national accelerator laboratory

FNAL-Pub-74/62-THY
June 1974

Gauge Theories and M-Spin Conservation

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ABSTRACT

M-spin conservation is shown to be a property of Salam-Weinberg gauge theories. Relations such as $A(\nu_e e^- \to \nu_e e^-) - A(\nu_\mu e^- \to \nu_\mu e^-) = A(\nu_\mu e^- \to \nu_e \mu^-)$ derived from M-spin invariance alone also hold for the gauge theories up to the breaking due to Higgs scalars.

*Supported in part by the U.S. Atomic Energy Commission.

Muon-electron universality is often built into gauge theories of weak and electromagnetic interactions by hand. The electronic part of the Langrangian is constructed according to the dictates of the specific theory being considered, and the muonic part is then obtained from it by the substitution

$$(e^-, \nu_e. E^+, E^o, \ldots) \rightarrow (\mu^-, \nu_\mu, M^+, M^o, \ldots) \tag{1}$$

This simple procedure ensures universality and the separate conservation of electron and muon numbers, but it also leads to a stronger symmetry, namely M-spin invariance for leptonic processes. In this paper we show that the Salam-Weinberg gauge theories[1,2] conserve M-spin up to the breaking due to Higgs scalars. Consequently lepton-lepton scattering amplitude relations derived from M-spin invariance, such as[3]

$$A(\nu_e e^- \rightarrow \nu_e e^-) - A(\nu_\mu e^- \rightarrow \nu_\mu e^-) = A(\nu_\mu e^- \rightarrow \nu_e \mu^-) \tag{2}$$

hold in the Salam-Weinberg gauge theories.

To demonstrate the point let us consider the original Weinberg model[2] for the spectrum of known leptons (e^-, ν_e, μ^-, ν_μ). Suppressing space-time indices, we can write the coupling between leptons and the gauge bosons $\underline{A}$ and B_o as

$$\mathcal{L}_1 = g[\bar{L}_e \underline{t}\, L_e + \bar{L}_\mu \underline{t}\, L_\mu]\cdot\underline{A} + \frac{1}{2} g'[\bar{L}_e L_e + \bar{L}_\mu L_\mu + 2\bar{R}_e R_e + 2\bar{R}_\mu R_\mu] B_o \tag{3}$$

where L_e and L_μ are the left-handed doublets (ν_e, e_L^-) and (ν_μ, μ_L^-) respectively, and R_e and R_μ are the right-handed singlets e_R^- and μ_R^-. This Lagrangian is obviously symmetric under the interchange

$$(e^-, \nu_e) \leftrightarrow (\mu^-, \nu_\mu) \tag{4}$$

In M-spin space,[3] e^- and μ^- behave as one doublet, call it M_1, ν_e and ν_μ behave as another doublet, M_2, and $\underline{A}$ and B_o are singlets. The terms in $\mathcal{L}_1$

can be re-arranged so that their dependence on lepton fields is given by a sum of the four quantities

$$\bar{M}_i M_j \quad (i, j = 1,2) \tag{5}$$

Each of these quantities is an M-spin invariant and so $\mathscr{L}_1$ itself is M-spin invariant.

As far as processes involving two leptons (e.g. β-decay) are concerned, M-spin conservation has no consequences beyond those of μ-e universality and lepton conservation; but for processes involving four or more leptons, it does. In fact we have shown elsewhere[3] that it yields the sum rule of Eq. (2) for lepton-lepton scattering. To the extent that the full Weinberg model is symmetric under the μ-e interchange of Eq. (4), it will conserve M-spin and its amplitudes for lepton-lepton scattering will satisfy Eq. (2) to all orders of perturbation theory.

The Weinberg model[2] contains a specific mechanism for breaking the symmetry between muons and electrons, namely the coupling of leptons to the doublet φ of Higgs[4] scalars:

$$\mathscr{L}_2 = g\left(\frac{m_e}{M_W}\right)\left(\bar{L}_e \varphi R_e + \bar{R}_e \varphi^+ L_e\right) + g\left(\frac{m_\mu}{M_W}\right)\left(\bar{L}_\mu \varphi R_\mu + \bar{R}_\mu \varphi^+ L_\mu\right) \tag{6}$$

where m_e and m_μ are the physical masses of the electron and muon respectively, and M_W is the mass of the charged gauge boson. Equation (2) is not an exact property of the Weinberg model; the deviations from it are engendered by diagrams involving the exchange of Higgs scalars. However, because the coupling constants of $\mathscr{L}_2$ involve the mass ratios (m_e/M_W) and (m_μ/M_W), it is not unreasonable to suppose that these deviations are quite small.

In order to generalize this result to other gauge models we note first that our demonstration of M-spin invariance in the Weinberg model is a special case of a general theorem due to Schechter, Ueda, and Okubo.[5] Among other things, these authors have shown that the combination of permutation symmetry for two objects plus one conservation law is equivalent to SU(2). In the Weinberg model the two objects are μ and e, the conservation law is that of lepton number, and the SU(2) symmetry is M-spin invariance. It follows that any coupling of gauge bosons to leptons which conserves lepton number, and which is symmetric with respect to the substitution of Eq. (1) and its inverse will indeed be M-spin invariant. Since the coupling of leptons to Higgs scalars generally breaks the permutation symmetry, M-spin conservation will not be an exact symmetry of the gauge theory. The extent to which it is broken will depend on the strength of the coupling of Higgs scalars to leptons.

The authors would like to thank J. D. Sullivan for raising the question that sparked this investigation. They would also like to thank Mary K. Gaillard, P. Kabir, B. W. Lee, and S. Pakvasa for useful conversations.

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