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Higher Order ϵ-Terms in the Renormalization Group Approach to Reggeon Field Theory[‡]

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ABSTRACT

We calculate the $0(\epsilon^2)$ terms in a Wilson ϵ-expansion of the scaling exponents in Reggeon field theory. We find that these corrections are comparable to the $0(\epsilon)$ terms.

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The technique of using the renormalization group[1] and the Wilson ϵ-expansion[2] to derive scaling properties of proper vertices in Reggeon field theory[3] was introduced by Migdal, Polyakov and Ter-Martirosyan,[4] and by Abarbanel and Bronzan.[5] In their work the behavior of the proper vertices in the infrared limit $j \approx 1$ and $t \approx 0$ was examined, and a number of conclusions were reached. The most important of these was a prediction that in a theory with a linear unrenormalized Pomeron trajectory and a triple-Pomeron coupling, the asymptotic behavior of the elastic amplitude is

$$T(s,t) = S(\ell n\ s)^{-\gamma}\ F[t(\ell n\ s)^{z}] \tag{1}$$

with $\gamma < 0$. This behavior arises from the coincidence at $j = 1$ and $t = 0$ of an infinite number of branch points, and the exponents γ and z are determinable in an ϵ-expansion. Here $\epsilon = 4 - D$ is the difference between the number D of transverse momentum components and the natural scaling dimension of the theory; we want answers for $\epsilon = 2$. Although ϵ is a large, it was shown[4,5] that to $0(\epsilon)$, $-\gamma = \epsilon/12 = 1/6$, $z = 1 + \frac{\epsilon}{24} = \frac{13}{12}$ and so it seemed that a natural expansion parameter was really $\epsilon/12$. Hence it was plansible that the ϵ-expansion might be rapidly convergent.

We have determined that[6]

$$-\gamma = \frac{\epsilon}{12} + \left[\ \frac{257}{12}\,\ell n\,\frac{4}{3} + \frac{37}{24}\right] \left(\frac{\epsilon}{12}\right)^2 + 0(\epsilon^3) \tag{2}$$

$$z = 1 + \frac{\epsilon}{24} + \left[\frac{155}{24} \ell n \frac{4}{3} + \frac{79}{48} \right] \left(\frac{\epsilon}{12}\right)^2 + 0(\epsilon^3) \quad .$$

Since the coefficients of the $(\epsilon/12)^2$ terms are 7.7 and 3.1, respectively, the $0(\epsilon^2)$ terms are larger than the $0(\epsilon)$ terms at $\epsilon = 2$. It would therefore seem that the ϵ-expansion is a questionable means of calculating γ and z at $\epsilon = 2$. Our results agree with those obtained independently by M. Baker.[7]

The Lagrangian we use for the Reggeon field theory is the same one used in Ref. 5:

$$= \frac{i}{2} \psi^{+} \frac{\overleftrightarrow{\partial}}{\partial t} \psi - \alpha_0' \vec{\nabla} \psi^{+} \cdot \vec{\nabla} \psi - \frac{ir_0}{2} \left[(\psi^{+})^2 \psi + \psi^{\dagger} (\psi)^2 \right] \tag{3}$$

In the ϵ expansion no mass or "gap" term is required. Proper renormalized vertex functions $\Gamma_R^{(n,m)}$ are expressed in terms of renormalized parameters r and α', which are chosen by the normalization conditions

$$\left. \frac{\partial i \Gamma_R^{(1,1)}}{\partial k^2} \right|_{P_A} = -\alpha'(E_N) \,, \quad \left. \Gamma_R^{(1,2)} \right|_{P_B} = \frac{r(E_N)}{(2\pi)^{(D+1)/2}} \,, \tag{4}$$

P_A: $E = -E_N$, $\vec{k} = 0$; P_B: $E_1 = -E_N$, $E_2 = E_3 = -\frac{1}{2} E_N$, $\vec{k}_i = 0$

Here $E = 1 - j$ and $t = -\vec{k}^2$ connect these variables to the usual variables of a t-channel partial wave amplitude. A renormalization constant Z is introduced so that

$$\left. \frac{\partial i \Gamma_R^{(1,1)}}{\partial E} \right|_{P_A} = 1. \tag{5}$$

Finally, we choose to replace r by the dimensionless coupling

$$g = \frac{r\, E_N^{-\epsilon/4}}{(\alpha')^{1-\epsilon/4}} \quad . \tag{6}$$

With these conventions, the proper vertices satisfy the renormalization group equation

$$\left[E_N \frac{\partial}{\partial E_N} + \beta \frac{\partial}{\partial g} + \zeta \frac{\partial}{\partial \alpha'} - \frac{\gamma}{2}(n+m)\right] \Gamma_R^{(n,m)}(E_i, \vec{k}_i, r, \alpha', E_N) = 0, \tag{7}$$

where

$$\beta = E_N \frac{\partial g}{\partial E_N}\Bigg|_{P_c}, \qquad \frac{\zeta}{\alpha'} = E_N \frac{\partial}{\partial E_N} \ell n \alpha' \Bigg|_{P_c}$$

$$\gamma = E_N \frac{\partial}{\partial E_N} \ell n Z \Bigg|_{P_c}, \qquad P_c:\ r_0, \alpha_0' \text{ fixed} \tag{8}$$

