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Asymptotic Behaviour of Form Factors Density of States and Total e^+e^- Annihilation Cross Section into Hadrons

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ABSTRACT

The quick decrease at large momentum transfers of form factor and the quick increase of hadronic level density at large energies are presumably both manifestation of hadron compositeness. We conjecture that a simple inequality relation holds between these two phenomenon, motivate over conjecture by considering e^+e^- annihilation potential and many particle models, and discuss some of the conclusions which can be drawn if the conjecture is indeed true.

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I. INTRODUCTION

It is generally accepted that the fall off at large momentum transfers of the elastic form factors $F_H(Q^2)$ reflects the compositeness of the hadron H. In particular a finite (i.e., non-zero limit) of $F_H(Q^2)$ as $Q^2 \to \infty$ is indicative of a finite wave function renormalization and the existence of an "elementary" component in H. More recently the relation

$$F_H(Q^2) \underset{Q^2 \to \infty}{\sim} \text{const}/(Q^2)^{n_H - 1} \tag{1}$$

was conjectured[1] for a hadron H which is made of n_H point-like constituents.

Another manifestation of hadron compositeness is the rapid increase of density of hadronic states (resonances) $\frac{dN}{d|Q^2|} \equiv \rho(|Q^2|)$ at the mass squared interval around $|Q^2|$. Thus, if the system H consists of n_H "elementary" components, the same total excitation m^2_H can be achieved by indepent excitations of each of these components and we would expect (after subtracting out the common cms motion) roughly

$$\frac{dN}{(dQ^2)} = [f(Q^2)]^{n_H - 1} \tag{2}$$

We would like to make in the following the conjecture that the general relation

$$|F_H(-Q^2)|^2 \approx |F_H(Q^2)|^2 \lesssim \frac{1}{\rho(Q^2)} \qquad (3)$$

where $\approx$ will always indicate that the relation is only true asymptotically when $Q^2 \rightarrow \infty$. (Better definitions of asymptotic Q^2 will be given below.)

The main motivation for our conjecture (3) (besides the suggestive similarity of Eqs. (1) and (2)) is the fact that it holds in the statistical bootstrap model[2,3].

We have been unable to prove the conjecture (3) in general. In the following section we will make plausible, by considering e^+e^- annihilation into hadrons, certain weaker versions of Eq. (3). There will involve in particular averaging of $|F_H(Q^2)|^2$ over a large range of masses m_H^2.

Only to the extent that $|F_H^2(Q^2)|^2$ does not tend to have systematically a stronger fall off in Q^2 for bigger m_H^2 values, will we be able to extract useful information on $|F(Q^2)|^2$ for the low lying states. In Sec. III we discuss this question in the framework of potential models and suggest that this possibility is quite unlikely and that form factors of all excited states $F_H(Q^2)$ have similar asymptotic bheaviours for $Q^2 >> m_H^2$. We next briefly discuss the conjectured relation for multiparticle systems. In Sec. IV we discuss the restriction which the relation (3) impososes on various models. Finally in Sec. V we speculate on the possibility that a violation of (3) in a finite Q^2 region may be the cause for the rise of $R(Q^2)$, the total e^+e^-, annihilation cross section into hadron relative to the point-like $e^+e^- \rightarrow \mu^+\mu^-$ cross section, observed recently at C.E.A.[4]

II. FORM FACTORS AND THE TOTAL COLLIDING e^+e^- CROSS SECTION

We assume the existence of a local electromagnetic current with well defined physical dimension (=3) under the scaling operation in the short distance limit.[5] The cross section for e^+e^- annihilation into hadrons -- via one photon -- can than be shown to satisfy[5,6]

$$\sigma_{tot}\ e^+e^- \rightarrow "\gamma" \rightarrow \text{hadrons} \sim \frac{C_m\ \alpha}{Q^2} \tag{4}$$

where C_m is a model dependent constant ($= \sum_i e_i^2$ when the sum extends over all the fundamental fields coupled to J^μem) and α is the fine structure constant.

