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Self-Consistent Pomeranchuk Singularities in Reggeon Field Theories*

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ABSTRACT

A self-consistency or bootstrap principle is suggested to determine the structure of the Pomeranchuk singularity in the neighborhood of $J=1$ and $t=0$. The ingredients in this are a Reggeon field theory which we require to be renormalizable and infrared free in the sense of the renormalization group. This guarantees that the input singularity reproduces itself near $J=1$, $t=0$ with small computable corrections. Several examples of physical significance are discussed.

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Reggeon field theories[1] provide a natural way to implement multiparticle crossed channel unitarity as concisely expressed in the discontinuity formulae across J-plane cuts.[2] In such a theory a Reggeon is a quasi-particle in two space and one time dimension carrying two momentum $\vec{q}$ and energy $E=1-J$, which "on shell" are related by $E(\vec{q}) = 1 - \alpha(t = -|\vec{q}\,|^2)$. The study of the infrared limit $E \rightarrow 0$, $\vec{q} \rightarrow 0$ provides the behavior of t-channel partial wave amplitudes in the $J = 1 - E \rightarrow 1$, $t = -|\vec{q}\,|^2 \rightarrow 0$ regime important for diffraction scattering. When the Reggeon is the Pomeranchuk singularity (called $\underset{\sim}{P}$), then the $E, \vec{q}$ relation is such that $E(\vec{q} = 0) = 1 - \alpha(0) = 0$, and we have a massless particle theory. One must then sum to an infinite order of perturbation theory to discover the full infrared structure of the interacting theory. This is most efficiently done using renormalization group methods.[3,4]

In formulating these field theories both the choice of the bare theory and the interaction is rather much at one's disposal. In part to reduce this ambiguity and in part to provide a technique to select out acceptable diffraction theories we explore here a self-consistency or bootstrap principle for Reggeon field theories. In particular we suggest that satisfactory theories must be (1) infrared free in the sense of the renormalization group and (2) renormalizable. After we discuss these properties we will examine several physically motivated examples to exhibit the power of the bootstrap.

The formulation of Reggeon, indeed most, field theories begins with a bare propagator G_0 which has poles in momentum space with an equivalent Lagrange density in configuration space. The unitarity relation for proper vertex functions of the theory exppesses them as integrals over Im G_0, a delta function for poles, and other proper vertices. We will specify our bare theories by $G_0(E, \vec{q})$ and some interaction of the underlying Reggeon field $\phi(\vec{x}, t)$. If G_0 does not consist of poles, then (1) one replaces the delta function in the unitarity relation by Im G_0 and (2) one must imagine that G_0 arises from the solution of an interacting field theory $L_0 = L_{FREE} + L_1$ where L_{FREE} does have poles in its propagator, and L_1 is whatever one requires to produce the chosen G_0. To this, some interaction L_I is now added. To be specific let us concentrate on adding a triple $\underset{\sim}{P}$ coupling

$$L_I = -\lambda_0 \; \phi(\vec{x}, t)^3 \tag{1}$$

or

$$L_I = -\lambda_0 \; (\nabla \phi)^2 \phi \tag{2}$$

With one coupling the n-point proper vertex functions, $\Gamma^{(n)}(E_i, \vec{q}_i)$, satisfy a renormalization group equation[4]

$$\left[\xi \frac{\partial}{\partial \xi} - \beta(y) \frac{\partial}{\partial y} + (\frac{n}{2} \gamma(y) - D_\Gamma)\right] \Gamma^{(n)}(\xi E_i, \vec{q}_i, y) = 0 \tag{3}$$

where y is the dimensionless coupling made from λ, D_Γ, the ordinary dimensions of $\Gamma^{(n)}$, and β and γ, the usual renormalization group functions. The solution to (3) involves $\tilde{y}(\eta)$ satisfying

$$\frac{d\tilde{y}(\eta)}{d\eta} = -\beta(\tilde{y}(\eta)) \tag{4}$$

and is

$$\Gamma^{(n)}(\xi E_i, \vec{q}_i, y) = \Gamma^{(n)}(E_i, \vec{q}_i, \tilde{y}(-\eta)) \exp \int_{-\eta}^{0} d\eta' \, [D_\Gamma - \frac{n}{2}\gamma(\tilde{y}(\eta'))], \tag{5}$$

with $\eta = \log \xi$. The first issue is the behavior of $\tilde{y}(-\eta)$ as $\eta \to -\infty$ or $\xi \to 0$. $\beta(y)$ is

