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Theories of Diffraction Phenomena: A Tour for Beginners and Others*

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ABSTRACT

A review is presented of several theories of diffraction phenomena in high energy hadron collisions: (1) Absorptive models, (2) multiperipheral bootstraps, and (3) Reggeon field theory models. The status of decoupling "theorems" for the Pomeron is discussed.

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The detailed structure of the mechanism responsible for processes, elastic and inelastic, characterized by cross sections which remain constant (up to logarithms in energy) for large collision energies persists as one of the intriguing theoretical challenges of hadronic physics.[1] This mechanism parades under the name Pomeron ($\underset{\sim}{P}$) and appears in many guises in the literature. Most physicists agree on many of this $\underset{\sim}{P}$ "thing" 's attributes (Fig. 1):

(1) The $\underset{\sim}{P}$ represents vacuum quantum number exchange in the crossed channel (t-channel). So for the $\underset{\sim}{P}$: $C = G = P = +1$, $S = B = I = 0$.

(2) If cross sections are to be pretty much constants the effective t-channel angular momentum associated with this $\underset{\sim}{P}$ must be approximately one.

(3) If $\underset{\sim}{P}$ is indeed responsible for $\sigma_T \sim$ constant, it cannot be represented by a simple pole , moving or stationary, in the angular momentum plane. This has its concise statement in the Pomeron decoupling results which we will address below.[2]

(4) As with most t-channel exchange mechanisms, $\underset{\sim}{P}$ exchange is sharply peaked near $t = 0$.

From this happy ground of agreement departures are made in most possible directions. Getting the disparate theoretical approaches together is likely to prove a difficult task from a purely experimental point of view. This is because various theories differ in the powers of log s that multiply the s^1 in diffraction amplitudes. From p_{lab} of 30 GeV/c through the CERN-ISR, log s varies from 4 to 8. So, alas, we are presented experimentally with a relatively blunt instrument for weeding out or slicing through competing theories. It is on this quicksand of precision that the remainder of my remarks firmly rest.

With no special preference given by their order, I will discuss three approaches to studying $\underset{\sim}{P}$. First, I will present some of the ideas and results that surround the concept of the multiperipheral bootstrap. The modern development of this has been at the hand of J.S. Ball and F. Zachariasen.[3] Second, I will consider various incarnations of the absorption model or eikonal models. The practitioners of this art are numerous, and I refer to the references of Ref. 4 to begin the list. Finally, I will come to the Reggeon field theory approach. Again the references of Ref. 5 will have to do in lieu of an unbounded list of authors. As de Tar so cogently remarked in his talk at this conference last year, theories in the first two categories emphasize the implementation of s-channel unitarity while the third focuses on t-channel unitarity. Some movement has been made in the direction of putting these unitarities together; we will come to this. After all is said on these three popular $\underset{\sim}{P}$'s, I will remind you of what we know the $\underset{\sim}{P}$ cannot be and how each of these approaches avoids the disaster of decoupling. Isn't it nice to know that the $\underset{\sim}{P}$ is back with us again?

A. MULTIPERIPHERAL BOOTSTRAP

The multiperipheral model (MPM) is an attempt to fulfill the requirements of multibody direct channel unitarity by approximating the $2 \to N$ production amplitude by a simple product of $2 \to 2$ or $2 \to$ (small cluster) amplitudes. The imaginary part of $T_{22}(s,t)$ representing $A + B \to C + D$ is given via unitarity (Fig. 2)

$$\mathrm{Im}T_{22}(s,t) = \sum_N \int d\Phi_N T(AB\to N)\ T(CD\to N)^* \tag{1}$$

where $d\Phi_N$ is N body phase space. In the simplest MPM one writes (Fig. 3)

$$T_{2\to N} = (g)^N \prod_{i=1}^{N-1} \frac{1}{m^2 - t_i} \quad . \tag{2}$$

Crucial here is the factorization along the MP chain and the damping in momentum transfers. This and a large class of basically similar models give rise to a leading simple pole in the t-channel partial wave amplitude

$$F(J,t) = \int_1^\infty ds\ s^{-J-1}\,\mathrm{Im}T_{22}(s,t) \tag{3}$$

$$\approx \frac{\gamma_{AC}(t)\,\gamma_{BD}(t)}{J-\alpha(t,g)}\ , \tag{4}$$

which factorizes, and whose position $\alpha(t,g)$ depends on t and g (Fig. 4).

The $t = 0$ intercept in these models $\alpha(0, g)$ need not be less than one by any feature of the model yet prescribed. However, having been instructed by Froissart, one requires $\alpha(0, g) \leq 1$ and inquires what occurs when the limit is reached. Life becomes very interesting then, for iteration of the basic T_{22} as depicted in Figs. 5 and 6 gives rise to branch points in J occuring also at $J = 1$. And this is the issue: How can this branch point and those further iterative branch points at $J = 1$ when $t = 0$ be put together to form an acceptable, hopefully simple, amplitude?

The criterion employed by MP bootstrappers is that of self-consistency: What goes in must be what comes out. More precisely one writes an integral equation for $F(J, t)$ looking similar to the Schrödinger equation (Fig. 5)

$$F = K + \int KF \tag{5}$$

where the "potential" K is taken to arise from elastic unitarity (Fig. 6)

$$K(J, t) = \int_1^\infty ds\, s^{-J-1} \int d\Phi_2 \left| T_{22} \right|^2 . \tag{6}$$

Recalling that $F(J, t)$ itself is given in terms of T_{22}, we see that we are faced with a hefty set of non-linear relations to satisfy. The idea is that whatever one puts into T_{22} to make K must also give F through the integral equation.

