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The Pomeranchuk Singularity in a Class of Reggeon Field Theories Suggested by Direct Channel Unitarity

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ABSTRACT

Within the context of a generalized Reggeon calculus we study the infrared ($J \to 1$, $t \to 0$) limit of a class of models whose "bare" structure arises from elastic amplitudes of the form $A(s, t) = s(\log s)^{\nu} J_{\nu}(a\sqrt{-t} \log s)$. Such amplitudes are suggested by the implementation of s-channel unitarity via eikonalization of a "Born term", via absorption models, and via the multiperipheral bootstrap. We employ the renormalization group to study the renormalized Pomeranchuk singularity when the interaction involves a triple coupling. Our major result is that for $\nu = 0$ these theories are infrared free. The total cross section behaves as $\sigma_T(s) \sim \gamma_1 - \gamma_2/(\log s)(\log \log s)^{\frac{1}{2}}$ where γ_1 factorizes. Scaling laws for the Reggeon proper vertex functions are given.

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I. INTRODUCTION

Constraints on hadron interactions coming from the unitarity relation are potentially powerful tools in isolating acceptable theories. The implications of direct channel unitarity (s-channel unitarity) range from the Froissart bound to much more model dependent statements about restrictions on this or that parameter. In a variety of models attempting to approximate s-channel unitarity one finds elastic scattering amplitudes of the form

$$\mathrm{Im}\, T_{el}(s,t) = \beta(t)\, s^{\alpha(t)} (\log s)^{\mu}\, \frac{J_\nu(a\sqrt{-t}\,\log s)}{(a\sqrt{-t})^{\nu}} \qquad (1)$$

as outputs. This is familiar from various versions of a multiperipheral bootstrap,[1] from eikonal production models,[2] from models with self-consistent absorption,[3] and perhaps others. Each of these is characterized by a J-plane singularity which has $t = 0$ intercept one and moves as $\sqrt{-t}$ in the neighborhood of that point.

Implementing crossed channel (t-channel) unitarity has been slightly more difficult. The tool which has emerged for carrying out some reasonable approximations to t-channel unitarity is that of the Reggeon field theories first proposed by Gribov.[4] Using the further analytical help of the renormalization group[5,6] one has learned how to study the details of Reggeon field theories in the interesting neighborhood of $J \approx 1$, $t \approx 0$.

In this paper we propose to weave these s-channel and t-channel amplitudes together. We will study as a generalized Reggeon field theory the elastic amplitudes

$$\mathrm{Im}\, T_{el}(s,t) = \frac{\Gamma(p+1/2)}{\Gamma(2p)}\, s(\log s)^{p-1/2}\, \frac{J_{p-1/2}(a \log s \sqrt{-t})}{(a\sqrt{-t}/2)^{p-1/2}} \tag{2}$$

whose Mellin transform

$$F(J,t) = \left((J-1)^2 - a^2 t\right)^{-p} \tag{3}$$

will play the role of the bare Green's function in our theories. This is clearly a simplification of the more general form (1) and represents the most straightforward starting point for the investigation in the t-channel of amplitudes motivated by s-channel considerations.

As discussed at some length in Ref. 6 there is a large freedom in Reggeon field theories both as to the bare theory which gives the amplitude with no Reggeon interactions and as to the interaction one chooses to abstract from hybrid Feynman graphs[4] or other considerations. Three choices for non-interacting theories seem to stand out. First, there is the bare linear trajectory

$$\alpha(t) = \alpha_0 + \alpha_0' t \tag{4}$$

which translates into the free Green's function[5,6]

$$G_0(E,\vec{q}) = i\left(E - \alpha_0' \vec{q}^{\,2} - (1-\alpha_0) + i\varepsilon\right)^{-1} \tag{5}$$

where $E = 1-J$ and $t = -|\vec{q}|^2$. The appeals of this beginning point are essentially simplicity and the guess of analyticity at $t = 0$ associated with the absence of Reggeon interactions. This has been studied in detail in Ref. 5 and 6.