$$\beta(y) = Ay + By^3 + \ldots \tag{6}$$

in perturbation theory. If $A > 0$, then the point $y=0$ is an infrared stable point and $\tilde{y}(-\eta) \to 0$ as $\eta \to -\infty$. This means that the infrared limit of $\Gamma^{(n)}$ can be evaluated in perturbation theory around the theory characterized by G_0. If $A = 0$, the sign of B may always be chosen positive because λ_0 may be pure imaginary. Indeed, Gribov's signature analysis indicates it must be.[1] We ask, then, that $A \geq 0$. This is the key to the <u>renormalization group bootstrap</u> which yields the input theory plus small computable corrections in the infrared limit:

$$G(\xi E, \vec{q}^{\,2}, y) \xrightarrow[\xi \to 0]{} G_0(E, \vec{q}^{\,2}) + 0(\tilde{y}(-\eta)^2) \tag{6}$$

Renormalizability is a more elusive aspect, since the theory has really been formulated in the neighborhood of $E=0, \vec{q}=0$ to begin with. We view it here as an economy which allows us to introduce no further parameters beyond what appears in G_0 and the coupling λ. Rigorous arguments against simply cutting off the theory at some "large" E and

$\vec{q}$ don't exist to my knowledge. That cutoff certainly detracts from the appeal of the theory and adds unwanted parameters. Although some experience[4] shows that elementary cutoff procedures do not essentially modify the infrared behavior, we will pursue the bolder tack of dispensing with them.

Finally a word about the triple $\underset{\sim}{P}$ interactions (1) and (2). Heuristic arguments based on phase space in the infrared limit and detailed calculations in Refs. 3 and 5 lead one to eliminate higher order polynomials in ϕ as less important as E, $\vec{q} \to 0$. High enough order polynomials are discarded on grounds of renormalizability. In any case the triple coupling is of enough physical interest that one must consider it first.

To determine the renormalizability of a theory we examine the superficial degree of divergence, δ, of its Feynman graphs. A diagram with $\mathscr{E}$ external lines, $\mathscr{I}$ internal lines, $\mathscr{L}$ loops, and $\mathscr{V}$ vertices has

$$\delta = d\mathscr{L} + a\mathscr{V} - p\mathscr{I} \tag{7}$$

where d is the dimension of the integration, a is the power of momenta at the vertices and p is the power of momenta in G_0^{-1}. Using the identities

$$2\mathscr{I} + \mathscr{E} = 3\mathscr{V} \text{ and } \mathscr{L} = \mathscr{I} + 1 - \mathscr{V} \tag{8}$$

we have

$$\delta = d + \mathscr{V}\left\{a + \frac{d-3p}{2}\right\} - \frac{\mathscr{E}}{2}(d-p). \tag{9}$$

For renormalizability we certainly must ask that

$$a + \frac{d-3p}{2} \leq 0, \tag{10}$$

and

$$d-p > 0 \tag{11}$$

so that only a finite number of proper vertices have $\delta \geq 0$ and need subtraction. Further we'll want $\delta(\mathscr{E} = 4) < 0$ for the minimum $\mathscr{V}$, so no quartic or higher counter terms are needed in $L = L_0 + L_I$.

Our first example is suggested by studies of s-channel unitarity of which the multiperipheral bootstrap[6] or the self-consistent absorption model[7] is a suggestive example. In s,t space we choose an amplitude with absorptive part

$$A(s,t) = s(\log s)^{\sigma} \frac{J_{\nu}(R_0\sqrt{-t}\log s)}{(R_0\sqrt{-t})^{\nu}}, \tag{12}$$

where a scale for s_0 has been omitted and R_0 is some constant. The total cross section coming from (12) behaves as $\sigma_T(s) \sim (\log s)^{\sigma+\nu}$. $G_0(E,\vec{q})$ is evaluated by the usual Mellin transform

$$G_0(E,\vec{q}) = \int_1^{\infty} d(\log s)\, e^{E \log s} \frac{A(s,t=-|\vec{q}|^2)}{s}. \tag{13}$$