Any progress at all in this problem is remarkable. One proceeds by expanding each of the functions above into a piece singular at $J = 1$, $t = 0$ plus terms analytic there. Remarkably enough running this ansatz through the non-linear machinery yields a form for $F(J, t)$

$$F(J,t) = \frac{1}{(-R_0^2 t)} \left\{ \frac{J-\alpha(t)}{\sqrt{\left[J-\alpha(t)\right]^2 - R_0^2 t}} - 1 \right\} \left[A+Bt\right]$$

$$+ \frac{1}{\sqrt{\left[J-\alpha(t)\right]^2 - R_0^2 t}} (C+Dt)$$

$$+ \frac{1}{(-R_0^2 t)} \left\{ J-\alpha(t) - \sqrt{\left[J-\alpha(t)\right]^2 - R_0^2 t} \right\} + \cdots \quad . \tag{7}$$

where A, B, C, D and R_0 are constants and $\alpha(t) = 1 + \alpha' t$. Translated into s, t space this gives

$$\mathrm{Im}T_{22}(s,t) = \beta_1(t) \left(\frac{s}{s_0}\right)^{\alpha(t)} \log \frac{s}{s_0} \frac{J_1\left[(R_0 \log s + R_1)\sqrt{-t}\right]}{(R_0 \log s + R_1)\sqrt{-t}}$$

$$+ \beta_0(t) \left(\frac{s}{s_0}\right)^{\alpha(t)} J_0\left[(R_0 \log s + R_1)\sqrt{-t}\right] + 0\left(\frac{1}{\log s}\right) \tag{8}$$

where the remaining terms are lower order in powers of log s , the natural expansion parameter in these models. The appearance of log s as the expansion parameter rather than, say, $(\log s)^p$, p non-integer, comes from the assumption of a Taylor series around $J = 1$ made in satisfying the MP bootstrap. Were one to expand the elements of the integral equation as $(J-1)^{\gamma}\sum_{n=1}^{\infty} c_n (J-1)^n$, non-integer powers of $\log s$ as corrections to the leading term might well appear. $\{$Just a reminder here that $J-1$ and $\log s$ are natural conjugate variables since

$$F(J,t) = \int_1^{\infty} d(\log s)\ e^{-(J-1)\log s}\left(\frac{\mathrm{Im}T_{22}(s,t)}{s}\right) \quad , \tag{9}$$

rewriting Eq. (3).$\}$

The leading term in the MP bootstrap expression may be understood in this way: go over to impact parameter space

$$F(s,b) = \int_{-\infty}^{0} dt\ J_0(b\sqrt{-t})\ T_{22}(s,t) \quad , \tag{10}$$

then the J_1 term in $\mathrm{Im}\,T_{22}$ gives a step function

$$F(s,b)\ \alpha i\,\theta(R_0 \log s - b) \tag{11}$$

which satisfies elastic unitarity

$$\mathrm{Im}\ F(s,b) = \left|F(s,b)\right|^2 + 0(1/s) \tag{12}$$

in an elegant fashion. This same observation applies to absorptive models.[4]

Features of this MP bootstrap which persist in all versions appear to be (1) shrinkage of the diffraction peak at very small t as $(\log s)^2$ going over to logs shrinkage because $\alpha(t) = 1 + \alpha' t$; (2) A cross section which rises, here as log s, and which factorizes; (3) A triple $\underset{\sim}{P}$ coupling in inclusive processes which vanishes linearly in t (shades of former analyticity), and (4) no pole in F(J, t) but only branch points which move around.

All of these features are attractive aspects of a theory of diffraction. From a theorist's point of view the major drawback to a full embrace of the MP bootstrap is the *ad hoc* nature of the starting point: Why bootstrap elastic unitarity down a multiperipheral chain? Do Reggeon interactions via the triple $\underset{\sim}{P}$ vertex and multiparticle t-channel unitarity sustain the attractive features of the amplitude.

Nevertheless the satisfaction of the non-linear strictures of even this toned down MP bootstrap is delightful. It gives a serious rationale for bessel function forms of $T_{22}(s,t)$ beyond impact parameter phenomenology. Further there is clearly a non-trivial amount of s-channel unitarity built into the model. If one can hold off his enthusiasm about the phenomenological brilliance of the theory and find both a more general context for the formulation of the results and a way to incorporate t-channel restrictions, one will have a most appealing model.

B. ABSORPTION MODELS

Models which study the effects of final state rescattering (absorption) on some basic amplitude have also proven a fruitful method to impose approximate s-channel unitarity on these input amplitudes. The problem one begins with is much the same as in the MP bootstrap. In the MP amplitude no one stops $\alpha(0, g)$ from growing larger than one. Absorption is an attempt to bring the effective $\alpha(0)$ back to one; the price one pays is usually several powers of log s (well, two: $(\log s)^2$).

