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$\Delta I = 1/2$ Rule for Nonleptonic Decays in Asymptotically Free Field Theories

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ABSTRACT

The effective nonleptonic weak interaction is examined assuming the Weinberg-Salam theory of weak interactions and an exactly conserved color gauge symmetry for strong interactions. It is shown that the octet part of the nonleptonic weak interaction is more singular at short distances than the $\underset{\sim}{27}$ part. The resulting enhancement of the octet term in the effective local weak Lagrangian, together with suggested mechanisms for the suppression of matrix elements of the $\underset{\sim}{27}$ operator, may be sufficient to account for the observed $|\Delta I| = 1/2$ rule.

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* $\Delta I = 1/2$ RULE FOR NONLEPTONIC DECAYS IN ASYMPTOTICALLY FREE FIELD THEORIES

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The purpose of this paper is to discuss the effectively local form of nonleptonic weak interactions in models in which weak interactions are described by a Weinberg-Salam type gauge theory[1] and strong interactions by an exactly conserved color gauge symmetry group,[2] and to comment on the origin of the $\Delta I = 1/2$ (or octet) rule observed in strangeness changing decays.

Our discussion is based on the operator product expansion of the product of two weak currents. In an asymptotically free field theory,[3] it is possible to compute the short distance behavior of coefficient functions in the operator product expansion; we find that the $\Delta I = 1/2$ part of the interaction is more singular at short distances. This is much as anticipated by K. Wilson.[4] (See also Mathur and Yen;[5] however, our conclusions differ substantially from theirs.)

In the following, we shall use the 't Hooft-Feynman gauge to describe both the weak bosons and the color gluons. In the 't Hooft-Feynman gauge, effects of (unphysical) Higgs scalar fields may be neglected, since they are of order $(m/m_W)^2$ compared to the W exchange, where m is a characteristic mass scale of hadrons and m_W the mass of the charged vector meson W.

It is useful to consider first the case of free quarks. The effective nonleptonic weak interaction is of the form

$$-f^2 \int d^4x \, D_F(x; m_W^2) T[\, j^{\dagger}_{\mu N}(x) \, j^{\mu}_{S}(0)] \qquad (1)$$

where $j_{\mu N}$ and $j_{\mu S}$ are strangeness-conserving, and -changing charged currents:

$$j_{\mu N} = (\bar{p} \cos\theta - \bar{p}' \sin\theta)\gamma_\mu(1-\gamma_5)n + \ldots \qquad (2)$$

and

$$j_{\mu S} = (\bar{p} \sin\theta + \bar{p}' \cos\theta)\gamma_\mu(1-\gamma_5)\lambda + \ldots \qquad (3)$$

In Eq. (2), p' denotes the fourth quark field associated with the proposal of Glashow, Iliopoulos and Maiani,[6] and θ is the Cabibbo angle. The time ordered product of two currents in (1) may be expanded in the form

$$T\,[j^{\dagger}_{\mu N}(x)\, j^{\mu}_{S}(0)] = \sum_{i,m} \mathscr{F}^{(i)}_{m}(x^2)\, \mathscr{O}^{(i)}_{m} \qquad (4)$$

where the $\mathscr{F}$ are c-number functions of the separation distance and the $\mathscr{O}^{(i)}$ are operators of (2i) quark fields.

Let us consider the operators bilinear in quark fields (one-body operators). The operators $\bar{n}\lambda$ and $n\gamma\cdot\partial\lambda$ will have coefficient functions which scale as x^{-2} at short distances. Such terms will have divergent coefficients in Eq. (1), but contribute only to mass and wave function renormalizations of the quark fields. Operators of dimension 6 or higher, such as $\bar{n}\gamma\cdot\partial\,\partial^2\lambda$, can in principle contribute to Eq. (1) to order m_W^{-2}, but these operators are automatically $\Delta I = 1/2$, and are expected to have coefficients proportional to $(m_{p'}^2 - m_p^2)/m_W^2$ because the contractions of p and p' quarks contribute to them with the opposite sign.