The renormalization procedure for our "massless" theories involves a normalization energy E_N[4]. Using this and remembering one must make separate dimensional considerations for E and $\vec{q}$ dimensions, the dimensionless coupling is found to be

$$y(E_N) = \frac{\lambda(E_N)^{a - \frac{3}{2}(\sigma+\nu)}}{(R)^{1+a}}$$

Since $\beta(y)$ is defined by

$$\beta(y) = E_N \frac{\partial}{\partial E_N} y(E_N) ,$$

we see that the A of Eq. (5) is

$$A = a - \frac{3}{2}(\sigma+\nu), \tag{16}$$

so

$$\sigma + \nu \leq \frac{2}{3} a \tag{17}$$

for infrared freedom. $d=3$ and $p = 1 + \sigma + \nu$ for this theory, so (10) and (11) require

$$\frac{2}{3} a \leq \sigma + \nu < 2 . \tag{18}$$

Amusingly enough the upper limit in (18) is just the Froissart bound! Together our constraints say

$$\sigma + \nu = \frac{2}{3} a, \tag{19}$$

and only that is acceptable. For the nonderivative coupling of Eq. (1) $a=0$, and $\sigma + \nu = 0$. The interesting particular case of this with $\sigma = \nu = 0$ has been explored in detail in Ref. 8. One finds that this renormalization group bootstrap leads to

$$\sigma^{A+B}_{\text{Total}}(s) \underset{s\to\infty}{\sim} \gamma_A \gamma_B - f_{AB}/\log s \,(\log\log s)^{\frac{1}{2}} + \cdots , \tag{20}$$

where f_{AB} need not factorize as the first term does. The derivative coupling of Eq. (2), which is crucial for the multiperipheral bootstrap,[6] is unacceptable as it stands, because proper vertices with up to nine external legs need subtractions!

Next consider scaling forms of $G_0(E,\vec{q})$ suggested by the work in Refs. 3 and 4:

$$G_0^{-1}(E,\vec{q}) = E^{p_1}(1 + b_0\vec{q}^{\,2}/E^{p_2}) \tag{21}$$

with $p_1, p_2 > 0$. Noting that the dimension of b_0 is $E^{p_2}|\vec{q}|^{-2}$, we find $d = 2 + 2/p_2$ and $p = 2p_1/p_2$. Infrared freedom requires

$$p_2 \geq \frac{3p_1 - 1}{1+a} \tag{22}$$

and renormalizability,

$$p_1 - 1 < p_2 \leq \frac{3p_1 - 1}{1 + a} \tag{23}$$

Again these yield a unique solution

$$p_2 = (3p_1 - 1)/1 + a \ . \tag{24}$$

The theory with a=0 (constant bare triple $\underset{\sim}{P}$) is fine for any $p_1, p_2 > 0$. In order to have $\sigma_T(s) \sim (\log s)^{p_1 - 1}$ not fall, we ask that $p_1 \geq 1$, so <u>no acceptable theory of the form (21) has a linear trajectory</u>. The theory with constant total cross section ($p_1 = 1$) must be

$$G_0(E,\vec{q}^{\,2}) = \frac{E}{E^2 + b_0\vec{q}^{\,2}} \ , \tag{25}$$

which has familiar $\sqrt{-t}$ pole trajectories. If the interaction has derivative couplings as in (2), then a = 2, and we learn

$$p_1 = p_2 + \frac{1}{3} \tag{26}$$

while the absence of counterterms not in L means $p_2 < 2/3$ and a falling $\sigma_T(s) \sim (\log s)^{-1/3}$ or faster; we reject this solution. It is clear that the renormalization group bootstrap is quite thorough in selecting out very limited sets of satisfactory theories.

Perhaps it is useful to end with a word of caution. In more elaborate theories than the simple models we have examined in this note, one may have more than one dimensionless coupling y. In that case the condition of infrared freedom and the power counting arguments become more intricate and are surely less definite in singling out a unique theory. Nevertheless their expression through the bootstrap will greatly restrict the class of allowed models. The attendant reduction of ambiguity in the formulation of models, the self-consistency of the input, and the ability to calculate corrections to whatever accuracy desired are indeed attractive. As a final virtue the attitude presented here enables one to depart from the problematic nature[9] of the ϵ-expansion resorted to in Refs. 3 and 4.

I have enjoyed fruitful discussions on these matters with B. W. Lee, F. Zachariasen, and all members of the NAL Reggeon Field Theory Workshop.

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[9] Preliminary calculations by M. Baker and J. B. Bronzan and J. W. Dash indicate that the corrections to leading order in ϵ are likely to be sizeable. A detailed consideration of the renormalization program for $\alpha(0) = 1$ by R. L. Sugar and A. R. White, demonstrates that nonanalytic terms of the form $\epsilon \log \epsilon$ play an important role in these corrections. I am indebted to each of these authors for valuable discussion on their work.