We note that there exists a more general "unitarity bound" $\sigma_{tot}\ e^+e^- \rightarrow \gamma \rightarrow \text{hadrons} \leq \frac{4\pi}{Q^2}$ which follows when we sum the series for the photon propagator with both lepton and hadron bubble insuctions. Since, however, we want to work consistently in lowest order in α the stronger bound (4) is required.

Let us focus our attention now on final hadronic states arising from the formation of a pair of conjugate resonances H^+H^- followed by the subsequent decay of H^+ and H^- into stable particles. We first consider the idealized situation in which the interaction responsible for this decay has been switched off[4] so that the resonances H have widths Γ_H smaller than the average spacings d between resonances. In this case there is no interference between amplitudes laeding to the same final state through different resonances and the interference with other possible non-resonant amplitudes is also suppressed. Remembering that the final states considered here form only a subset of all possible final states[8] we have[9]

$$\sum_H |F_H(Q^2)|^2 \cdot \frac{q_{cm}(H)}{\sqrt{Q^2}} \leq C_m \tag{5}$$

It is convenient to restrict ourselves to H^+H^- states with

$$m_H^2 \lesssim C(Q^2)^{1-\epsilon}, \quad \epsilon > 0 \tag{6}$$

In which case H^+ and H^- are for large Q^2, super relativistic in the overall cms frame and the two body phase space factor in Eq. (5) can be ignored. Equation (5) yields then the relation

$$\overline{|F_H(Q^2)|^2} \lesssim \frac{1}{N[(Q^2)^{1-\epsilon}]} \tag{7}$$

where $N[(Q^2)^{1-\epsilon}] = \int^{(Q^2)^{1-\epsilon}} \rho(m^2)\, dm^2$ is the total number of states H with $m_H^2 \leq (Q^2)^{1-\epsilon}$, and the bar indicates averaging over all states in this range. This result is quite close to the conjectured relation.[3]

In view of the quick increases of $\rho(m^2)$ and possibly also the width Γ_H of the high lying resonances the condition $\Gamma << d$ is likely to be strongly violated in all cases of physical interest, and many interfering H^+H^- amplitudes contribute to the same final state.

We note that if the resonances have finite widths $\Gamma_H \lesssim$ const and condition 6 holds the states H^+H^- are strongly boosted in the over-all cms, and decay only when a large distance $\sim Q^\epsilon$ apart. The decay processes of H^+ and H^- are, therefore, dynamically independent and should be described by separate factorizing unitary matrices.[10]

If we assume only $\Gamma_H \lesssim$ const. m_H {for $\Gamma_H >> m_H$, it would make little sense to talk about the "particle H," and in the following ρ will specifically be restricted to states with $\Gamma_H \leq cm_H$} then the above boost and time dilation argument would apply providing however that we restrict ourselves for given Q^2 to states H satisfying

$$m_H{}^2 \lesssim C(Q^2)^{\frac{1}{2}-\epsilon} \tag{6}$$

Notice this last stronger constraint also insures a complete kinematical separation of the jets of stable final state particles which emerge from the decay of H^+ H^-. Indeed even in the extreme case when H^+, say, decays into two pions which fly exactly along the same direction as the H^+ H^-, in the overall cms, the backward moving pion will not appear in the opposite hemisphere as long as (6') holds and $m_\pi > 0$.

While the above arguments suggest that when condition 6 (or 6') holds, we do not have to worry about interference between different pairs of backward-forward moving resonances leading to the same final state it is still true that for any given specific final forward jet state several forward (H^+-s say) resonances could contribute.

To the extent that the basic picture which we have assumed (see Fig. 1) about breaking the process into two stages well separated in space time of a fast production process followed by a slower independent decays of H^+ and H^-, we do not have to worry about such interferences. The reason is that we are not interested in one particular final state, but rather in computing the total e^+e^- annihilation cross sections summing over all states. Since the evolution of H^+ or H^- is given by a unitary matrix the possible interference terms vanish because of the orthogonality of the different states H_m, H_n

$$\sum_f \langle H_m | U | f\rangle \langle f | U^+ | H_n\rangle = \langle H_m | UU^+ | H_n\rangle$$
$$= \langle H_m | H_n\rangle \sim \delta_{mn}$$

Rapidly fluctuating couplings of consecutive resonances to a given final state are expected from simple potential (R matrix) considerations. This fact has been used by Margolis and coworkers[11] to argue away interference terms even for transitions to a given final state. In the above we have not used such an argument (obtaining admittedly weaker results).

