FNAL-Pub-74/64-THY
July, 1974

Comments on Proposed Explanations for the μ-Mesic Atom X-ray Discrepancy

Stephen L. Adler and Roger F. Dashen
National Accelerator Laboratory, Batavia, Illinois 60510
and The Institute for Advanced Study, Princeton, New Jersey 08540

and

S. B. Treiman*
National Accelerator Laboratory, Batavia, Illinois 60510

ABSTRACT

Two proposed explanations for the apparent μ-mesic atom X-ray discrepancy are the possible existence of nonperturbative vacuum polarization modifications and the possible existence of a weakly coupled light scalar boson. We show that a nonperturbative decrease in the vacuum polarization spectral function implies a reduction in the vertex for a time-like photon to couple to an electron-positron pair. This would lower by a few percent the rate for π^0 Dalitz decay and suggests observable effects in the colliding beam reactions $e^+e^- \to e^-e^+$, $e^+e^- \to \mu^\pm\mu^\mp$. Turning to the scalar boson hypothesis, we use neutron-electron and electron-deuteron scattering data to show that a scalar particle with mass lighter than about 0.6 MeV cannot be invoked to explain the μ-mesic discrepancy. We conclude by discussing the useful role which isotope effects and π-mesic atom experiments might play in determining the phenomenological structure of the extra potential implied by the discrepancy.

* Permanent address: Princeton University, Princeton, N.J. 08540

Recent studies of the X-ray spectra in μ-mesic atoms have shown persistent discrepancies between theory and experiment.[1] These discrepancies, if confirmed in future measurements of higher resolution, will require modification of the usual quantum electrodynamic theory used for calculating the μ-mesic atom energy levels. Phenomenologically, the required modification takes the form of an additional repulsive potential $\delta V(r)$ seen by the orbiting muon, which if written as a superposition of Yukawa potentials

$$\delta V(r) = \int d\sigma\, w(\sigma) \frac{e^{-\sigma r}}{r} \tag{1}$$

can involve masses σ in the range from 0 to $\sim$ 22-30 MeV.[1] At a fundamental level, the potential of Eq. (1) could arise from various sources. One possible origin[2] would be the presence of nonperturbative vacuum polarization modifications, which would change the usual vacuum polarization potential given by the lowest two orders of perturbation theory,[3]

$$V_{VP}(r) = -Z \frac{\alpha^2}{3\pi} \int_{4m_e^2}^{\infty} \frac{dt}{t} [\rho_e^{(0)}(t) + \rho_\mu^{(0)}(t) + \rho_e^{(1)}(t) + \rho_\mu^{(1)}(t) + \rho_{e\mu}^{(1)}(t)] \frac{e^{-t^{\frac{1}{2}} r}}{r} , \tag{2}$$

Z = nuclear charge,

$$\rho_e^{(0)}(t) = \left(1 + \frac{2m_e^2}{t}\right)\left(1 - \frac{4m_e^2}{t}\right)^{\frac{1}{2}} , \quad \rho_e^{(1)}(t) = \dots ,$$

to read $V_{VP} \to V_{VP} + \delta V$,

$$\delta V(r) = -Z \frac{\alpha^2}{3\pi} \int_{4m_e^2}^{\infty} \frac{dt}{t} \, \delta\rho(t) \, \frac{e^{-t^{\frac{1}{2}} r}}{r} \, . \tag{3}$$

Here $\delta\rho(t)$ (which must be negative to produce a repulsive potential) is a non-perturbative change in the vacuum polarization spectral function, of a magnitude much larger than one's naive estimate of the sixth and higher order perturbation theory terms omitted in Eq. (2). Another possible origin would be the existence of a weakly coupled light scalar, isoscalar boson ϕ coupling both to the μ and to nucleons,[4] which would produce a pure Yukawa potential

$$\delta V(r) = \frac{-g_{\phi\mu\bar{\mu}} \, g_{\phi N\bar{N}}}{4\pi} \, A \, \frac{e^{-M_\phi r}}{r} \, , \tag{4}$$

$$A = \text{nuclear mass number} \, .$$

One way of distinguishing between the vacuum polarization modification and scalar meson mechanisms is through the muon $g_\mu - 2$ experiment, where very different effects are predicted at the level of accuracy to be attained in forthcoming experiments.[2] In this paper we discuss some additional phenomenological aspects of the two mechanisms, with the aim of providing further means for distinguishing the fundamental origin of the potential of Eq. (1).

We begin by considering some implications of a non-perturbative modification in the vacuum polarization spectral function $\rho(t)$. In particular, let us focus on a process in which a time-like photon (of mass smaller than the mass of a μ pair) is emitted from a "black box" and decays into an electron-positron pair (Fig. 1). Neglecting radiative corrections, the rate Γ for this process is given by

$$\Gamma_{(e^+e^-)} = K \int dt \, \frac{1}{t} \, \gamma_B(t) \, \rho_e^{(0)}(t) \quad , \tag{5}$$

with K a kinematic constant. The first two factors $1/t$ and $\gamma_B(t)$ are, respectively, the photon propagator and a function describing the behavior of the "black box." The final factor $\rho_e^{(0)}(t)$, which arises from the phase space integral for the electron-positron pair, is just the lowest order photon spectral function defined in Eq. (2). Let us next consider the effect of first order radiative corrections.[5] If photons emitted from the "black box" are neglected, these consist of the interference of the loop diagrams in Fig. 2(a) with the lowest order diagram of Fig. 1, and the square of the bremsstrahlung diagrams of Fig. 2(b). Carrying out the phase space integrals for these gives

