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A General Formulation of Pionic Decays in the Quark Model*

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ABSTRACT

Predictions for pionic decays of resonances to the low-lying baryon 56 or meson 35 are shown to follow from general model-independent algebraic properties. All models in which transition operators change the state of only a single quark allow only two partial waves for the decay pion and give $SU(6)_W$ relations between decays in each partial wave, while not necessarily relating different partial waves.

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There have recently been a number of treatments of pionic decays of hadron resonances using modified quark models.[1] The failure of the simplest version[2] to predict experimental decays of hadrons which contain orbital excitation[3] has led to a search for symmetry-breaking mechanisms. However, different formulations have led to very similar results, suggesting that there is some underlying principle common to all approaches. The purpose of this paper is to demonstrate this underlying principle explicitly and to show that all models having the same basic algebraic properties must give the same results.

We consider the decay of a hadron classified in a representation[4] of $SU(6) \times O(3)$ with orbital angular momentum L to a hadron state in the low-lying 35 or 56 supermultiplets with $L = 0$ by emission of a pion with angular momentum J. We consider constituent quarks[5] only in our analysis and assume that hadrons are classified in pure representations of $SU(6) \times O(3)$, as is common in all treatments based on constituent quarks. Our results thus apply to all treatments in which the final calculation is done in the constituent quark basis.

The interesting cases where difficulties arise are those where more than one partial wave is allowed for the decay pion; i.e., more than one value of J can occur. Three kinds of experimental predictions have been obtained:

1. Relations between decays having the same value of J from different members of the same $SU(6) \times O(3)$ supermultiplet.

2. Relations between decays having different values of J, in particular relations between different partial waves in the same decay. This includes relative phases of different partial wave amplitudes, which are measured directly by helicity amplitudes.

3. Selection rules forbidding certain partial waves.

It is the second type of prediction, relations between partial waves, which has led to disagreement with experiment. The first cases where two partial waves are allowed in the same decay arose in the decays of the axial vector mesons,[3] A1 and B. Here the simplest version of the quark model gave strong disagreement with experiment. There have been many prescriptions to give additional freedom to the model to avoid this disagreement. However, all of them still seem to give identical predictions of the first type, which relate decays having the same value of J. Predictions of the third type have not been considered seriously, because the selection rules are trivially satisfied in the decays of low-lying states. Nontrivial tests of these selection rules for higher resonances are discussed below.

We shall now see how different theoretical models all give the same predictions of the first type and the same selection rules. The basic assumption common to all approaches is the Levin-Frankfurt[6] assumption that the transition matrix element is given by additive contributions from individual quarks. Such an additive single-quark operator transforms[7] under SU(6) like a member of a 35. The particular member of the 35 is determined uniquely by the charge of the emitted pion and the following spin considerations. A single quark operator can flip only one quark spin and must transform under rotations in quark spin space only like a scalar or a vector, but not like any higher tensor. Parity conservation excludes the scalar, as shown below. This quark spin restriction can be called the $\Delta S = 1$ rule. The contributions of different components of the spin vector are determined by the coupling of the spin part of the transition operator to the orbital part to obtain the total angular momentum J. In this discussion we are using ordinary S-spin, not W-spin. The consequences of this SU(6) structure and $\Delta S = 1$ rule, when combined with the constraints from angular momentum and parity conservation and the SU(6) wave functions used in all treatments, are conveniently expressed by the following theorem:

Theorem: If the pionic decays of resonances classified in a given $SU(6) \times O(3)$ supermultiplet with orbital angular momentum L to a 35 or 56 supermultiplet with $L = 0$ are described by an operator which transforms like a vector in quark spin space and a 35 in SU(6).

1. Only two partial waves are allowed for the decay pion, $J = L + 1$ and $J = L - 1$.

2. There are only two independent transition matrix elements for all the decays of the entire supermultiplet, one for each partial wave.

