CALT-68-423
AEC RESEARCH AND
DEVELOPMENT REPORT

Spontaneous Symmetry Breaking in Reggeon Field Theory*

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ABSTRACT

We consider a Reggeon field theory when the bare or input Regge intercept α_0 is greater than one. This corresponds to a negative mass squared term in conventional field theory and allows for a spontaneous symmetry breakdown. A theory with Regge intercept at one emerges, restoring the Froissart bound by t-channel considerations alone. In our elementary example the resulting bare trajectory is nearly of the square root variety familiar from s-channel eikonalization of models which violate the Froissart bound.

*Work supported in part by the U.S. Atomic Energy Commission. Prepared under Contract AT(11-1)-68 for the San Francisco Operations Office, U.S. Atomic Energy Commission.

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By abstracting properties of Reggeon exchange amplitudes in hybrid quantum field theories, Gribov[1] has established rules for a Reggeon calculus or Reggeon field theories. These field theories provide a convenient framework in which to study the interaction of poles and branch cuts in the angular momentum plane. By the nature of field theories one is assured of satisfying the discontinuity relations across Reggeon cuts[2] in whatever approximation one uses to study the theory. In this brief note we would like to point out an interesting property of an elementary Reggeon field theory which is the direct analogue of spontaneous symmetry breaking in conventional quantum field theories[3].

We recall that a Reggeon is described as a quasi-particle or elementary excitation in two dimensions of space and one of time. These are conjugate respectively to the two-momentum $\vec{q}$ and $E = 1 - \ell = 1 -$ angular momentum carried by the Reggeon. The basic problem of the Reggeon calculus is to choose an energy, E, momentum, $\vec{q}$, relationship for the "bare" Reggeon and then choose an interaction among the Reggeons to discover the modifications of partial wave amplitudes due to these interactions. We will study the elementary but instructive example of a bare linear trajectory

$$\alpha(t) = \alpha_0 + \alpha_0' \, t. \tag{1}$$

In E, $\vec{q}$ language this means the energy-momentum relation of the non-interacting Reggeon is

$$E = \alpha_0' \, \vec{q}^{\,2} + \Delta_0 \tag{2}$$

remembering that $t = - |\vec{q}|^2$ and calling $\Delta_0 = 1 - \alpha_0$. The Lagrangian for (2) reads

$$L_0(\vec{x},t) = \frac{i}{2} \, \phi^+(\vec{x},t)\overleftrightarrow{\partial}_t\phi(\vec{x},t) - \alpha_0'\nabla\phi^+(\vec{x},t) \cdot \nabla\phi(\vec{x},t)$$

$$- \Delta_0\phi^+(\vec{x},t)\phi(\vec{x},t) \quad , \tag{3}$$

with $\phi(\vec{x},t)$ the Reggeon field operator. As an interaction we take the renormalizable ϕ^4 coupling

$$L_I(\vec{x},t) = -\frac{\lambda_0}{4}\,[\phi^+(\vec{x},t)\phi(\vec{x},t)]^2. \tag{4}$$

Our main observation is that for $\Delta_0 \geq 0$, that is the input or bare Reggeon intercept $\alpha_0 \leq 1$, this theory is rather much an ordinary field theory in which $\Delta_0\phi^+\phi$ acts like a mass term. When $\Delta_0 = 0$, we have a "massless" theory in the sense that the (bare) energy momentum relation is such that there is no energy gap at $\vec{q}^{\,2} = 0$: $E(0) = 0$. Such a massless theory has been thoroughly investigated in the infrared region, E, $\vec{q} \simeq 0$ which is relevant for diffraction scattering, and gives an asymptotically free theory and an asymptotically constant total cross section[4].

In many models of hadronic collisions, of which the multiperipheral model is a standard example, the intercept of the bare Reggeon α_0 depends on a strength or coupling parameter in such a way that as that coupling is increased α_0 can reach and exceed one with no instruction from the theory to the contrary. This is in apparent violation of the Froissart bound, and one often invokes some s-channel unitarity argument implemented by an eikonalization procedure to restore this bound.