$$\Gamma_{(e^+e^-)+(e^+e^-\gamma)} = K \int dt \, \frac{1}{t} \, \gamma_B(t) \left[\rho_e^{(0)}(t) + \rho_e^{(1)}(t) + \rho_{e\mu}^{(1)}(t) \right] \quad , \tag{6}$$

with $\rho_e^{(1)}(t)$ and $\rho_{e\mu}^{(1)}(t)$ the order α corrections defined above.[3] The generalization of this argument to all orders[5] states that the rate for the "black box" to decay, via one virtual photon exchange, into a general electromagnetic final state containing electrons, positrons, and photons but excluding the one-photon state, is given by

$$\Gamma_{(EM)} = K \int dt \, \frac{1}{t} \, \gamma_B(t) \, \rho(t) \, , \tag{7}$$

with $\rho(t)$ the exact photon spectral function. Let us now use the fact that the various independent final states (e^+e^-, e^+e^- soft γ's, e^+e^- hard γ, 3γ, $e^+e^-\,e^+e^-$, ...) all make positive contributions to $\Gamma_{(EM)}$. Hence we can write

$$\Gamma_{(EM)} = \Gamma_{(e^+e^-) + (e^+e^- \text{ soft } \gamma\text{'s})} + \Gamma_{\text{other}} \, , \tag{8}$$

$$\Gamma_{\text{other}} \geq 0 \, ,$$

which combined with Eq. (7) gives the inequality

$$\Gamma_{(e^+e^-) + (e^+e^- \text{ soft } \gamma\text{'s})} \leq K \int dt \, \frac{1}{t} \, \gamma_B(t) \, \rho(t) \, . \tag{9}$$

Now let us introduce the assumption that $\rho(t)$, rather than being well-approximated[6] by $\rho_e^{(0)}(t) + \rho_e^{(1)}(t) + \rho_{e\mu}^{(1)}(t)$, is given by $\rho_e^{(0)}(t) + \rho_e^{(1)}(t) + \rho_{e\mu}^{(1)}(t) + \delta\rho(t)$, with $\delta\rho$ a (negative) non-perturbative modification. We then find

$$\Gamma_{(e^+e^-) + (e^+e^- \text{ soft } \gamma\text{'s})} \leq K \int dt \, \frac{1}{t} \, \gamma_B(t) \left[\rho_e^{(0)}(t) + \rho_e^{(1)}(t) + \rho_{e\mu}^{(1)}(t) + \delta\rho(t)\right] , \tag{10}$$

indicating that <u>postulating a non-perturbative vacuum polarization modification to explain the μ-mesic X-ray discrepancy predicts a corresponding reduction in the (radiative corrected) rate for pair production by a time-like photon</u>. To roughly estimate the magnitude of the expected effect, we recall that in Ref. 2, fits were made to the μ-mesic X-ray discrepancy using various forms for $\delta\rho(t)$, subject to the technical assumption that the magnitude of $\delta\rho$ is a monotonically increasing function of t (as might be expected if we are just entering a new regime where the discrepancy $\delta\rho$ is appearing). All monotonic forms giving good fits were found to satisfy

$$\delta\rho \lesssim -0.03 \;, \quad t \gtrsim (180\, m_e)^2 \sim 0.01\, (\mathrm{GeV}/c)^2, \tag{11}$$

with some representative fits given by

$$\text{(i)} \quad \delta\rho = -0.032\;\theta(w-2.5) \;, \tag{12a}$$

$$\text{(ii)} \quad \delta\rho = -0.071\;\theta(w-3)\left(\frac{w-3}{w}\right)^{0.2} \;, \tag{12b}$$

$$\theta = \text{step function} \;, \qquad w = \ln\left(\frac{t}{4m_e^2}\right) .$$

We conclude from Eqs. (11) - (12) that reductions in pair production rates of the order of a few percent are to be expected.

Let us now apply these remarks to some specific cases. We consider first π^0 Dalitz decay, to which the "black box" argument is directly applicable[5] and for which Eq. (10) becomes

$$B \equiv \Gamma^{-1}_{(\pi^0\to 2\gamma)}\left[\Gamma_{(\pi^0\to\gamma e^+e^-)+(\pi^0\to\gamma e^+e^-\ \text{soft}\ \gamma\text{'s})}\right]$$

$$\le \frac{4\alpha}{3\pi}\int_{4m_e^2}^{m_\pi^2} dt\,\frac{1}{t}\,\gamma_{\pi^0}(t)\left[\rho_e^{(0)}(t)+\rho_e^{(1)}(t)+\rho_{e\mu}^{(1)}(t)+\delta\rho(t)\right] ,$$

$$\gamma_{\pi^0}(t) = \left(1-\frac{t}{m_\pi^2}\right)^3 (1+2at) \;. \tag{13}$$

The term $1 + 2at$ arises from the hadronic electromagnetic form factor for time-like photon emission. In the approximation of ρ-dominance, a is given by

$$a \approx \frac{1}{M_\rho^2} \approx \frac{0.03}{m_\pi^2} , \tag{14}$$

but could possibly deviate from Eq. (14) by up to a factor of two in either direction. Neglecting products of small quantities and carrying out the integrals in Eq. (13) gives

$$\frac{4\alpha}{3\pi} \int_{4m_e^2}^{m_\pi^2} dt \, \frac{1}{t} \left(1 - \frac{t}{m_\pi^2}\right)^3 \rho_e^{(0)}(t) = 1.186 \cdot 10^{-2} \equiv B_0 ,$$