Second, one may consider beginning with the scaling forms

$$G_0(E, \vec{q})^{-1} = E^A \left(1 - a_0 \vec{q}^{\,2}/E^B\right) \tag{6}$$

derived in Ref. 5 and 6 and found by Gribov and Migdal.[7] Finally one may begin with the $\sqrt{t}$ type singularities given by (3)

$$G_0(E, \vec{q})^{-1} = \left(E^2 + a^2 \vec{q}^{\,2}\right)^p. \tag{7}$$

This is the subject of this present paper. Other choices are clearly possible but appear to us to lack significant physical motivation.

Before proceeding further, one should take note of the sobering thought that none of the above forms for G_0 satisfy the requirement of exponential decrease as $\mathrm{Re}\, j \to \infty$. This exponential decrease derives from the Froissart-Gribov formula, and is closely connected with the existence of s-channel thresholds. In this case the threshold is the diffractive threshold; that is, that energy Δ above which one expects that a reasonable approximation to the energy dependence of the elastic amplitude is given by a simple diffractive term, unrenormalized or otherwise. These diffractive thresholds can in principle modify results for the $\Delta \to \infty$ behavior of the theory. This certainly happens in strong coupling ladder models.[8]

A second point that occurs in conjunction with the above remarks is that at finite s (e.g. $\ln s < 10$) , the subenergies in most graphs are certainly not asymptotic (e.g. $\ln s < 4$) . For consistency the bare G_0 must therefore be capable of describing the diffraction scattering at corresponding values of s ($\ln s < 4$ or $p_{lab} < 30$ GeV/c) Phenomenologically, it is known that this probably entails a rather large real part for the diffractive amplitude in this region. Most choices of G_0 will not satisfy this physical requirement, although it can be realized by a bare linear trajectory with $\alpha_0 \approx 0.85$.[9] We will have to assume that the phase generated by our elastic bare amplitude Eq. (3) also has this feature.

After settling on (7) as a free Green's function one must choose an interaction. Now, except for $p = 1$, the form of G_0 in Eq. (7) does not represent <u>poles</u> in E and in the conventional view this means some form of interaction has already been accounted for giving rise to the theory around which we wish to perturb. The better our choice of G_0 the less the perturbation will play a role and, indeed, if we choose cleverly in the neighborhood of $E = 0$, $\vec{q} = 0$ the perturbation will be weak. This is the idea behind the renormalization group bootstrap: find a G_0 which will reproduce itself via t-channel unitarity, at least in the region $E \approx 0$, $\vec{q} \approx 0$, plus computable small corrections due to interaction. This will be elaborated on in another paper.[10]

In this paper we imagine that our G_0 has come from the solution (no doubt approximate) of an interacting field theory with Lagrangian density

$$\mathcal{L}_0 = \mathcal{L}_{Free} + \mathcal{L}_1 , \tag{8}$$

where $\mathcal{L}_{Free}$ represents poles in E and $\mathcal{L}_1$ is whatever one needs to produce (8). To this we add the further interaction of the field $\phi(\vec{x}, t)$

$$\mathcal{L}_I = - \frac{\lambda_0}{6} \phi(\vec{x}, t)^3 , \tag{9}$$

where $\vec{x}$ and t are the conjugate variables to $\vec{q}$ and E. With this propagator and this $\mathcal{L}_I$ we begin our discussion.

By starting at this intermediate stage we must modify the unitarity relation that is satisfied by such a theory. In a theory which begins with poles one puts $\delta\{E - [1-\alpha(\vec{q})]\}$ for the cutting of any line entering the unitarity relation for any proper vertex function. We need here a generalized unitarity where $\mathrm{Im}\, G_0(E, \vec{q})$ replaces the insertion of a delta function. This generalization and some of its properties have been discussed in the literature,[11] and we refer the reader to that.