A neat formulation of absorption is given by Schwimmer in Ref. 4. One expresses the $T_{2\to N}$ entering in s-channel unitarity in impact parameter, $\underset{\sim}{b}$, and rapidity, Y, space (Fig. 7) taking into account two particle rescattering by $S_{22}(\Delta\underset{\sim}{b}, \Delta y)$ for particles separated by impact parameter $\Delta\underset{\sim}{b}$ and rapidity Δy. So one writes (Fig. 8)

$$|T_{2\to N+1}(Y, \underset{\sim}{b})|^2 = g^{N-1} S_{22}(Y, \underset{\sim}{b})$$

$$\int \prod_{j=1}^{N} dy_j \, d^2b_j \, \delta\left(Y - \sum_{j=1}^{N} y_j\right) \delta^2\left(\underset{\sim}{b} - \sum_{j=1}^{N} \underset{\sim}{b}_j\right)$$

$$|T_{22}(y_j, \underset{\sim}{b}_j)|^2 \prod_{1\le j\le k\le N+1} S_{22}(y_k - y_j, \underset{\sim}{b}_k - \underset{\sim}{b}_j) \quad . \tag{13}$$

The "basic amplitude" as I have called it is gotten by putting $S_{22} = 1$;

that is called the multiperipheral model. {By the way, except for eventual disagreement on the sign of the two $\underset{\sim}{P}$ cut, everyone begins with the MPM . Indeed, it is good physics to do so.}

Leaning on past experience[4] one makes the ansatz that

$$T_{22}(y, \underset{\sim}{b}) = i\,\theta\,(R_0^2 y^2 - b^2) \quad , \tag{14}$$

much the same as the result of the MP bootstrap. This expanding black disc can be quite naturally be made to reproduce itself. From the details of the model one finds that because of the complete absorption in a disc, the final state rescatterings cut down the produced multiplicity of particles to a finite number at all s . Thus $\langle n \rangle$, $f_2(s) = \langle n(n-1) \rangle - \langle n \rangle^2$, and other interesting moments of $\sigma_n(s)$ remain finite. An amusing result which does not apparently depend on the details of the T_{22} chosen is that the sum of elastic plus diffractive inelastic events must be $\frac{1}{2}$ the total cross section

$$\sigma_{\text{elastic}} + \sigma_{\text{Diffraction}} = \tfrac{1}{2}\,\sigma_{\text{Total}} \quad . \tag{15}$$

The evaluation of absorption models from a theoretical point of view is very difficult. It is all too easy to smilingly point to the relatively *ad hoc* nature of the input: only $2 \to 2$ amplitudes and only two body final state rescattering. A continuation of the smile is at present the only answer one can give to the questions naturally raised by the model: what about cluster effects? (that's actually not too hard) or multiple rescattering, etc. Again let me point out that a non-trivial account of s-channel unitarity has been provided in these models. Although it may prove impossible to defend that the absorptive effects presently included are the most important ones, surely they are very important. One lesson I feel we may profitably extract from these studies is the appearance once again of a step-function-like $\underset{\sim}{b}$ structure and the resulting shrinkage as $(\log s)^2$ at very tiny t.

C. REGGEON FIELD THEORIES

Now we make a 90° rotation and begin thinking in t-channel terms. The exchange of n Regge poles with trajectories $\alpha(t)$ give rise in the t-channel to a branch point at

$$\alpha^{(n)}_{B.P.}(t) - 1 = n\left[\alpha\left(\frac{t}{n^2}\right) - 1\right] , \tag{16}$$

and the discontinuity across this branch line is

$$\begin{aligned} \mathrm{disc}_J F(J,t) = \int \prod_{k=1}^{n} d^2 q_k \quad & \delta^2\left(\vec{q} - \sum_{k=1}^{n} \vec{q}_k\right) \\ \delta\left\{1 - J - \left[1-\alpha(\vec{q}_1)\right] - \cdots - \left[1-\alpha(\vec{q}_n)\right]\right\} & A(J,\vec{q},\vec{q}_1 \cdots \vec{q}_n) \\ & B(J,\vec{q},\vec{q}_1 \cdots \vec{q}_n) , \end{aligned} \tag{17}$$

where each Reggeon is parametrized by a two momentum $\vec{q}_k$ whose length is $t_k = -|\vec{q}_k|^2$, the invariant $(\text{mass})^2$ of the Reggeon. A and B are some functions describing the two particle $\to$ n Reggeon transition. This t-channel unitarity formula suggests that momentum $\vec{q}$ and "energy" = $1 - J$ are conserved in Reggeon theories. So the Reggeon is profitably viewed as a quasi-particle with momentum $\vec{q}$ and "on shell" energy $E = 1 - \alpha(\vec{q})$. When $\alpha(0) = 1$, we have an $E, \vec{q}$ relation which vanishes at $\vec{q} = 0$ which is reminiscent of a massless particle in usual quantum theory. The conjunction of all the branch points in (16) at $t = 0$ when $\alpha(0) = 1$ is then just the familiar coincidence of branch points in an infrared problem.

Now several years ago Gribov[6] called attention to the $E, \vec{q}$ space where Reggeons live and argued that a field theory over that space could be a very useful tool for studying the interaction, propagation, emission, and absorption of Reggeons. In particular one might utilize the field theory to learn how the conjunction of an infinite number of branch points finally yielded up a $\underset{\sim}{P}$ consistent with t-channel unitarity as given in (17).