$$\left.\begin{aligned}
&\frac{4\alpha}{3\pi} \int_{4m_e^2}^{m_\pi^2} dt \, \frac{1}{t} \left(1 - \frac{t}{m_\pi^2}\right)^3 \rho_e^{(1)}(t) = 1.05 \cdot 10^{-4} = 8.85 \cdot 10^{-3} B_0 , \\
&\frac{4\alpha}{3\pi} \int_{4m_e^2}^{m_\pi^2} dt \, \frac{1}{t} \left(1 - \frac{t}{m_\pi^2}\right)^3 \rho_{e\mu}^{(1)}(t) \ll 1.05 \cdot 10^{-4} , \\
&\frac{4\alpha}{3\pi} \int_{4m_e^2}^{m_\pi^2} dt \, \frac{1}{t} \left(1 - \frac{t}{m_\pi^2}\right)^3 2at \, \rho_e^{(0)}(t) = 2.32 \cdot 10^{-5} = 1.96 \cdot 10^{-3} B_0 , \quad a = \frac{0.03}{m_\pi^2} , \\
&\frac{4\alpha}{3\pi} \int_{4m_e^2}^{m_\pi^2} dt \, \frac{1}{t} \left(1 - \frac{t}{m_\pi^2}\right)^3 \delta\rho(t) = \begin{cases} -2.69 \cdot 10^{-4} = -2.27 \cdot 10^{-2} B_0, & \delta\rho = \text{Eq. (12a)}, \\ -4.46 \cdot 10^{-4} = -3.76 \cdot 10^{-2} B_0, & \delta\rho = \text{Eq. (12b)}. \end{cases}
\end{aligned}\right\} \tag{15}$$

From the final line, we see that $\delta\rho$ leads to a reduction in the π^0 Dalitz rate in the range of 2 - 4%, as anticipated;[7] comparing with the third line, we see that the effect is an order of magnitude larger than possible uncertainties in the rate arising from uncertainties in the hadronic structure constant a. The current experimental status of π^0 Dalitz decay, and of double Dalitz decay (where both photons convert and our reasoning suggests a 4 - 8% rate reduction) are[8]

$$\begin{array}{llll} \pi^0 \to \gamma e^+ e^- & \text{conventional theory:} & B = & 1.186 \cdot 10^{-2} + 1.05 \cdot 10^{-3} = 1.196 \cdot 10^{-2} \\ & \text{experiment:} & B = & (1.166 \pm 0.045) \cdot 10^{-2} \\ \pi^0 \to e^+ e^- e^+ e^- & \text{conventional theory:} & B = & 3.47 \cdot 10^{-5} \\ & \text{experiment:} & B = & (3.18 \pm 0.30) \cdot 10^{-5}, \end{array} \qquad (16)$$

entirely compatible with reductions of the magnitude which we have been considering. Evidently, an experiment to measure the Dalitz decay rate to an accuracy of better than 1% would be of considerable interest.

We consider next the electron-positron colliding beam reaction $e^+e^- \to \mu^\pm \mu^\mp$, which has been extensively studied for t values above $2(\mathrm{GeV}/c)^2$. In this case we cannot establish precise inequalities including radiative corrections as we did via the "black box" argument for π^0 Dalitz decay. However, since the "black box" argument indicates that the coupling of a time-like photon to a pair is reduced, we qualitatively expect

$$\frac{\sigma_{e^+e^- \to \mu^\pm \mu^\mp}(\text{with } \delta\rho)}{\sigma_{e^+e^- \to \mu^\pm \mu^\mp}(\text{conventional theory})} \approx (1+\delta_e)(1+\delta_\mu), \qquad (17)$$

with $1+\delta_e$, $1+\delta_\mu$ reduction factors describing changes in the coupling of the time-like photon to the electron, muon pair respectively. If we assume that we can extrapolate the order of magnitude of δ from the μ-mesic atom fits at $t < 0.01\,(\mathrm{GeV}/c)^2$ to the vastly larger t values seen in colliding beam experiments, then we might expect to see a 6 - 10% reduction from the conventional theoretical prediction in the cross section for μ pair production. Present experimental results for this process are

Frascati[9]
$t \sim 2.5 - 4(\mathrm{GeV}/c)^2$

$$\frac{\left[\dfrac{\sigma_{e^+e^-\to\mu^\pm\mu^\mp}\ (\text{experiment})}{\sigma_{e^+e^-\to e^\pm e^\mp}\ (45^\circ<\theta<135^\circ\ \text{experiment})}\right]}{\left[\dfrac{\sigma_{e^+e^-\to\mu^\pm\mu^\mp}\ (\text{conventional theory})}{\sigma_{e^+e^-\to e^\pm e^\mp}\ (45^\circ<\theta<135^\circ\ \text{conventional theory})}\right]} = 0.98 \pm 0.08, \tag{13}$$

SPEAR[10]
$t \sim 27(\mathrm{GeV}/c)^2$

$$\frac{\left[\dfrac{\sigma_{e^+e^-\to\mu^\pm\mu^\mp}\ (\text{experiment})}{\sigma_{e^+e^-\to e^+e^-}\ (\theta\sim 3.7^\circ\ \text{experiment})}\right]}{\left[\dfrac{\sigma_{e^+e^-\to\mu^\pm\mu^\mp}\ (\text{conventional theory})}{\sigma_{e^+e^-\to e^+e^-}\ (\theta\sim 3.7^\circ\ \text{conventional theory})}\right]} = 0.92 \pm 0.11,$$

both of which are evidently compatible, within errors, with a time-like vertex reduction in the 3 - 5% range. More precise experiments here would again be of considerable interest.