3. All decays for a given value of J are related by $SU(6) \times O(3)$ Clebsch-Gordan coefficients.

The proof of the theorem is straightforward. We write the transition operator as a product of a factor which acts on the SU(6) degrees of freedom and a factor which acts on the orbital degrees of freedom,

$$M_0^J = \sum_m (1mL' - m|J0)\ U_m T_{-m}^{L'} \tag{1}$$

where M_0^J describes the emission of a pion with angular momentum J and projection zero on the z-axis, U_m is an operator which acts in the SU(6) degrees of freedom and transforms under rotations like a vector with projection m on the z-axis, and $T_m^{L'}$ is an operator which acts in the orbital space and transforms under rotations like an irreducible tensor of degree L' and projection m. The operators U and T are coupled with Clebsch-Gordan coefficients to a total angular momentum of J to insure conservation of angular momentum in the emission of a pion of angular momentum J.

Since the initial state has orbital angular momentum L and the final hadron has zero orbital angular momentum only the single value $L' = L$ can contribute to the transition. Conservation of total angular momentum then gives the triangular inequality,

$$|J - L| \leq 1. \tag{2}$$

All known SU(6) $\times$ O(3) multiplets have natural orbital parity.[8] Thus the parity change in the transition is even or odd when L is even or odd, respectively. Since the outgoing pion wave always has unnatural parity, conservation of parity excludes J = L. Thus there are only two allowed values of J for the transition operator (1); namely, J = L + 1 and J = L - 1.

The value of the transition matrix element of an operator of the form (1) for a given value of J between two states which are members of SU(6) × O(3) supermultiplets is given by the Wigner-Eckart theorem in terms of a reduced matrix element and Clebsch-Gordan coefficients for SU(6) × O(3). There is only one reduced matrix element for each value of J. Since only two values of J are allowed, only two independent reduced matrix elements arise in the consideration of all possible decay modes for all states in a given SU(6) × O(3) supermultiplet to all states in a given SU(6) × O(3) supermultiplet with $L = 0$. This completes the proof of the theorem.

In the simplest formulations,[2] only one independent transition matrix element appears. Such formulations either assume W spin conservation, separate conservation of L_z and S_z, or neglect "recoil" terms[9] in the quark description. In all these cases this additional assumption relates the matrix elements for the two allowed partial waves. The relation is that imposed by conservation of L_z as an additional assumption.

In the W-spin formulation, there is only a single reduced matrix element, because the transition operator is required by W-spin conservation to transform under W-spin like the emitted pion. The pion has $W = 1$, $W_z = 0$; thus the transition operator has $S_z = 0$ and therefore $L_z = 0$ since it must have $J_z = 0$. This chooses a particular mixture of the two allowed partial waves in cases where both are allowed by angular momentum conservation.

When these treatments led to disagreement with experiment, showing that L_z conservation was violated, attempts were made to relax the assumptions of the simple model to allow for violation of L_z conservation. However, all these new treatments, as well as the old formulation which included recoil terms[9] satisfy the conditions of the above theorem. They therefore lead to the same results; namely that two partial waves are allowed, and that their matrix elements are independent of one another. However, the matrix elements for each partial wave are still related in the same manner as in the simple treatment.

The treatment by Faiman and Plane[1] which assumed SU(6)W relations for each of the two partial waves independently and dropped the relation between partial waves imposed by SU(6)W is seen to be the most general treatment which satisfies the condition of the theorem. This is because the theorem defines a unique set of relations between transition matrix elements for each allowed partial wave. Any formulation which satisfies the conditions of the theorem must predict the same set of relations. Note that our treatment considers only the transition matrix element and does not include phase space and barrier factors which reflect symmetry breaking due to mass differences. Differences between recipes for treating these kinematic factors can give rise to differences between results in different treatments. The transition matrix elements, however, must satisfy the conditions imposed by the theorem.

There have been attempts to find smaller groups than SU(6) under which the transition amplitude might be invariant. However, as long as the transition operator is a single quark operator and has no components which transform under spin rotations like higher order tensors than vectors, the above theorem still applies. The restriction of the domain of validity to a smaller subgroup of SU(6) $\times$ O(3) implies breaking of SU(6) $\times$ O(3) in the hadron wave functions, so that only the classification in the smaller subgroup is valid. The only effects of the smaller group would be (1) restriction of the use of the Wigner-Eckart theorem to the smaller supermultiplets of the subgroup, or (2) introduction of a relation between the two allowed partial wave amplitudes which might be different from the one given by SU(6)W. However, the transition matrix elements for each partial wave remain related within the supermultiplets of the smaller group in exactly the same way as in all other treatments satisfying the conditions of the above theorem; e.g., the SU(6)W treatment.