The infrared nature of the problem indeed enables one to solve it. Using the renormalization group as a tool, groups in the U.S. and U.S.S.R.[5] have presented solutions. There is a relatively large freedom in these solutions having to do with what one chooses for the input or bare field theory and what one chooses for the interaction among the $\underset{\sim}{P}$'s. Many models are under active study, each employing the same technique to solve the infrared problem. Rather than leap into any details of these models I will summarize the general features that emerge:

1. Beginning with a linear trajectory and a triple coupling one finds the full $\underset{\sim}{P}$ to be a branch point at $J = 1$ times a moving pole

$$\alpha(t) = 1 + at^{\nu}, \tag{18}$$

where $\nu \ll 1$ so the trajectory has a cusp. The total cross section ($AB \rightarrow$ anything) rises very slowly (σ is small):

$$\sigma_T^{AB}(s) \sim (\log s)^{\sigma} \gamma_A \gamma_B - f_{AB} (\log s)^{\sigma - \frac{1}{2}} \tag{19}$$

and has a leading term which factorizes. (Fig. 10)

2. It is possible to find theories which reproduce the input when run through the <u>t-channel unitarity</u> mill. One attractive theory of this sort gives

$$\mathrm{Im} T_{22}(s,t) = \beta(t) s \, J_0(R_0 \sqrt{-t} \log s) + \cdots \quad , \tag{20}$$

which is reminiscent of results from <u>s-channel unitarity</u>. Such a renormalization group bootstrap may hold out the way to locate approximate $F(J,t)$ which satisfy both unitarities. This would be an explicit realization of the observation of de Tar and others[1] that summing t-channel multiple Reggeon exchanges will bring s-channel unitarity (approximately anyway) into line.

3. In most examples the full triple $\underset{\sim}{P}$ vertex vanishes, but not analytically in t. This is in marked contrast to "planar" theories, such as the 6 point dual model or planar ladders, in which full $\underset{\sim}{P}$ interaction is absent.

4. Factorization of σ_{Total} and other inclusive cross sections seems inevitable although the $\underset{\sim}{P}$ singularity is usually not a pole but some sort of branch point. Since the next leading, non-factorizing term is down by at most one power of $\log s$, the observable onset of this factorization must be expected to be slow.

At this point I will express a strong personal bias: I feel that Reggeon field theories are precisely the framework in which to determine theoretically the structure of the $\underset{\sim}{P}$ singularity in the J plane. The full content of these theories has just begun to be explored.

D. DECOUPLING "THEOREMS"

Starting from the elementary attractive premise that the $\underset{\sim}{P}$ is a simple J plane pole with trajectory $\alpha(t) = 1 + \alpha' t + \cdots$, it was shown two years ago[2] that such a $\underset{\sim}{P}$ could not couple to total cross sections. The assumptions in these theorems (all theorems have assumptions) blatantly and openly neglected the effects of unitarity (s or t) on the $\underset{\sim}{P}$ structure. Their real importance, then, was to show that unitarity is ignored at one's own risk.

Now none of the theories in sections A, B, or C satisfy the assumptions of the theorems. Each in its own way puts in enough from the unitarity relation to preserve the $\underset{\sim}{P}$ coupling in σ_{Total}. What more can one say: <u>the $\underset{\sim}{P}$ rides again!</u>

One remark of some phenomenological import: the beginning step down the primrose path to the decoupling of the $\underset{\sim}{P}$ was the demonstration that the triple $\underset{\sim}{P}$ vertex measured in inclusive processes must vanish when all legs have zero t.[7] The argument was almost kinematic. In models of type A or C, above, at least, this triple coupling still vanishes at the $t = 0$ point, but the precise manner in which it occurs is now a dynamical issue. If the triple $\underset{\sim}{P}$ vertex proves to vanish, then the <u>way</u> it does is one, hopefully useful, experimental method of picking out one theory over others.

E. CLOSING REMARKS AND SOME ASSUMPTIONS

Although I have exhibited my prejudice explicitly above, I would like to emphasize that theories beginning with s-channel unitarity are very important and ought to be vigorously pursued. There is no doubt that the $\underset{\sim}{P}$ knows about both s and t unitarity; we would be remiss in forgetting either.

The satisfying of unitarity in either the s or t channel is something that everyone would agree is a "good thing". The Reggeon field theories provide a natural framework within which to meet the requirements of t-channel unitarity. They do this by building t-channel partial wave amplitudes which obey the discontinuity relation in (17) as a kinematic feature. In terms of (17) only the functions A and B are computed in the field theories, the rest of the equation comes free. Those functions are evaluated in perturbation theory in some triple $\underset{\sim}{P}$ or higher number of $\underset{\sim}{P}$ couplings, but so it is with field theories. Only in the infrared limit can one transcend perturbation theory; indeed, one must.

The requirements of s-channel unitarity are as compact to state as Eq. (17); in fact, Eq. (1) is the compact statement. If one satisfies the phase space structure of (1) by dealing with intermediate states of particles in three space and one time dimension (what else), then only $T_{2\to N}$ need be evaluated, more or less exactly and more or less convincingly, to satisfy s-channel unitarity. Clearly computing $T_{2\to N}$ is the analogue of computing A and B in the previous paragraph.