We consider finally the reaction $e^+e^- \to e^\pm e^\mp$. Here the cross section contains three terms, involving respectively space-like photon exchange, time-like photon exchange and space-like-time-like interference. In the only precision experiment reported to date (done at Frascati) the sign of the final e^+ and e^- could not be distinguished, and hence it was not possible to select a kinematic region where the time-like vertices dominate. The Frascati result[11] gives

$$\frac{\sigma_{e^+e^-\to e^\pm e^\mp}\left(\begin{array}{c}45^\circ < \theta < 135^\circ \\ \text{experiment}\end{array}\right)}{\sigma_{e^+e^-\to e^\pm e^\mp}\left(\begin{array}{c}45^\circ < \theta < 135^\circ \\ \text{conventional theory}\end{array}\right)} = 1.00 \pm 0.02\,; \qquad (19)$$

to determine the sensitivity of this experiment to the time-like vertex we replace the time-like vertex in the usual Bhabba formula by $(1+\delta_e)$ and integrate over the angular range $45^\circ < \theta < 135^\circ$, giving

$$\frac{\sigma_{e^+e^-\to e^\pm e^\mp}\left(\begin{array}{c}45^\circ < \theta < 135^\circ \\ \text{timelike vertex} \times (1+\delta_e)\end{array}\right)}{\sigma_{e^+e^-\to e^\pm e^\mp}\left(\begin{array}{c}45^\circ < \theta < 135^\circ \\ \text{conventional theory}\end{array}\right)} = (1 - 0.08\,\delta_e)\,. \qquad (20)$$

Hence a δ_e of -0.03 to -0.05 is perfectly compatible with Eq. (19). Accurate experiments emphasizing the time-like vertex contribution (that is, measurements of $e^+e^- \to e^-e^+$) would evidently be desirable.

Let us now turn our attention to the possible existence of a weakly coupled scalar, isoscalar boson ϕ as an explanation for the μ-mesic X-ray discrepancy. Fitting Eq. (4) to the μ-mesic atom data gives a coupling

$\sim(g_{\phi\mu\bar{\mu}}\, g_{\phi N\bar{N}})/4\pi$ ranging from $\sim 1.3 \cdot 10^{-7}$ for $M_\phi \leq 1$ MeV to $\sim 2.5 \cdot 10^{-6}$ for $M_\phi = 22$ MeV.[1,2] Using these empirical strengths, and making the additional assumption

$$g_{\phi e\bar{e}} = \frac{m_e}{m_\mu}\, g_{\phi\mu\bar{\mu}} , \tag{21}$$

as suggested by the structure of ϕ couplings in unified gauge theories of the weak and electromagnetic interactions, it is possible to give arguments ruling out the existence of a ϕ in most of the allowed mass range between 0 and 30 MeV. First of all, Resnick, Sundaresan and Watson[12] have suggested looking, via the $\phi \to e^+e^-$ decay mode, for ϕ mesons emitted in $0^+ \to 0^+$ nuclear transitions. Such an experiment has been reported recently by Kohler, Becker and Watson[13] using the transitions between the excited ^{16}O(6.05 MeV) and ^4He(20.2 MeV) 0^+ levels and the corresponding 0^+ ground states, with a negative result which rules out the existence of a ϕ with mass between 1.030 and $\sim$18 MeV. This method obviously cannot be used for $M_\phi < 1.022$ MeV, where the e^+e^- decay mode is kinematically forbidden. We will now present a second argument, using neutron-electron and electron-deuteron scattering data, which rules out the existence of a ϕ meson in the mass range between 0 and 0.6 MeV.

The argument is based on the fact that the scattering of thermal neutrons from atomic electrons can be disentangled from the neutron-nucleus interaction, for example by using the fact that the former is modulated by the rapidly varying atomic electron form factor $F_A(t)$.[14]

Now if a scalar meson ϕ is present which couples to both neutrons and electrons, the usual electron-neutron Coulomb interaction is modified to read

$$i\left[-\frac{e^2 G_E^N(t)}{t} - \frac{g_{\phi e\bar{e}}\, g_{\phi N\bar{N}}}{t - M_\phi^2}\right], \tag{22}$$

with $G_E^N(t)$ the neutron charge form factor. Substituting

$$G_E^N(t) = -a_N t\ , \tag{23}$$

with a_N the neutron form factor slope, and using

$$e^2 = 4\pi\alpha\ ,$$

$$-g_{\phi e\bar{e}}\, g_{\phi N\bar{N}} = 4\pi \frac{m_e}{m_\mu}\, 1.3\cdot 10^{-7}\ ,\quad M_\phi \lesssim 1\ \mathrm{MeV}\ , \tag{24}$$

the $t \to 0$ limit of Eq. (22) which is appropriate to the scattering of thermal neutrons becomes

$$i\ 4\pi\left[\alpha\, a_N - \frac{m_e\, 1.3\cdot 10^{-7}}{m_\mu\, M_\phi^2}\right] \equiv i\ 4\pi\left[\alpha\ a_{EFF}\right]\ . \tag{25}$$

That is, in the presence of a scalar meson, the thermal neutron method measures not the true neutron form factor slope a_N, but rather the effective slope

$$a_{EFF} = a_N - \frac{1}{\alpha}\ \frac{m_e\, 1.3\cdot 10^{-7}}{m_\mu\ M_\phi^2}\ . \tag{26}$$