The solutions of the renormalization group equation lead to the scaling laws discussed in Refs. 4 and 5, and to Eq. (1). The exponents γ and z are determined by

$$\beta(g_1) = 0 \quad (\text{with } \beta'(g_1) > 0)$$

$$\gamma = \gamma(g_1), \quad z = 1 - \frac{\zeta}{\alpha'}(g_1) \ . \tag{9}$$

We have calculated β, γ, and ζ/α' to the accuracy required to obtain Eq. (2),

$$\beta(g) = -\frac{\epsilon}{4} g + \left[\frac{3}{2} + \frac{\epsilon}{4}\left(-3\gamma_{em} + 3\ell n\pi + \frac{15}{4} + 5\ \ell n\ 2\right)\right] \left(\frac{g}{8\pi}\right)^2$$

$$-\left[\frac{157}{32}+\frac{149}{16}\ell n\frac{4}{3}\right]\left(\frac{g}{8\pi}\right)^5 , \qquad (10)$$

$$\gamma(g) = \left[-\frac{1}{2}+\frac{\epsilon}{4}(\gamma_{em}-\ell n\,\pi-3\ell n\,2)\right]\left(\frac{g}{8\pi}\right)^2+\left[-\frac{5}{2}\ell n\,2+\frac{9}{4}\ell n\,3-\frac{5}{8}\right]\left(\frac{g}{8\pi}\right)^4$$

$$\frac{\zeta}{\alpha'}(g) = \left[-\frac{1}{4}+\frac{\epsilon}{8}(\gamma_{em}-\ell n\,\pi-3\ell n\,2)\right]\left(\frac{g}{8\pi}\right)^2+\left[\frac{7}{8}\ell n\,2+\frac{1}{16}\ell n\,3-\frac{17}{32}\right]\left(\frac{g}{8\pi}\right)^4$$

γ_{em} is the Euler-Mascheroni constant.

Solving the first equation, to order ϵ^2

$$\left(\frac{g_1}{8\pi}\right)^2 = \frac{\epsilon}{6}+\left[\gamma_{em}-\ell n\,\pi+\frac{356\,\ell n\,2-298\,\ell n\,3-23}{144}\right]\frac{\epsilon^2}{12}+0(\epsilon^3), \qquad (11)$$

and we obtain Eq. (2) by substitution.

The careful reader will note that, at $\epsilon = 2$, $\left(\frac{g_1}{8\pi}\right)^2 \approx -0.09$. This does not mean that the Gell-Mann-Low zero of β has disappeared. To see this, suppose we define a new coupling constant

$$G = (1 + A\epsilon)g \ . \qquad (12)$$

This substitution does not change the scaling exponents, as given by Eq. (2), but by adjusting A we can change the coefficients in $\tilde{\beta}(G)$ so that $(G_1/8\pi)^2$ is positive. We learn by this example that the value of $(g_1/8\pi)^2$ carries no information about the existence of a Gell-Mann-Low zero <u>when the coefficients of β are evaluated in the ϵ-expansion and ϵ is finite.</u> Similarly, setting $\epsilon = 2$ and counting real zeroes yields no information about the existence of a Gell-Mann-Low zero because the number of real zeroes can change with A. All we know for certain is that for sufficiently small ϵ, β has a real zero which approaches the origin.

The choice $A = \frac{1}{2}(\gamma_{em} - \ln 8\pi)$ illustrates these points. Now $(G_1/8\pi)^2$ is positive $\widetilde{\beta}(G)$ has only one real zero at $\epsilon = 2$, as compared with the five real zeroes of $\beta(g)$.

The proper vertices $\Gamma^{(2,2)}$ and $\Gamma^{(1,3)}$, with four external Pomerons, are $0(\epsilon^2)$ at the Gell-Mann-Low eigenvalue. (Vertices with more legs are smaller.) One might suppose that four Pomeron terms should be added to Eq. (3) to ensure that the $0(\epsilon^2)$ terms of Eq. (2) are correct. However, Wilson[2] has pointed out that the two-charge theory will have the same scaling exponents as the theory we examine, basically because the induced four-Pomeron vertices simulate the bare coupling when energies and momenta are scaled down.

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Eq. (10) should read:

$$\beta(g) = -\frac{\epsilon}{4} g + \left[\frac{3}{2} + \frac{\epsilon}{4}(-3\gamma_{em} + 3\ell n\,\pi + \frac{15}{4} + 5\ell n\ 2)\right] \frac{g^3}{(8\pi)^2}$$

$$- \left[\frac{157}{32} + \frac{149}{16}\ \ell n\ \frac{4}{3}\right] \frac{g^5}{(8\pi)^4}$$