In the next section we establish our field theory and renormalization group equations. After that we study the equations in perturbation theory and learn that for $p = \frac{1}{2}$ the theory is free in the infrared limit in the physical number of dimensions, D = 2 . We then study the scaling laws this imposes on the Reggeon proper vertex functions and by coupling in particles find that the total cross section for $A + B \to$ anything behaves as

$$\sigma_T(s) \underset{s \to \infty}{\sim} \gamma_A \gamma_B \left[1 + c/(\log \log s)^2\right] - f^{AB}/(\log s)(\log \log s)^{1/2} + \cdots . \tag{10}$$

The leading term factorizes while the secondary terms need not. This approach of total cross sections to a constant asymptotic limit from below is consistent with the observed behavior of proton-proton cross sections at the CERN-ISR.[12.]

II. REGGEON GRAPH RULES AND RENORMALIZATION GROUP EQUATIONS

We begin our discussion by choosing the Green's function in momentum space for the non-interacting theory to be

$$G_0(E, \vec{q})^{-1} = (E^2 + a^2 \vec{q}^{\,2})^p \tag{11}$$

where $E = 1 - J$ and $\vec{q}$ is a D dimensional vector. Physics takes place at $D = 2$. The theory specified by (11) has a Euclidian invariance in $D + 1$ dimensions with the length of a Euclidian vector

$$q_\mu = (E, \vec{q}) \tag{12}$$

given by

$$q^2 = E^2 + a^2 \vec{q}^{\,2} . \tag{13}$$

The parameter a plays the role of the speed of light and is not renormalized when the interaction is chosen to respect the $SO(D+1)$ invariance. We will choose the interaction to be a cubic in the hermitean scalar field $\phi(\vec{x}, t)$ defined in D space dimensions and one time

$$\mathcal{L}_0(\vec{x}, t) = -\frac{\lambda_0}{6}\phi(\vec{x}, t)^3. \tag{14}$$

In this expression λ_0 need not be real.

We pick this interaction on several grounds: (1) it is the simplest non-trivial Reggeon interaction; (2) it is renormalizable for an interesting range of our parameter p; and (3) on the basis of previous calculations with linear trajectories[1] we may argue that in the limit we shall probe, $E \to 0$, $\vec{q} \to 0$ - the infrared limit - other polynamials in ϕ are either not renormalizable or give less singular behavior in the infrared region. None of these reasons is compelling; each is suggestive.

Using $G_0(q^2)$ and $\mathcal{L}_I$ we may derive the following rules for evaluating the unrenormalized Green's functions $G_U^{(n)}(q_1, \cdots q_n)$ in perturbation theory in λ_0 :

1. Draw all topologically distinct graphs with n external legs.

2. Integrate $d^{D+1}q$ around each loop.

3. At each vertex put the factor $\lambda_0/(2\pi)^{(D+1)/2}$.

4. For each Reggeon of momentum q_i use the propagator

$$G_0(q_i) = (q_i^2)^{-p}.$$

5. For each loop with only two Reggeons multiply by $\frac{1}{2}$.

6. Conserve D + 1 momentum at each vertex.

From $G_U^{(n)}(q_1, \cdot\cdot q_n)$ it is convenient to form the proper vertex functions

$$\Gamma_U^{(n)}(q_1, \cdots q_n) = \prod_{j=1}^{n} G_U^{(2)}(q_j)^{-1} \quad G_U^{(n)}(q_1, \cdots q_n) , \qquad (15)$$

and we concentrate on these quantities.

To determine whether the theory we have established is renormalizable in any conventional sense we now study the degree of divergence of the graphs computed by the rules above. Suppose we have a graph with $\mathscr{E}$ external lines, $\mathscr{I}$ internal lines, $\mathscr{V}_3$ three point vertices, and $\mathscr{L}$ loops. These quantities are connected by

$$2\mathscr{I} + \mathscr{E} = 3\mathscr{V}_3 , \qquad (16)$$

and

$$\mathscr{L} = \mathscr{I} + 1 - \mathscr{V}_3 . \qquad (17)$$

When each vertex carries zero powers of derivatives and each propagator carries 2p powers of q , the superficial degree of divergence, δ of graphs is