In Reggeon field theory when α_0 exceeds one, Δ_0, the mass parameter becomes negative. This is the classic situation in which spontaneous symmetry breaking occurs. The potential term of the full Lagrangian

$$V(\phi^+\phi) = \Delta_0\phi^+\phi + \frac{\lambda_0}{4}\,(\phi^+\phi)^2 \quad , \tag{5}$$

developes a minimum at

$$\phi^+\phi \quad = -\frac{2\Delta_0}{\lambda_0} \tag{6}$$

which for $\Delta_0 < 0$ is positive. This minimum corresponds to the new vacuum. The field ϕ develops a vacuum expectation value in this situation

$$<\phi>_0 = \sqrt{\frac{2\delta_0}{\lambda_0}} \tag{7}$$

with $\delta_0 = -\Delta_0 > 0$. Define, as usual, a new field $\chi = \phi - <\phi>_0$ and express the Lagrangian in terms of χ:

$$\begin{aligned} L(\vec{x},t) &= \frac{i}{2}\chi^+ \overleftrightarrow{\partial}_t \chi - \alpha_0' \nabla\chi^+ \cdot \nabla\chi \\ &- \delta_0 \chi^+\chi - \frac{\delta_0}{2}(\chi^2 + \chi^{+2}) \\ &- \sqrt{\frac{\delta_0\lambda_0}{2}}\,[(\chi^+)^2\chi + \chi^+\chi^2] - \frac{\lambda_0}{4}(\chi^+\chi)^2 \quad , \end{aligned} \tag{8}$$

where we have discarded an irrelevant constant and a harmless total derivative with respect to time. The Euler equations of motion are

$$(i\partial_t + \alpha_0' \nabla^2 - \delta_0)\chi(\vec{x},t) = \delta_0 \chi^+(\vec{x},t) \tag{9}$$

$$(i\partial_t - \alpha_0' \nabla^2 + \delta_0)\chi^+(\vec{x},t) = -\delta_0 \chi(\vec{x},t) \tag{10}$$

for the free fields χ and χ^+. This gives

$$(\partial_t^2 + (\alpha_0')^2\nabla^4 - 2\alpha_0'\delta_0\nabla^2)\chi(\vec{x},t) = 0 \quad , \tag{11}$$

or the free energy momentum relation

$$E^2(\vec{q}^{\,2}) = (\alpha_0')^2(\vec{q}^{\,2})^2 + 2\alpha_0'\delta_0(\vec{q})^2 \quad . \tag{12}$$

This is our key result.

First we note that $E(0) = 0$ so that by pushing α_0 above one we have restored a massless theory by the spontaneous breaking of the symmetry. That is, the

Froissart bound is automatically respected by the interacting Reggeon field theory in (3) and (4). This is amusing since the formulation of the Reggeon calculus is in the t-channel and it would thus appear that t-channel arguments alone are sufficient to maintain the Froissart bound. This observation is compatible with Bronzan's work on the Reggeon calculus for $\alpha_0 > 1$[5]. Second, the energy momentum relation in (12) for small $\vec{q}^{\,2}$ (that is $\alpha(t)$ for small t) is a pair of Schwarz-like[6] square-root trajectories. This is the kind of angular momentum structure that seems inevitably to emerge when α_0 becomes larger than one[7]. Finally note that the Lagrangian (8) has very naturally developed a triple Reggeon coupling, a quantity of direct physical interest.

One can build and discuss a wide variety of model Reggeon field theories beginning with different bare $E, \vec{q}^{\,2}$ relations and having different interactions. The ones I have looked at possess in a more or less complicated fashion, the general features of the elementary example discussed here. One may imagine, though pretentions to rigor are certainly foresworn, that the basic features we have found will persist in general.

I am grateful to A. R. White for a discussion which initiated this investigation and to B. W. Lee for conversations during it.

References

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3. An excellent pedagogical discussion of this and related subjects as well as an extensive set of references may be found in the "Lectures on Gauge Theories," E. S. Abers and B. W. Lee, _Physics Reports_, to be published.

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6. J.H. Schwarz, _Phys. Rev._ _167_, 1342 (1968).

7. See reference 5, and for a review of other models consult the lectures by F. Zachariasen at the Canadian Institute of Particle Physics Summer School, Montreal, September, 1973 (to be published at McGill University).