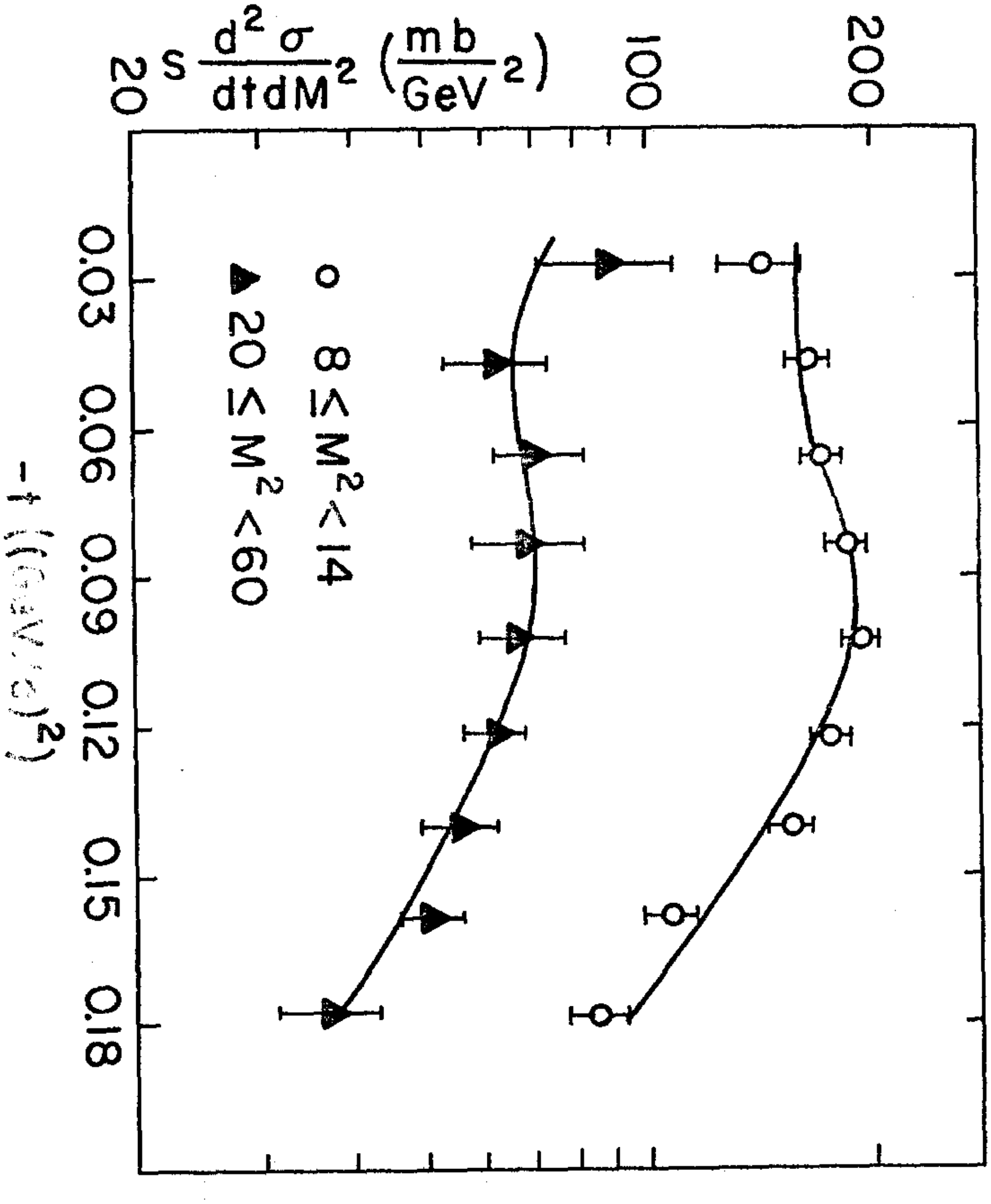


FIGURE 16



$s\frac{d^2\sigma}{dtdM^2}\left(\frac{mb}{GeV^2}\right)$
200
100
20
0.03
0.06
0.09
0.12
0.15
0.18
○ $8 \le M^2 < 14$
▲ $20 \le M^2 < 60$
$-t\ ((GeV/c)^2)$