We can now distinguish between experimental predictions which test the details of a given model and those which follow only from the SU(6) structure and $\Delta S = 1$ rule and are obtained from all models. The only predictions which do not follow from the SU(6) structure and $\Delta S = 1$ rule are those relating the relative magnitude and relative phase of the two partial waves. Thus any new model has only this single complex number to fit; i.e., two real parameters for each SU(6) $\times$ O(3) supermultiplet. There is therefore no point in presenting extensive tables of predictions and data with each new model; the main part of the information in the table merely tests the SU(6) structure and $\Delta S = 1$ rule. The best test is to combine the data to get the best value for the two parameters and compare these with the predictions of the model. A more detailed model may also predict relations between decays from different $SU(6) \times O(3)$ supermultiplets. These predictions also go beyond the SU(6) structure and $\Delta S = 1$ rule and test the model.

There is also interest in testing the SU(6) structure and $\Delta S = 1$ rule itself, particularly since serious disagreement here can throw out all models. In addition to the SU(6)W relations relating all decays of a given partial wave, there are also predictions of relative phases of the two partial waves in the same decay. The relative phase between the two partial waves is a single parameter left free by the SU(6) structure and $\Delta S = 1$ rule. Once it is determined in a single decay, it must be the same for all decays. Thus if the relative phases of the two partial waves are known experimentally for several decays, checking whether they are all fit by the same parameter tests the SU(6) structure and $\Delta S = 1$ rule.

The selection rule restricting decays to two partial waves can also test the $\Delta S = 1$ rule. This selection rule is trivial for meson decays, where more than two partial waves are not possible in any case. For baryon decays the selection rule is nontrivial for decays to $J = \frac{3}{2}$ final states from initial states with $J = L + \frac{3}{2}$ and $J = L - \frac{3}{2}$. The decays of these states by emitting a pion with $J = L + 3$ and $L - 3$, respectively, are forbidden by this selection rule. For $L = 1$ decays, $J = 0$ and 2 (s-waves and d-waves) are allowed for the decay pion by this selection rule. The only case where another J is allowed by angular momentum conservation is in the decay of a $\frac{5}{2}^-$ baryon to a $\frac{3}{2}^+$ state, where J = 2 and J = 4 are both allowed. The selection rule forbids J = 4. This case is not a very sensitive test of the model, since centrifugal barriers also suppress J = 4. The L = 3 case would provide the first interesting test of the selection rule. Here the s-wave decay $\frac{3}{2}^- \to \frac{3}{2}^+$ is forbidden, while the d-wave decay is allowed, and the centrifugal barrier suppresses the allowed decay mode.

A similar approach can be used for photon decays and excitations of resonances in the quark model. However, there are more partial waves for an emitted photon than for an emitted pion because of the spin of the photon. For a resonance having a given value of L, the angular momentum of the photon can be J = L - 1, L, or L + 1 in a single quark transition. The value J = L is no longer excluded as in the pion case by parity conservation. For J = L there are single quark transitions with $\Delta S = 0$ allowed as well as $\Delta S = 1$. Thus there are four independent reduced matrix elements for decay by photon emission, three $\Delta S = 1$ with J = L -1, L and L + 1 and one $\Delta S = 0$ with J = L. These may be reduced to a smaller number by additional assumptions from specific models. However, the SU(6) structure allows all four.

The same approach can be extended to treat the case of decays to states having $L \neq 0$. The angular momentum couplings are more complicated and there are more allowed partial waves and reduced matrix elements. The general case for pionic decays to states of arbitrary L has been considered by Gilman, Kugler and Meshkov[5] in the context of their specific model. Their results should hold for all models having the same SU(6) structure and satisfying the $\Delta S = 1$ rule.

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References and Footnotes

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