According to the most recent thermal neutron experiments,[15]

$$a_{EFF} = (0.51 \pm 0.02)/(\mathrm{GeV}/c)^2\ , \tag{27}$$

while if $M_\phi \leq 0.6$ MeV, the second term on the right-hand side of Eq. (26) is

$$\frac{1}{\alpha} \frac{m_e}{m_\mu} \frac{1.3 \cdot 10^{-7}}{M_\phi^2} \geq \frac{1}{\alpha} \frac{m_e}{m_\mu} \frac{1.3 \cdot 10^{-7}}{(0.6 \cdot 10^{-3})^2} \frac{1}{(\mathrm{GeV}/c)^2} = \frac{0.24}{(\mathrm{GeV}/c)^2} , \quad (28)$$

implying that a_N exceeds a_{EFF} by 50% or more. On the other hand, electron-deuteron scattering experiments have been performed at momentum transfers in the range $t = 0.01 - 0.1\ (\mathrm{GeV}/c)^2$, where the scalar meson term in Eq. (22) is negligible relative to the Coulomb term and so the true slope a_N is measured. These experiments[16] are incompatible with an a_N which exceeds the thermal neutron value by 50% -- if anything, they indicate a slightly smaller value of a_N than the thermal neutron value, most likely[17] as a result of uncertainties in the deuteron corrections. So we conclude that the μ-mesic X-ray discrepancy cannot be caused by a scalar meson of mass lighter than 0.6 MeV. Closing the gap between 0.6 MeV and 1.022 MeV would require a determination of a_N in electron-deuteron scattering to better than 15% accuracy, a difficult but perhaps achievable goal.

We conclude by discussing two additional experiments which could be useful in determining the phenomenological structure of the extra potential of Eq. (1). One obvious distinction between Eqs. (3) and (4), quite independent of their detailed r-dependence, is that Eq. (3) couples to the nuclear charge Z, whereas Eq. (4) couples to the nucleon mass number A. Clearly, a potential coupling to Z will produce the same X-ray discrepancy in all isotopes of a given element, whereas a potential coupling to A will show a change in the discrepancy from isotope to isotope. Perhaps the most likely candidates[18] for observation of this effect are the μ-mesic helium ions $[{}^3\mathrm{He}, \mu]^+$ and $[{}^4\mathrm{He}, \mu]^+$. In Ref. 2 we showed that any potential of the form of Eq. (1), which fits the observed μ-mesic X-ray discrepancies and which does not involve masses σ lighter than 1 MeV, will reduce the $2p \rightarrow 2s$ Lamb shift in $[{}^4\mathrm{He}, \mu]^+$ by about -0.027 eV. The corresponding effect in $[{}^3\mathrm{He}, \mu]^+$ will thus be -0.027 eV for a potential coupling to Z and $3/4 \times (-0.027\ \mathrm{eV}) = -0.020\ \mathrm{eV}$ for a potential coupling to A. To give an idea of the energy scale involved, we note that the difference between the two cases of -0.007 eV is rougly a factor of two smaller than the fourth order vacuum polarization shift[19] of -0.012 eV. For the effect to be observable in practice, current uncertainties[19] in the nuclear charge radius and nuclear polarizability will have to be reduced, particularly in the case of ^{3}He. Obviously, the same reasoning applies to isotopic partners of higher atomic number, but the attainable fractional change in the μ-mesic discrepancy is not as high in these cases as in the helium system.

Clearly, a second general phenomenological question which can be asked about the potential of Eq. (1) is that of determining the nature of the coupling to the orbiting particle. Obviously, the vacuum polarization potential of Eq. (3) is the same for all negatively charged orbiting elementary particles, whereas the scalar meson potential analogous to Eq. (4) could be rather different if the orbiting particle were a hadron rather than a muon. A natural way to search for such differences in couplings would be to study the transition X-rays in π-mesic atoms. In order for the small energy shift produced by Eq. (1) not to be masked by the rather uncertain energy shifts resulting from the strong π-nuclear interactions, it is necessary to consider pions in large circular orbits ($\ell = n - 1$, n large) in which the pion spends a very small amount of time inside the nucleus. At the same time, the orbits cannot be chosen too large, lest uncertainties in the electron screening correction (typically about 5% of the screening correction itself) mask the energy shift being searched for. We have systematically surveyed all π-mesic atom circular orbits in the range $Z = 15$-90, $n = 2$-9 and have located a number of cases, tabulated in Table I, where the expected energy shift from Eq. (1) is substantially larger than both the nuclear energy shift and the estimated few percent uncertainty in the screening correction. (The formulas used for making these estimates are listed in the Appendix.) The transitions of interest all lie in the energy range 75 - 200 keV, which may permit high resolution crystal spectrometer studies[20] using the high pion fluxes expected to be available at meson factories. We emphasize that on the theoretical side, the pursuit of this idea will require a careful determination of all normal energy shifts which affect the π-mesic levels, much as has already been done in the corresponding μ-mesic cases.

ACKNOWLEDGEMENTS

We wish to thank S. Brodsky, E. B. Hughes, B. Lautrup, B. W. Lee, V. Telegdi, W. I. Weisberger, and R. Wilson for helpful conversations.