Next we consider the operators quartic in quark fields. There are two of them:

$$\mathcal{O}_1^{(2)} = :\bar{n}\gamma_\mu(1-\gamma_5)p\,\bar{p}\gamma^\mu(1-\gamma_5)\lambda:$$

and

$$\mathcal{O}_2^{(2)} = :\bar{n}\gamma_\mu(1-\gamma_5)p'\,\bar{p}'\gamma^\mu(1-\gamma_5)\lambda: \qquad (4)$$

The second is automatically $\Delta I = 1/2$ and the first a mixture of $\Delta I = 1/2$ and $3/2$. The associated c-number functions $\mathcal{F}_1^{(2)}$ and $\mathcal{F}_2^{(2)}$ are constants, so that the operators $\mathcal{O}_1^{(2)}$ and $\mathcal{O}_2^{(2)}$ contribute to Eq. (1) with the coefficient

$$\sim -f^2 \int d^4x\, D_F(x^2;m_W^2) = f^2/M_W^2 = G_F/\sqrt{2}$$

The quartic operators (or two-body operators) of higher dimensions such as $:\{\partial^2[\bar{n}\gamma^\mu(1-\gamma_5)p]\}\bar{p}\gamma_\mu(1-\gamma_5)\lambda:$ will have coefficients of order $G_F m_W^{-2}$ and may be neglected in Eq. (1). Thus the effective local form of the interaction (1) to leading order in m_W^{-2} is

$$[\mathcal{L}_W]_{\rm effective} = (G_F/\sqrt{2}) : j^\dagger_{\mu N}(0)\, j^\mu_S(0): + \text{h.c.} \qquad (5)$$

We shall now extend similar considerations to models in which strong interactions are mediated by unbroken color gluons. For definiteness we consider the model of color SU(3). We write the color quark fields as column matrices. Strong interactions are described by the Lagrangian

$$\mathscr{L}_S = \sum_{i=p,n,\lambda,p'} \bar{q}_i \, i\gamma_\mu (\partial^\mu - \frac{i}{2} g \underset{\sim}{\lambda} \cdot \underset{\sim}{G}^\mu) q_i \qquad (6)$$
$$+ \text{ mass terms}$$

where $\underset{\sim}{G}^\mu$ are the octet of color gluon fields. The comments made for the free quark model on the one-body operators $\mathscr{O}_m^{(1)}$ that appear on the right hand side of Eq. (4) also apply to the color (or singlet) vector gluon scheme; in particular, as shown by Weinberg,[7] the terms containing the operators $\bar{n}\lambda$ and $\bar{n}\gamma\cdot\partial\lambda$ in Eq. (1) are "transformed away" by the mass and amplitude renormalizations of quark fields: the remaining operators are $\Delta I = 1/2$, but their coefficients are in general of order $G_F(m_{p'}^2 - m_p^2)/m_W^2$.

We need only consider two-body operators of lowest dimensions which conserve charm and which are antisymmetric in p and p'. There are two operators we must deal with

$$\mathscr{O}_{\mp}^{(2)} = \frac{1}{2} N \, [(\bar{n}p)(\bar{p}\lambda) \mp (\bar{n}\lambda)(\bar{p}p) - (\bar{n}p')(\bar{p}'\lambda) \pm (\bar{n}\lambda)(\bar{p}'p')] \qquad (7)$$

where we have dropped the reference to the spinor structure; thus $(\bar{A}B)(\bar{C}D) = \bar{A}\gamma_\mu(1-\gamma_5)B\,\bar{C}\gamma^\mu(1-\gamma_5)D$. The symbol N stands for Zimmermann's normal ordering,[8] except that the subtraction point should be chosen in the Euclidean region of the external momenta to avoid infrared difficulties. The combinations (7) are chosen so that each one transforms like the U_x component of a U-spin triplet.[9] That the two combinations form a complete set of operators is a consequence of the Fierz identity. Note further that the first operator in (7) is pure $\Delta I = 1/2$, while the second is a mixture of $\Delta I = 1/2$ and $3/2$. The first operator $\mathcal{O}_-^{(2)}$ is anti-symmetric with respect to color indices of $\bar{n}_i$ and $\bar{p}_j$ (or p_i and λ_j) while the second is symmetric. To lowest order in g^2, the two operators in Eq. (7) are renormalized multiplicatively.

The associated functions $\mathcal{F}^{(2)}_{8,8',27}$ may be evaluated for small x^2, $m^2x^2 << 1$ by the method of Christ, Hasslacher and Mueller,[10] using the Gell-Mann, Low,[11] Callan, Symanzik[12] equation. We consider the irreducible vertex

$$<T[j^\dagger_{\mu N}(x)\, j^\mu_S(0)\, q(p_1)q(p_2)\bar{q}(p_3)\bar{q}(p_4)]>^{irr}$$

in the limit of vanishing quark mass, and choose $p_1^2 = p_2^2 = p_3^2 = p_4^2 = -\mu^2$, $s = t = u = -4\mu^2/3$ as the subtraction point. The renormalized matrix element

$$<T[\mathcal{O}^{(2)}_m\, q_1q_2\bar{q}_3\bar{q}_4]>^{irr}$$

has the form, as far as the dependence on μ is concerned.

$$\{1 + \gamma_m[\ell n\, \mu + \ldots.]\}$$

where γ_m is a number which can be computed from one gluon exchange diagrams in perturbation theory. As discussed explicitly by Georgi and Politzer,[13] the functions $\mathcal{F}^{(2)}_m$ satisfy the Callan-Symanzik equations;