$$\delta = (D+1)\mathscr{L} - 2p\mathscr{I} \qquad (18)$$

$$= D+1 + \mathscr{V}_3\left(\frac{D+1-6p}{2}\right) - \mathscr{E}\left(\frac{D+1-2p}{2}\right), \qquad (19)$$

using the identities above. In order to be renormalizable we must require that the coefficient of $\mathscr{V}_3$ be less than or equal to zero

$$p \geq \frac{D+1}{6}\ , \tag{20}$$

$$\text{or} \quad p \geq \frac{1}{2}\ , \tag{21}$$

for the physical number of dimensions. In the limiting case $p = \frac{1}{2}$ (also the most interesting case as it will turn out) only graphs with $\mathscr{E} = 2$ and $\mathscr{E} = 3$ are divergent. So one must add ϕ^2 and ϕ^3 counter terms to our $\mathscr{L}_I$. Equivalently one may specify the renormalized theory by making "subtractions" in $\Gamma^{(2)}$ and $\Gamma^{(3)}$. We will carry out subsequent calculations for general D and p. The conditions (20) and (21) will make their role felt.

The $\Gamma_U^{(n)}$ are functions of the q_i, of λ_0, p, D and possibly some cutoff to regularize the integrals. We will employ D, continued away from D = 2, as this regulator. The renormalized proper vertex functions $\Gamma^{(n)}$ depend on the q_i, and on p, D, and a renormalized coupling λ. The two vertex functions are related in the usual manner

$$\Gamma^{(n)}(q_i) = Z^{n/2}\ \Gamma_U^{(n)}(q_i). \tag{22}$$

To specify the renormalized theory we must choose the value of certain vertex functions at appropriate points. Since we are dealing with a "massless" theory: $G_0^{-1}(q^2) = 0$ at $q^2 = 0$; we require a normalization momentum q_N^2 to define quantities like λ away from the infrared singular point $q^2 = 0$. Noting that

$$\Gamma_0^{(2)}(q^2) = G_0(q)^{-1} = (q^2)^p \,, \tag{23}$$

we choose to normalize our theory by

$$\Gamma^{(2)}(q^2)\Big|_{q^2=0} = 0 \,, \tag{24}$$

$$\frac{\partial}{\partial q^2}\Gamma^{(2)}(q^2)\Big|_{q^2=q_N^2} = p\,(q_N^2)^{p-1} \,, \tag{25}$$

and

$$\Gamma^{(3)}(q_1, q_2, q_3)\Big|_{\substack{q_1^2=0 \\ q_2^2=q_3^2=q_N^2}} = \lambda(q_N^2)/(2\pi)^{\frac{D+1}{2}} \,. \tag{26}$$

The first condition assures us that whatever the detailed nature of the renormalized singularity it occurs at $q^2 = 0$. This is tantamount to assuring that the renormalized theory remain massless.[14] The second equation will serve to define Z via (22), and the third gives $\lambda(q_N^2)$, which parametrizes all $\Gamma^{(n)}$.

Before writing the renormalization group equations it is useful to carry out some conventional dimensional analysis. Since a only appears in $q^2 = E^2 + a^2|\vec{q}|^2$, we may identify E and $|\vec{q}|$ dimensions. Calling the dimension of q

$$[q] = q \,, \qquad (27)$$

we find

$$[\phi] = q^{\frac{D+1}{2} - p} \,, \qquad (28)$$

and

$$[\lambda] = q^{3p - \frac{D+1}{2}} \,. \qquad (29)$$

We now trade in λ as an argument of $\Gamma^{(n)}$ for the dimensionless quantity

$$y(q_N^2) = \lambda(q_N^2)\,(q_N^2)^{\frac{D+1}{4} - \frac{3p}{2}} \,. \qquad (30)$$

Ordinary dimensional analysis now allows us to write

$$\Gamma^{(n)}(q_i, y, q_N^2) = (q_N^2)^{\frac{D+1}{4}(2-n) + \frac{np}{2}}\, \psi_n\left(\frac{q_i \cdot q_j}{q_N^2}, y\right), \qquad (31)$$

so

$$\Gamma^{(n)}(\xi^{1/2} q_i, y, q_N^2) = \xi^{\frac{D+1}{4}(2-n) + \frac{np}{2}}\, \Gamma^{(n)}(q_i, y, q_N^2/\xi) . \qquad (32)$$