APPENDIX:

Formulas for π-Mesic Atom Estimates

We outline here the formulas used in making the estimates of Table I. We consider throughout circular pionic orbits ($\ell = n-1$) and evaluate matrix elements of the various perturbing potentials using non-relativistic hydrogenic wave functions for the pion. The unperturbed pion energy is[21]

$$E_{n\,n-1} \approx -\frac{Z^2\alpha^2}{2n^2}\, m_\pi \left[1 + \frac{(Z\alpha)^2}{n^2}\left(\frac{n}{n-\frac{1}{2}} - \frac{3}{4}\right)\right], \tag{A.1}$$

giving for the X-ray transition energy

$$E_\gamma \equiv E_{n\,n-1} - E_{n-1\,n-2} \approx 3.72\,\text{keV}\; Z^2 \left\{ \frac{1}{(n-1)^2} - \frac{1}{n^2} + (Z\alpha)^2 \left[\frac{1}{(n-1)^3(n-\frac{3}{2})} - \frac{1}{n^3(n-\frac{1}{2})} - \frac{3}{4}\left(\frac{1}{(n-1)^4} - \frac{1}{n^4}\right)\right]\right\}. \tag{A.2}$$

To estimate the nuclear energy shift we use the approximate formula[22]

$$\left|\frac{\Delta E}{E}\right| \sim \frac{\left(\frac{2Z\alpha\, R\, m_\pi}{\ell+1}\right)^{2\ell+1}}{(2\ell+1)!}\; C\,, \tag{A.3}$$

with C a complex expression which is numerically of order unity (and for which we take the value 1 in the numerical work), with ℓ the orbital angular momentum and R the nuclear radius. Taking

$$|\Delta(E_n - E_{n-1})|_{\text{nuclear}} \approx |\Delta E_{n-1}|_{\text{nuclear}}\,,$$

$$E_{n-1} \approx 3720\,\text{eV}\; \frac{Z^2}{(n-1)^2}\,, \tag{A.4}$$

$$m_\pi R \approx A^{1/3} ,$$

we get the final formula

$$|\Delta (E_n - E_{n-1})|_{nuclear} \approx 3720 \text{ eV} \frac{Z^2}{(n-1)^2} \frac{\left(\frac{2 Z\alpha A^{1/3}}{n-1}\right)^{2n-3}}{(2n-3)!} . \tag{A.5}$$

In evaluating A as a function of Z we use the approximate relation

$$\begin{aligned} A &= 2Z & Z &\le 22 , \\ A &= Z + 22 + 1.75 (Z - 22) , & Z &\ge 22 . \end{aligned} \tag{A.6}$$

To estimate screening corrections we use the parameterization for the screening potential given by Vogel,[23]

$$V_{SCR}(r) \approx -C r^K e^{-\beta r} , \tag{A.7}$$

with C (in 10^{-2} eV for r in fm), K and β (in fm^{-1}) as follows for the transitions of greatest interest,

Z	30	35	50	55	60	65
C	0.0605	0.112	0.533	0.867	1.389	2.202
K	1.9319	1.9166	1.8606	1.8380	1.8142	1.7897
100 β	0.04468	0.05245	0.07632	0.08433	0.09300	0.10194

(A.8)

Vogel actually tabulated the parameters C, K and β for Z between 35 and 90; in extrapolating his table to smaller Z we assume a linear variation of K and β (as fixed by his Z = 35 and Z = 40 values) and a $(Z/35)^4$ variation of C from Z = 35 downwards. Evaluating matrix elements of

Eq. (A. 7) (and using $M_\pi^{-1} \approx \sqrt{2}$ fm to absorb the dimensional factors) gives the final formula

$$\Delta(E_n - E_{n-1})_{\text{screening}}$$
$$\approx -C\, 10^{-2} \text{eV} (\sqrt{2})^K \left\{ \frac{\Gamma(2n+1+K)}{\Gamma(2n+1)} \frac{\left(\frac{n}{2Z\alpha}\right)^K}{\left[1 + \frac{n}{2Z\alpha}\sqrt{2}\,\beta\right]^{2n+1+K}} \right.$$
$$\left. - \frac{\Gamma(2n-1+K)}{\Gamma(2n-1)} \frac{\left(\frac{n-1}{2Z\alpha}\right)^K}{\left[1 + \frac{(n-1)}{2Z\alpha}\sqrt{2}\,\beta\right]^{2n-1+K}} \right\} . \tag{A. 9}$$

We have not tabulated nuclear polarizability corrections because detailed curves for these have been given by Ericson and Hüfner.[24] For the transitions of interest the shifts resulting from this effect are considerably smaller than the shift resulting from the pion-nuclear strong interaction. Similarly, pion polarization effects and energy uncertainties arising from uncertainties in the nuclear and pion charge radii should be unimportant.

To evaluate the energy shifts produced by the perturbing potentials of Eqs. (3) and (4) we employ the calculations of Ref. 2. From the vacuum polarization modification of Eq. (3) we find $[5.75\ \text{eV} = (\alpha^3/3\pi)\, m_\pi]$

$$\delta(E_n - E_{n-1})_{\substack{\text{vacuum polarization}\\ \text{modification}}}$$
$$= 5.75\ \text{eV}\ Z^2 \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] \int_0^\infty dw\, f_\gamma\, [4m_e^2\, e^w]\ \delta\rho(w) , \tag{A. 10}$$

$$f_\gamma[t] = \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]^{-1} \left\{ \frac{1}{(n-1)^2} \left[1 + \left(\frac{t}{4m_\pi^2}\right)^{\frac{1}{2}} \frac{n-1}{Z\alpha}\right]^{-2(n-1)} - \frac{1}{n^2}\left[1 + \left(\frac{t}{4m_\pi^2}\right)^{\frac{1}{2}} \frac{n}{Z\alpha}\right]^{-2n} \right\} ,$$

while from the scalar meson potential of Eq. (4) we get

$$\delta(E_n - E_{n-1})_{\text{scalar meson}} \tag{A.11}$$

$$= \left(\frac{A}{2Z}\right) 5.75\,\text{eV}\, Z^2 \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] 2.82 \cdot 10^4\, g_{\phi\mu\bar{\mu}}\, g_{\phi N\bar{N}}\, f_\gamma[M_\phi^2]\, \frac{g_{\phi\pi\pi}}{g_{\phi\mu\bar{\mu}}} .$$