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - 4\gamma + \gamma_m\right]\mathcal{F}^{(2)}_m(\mu^2x^2, g) = 0 \qquad (8)$$

with the usual meanings for β and γ.[12]

In asymptotically free theories, the asymptotic form of $\mathscr{F}^{(2)}_m$ can be reliably estimated. Denoting

$$\beta(g) = -(g^3/16\pi^2)\,[\,b + 0(g^2)] \qquad (9)$$
$$\gamma_m = (g^2/16\pi^2)[\,d_m + 0(g^2)]$$

and recalling that $\gamma = (g^2/16\pi^2)(4/c)^2$ we find that

$$\mathscr{F}^{(2)}_m(\mu^2x^2, g) \xrightarrow[\substack{\mu^2x^2 << 1 \\ x^2 < 0}]{} \mathscr{F}^{(2)}_m(1,0)\left[1 - \frac{g^2}{8\pi^2}\, b\ell n\,(\mu\sqrt{-x^2})\right]^{d_m/2b} \qquad (10)$$

where

$$(\cos\,\theta\,\sin\,\theta)^{-1}\mathscr{F}^{(2)}_m(1,0) = (1/\sqrt{2},\ 1/\sqrt{10},\ 2/\sqrt{10})$$

for m = $\underset{\sim}{8}$, $\underset{\sim}{8}'$ and $\underset{\sim}{27}$, respectively.

In the three-color quartet scheme

$$b = 25/3 \quad ^2 \qquad (11)$$

and

$$d_m = 8 \text{ for } 8 \qquad (12)$$
$$= -4 \text{ for } \underset{\sim}{8}' \text{ and } \underset{\sim}{27}$$

[If the color gauge group is SU(N), we have $b = (11N - 8)/3$, $d_8 = 6(N+1)/N$, $d_{\underset{\sim}{8}', \underset{\sim}{27}} = -6(N-1)/N$.] Thus, we see that the short distance behavior associated with $\mathcal{O}^{(2)}_8$ is more singular than those associated with $\mathcal{O}^{(2)}_{8', 27}$: the former is of the form $(\ell n\,|x|)^{0.48}$, the latter $(\ell n\,|x|)^{-0.24}$

This difference in the short distance behavior is reflected in the coefficients of the operators $\mathcal{O}^{(2)}_m$ in the effective local Lagrangian for nonleptonic weak interactions. Substituting Eqs. (4) and (10) in Eq. (1), and performing the integration over x after the Wick contour rotation, we obtain

$$\mathscr{H}_W = \frac{G_F}{\sqrt{2}} \cos\theta \sin\theta \left\{ c_1 \mathscr{O}^{(2)}_{8} + c_2 \mathscr{O}^{(2)}_{8'} + c_3 \mathscr{O}^{(2)}_{27} + \text{h.c.} \right\} \tag{13}$$

where

$$\mathscr{O}^{(2)}_{\underset{\sim}{8}} = N[(\bar{n}p)(\bar{p}\lambda) - (\bar{n}\lambda)(\bar{p}p)]/\sqrt{2}$$

$$\mathscr{O}^{(2)}_{\underset{\sim}{8'}} = N[(\bar{n}p)(\bar{p}\lambda) + (\bar{n}\lambda)(\bar{p}p) + 2(\bar{n}\lambda)(\bar{n}n) + 2(\bar{n}\lambda)(\bar{\lambda}\lambda)]/\sqrt{10}$$

$$\mathscr{O}^{(2)}_{\underset{\sim}{27}} = N[2(\bar{n}p)(\bar{p}\lambda) + 2(\bar{n}\lambda)(\bar{p}p) - (\bar{n}\lambda)(\bar{n}n) - (\bar{n}\lambda)(\bar{\lambda}\lambda)]/\sqrt{10}$$

and

$$c_1 \simeq \frac{1}{\sqrt{2}} \left[1 + \left(\frac{g^2}{4\pi}\right) \frac{b}{2\pi} \ln \frac{m_W}{\mu}\right]^{0.48}$$

$$\sqrt{10}\, c_2 = \sqrt{5/2}\, c_3 \simeq \left[1 + \left(\frac{g^2}{4\pi}\right) \frac{b}{2\pi} \ln \frac{m_W}{\mu}\right]^{-0.24} \tag{14}$$

To get some idea of the magnitude of these constants, we choose μ to be the onset of scaling of the form (10), which we assume optimistically to be $\simeq 1$ GeV.[14] The gluon-quark coupling at that subtraction point is assumed to be $(g^2/4\pi) \simeq 1$. Taking $m_W \simeq 100$ GeV, we have

$$1 + (g^2/4\pi)(b/2\pi) \ln(M_W/\mu) \simeq 7 \tag{15}$$

so that $c_1/c_3 \simeq 5$.