Our considerations above strongly suggest the relation

$$|F_H(Q^2)| \text{ average for } m_H^2 < Q^{2\frac{(1-\epsilon)}{2}} \lesssim \frac{1}{N[(Q^2)^{\frac{(1-\epsilon)}{2}}]} \qquad (7')$$

or the stronger result (7) when resonances of finite width are concerned.

III. CONSIDERATIONS OF SIMPLE POTENTIAL MODELS

The main shortcoming of either Eq. (7) or (7') is that it involves averaging $|F_H(Q^2)|^2$ over a large interval of resonance masses which grows with Q^2. Indeed both relations seem to be a statement on the behaviour of $F_H(m^2, Q^2)$ as a function of m^2, the mass of H, rather than as a function of Q^2, with Q^2 having the role of allowing higher m^2 values to be probed. We rewrite Eq. (5) in a continuous form

$$\int^{(Q^2)^a} dm^2 \ \rho(m^2) \ |\hat{F}(m^2, Q^2)|^2 \lesssim c_m \tag{5'}$$

where $\hat{F}$ is the average of $|F_H(Q^2)|^2$ for hadrons of mass m^2.

The simplest way to satisfy the bound is to assume $|F(m^2, Q^2)|^2 \sim \frac{1}{\rho(m^2)} g(Q^2)$. Our choice of charge conjugate pairs (H^+ and H^- instead of H^+ and G^-) excludes this simple possibility since for all m_H^2 we have $F_H(Q^2)|_{Q^2=0} = 1$, and thus 7 or 7' have to imply a fast fall off with Q^2 for the massive states.

To the extent that this fall off may be systematically faster for hadrons with larger m^2 the averaged relations (7) or (7') would not constrain at all $|F_o(Q^2)|^2$ the form factor for the ground state or any fixed low lying state. By appealing to some specific examples as well as to certain general considerations in potential theory we would like to argue that the last possibility is quite unlikely, and all states satisfying $m_H{}^2 \rightarrow E_H < (Q^2)^a$, with $a \leq 1$ being model (potential) dependent, have essentially the same asymptotic behaviour with Q^2.

This contention is based on the fact that for $Q^2 \rightarrow \infty$ the behaviour of

$$F_H(Q^2) \equiv \int d\vec{r}\, e^{i\vec{Q}\cdot\vec{r}} \, |\psi_H(\vec{r})|^2 \tag{9}$$

depends only on $|\psi_H(\vec{r})|^2$ near $r \rightarrow 0$ which in turn reflects behaviour of the potential $V(r)$ at $r \rightarrow 0$[12] and is, therefore, essentially independent of the excitation E_H of H.

It is quite instructive to try and follow $F_H(Q^2)$ for an excited state as Q^2 increases from zero where $F_H(0) = 1$ towards asymptotic values. The values of the excited states' $r^2{}_H \equiv 6 \left.\frac{\partial F_H(Q^2)}{\partial Q^2}\right|_{Q^2=0}$ tends to be larger than $r^2{}_0$ the radius of the ground state because, roughly speaking, the particle lies higher up in the attractive potential $v(r)$ and its classical turning points are spaced farther apart. Also because $|\psi_H(r)|^2$ has n_r radial notes. $F_H(Q^2)$ also tends to have n_r oscillations before settling on its asymptotic fall off.

A particularly prominent peak results when $|Q|$ is matched to the dominant frequency of $|\psi_H(r)|^2$ which for smooth potentials is related to the wave number near the bottom of the well which is taken as the zero of the energy scale, i.e., for $Q^2 \simeq 2mE_H$. This is the counterpart of the quasi-elaptic peak in the transition $0 \rightarrow n$.