The so called renormalization group equations are constraints on $\Gamma^{(n)}(q_i)$ which guarantee that the $\Gamma_U^{(n)}$ are independent of q_N^2. They follow from

$$q_N^2 \frac{\partial}{\partial q_N^2} \Gamma_U^{(n)} = 0, \qquad (33)$$

the chain rule, and (32):

$$\left\{ \xi \frac{\partial}{\partial \xi} - \beta(y)\frac{\partial}{\partial y} + \frac{n}{2}\gamma(y) - \left(\frac{D+1}{4}(2-n) + \frac{np}{2} \right) \right\} \Gamma^{(n)}(\xi^{1/2} q_i, y, q_N^2) = 0. \quad (34)$$

In this we have defined

$$\beta(y) = q_N^2 \frac{\partial}{\partial q_N^2} y(q_N^2) \Big|_{\lambda_0, D, p \text{ fixed}} , \quad (35)$$

and $$\gamma(y) = q_N^2 \frac{\partial}{\partial q_N^2} \log Z \Big|_{\lambda_0, D, p \text{ fixed}} . \quad (36)$$

The solution to Eq. (34) is given in terms of an effective coupling constant $\tilde{y}(\eta)$ satisfying

$$d\tilde{y}(\eta)/d\eta = -\beta(\tilde{y}(\eta)). \quad (37)$$

With $\eta = \log \xi$ this solution is

$$\Gamma^{(n)}(\xi^{1/2} q_i, y, q_N^2) =$$

$$\Gamma^{(n)}(q_i, \tilde{y}(-\eta), q_N^2) \exp \int_{-\eta}^{0} d\eta' \left[\frac{D+1}{4}(2-n) + \frac{np}{2} - \frac{n}{2}\gamma(\tilde{y}(\eta')) \right]. \quad (38)$$

Solving (37) with the boundary condition $\tilde{y}(0) = y$ would enable us to determine the detailed behavior of $\Gamma^{(n)}(\xi^{\frac{1}{2}} q_i)$ as $\xi \to 0$ or $\eta \to -\infty$. We do not know $\lambda(q_N^2)$ and, thus, $\beta(y)$ except in perturbation theory. If we are to make any headway in determining $\Gamma^{(n)}(\xi^{\frac{1}{2}} q_i)$ for small ξ we must hope that $\tilde{y}(-\eta)$ is small as $\eta \to -\infty$. Then we know $\beta(\tilde{y})$ consistently in that regime and may use (38) fruitfully.

III. RENORMALIZATION GROUP EQUATIONS IN PERTURBATION THEORY

Before we launch into the perturbative determination of $\beta(y)$ and $\gamma(y)$ we make a few observations about the crucial function β. From its definition and the definition of y in Eq. (30) we see that

$$\beta(y) = \frac{1}{4}\left[D+1-6p\right] y + O(y^3) . \qquad (39)$$

In any number of dimensions where

$$p < \frac{D+1}{6} \qquad (40)$$

$\beta(y)$ will have a zero with positive slope at the origin. Such a zero is an infrared stable point of $\tilde{y}$; that is, as $\eta \to \infty$, $\tilde{y}(\eta)$ goes to such a zero. When

$$p = \frac{D+1}{6} \qquad (41)$$

then the sign of $\beta'(y)\Big|_{y=0}$ is determined by the coefficient of the y^3 term in $\beta(y)$. The advantage of the infrared free theory, a stable point at the origin, is that the Green's function we begin with $G_0(q^2)$ reproduces itself in the infrared limit with small computable corrections.