Suppose one takes the view that he ought to proceed by satisfying some unitarity as well as possible. Then t-channel unitarity stands out as the easier of the two. Just use a Reggeon field theory to evaluate $F(J,t)$ and you have done it! Now comes the hard question: What restrictions are there on $F(J,t)$ - the t-channel partial wave amplitude - from s-channel unitarity? Well, we know that at $t = 0$, $F(J, 0)$ must be no more singular than $(J-1)^{-3}$ which exactly saturates the Froissart bound. Is that all? What about satisfying details of multibody unitarity? Frankly, I don't know the answer to these questions. I suspect there must be more. Perhaps at the next meeting of this conference we will know.

Along the way in this talk I have made a number of assumptions. Let me expose several:

(1) $\alpha(0) = 1$. Anyone not accepting this is forced to the position that $1-\alpha(0) = \epsilon$ where ϵ is a very small, time dependent number. It gets smaller as time increases. If this is really the case, the important issue in the physics of hadron collisions is the smallness of ϵ: what sets the scale?

(2) I have presumed that multiple $\underset{\sim}{P}$ exchange as would occur in $pp \rightarrow pp\pi^+\pi^-$ (see Fig. 11) does occur. Clean unambiguous experimental evidence on this account, hard as it is to come by, is most significant. If it does not occur, throw away theories of type A, B, and C above and return to your starting blocks.

(3) Finally, I have presumed that the naïve observation suggested by unitarity that if $\underset{\sim}{P}$'s exist, they interact. In particular a triple $\underset{\sim}{P}$ coupling (vanishing or not) must appear in inclusive processes. The evidence on this is certainly encouraging;[8] how nice it would be to have it less ambiguous.

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[8]See the review by G. Fox, Cal Tech Preprint 68-413, to be published in the proceedings of the 1973 Stony Brook Conference.

FIGURE CAPTIONS

Fig. 1 Exchange of a Cheshire Cat known as the Pomeron or $\underset{\sim}{P}$.

Fig. 2 Multiparticle s-channel unitarity.

Fig. 3 The multiperipheral approximation to the $2 \rightarrow N$ amplitude.

Fig. 4 J-plane structure resulting in the multiperipheral model (MPM).

Fig. 5 The multiperipheral integral equation.

Fig. 6 Elastic s-channel unitarity employed to determine the "potential" in a MPM.

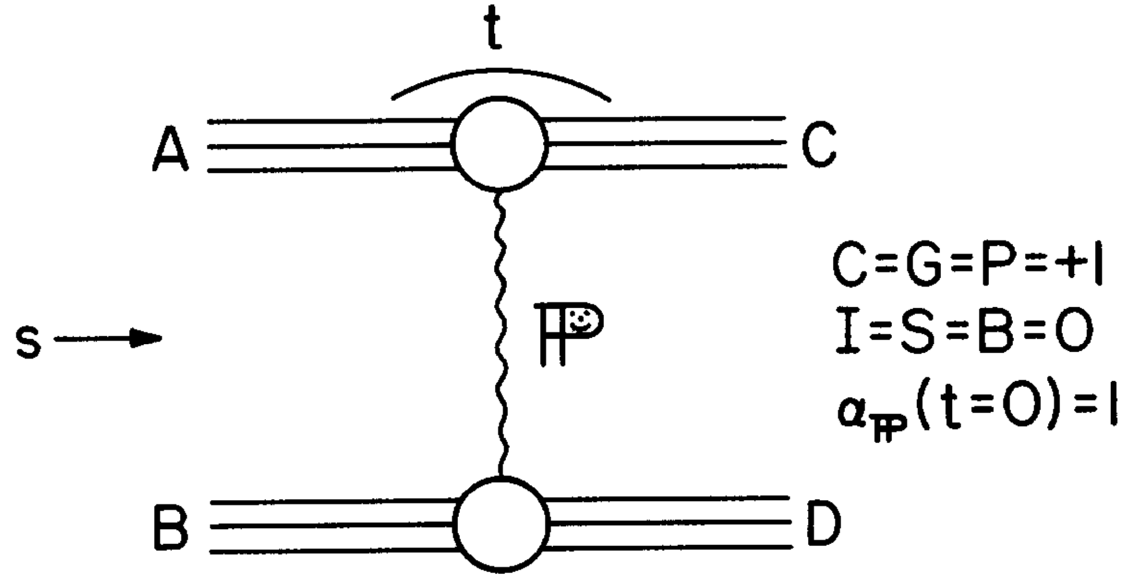
Fig. 7 s-channel multiparticle unitarity in rapidity, impact parameter space.

Fig. 8 Absorption model approximation to $T_{2 \rightarrow N}$ entering unitarity. T_{22} enters as the basic amplitude and the wiggly line is the $2 \rightarrow 2$ S matrix.

Fig. 9 The discontinuity in the t-channel across the n Reggeon branch cut. Reggeon field theories automatically satisfies this for each n.