The product $g_{\phi\mu\bar{\mu}}\, g_{\phi N\bar{N}}$ is taken from Table VIII of Ref. 2, while for $g_{\phi\pi\pi}/g_{\phi\mu\bar{\mu}}$ we use the value $m_\pi/(2m_\mu) = 0.66$ taken by this ratio in the simplest gauge models, in which there is only one physical scalar meson and in which the chiral $SU(3) \otimes SU(3)$ symmetry breaking term in the strong interaction Lagrangian transforms as pure $(3,\bar{3}) \oplus (\bar{3},3)$. The reasoning leading to the coupling ratio which we use is as follows. If only one scalar ϕ develops a vacuum expectation value λ, then the ϕ couplings to muons and to hadrons are respectively

$$\mathcal{L}_{\phi\mu\bar{\mu}} = \frac{\phi}{\lambda} m_\mu \bar{\psi}_\mu \psi_\mu ,$$
$$\mathcal{L}_{\phi\,\text{hadron}} = \frac{\phi}{\lambda} \delta\mathcal{L}_{\text{chiral breaking}} . \tag{A.12}$$

For muons at rest we find

$$< \mu | m_\mu \bar{\psi}_\mu \psi_\mu | \mu > = m_\mu , \tag{A.13}$$

while for pions at rest we find, in the $(3,\bar{3}) \oplus (\bar{3},3)$ model[25] for $\delta\mathcal{L}_{\text{chiral breaking}}$,

$$< \pi | \delta\mathcal{L}_{\text{chiral breaking}} | \pi > = \frac{1}{2m_\pi} m_\pi^2 = \tfrac{1}{2} m_\pi , \tag{A.14}$$

giving a coupling ratio of $m_\pi/(2m_\mu)$. Heuristically, the factor of $\frac{1}{2}$ arises from the fact that in a free Dirac Hamiltonian $\mathcal{H}_D = \psi^\dagger(-i\,\vec{\alpha}\cdot\vec{\nabla} + \beta m)\psi$, all of the particle mass in the rest frame arises from the chiral symmetry breaking term $m\bar{\psi}\psi$, whereas in a free Klein-Gordon Hamiltonian $\mathcal{H}_{KG} = \frac{1}{2}(\dot{\varphi})^2 + \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{1}{2}m^2\varphi^2$, only half of the particle mass in the rest frame arises from the chiral breaking term $\frac{1}{2}m^2\varphi^2$, the other half arising from the term $\frac{1}{2}(\dot{\varphi})^2$ which belongs to the chiral symmetric kinetic energy term.[26]

TABLE I.

Pi-Mesic Atom Transitions Where the Potential of Eq. (1) May be Visible

Z	$n \to n-1$	E_γ (keV)	$\lvert\Delta E\rvert_{nuclear}$ (eV)	$\Delta E_{screening}$ (eV)	δE (scalar meson) $M_\phi = 1$ MeV (eV)	δE (scalar meson) $M_\phi = 22$ MeV (eV)	δE (vacuum polarization discrepan $\delta\rho$ = Eq. (12a) (eV)	δE (vacuum polarization discrepan $\delta\rho$ = Eq. (12b) (eV)
30	$5 \to 4$	75	0.14	-7.1	-3.5	-0.16	-1.5	-1.2
35	$5 \to 4$	103	0.85	-9.0	-5.1	-0.46	-2.8	-2.3
50	$6 \to 5$	114	0.054	-26	-6.0	-0.26	-2.6	-2.1
55	$6 \to 5$	138	0.21	-31	-7.5	-0.51	-3.8	-3.1
60	$6 \to 5$	164	0.74	-37	-9.1	-0.92	-5.2	-4.5
65	$6 \to 5$	193	2.3	-44	-10.9	-1.6	-6.9	-6.2

REFERENCES

1. A good review of the relevant experiments and theory is given by P. J. S. Watson and M. K. Sundaresan, Discrepancy between Theory and Experiments in Muonic X-rays - A Critical Discussion, Carleton University preprint.

2. S. L. Adler, Some Simple Vacuum Polarization Phenomenology: $e^+e^- \to$ Hadrons; the μ-Mesic Atom X-ray Discrepancy and $g_\mu - 2$, Institute for Advanced Study preprint.

3. In Eq. (2) and subsequent equations, $\rho_e^{(1)}(t) = (3\pi/\alpha)\ \pi^{(1)}(-t)$, with $\pi^{(1)}$ the fourth order vacuum polarization spectral function calculated by G. Källén and A. Sabry, Dan. Mat. Fys. Medd. 29, no. 17 (1955). The muon contributions to the spectral functions $\rho_\mu^{(0)}(t)$ and $\rho_\mu^{(1)}(t)$ are obtained from their electron analogs by the replacement $m_e \to m_\mu$, while $\rho_{e\mu}^{(1)}(t)$ is the piece of the spectral function deriving from the photon propagator diagram containing a lowest order electron loop joined to a lowest order muon loop.

4. M. K. Sundaresan and P. J. S. Watson, Phys. Rev. Letters 29, 15 (1972); R. Jackiw and S. Weinberg, Phys. Rev. D5, 2396 (1972).