In our case the only allowed value of p which is both renormalizable <u>and</u> infrared free is $6p = D + 1$ or $p = \frac{1}{2}$ at $D = 2$. From the point of view of physics this is also the only value of interest since at $D = 2$ the total cross section behaves as

$$\sigma_T(s) \sim (\log s)^{2p-1}, \qquad (42)$$

and $p < \frac{1}{2}$ means a falling cross section. Several arguments then single out $p = \frac{1}{2}$ as the sole significant physical example of the theories we consider here.[15] The constraints of infrared freedom and renormalizability are quite restrictive. Attention now focuses on the y^3 term in β .

To compute β and γ in lowest order of perturbation theory we evaluate the graphs shown in Fig. 1 for $\Gamma^{(2)}$ and $\Gamma^{(3)}$. For the wave function renormalization constant we find from (25)

$$\frac{1}{Z} = 1 + \frac{\lambda_0^2 \left(g_N^2\right)^{\frac{D+1}{2}-3p} \pi^{\frac{D+1}{2}} \Gamma\left(1+2p-\frac{D+1}{2}\right) B\left(\frac{D+1}{2}-p, \frac{D+1}{2}-p\right)}{2p\,\Gamma(p)^2 \,(2\pi)^{D+1}}, \quad (43)$$

where B(a,b) is the ordinary beta function. From this we learn

$$\gamma(y) = y^2 \frac{\pi^{D+1/2} B\left(\frac{D+1}{2}-p, \frac{D+1}{2}-p\right)\left(3p-\frac{D+1}{2}\right)\Gamma\left(1+2p-\frac{D+1}{2}\right)}{2p\,\Gamma(p)^2}. \quad (44)$$

It is amusing to note that at $p = \frac{D+1}{6}$, $\gamma(y) = 0$ to this order of perturbation theory.

Next we evaluate $\beta(y)$ finding

$$\beta(y) = \frac{1}{4}\left[D+1-6p\right] y - \quad (45)$$

$$-\frac{y^3 \pi^{D+1/2}}{(2\pi)^D \Gamma(p)^2}\left[\frac{\Gamma\left(3p+1-\frac{D+1}{2}\right) B(p,p)\, B\left(\frac{D+1}{2}-p, \frac{D+1}{2}-2p\right)}{\Gamma(p)} - \frac{3}{4p}\left(\frac{D+1}{2}-3p\right)\Gamma\left(2p+1-\frac{D+1}{2}\right) B\left(\frac{D+1}{2}-2p, \frac{D+1}{2}-2p\right.\right.$$

At $D = 2$, $p = \frac{1}{2}$ this becomes $\beta(y) = -y^3/4\pi^2$. (46)

For $y = ig$ with g a real number and $\beta(g)$ defined to be the coefficient of $\partial/\partial g$ in Eq. (34) we have

$$\beta(g) = g^3/4\pi^2 . \tag{47}$$

This definition of y (or λ) as pure imaginary is suggested by Gribov's[4] signature analysis for singularities lying at $J = 1$ at $t = 0$.

Now $g = 0$ is an infrared stable point of the renormalization group equations. Solving for $\tilde{g}(\eta)$ we find

$$\tilde{g}(\eta) = g\left[1 + g^2\eta/2\pi^2\right]^{-1/2} \tag{48}$$

and

$$\tilde{g}(\eta) \underset{\eta\to\infty}{\sim} 1/\eta^{1/2} . \tag{49}$$

So we have learned that when $D = 2$, $p = \frac{1}{2}$ our theory is infrared free with a purely imaginary ϕ^3 coupling constant. Looking back at the solution to the renormalization group equations as given in Eq. (38) we see

$$\Gamma^{(n)}\left(\xi^{1/2} q_i, g, q_N^2\right) = \xi^{\frac{3-n}{2}}\, \Gamma^{(n)}\left(q_i, \tilde{g}(-\eta), q_N^2\right) \tag{50}$$

in the regime of $\xi \to 0$, $\eta \to -\infty$ where the approximation of keeping only lowest order terms in $\beta(y)$ and $\gamma(y)$ has now been justified.