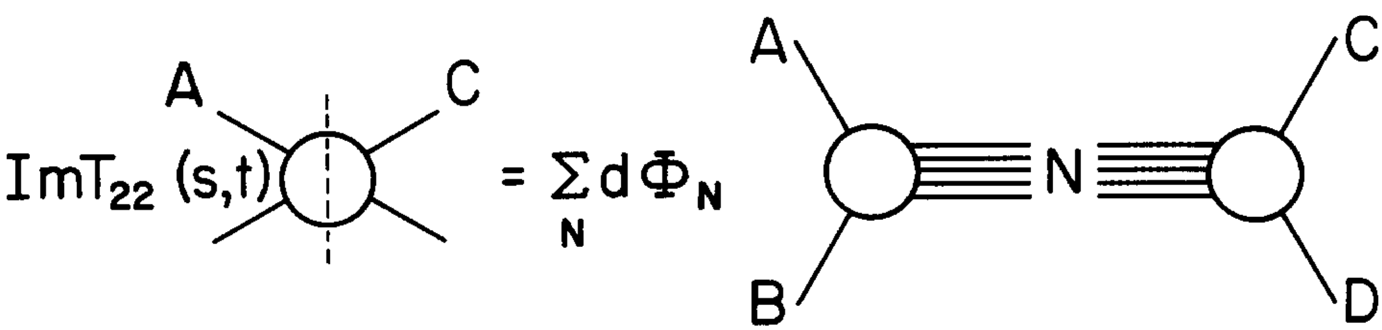
Fig. 10 The heirarchy of contributions to σ_{Total} which emerges in Reggeon field theories.

Fig. 11 Double $\underset{\sim}{P}$ exchange in $pp \rightarrow pp\pi^+\pi^-$. Its existence would be nice to establish experimentally.



t
A
C
B
D
s
C=G=P=+1
I=S=B=0
$\alpha_{\mathbb{P}}(t=0)=1$

FIG. 1



A
C
B
D
N
$\mathrm{Im}T_{22}(s,t) = \sum_N d\Phi_N$

FIG. 2

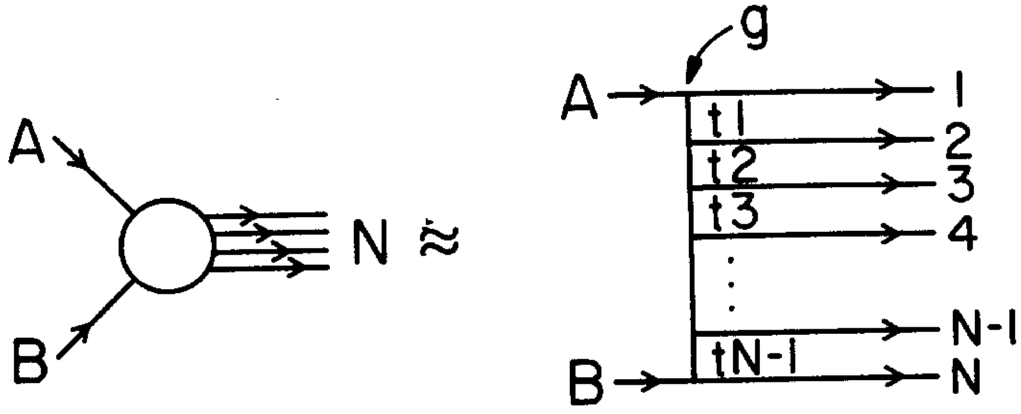
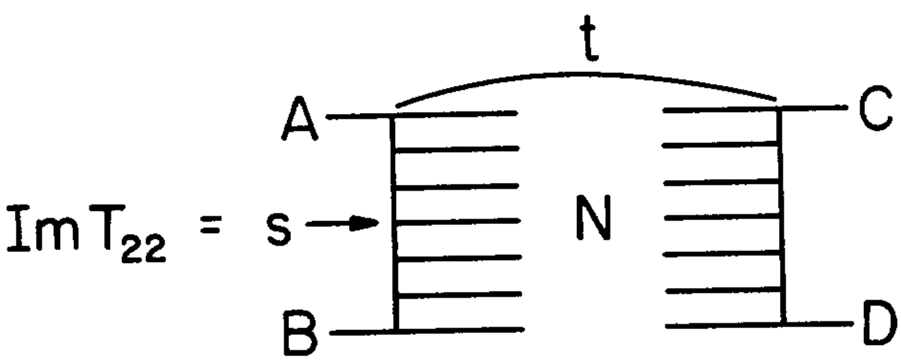


FIG.3



$$\Rightarrow \quad F(J,t) = \frac{\gamma_{AC}(t)\,\gamma_{BD}(t)}{J-\alpha(t,g)}$$