Once $Q > \frac{\Lambda}{<d>} \approx cn_r$ where $<d>$ is the average spacing between nodes we can expect, for general potentials, that each of the n_r humps in $|\psi_H(r)|^2$ scatters almost incoherently. If for $Q \approx 1/d$ we can still assume that the charges inside each hump still scatter coherently then we have

$$|F_H(Q \approx cn_r)|^2 = \sum^{n_r} e_i^2 \approx \frac{1}{n_r} \tag{10}$$

where in the last step we assumed roughly equal humps. Since $E = n_r^2$ for square-well-like potentials, the value of Q^2 at which this incoherent hump scattering sets in is $Q^2 \geq E_H (\approx m_H^2)$. Omitting again $\ell \neq 0$ state, n_r is the total number of levels H with energy $\lesssim E_H$, and we reproduce in this way the earlier results 7.

The crucial point, however, is that once the incoherent scattering and the presumably smooth asymptotic drop of $F_H(Q^2)$ have set in, $F_H(Q^2)$ tends to lie on or close to the same asymptotic curve as $F_o(Q^2)$ and <u>not</u> systematically lower -- so that the bounds (7) and (7') are indeed bounds for $F_o(Q^2)$, (see Fig. 2). We obviously cannot prove this statement in general. Let us rather examine three cases Coulomb, Harmonic oscillator and square wells potentials for which closed expressions for $F_H(Q^2)$ exist.

<u>Coulomb Potential</u>

Writing the hydrogen function as $R_{n\ell}(r) = C_{n,\ell}\, e^{-\frac{1}{2}(\rho)} \rho^{\ell} L^{2\ell+1}_{n+\ell}(\rho)$ where $\rho \approx r/n$ we can readily find the form factor by summing a series of successive derivative of the ground state form factor. Since the latter behaves like an inverse power the asymptotically leading term is obtained by taking no derivatives (i.e., the zeroth order term in the polynomial). The coefficient of this term which gives us then the asymptotic ratio of $F_n(Q^2)/F_o(Q^2)$ is simply the value of $R_{no}(0)$. Using the explicit expression it is readily found to be $\frac{1}{(n)^3}$ and hence:

$$\left| F_n^{\text{coul}}(Q^2)/F_o^{\text{coul}}(Q^2)\right|^2 = \frac{1}{n^6} \qquad (11)$$

The last result confirms our expectation that all form factors have the same asymptotic behaviour. The rather fast drop off of the ratio with n reflects the fact that the nth state keeps expanding out very fast so that the value of the wave function at the origin drops accordingly.

In the present case, however, the inequality does not really constrain the asymptotic behaviour of $F_H(Q^2)$ for large Q^2. Indeed it does not provide really an adequate test for our conjecture on the connection between $F_H(Q^2)$ as $Q^2 \to -\infty$ and $\rho_n(Q^2)$ as $Q^2 \to \infty$ simply because there are no bound states in this limit. Nonetheless, the fact that the discrepancy between $F_n(Q^2)$ and $F_o(Q^2)$ at asymptotic Q^2 could be completely ascribed to the expanding volume of the wave function is encouraging.

The above example suggests that we consider potentials which can yield infinite binding, to which we proceed next.

Harmonic Oscillator Potentials (One Dimensional)

The wave functions are $U_n(X) = \left(\frac{1}{\pi^{\frac{1}{2}} 2^n n!}\right)^{\frac{1}{2}} H_n(X) e^{-\frac{1}{2}X^2}$.
The form factors $F_n(Q^2)$ can again be written as a polynomial in $\frac{\partial}{\partial Q^2}$ operating on the form factor of the ground state $F_o(Q^2) = e^{-Q^2}$.
In this case the leading asymptotic term is obtained by taking the maximal number of derivatives (n) corresponding to the term $\frac{(X^2)^n}{\pi^{\frac{1}{2}} 2^n n!}$ in $|U_n(X)|^2$, $\left(|H_n(X)|^2 = 2^{2n}(X^2)^n + \ldots.\right)$. This gives

$$F_n(Q^2) \approx \frac{(Q^2)^n}{2n!} e^{-Q^2} \tag{12}$$

In this particular case $F_n(Q^2)/F_o(Q^2)$ is actually rising asymptotically. It is amusing to note that $F_o(Q^2) \approx F_n(Q^2)$ for $Q^2 \approx n \approx E_n$.