IV. SCALING LAWS AND SOME PHYSICAL CONSEQUENCES

Combining the solution to the renormalization group equation given in Eq. (50) with the dimensional analysis embodied in Eq. (31) puts constraints on the form $\Gamma^{(n)}$ may have. Defining

$$\tilde{\psi}_n(x, w) = \psi_n(x, g) \quad , \quad w = \exp - 1/g^2 \, , \tag{51}$$

we obtain

$$\tilde{\psi}_n\left(\xi \frac{q_i \cdot q_j}{q_N^2}, w\right) = \xi^{\frac{3-n}{2}} \, \tilde{\psi}_n\left(\frac{q_i \cdot q_j}{q_N^2}, \xi^{1/2\pi^2} w\right), \tag{52}$$

noting

$$\tilde{g}(-\eta)^{-2} = g^{-2} - \eta/2\pi^2 . \tag{53}$$

This requires

$$\tilde{\psi}_n\left(\frac{q_i \cdot q_j}{q_N^2}, w\right) = \left(q^2/q_N^2\right)^{\frac{3-n}{2}} X_n\left(\frac{q_i \cdot q_j}{q^2}, \left(\frac{q^2}{q_N^2}\right)^{1/2\pi^2} w\right), \tag{54}$$

where X_n is undetermined, and q is some convenient momentum, say

$$q = \sum_{\text{incoming}} q_i \, . \tag{55}$$

From perturbation theory we know that the lowest order contribution to $\Gamma^{(n)}$ is proportional to g^{n-2}. Using Eqs. (50), (53) we see that as $q^2 \to 0$

$$X_n \sim \left(- \log q^2/q_N^2\right)^{\frac{n-2}{2}} \tag{56}$$

Now we choose to couple particles into the theory using the procedure described in Ref. 6. The contribution to the elastic amplitude

A + B → A + B of the n Reggeon in, m Reggeon out graph in Fig. 2 is

$$I_{n,m}(q^2) = N_n^A\, N_m^B \int d^3q_1 \cdots d^3q_{n+m}\, \delta^3\Big(q - \sum_{i=1}^{n} q_i\Big) \times$$

$$\delta^3\Big(q - \sum_{j=n+1}^{n+m} q_j\Big)\, G^{(n+m)}(q_1, \cdots q_{n+m}) \quad , \tag{57}$$

where N_k^A is a number describing the amplitude for k Reggeons to be emitted or absorbed from particle A . According to Gribov's signature analysis N_1^A is real, N_2^A is pure imaginary, N_3^A is real, etc.

This gives rise to a heirarchy of contributions to $\sigma_T(s)$ which follow from (56)

$$\sigma_T^{AB}(s) \underset{s\to\infty}{\sim} \gamma_A \gamma_B - f_{AB} \big/ (\log s)(\log \log s)^{1/2}$$
$$+ h_{AB} \big/ (\log s)^2 (\log \log s) - \cdots \tag{58}$$

Corrections to the leading constant term [of 0 $(\log \log s)^{-2}$] from the expansion of $\Gamma^{(2)}$ in $\tilde{g}$ are order $\tilde{g}^4$, there being an amusing cancellation of the $\tilde{g}^2$ terms at D = 2, $p = \frac{1}{2}$. In passing we note that the renormalized triple Pomeron vertex $\Gamma^{(3)}$ vanishes as $J \to 1$, $t \to 0$, albeit only as $(\log q^2)^{-\frac{1}{2}}$.

V. DISCUSSION

In this paper we have studied the modification to a Reggeon Green's function

$$G_0^{-1}(J,t) = \left((J-1)^2 - a^2 t \right)^p$$

arising from a triple Reggeon coupling. To perform this job we first generalized the usual Reggeon field theories to take account of cuts in the J-Plane in the non-interacting Green's function. Next we employed the renormalization group equations[6] to argue that in the infrared region $J \to 1$, $t \to 0$, the theory is infrared free in the very amusing case of $p = \frac{1}{2}$ in D = two transverse dimensions, the number occuring in real physics. Since the theory is renormalizable only for p [illegible] $\frac{1}{2}$ at $D = 2$, this value of p is certainly picked out on a number of grounds. Indeed, this constitutes an example of the renormalization group bootstrap[10] of serious physical interest.