FIGURE 17

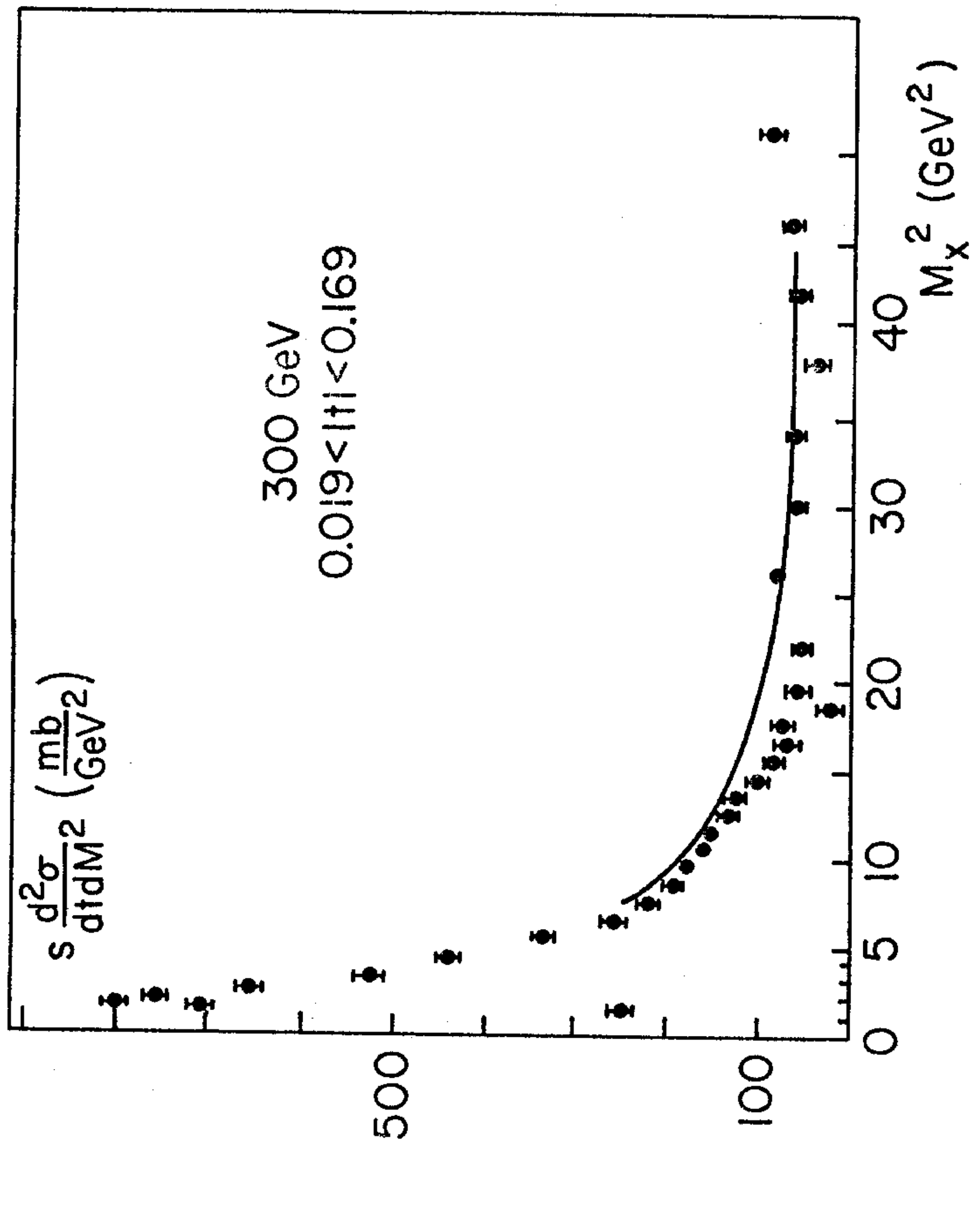

300 GeV
0.019<|t|<0.169
$s \frac{d^2\sigma}{dt\,dM^2} \left(\frac{mb}{GeV^2}\right)$
M_x^2 (GeV^2)
500
100
0
5
10
20
30
40

FIGURE 18

ERRATUM

Some Applications of f/P Universality to Inclusive Processes in the Triple-Regge Region, Louis A. P. Balázs

The last four sentences of the last paragraph of Sec. III should be replaced by

> Now $g_{fff} = g_{\omega\omega f}$ by exchange degeneracy. Since g_{acf}/g_{acP} is independent of a, c by f/P universality, it follows that we also have $g_{ffP} = g_{\omega\omega P}$.

In the sentence preceding Eq. (4.7),

> + vertex symmetry result $g_{\omega\omega k} = -g_{ffk}$

should read

> result $g_{\omega\omega k} = g_{ffk}$.