The relative enhancement of an octet piece in Eq. (13) is not by itself sufficient to account for the observed small violation of the

$\Delta I = 1/2$ rule of about 5% (in amplitude) in nonleptonic decays of hyperons and K-mesons. [For a review of this and related subjects, see for instance (9).] It has often been remarked that the effective strength of $|\Delta S| = 1$ nonleptonic transitions is of order G_F, rather than $G_F \sin\theta$, and that some enhancement of the $\Delta I = 1/2$ part by a dynamical mechanism is necessary.[15] Since we expect $c_1 \sin\theta \simeq 1$ in Eq. (13) we do have such a mechanism for enhancing the $\Delta I = 1/2$ part. However, if the typical hadronic matrix element of $\mathcal{O}^{(2)}_{\underset{\sim}{27}}$ is comparable to that of $\mathcal{O}^{(2)}_{\underset{\sim}{8}}$, one expects only a 20% validity for the $\Delta I = 1/2$ rule, which is not sufficient to explain the data. Thus to explain the observed $\Delta I = 1/2$ rule, it is further necessary that the matrix elements of $\mathcal{O}^{(2)}_{\underset{\sim}{27}}$ are suppressed by a factor of ~ 4 relative to those of $\mathcal{O}^{(2)}_{\underset{\sim}{8}}$. Such a suppression may come about in a number of ways; for example, the duality considerations of Nussinov and Rosner[16] indicate the suppression of the $\underset{\sim}{27}$ operator. Pati and Woo[17] have shown in the context of free color quarks that the baryon to baryon matrix elements of the $\underset{\sim}{27}$ operator vanish. This result, which is a consequence of the color symmetry of $\mathcal{O}_{\underset{\sim}{27}}$ in Eq. (7), remains true for quarks coupled to color gauge gluons, provided the SU_6 ground state wave function is assumed. It then follows from the usual lore of current algebra that the $\Delta I = 3/2$ amplitudes vanish in the soft pion limit for both baryon and K decays. (Of course, the continuation in pion momenta is problematic as always!) Consequences of our results for $\Delta S = 0$ transitions, the decays of charmed particles and Ω^- decay, and a detailed account of the results discussed in this letter will be presented elsewhere.

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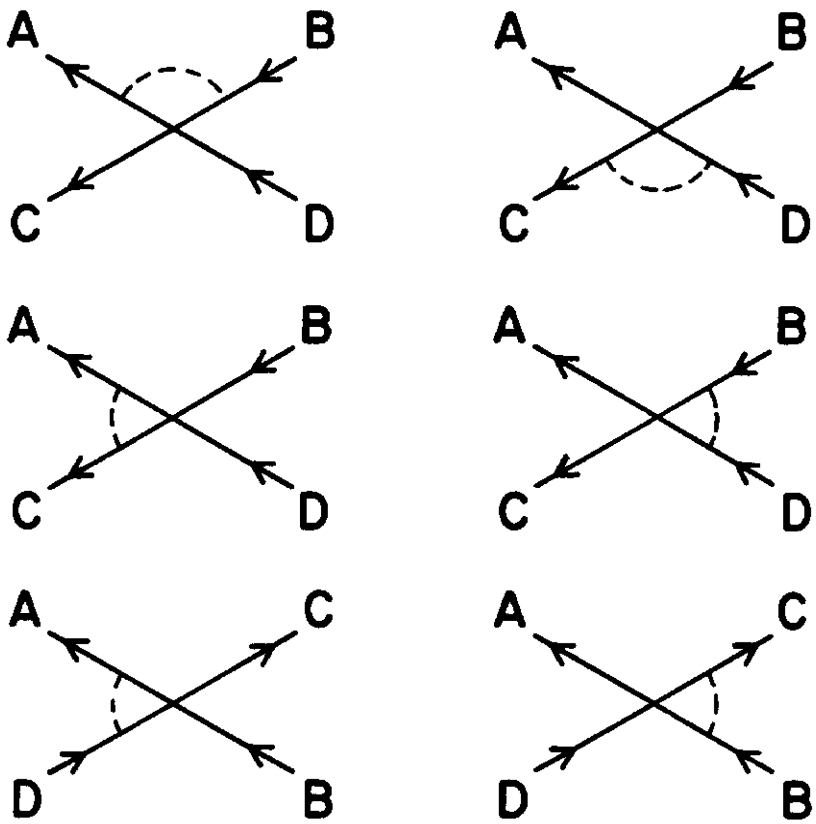

FIG. 1