FIG.4

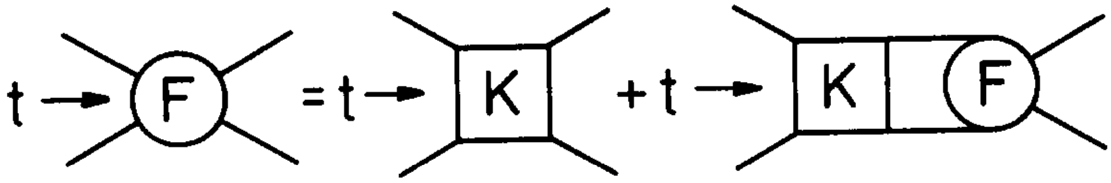


FIG.5

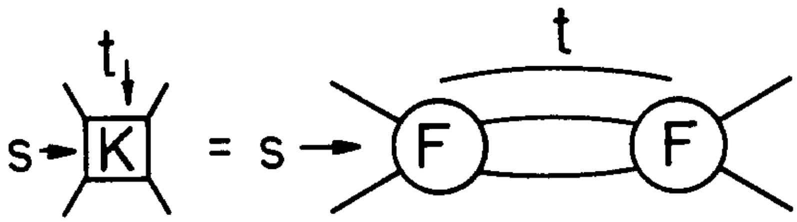


FIG. 6

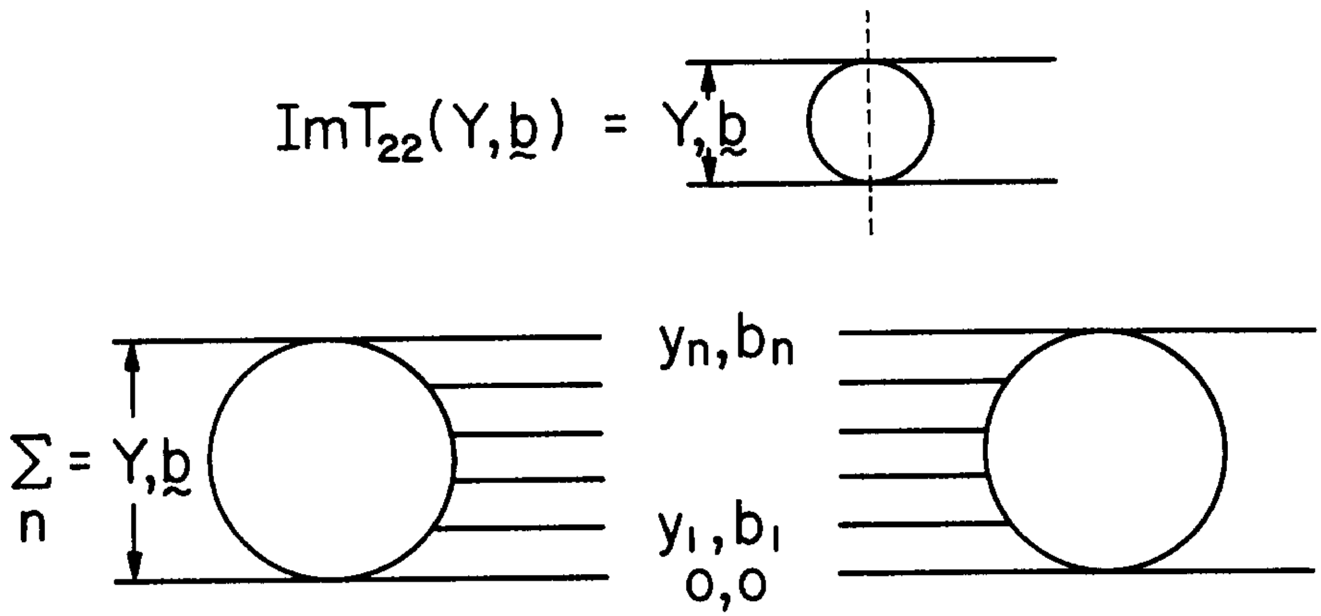


FIG. 7

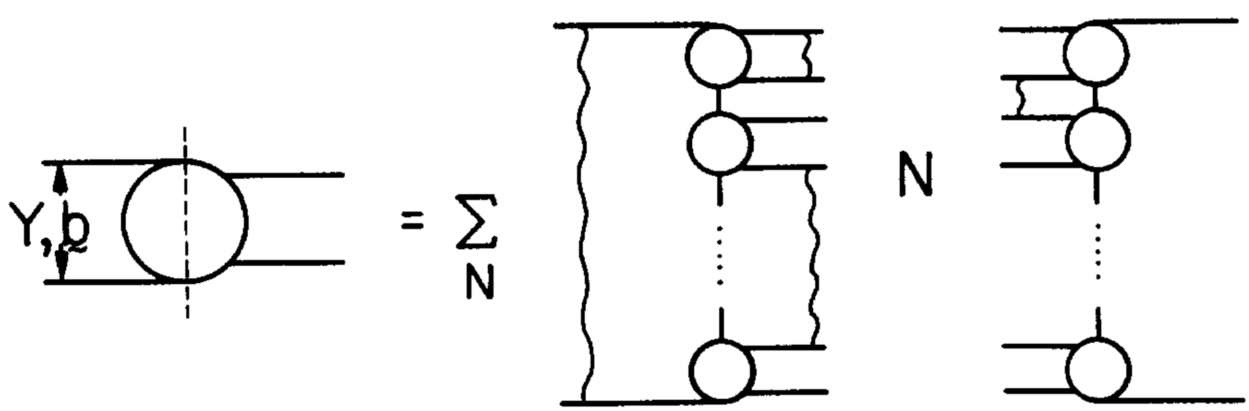
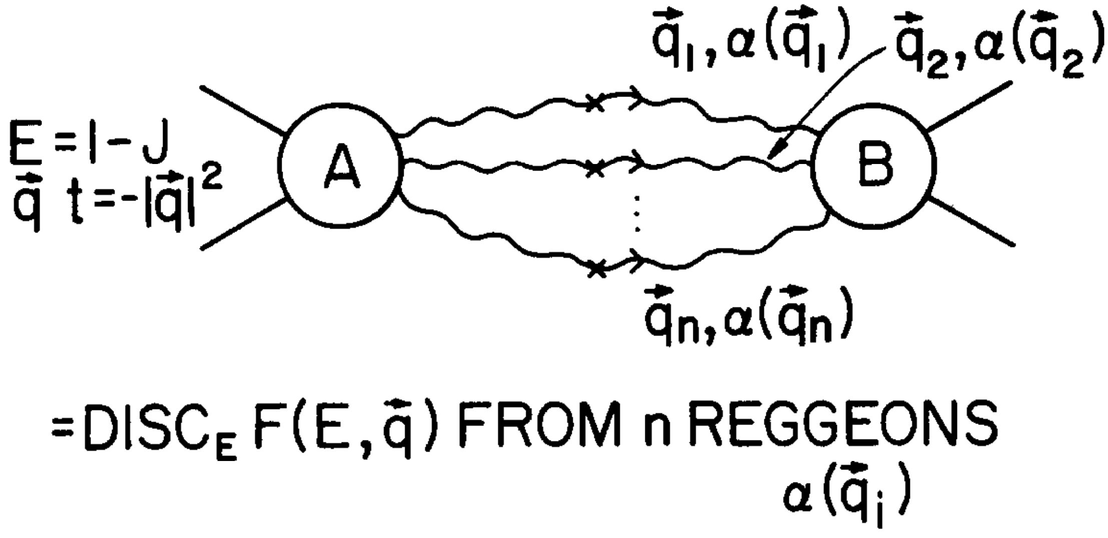
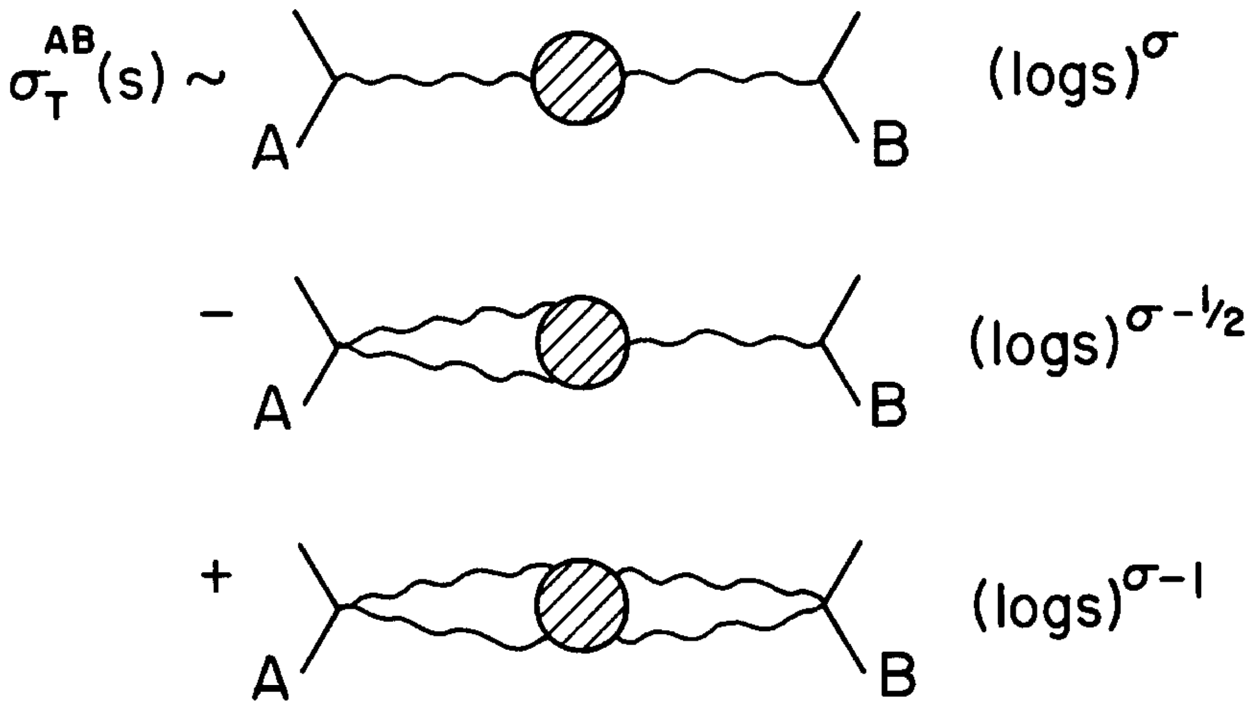


FIG. 8



$\vec{q}_1, \alpha(\vec{q}_1)$
$\vec{q}_2, \alpha(\vec{q}_2)$