Using the infrared freedom we determined that there is a heirarchy of contributions to the total cross section $A + B \to$ anything of the form

$$\sigma_T^{AB}(s) \sim \gamma_A \gamma_B + \sum_{n=1}^{\infty} f_{AB}^{(n)} (-1)^n \left[\log s \, (\log \log s)^{1/2} \right]^{-n}, \qquad (59)$$

with computable corrections to each term of $0\left[(\log \log s)^{-1} \right]$.

Since the original Green's function was chosen to be representative of partial wave amplitudes arising from considerations of s-channel unitarity, it is most pleasing to find a physically very attractive example which, in the manner explained in Sec. I and Ref. 6, also satisfies t-channel unitarity with small computable modifications. That expressions like (59) are also acceptable respresentations of the very high energy proton-proton data[1[illegible]] is also satisfying.

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[11] See the lectures by G. Mack in Lecture Notes in Physics: Strong Interaction Physics, ed. W. Rühl and A. Vancura (Springer Verlag,

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[12]For a review of ISR results consult the lectures by M. Jacob, "Multi-Body Phenomena in Strong Interactions", CERN preprint TH-1683.

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[14]A detailed discussion of this condition has been carried out in the context of the Reggeon calculus by R. L. Sugar and A. R. White, private communication. See also K. Symanzik, DESY preprint 73/58, December, 1973.

[15]One might well imagine studying theories with $1/2 < p < 3/2$ (the 3/2 comes from the renormalizability requirement that the coefficient of $\mathscr{E}$ in Eq. (19) be positive) using the ϵ-expansion methods of Ref. 5 and 6. One can demonstrate, however, that the sign of the coefficient of y^3 in $\beta(y)$ depends on the renormalization conventions at least in the region $3/4 < p < p_0 < 1$. This indicates that such an expansion is likely to be unreliable except in the very immediate neighborhood of $p = 1/2$ where the coupling λ is dimensionless and the theory possesses a scale invariance.

FIGURE CAPTIONS

Fig. 1 a. The lowest order correction to the propagator which yields Z and $\gamma(y)$.

b. The lowest order correction to the three point proper vertex which yields $\beta(y)$.

Fig. 2 The n Reggeon to m Reggeon contribution to the elastic amplitude $AB \rightarrow AB$. Using the behavior of $\Gamma^{(n+m)}$ given in the text we find this gives a contribution to $\sigma^{AB}_{\text{Total}}(s)$ of order $\left[\log s\,(\log\log s)^{\frac{1}{2}}\right]^{2-n-m}$.

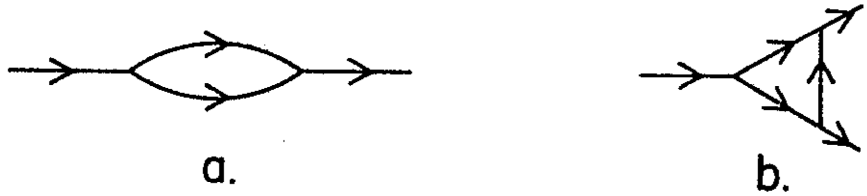

FIG. 1

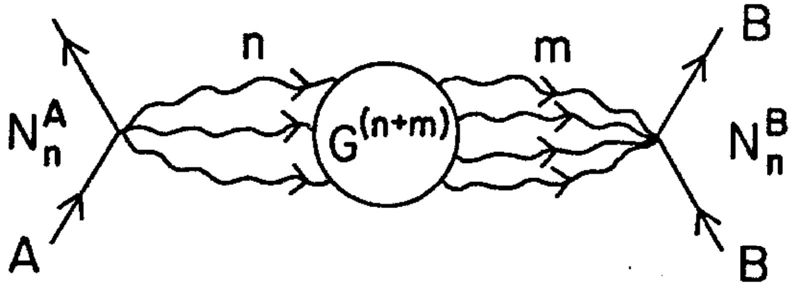

FIG. 2