Infinite Square Well Potential (One Dimensional)

The form factor for the nth excited state is (we take the well to extend between $\frac{-\pi}{2}$ and $\frac{+\pi}{2}$)

$$F_n(Q) = \frac{2}{\pi}\left\{\frac{1}{2n+Q} \sin(Q+2n)\frac{\pi}{4} + \frac{1}{2n-Q} \sin(2n-Q)\frac{\pi}{4} + \frac{2}{Q} \sin Q\frac{\pi}{4}\right. \tag{13}$$

and we realize that all $F_n(Q)$ have again the same asymptotic behaviour as $Q^2 >> n^2 = CE_n$.

A feature which seems to be peculiar to this case is that the form factors keep oscillating indefinitely. It presumably reflects the infinitely sharp rise in the potential and is not expected to be true in more physical models.

IV. MANY PARTICLE SYSTEMS

Our discussions in the framework of potential models did not incorporate the important feature that hadrons may be made of a large (or even indefinite) number of constituents. It is very difficult to test for the correctness of our conjecture (3) in a genuine many body (i.e., field theoretic) framework. We will attempt, however, to see qualitatively by considering a very simple example how the density of states $\rho(E)$ and the form factor $F(Q^2)$ evolve as we go to systems with more and more constituents.

Consider a nonrelativistic three-body system with a given total energy E. If a two-body subsystem (say, a and b) has a relatively stronger binding then we could roughly describe the system as an ab cluster bound to the remaining particle c:

$$\Psi(\vec{a},\vec{b},\vec{c}) = h_i^{(1)}(\vec{a}-\vec{b})\cdot h_j^{(2)}(\vec{ab}_{cm} - \vec{C}) \qquad (14)$$

where $\vec{ab}_{cm}$ is the coordinate of the center of mass of the ab system. Any of the above wave functions corresponds to a partition

$$E = E_i^{(1)} + E_j^{(2)}$$

of the available energy into excitation of the ab ($h^{(1)}$)cluster and the (ab)-c($h^{(2)}$) system. The level density of the 3 body system is

$$\rho(E) = \int dE^{(1)}\ dE^{(2)}\ \rho_1(E^{(1)}\ \rho_2(E^{(2)})\ \delta(E^{(1)} + E^{(2)} - E) \quad (15)$$

where ρ_1, ρ_2 are the level densities for the two subsystems. Assuming that the integrand peaks strongly for a particular division $E^{(1)} = E_o$ $E^{(2)} = E - E^{(0)}$ we can roughly estimate

$$\rho(E) \simeq \rho^{(1)}(E^0)\ \ \rho^{(2)}(E-E^0)\,\Delta E \quad (16)$$

where the ratio $E_o / E-E_o$ will depend on the specific $\rho^{(1)}$ and $\rho^{(2)}$.

Let us next turn to the form factor of the 3 body system. Since we are interested in the additional decrease of form factors because of the compositeness of the ab system we assume that the charge resides in, say, a. Also to eliminate recoil, we assume c to be infinitely heavy. The charge distribution is given from Eq. (16) by the convolution of two distributions, $|h^{(1)}|^2$ and $|h^{(2)}|^2$ and hence the form factor is given by a product

$$|F(Q^2)|^2 \sim |F^{(1)}(Q^2)|^2 \cdot |F^{(2)}(Q^2)|^2 \quad (17)$$

The last equation is the familiar expression for the form factors of the deutron. It simply says that in order for the three body system not to break when particle (a) is hit we have to first make sure that the (ab) system does not dissociate (probability $|F^{(1)}(Q^2)|^2$ for that) and next treating the (ab) system as a single cluster we have to make sure that it does not tear away from (c), and the probability for that is $|F^{(2)}(Q^2)|^2$.

Suppose now that the conjectured inequality (3) holds for each of the subsystems (1) and (2) separately. Equations (16) and (17) then imply that $\frac{|F(Q^2)|^2}{\rho(Q^2)}$ will be smaller yet for the complete three-body system. The inequality does, therefore, pass this simple consistency check as we increase the complexity of our system by joining simpler systems.