5. D. W. Joseph, Nuovo Cimento 16, 997 (1960); B. E. Lautrup and J. Smith, Phys. Rev. D3, 1122 (1971), Sec. III.

6. Since we are below the μ-pair threshold, the spectral function contributions $\rho_\mu^{(0)}$ and $\rho_\mu^{(1)}$ are zero.

7. Alterations in the Dalitz pair spectrum would also be expected but would not correspond directly to the integrand of Eq. (13), for reasons discussed in K.O. Mikaelian and J. Smith, Phys. Rev. D5, 2890 (1972).

8. Particle Data Group, Reviews of Modern Physics 45, 51 (1973).

9. V. Alles Borelli et al., Lettere al Nuovo Cimento 2, 376 (1971).

10. B.L. Beron et al., (to be published).

11. V. Alles Borelli et al., as summarized by A. Zichichi, Why e^+e^- Physics is Fascinating, CERN (unpublished).

12. L. Resnick, M.K. Sundaresan and P.J.S. Watson, Phys. Rev. D8, 172 (1973).

13. D. Kohler, J.A. Becker and B.A. Watson, Bull. Am. Phys. Soc. Series II V19 #4, 514 (1972).

14. E. Fermi and L. Marshall, Phys. Rev. 72, 1139 (1947). See also W.W. Havens, Jr., I.I. Rabi and L.J. Rainwater, Phys. Rev. 72, 634 (1947).

15. E. Melkonian, B.M. Rustad and W.W. Havens, Jr., Phys. Rev. 114, 1571 (1959); D.J. Hughes et al., Phys. Rev. 90, 497 (1953); V.E. Krohn and G.R. Ringo, Phys. Rev. 148, 1303 (1966).

16. R. Felst, Phenomenological Fits to the Nuclear Electromagnetic Form Factors Based on Vector-Meson-Dominance, DESY 73/56, 1973.

17. R. Wilson (private communication).

18. For atomic physics reasons, experiments on μ-hydrogen do not seem feasible. See E. Campini, Ref. 19.

19. E. Campini, Lettere al Nuovo Cimento 4, 982 (1970); P. J. S. Watson and M. K. Sundaresan, Ref. 1.

20. E. H. S. Burhop, Mesonic Atoms, in E. H. S. Burhop, ed., High Energy Physics VIII (Academic Press, N. Y. 1969), p. 141-143.

21. E. H. S. Burhop, Ref. 20, p. 143.

22. E. H. S. Burhop, Ref. 20, p. 198. Eq. (A. 3) multiplies a complex expression which is of order unity numerically. The simple formula in Eq. (A. 3) with C = 1 agrees well with the 4f state energy shifts plotted in Fig. 23 on p. 202 of Ref. 20.

23. P. Vogel, Electron Screening in Mesic Atoms, California Institute of Technology preprint CALT-63-191, 1973.

24. T. E. O. Ericson and J. Hüfner, Nucl. Phys. B47, 205 (1972).

25. M. Gell-Mann, R. J. Oakes and B. Renner, Phys. Rev. 175, 2195 (1968).

26. See Eqs. (12. 5) and (13. 46) of J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields, (McGraw-Hill, N. Y., 1965).

FIGURE CAPTIONS

Fig. 1 "Black box" emitting a time-like virtual photon, which converts to an electron-positron pair.

Fig. 2 First order radiative corrections to the process of Fig. 1 (neglecting diagrams in which additional photons are emitted from the "black box").

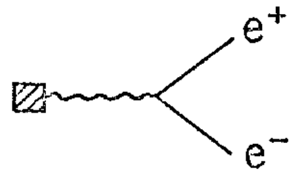

FIG. 1

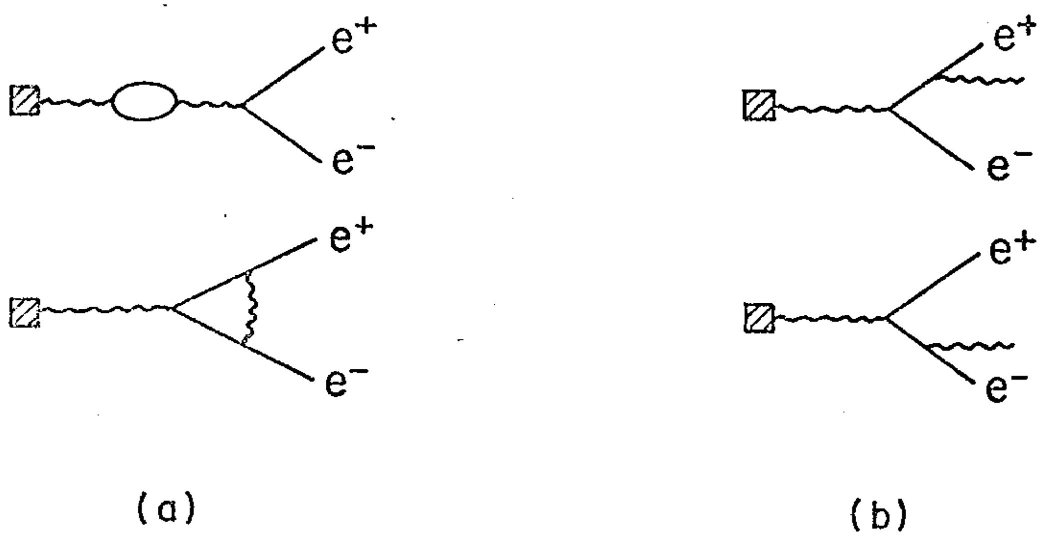

FIG. 2