$E = 1 - J$
$\vec{q}\ \ t = -|\vec{q}|^2$
A
B
$\vec{q}_n, \alpha(\vec{q}_n)$
$= \mathrm{DISC}_E\, F(E, \vec{q})$ FROM n REGGEONS
$\alpha(\vec{q}_i)$

FIG.9



$\sigma_T^{AB}(s) \sim$
A
B
$(\log s)^{\sigma}$
$-$
A
B
$(\log s)^{\sigma - 1/2}$
$+$
A
B
$(\log s)^{\sigma - 1}$

FIG.10

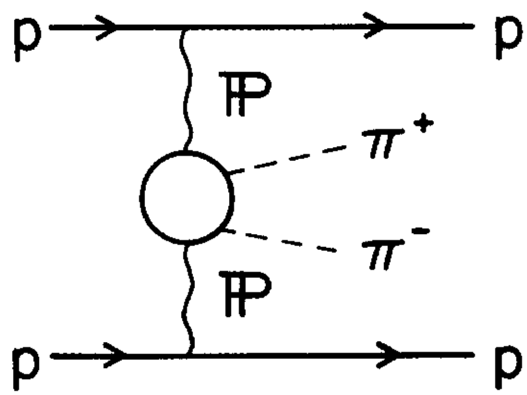


FIG. 11