At this junction we would like to make an amusing observation on the large t behaviour of hadron collisions. An important distinction between this case and the electromagnetic form factor with which we were dealing so far is that now the large momentum Q can be imparted to the composite hadron H in more than one step. Let us consider again an idealized situation in which H consists of two subsystems A and B which are only weakly linked together so that we have $m_H \approx m_A + m_B$. Again the most likely partition will be that which maximizes $\rho_A(m_A)\ \rho_B(m_B) \approx \rho(m)$. Assume that Q the momentum transfer is surely transverse. If we want the hadron H to remain intact in the scattering we should impart to A and B the same transverse boosts (see Fig. 3). The momentum transfers to the subsystems should, therefore, satisfy

$$\frac{Q_A}{m_A} = \frac{Q_B}{m_B}$$

since

$$Q_A + Q_B = Q$$

and

$$F_H(Q) = F_A(Q_A)\ F_B(Q_B)$$

We see that the evolution of density of states and form factors as we go to more composite states ("A+B→H") is such that the conjectured inequality (3) may actually be an asymptotic equality for "hadronic" form factors.

V. CONSEQUENCES FOR MODELS

The assumption that the inequality (3) (or even the weaker version (4')) apply asymptotically has several applications in general and in the context of various particular models which we will enumerate.

Dual Models

As is well known, one of the important predictions of dual models[13] -- which holds in a very large class of statistical models[2,3] -- is the exponential rise

$$\rho(m^2) = m^\alpha e^{m/T_c} \tag{18}$$

of density of states. It reflects the large number of ways in which we can partition the total excitation of the system between the different modes of an harmonic string

$$m^2 \approx \sum_{k=0}^{\infty} n_k\, k + 0(1) \tag{19}$$

where n_k are non-negative integers and m^2 (rather than m) is the relevant quantity because of the linearly rising trajectories. Thus, even if we adopt only the weaker bound suggested by (7')

$$F(Q^2) \lesssim \frac{1}{\rho[(Q^2)^{\frac{1}{2}-\epsilon}]} = e^{-\lambda(Q^2)^{\frac{1}{4}-\epsilon}}$$

we see that simple power fall-off of form factors suggested by simple minded applications of the beta function expression to form factor calculations are inconsistent. A particularly intriguing possibility is to speculate that this inconsistency is related to the failure of dual models to incorporate one basic ingredient of the above-discussion--namely, the existence of local currents.[(14)]

Parton Models

Asymptotic power behavior of form factors can quite naturally be incorporated into parton models.[12] Our conjecture then suggests that such models have power behaved asymptotic density of states. This is readily verified for the simple prototype of parton models where the system of energy m^2 consists of $\ell n(m^2)$ partons distributed roughly uniformly along the rapidity axis.[16] By the dilation arguments the level spacing for successive partons tends therefore to increase in a geometric proportion[12] with roughly a fixed ratio $r = e^{\Delta y}$ where $\Delta y \approx \langle y_n - y_{n-1}\rangle$. The number of levels is therefore, given by a modified partitioning problem

$$m^2 = \sum n_k r^k \qquad r > 1, \; n_k = 0, 1, 2, \ldots . \tag{20}$$

The number of combinations in this case is a power of m^2.

The Asymptotic Lower Bound on $F(Q^2)$

Elementary arguments based on the analytic properties of $F(Q^2)$(in particular,the fact that it has only a right hand cut) imply the asymptotic lower bound

$$F_H(Q^2) > c \; e^{-\lambda\sqrt{Q^2}} \tag{21}$$

It has been subsequently shown by A. Jaffe that this bound follows actually from axiomatic field theory.

It is amusing to note that our conjectured relation (3) correlates this limit with the following upper bound on the density of states

$$\rho_m(m^2) \lesssim c \; e^{\lambda\sqrt{(m^2)}} \tag{22}$$

This is indeed the situation in the case of the statistical model where both relations are satisfied with = sign (within powers of m^2 or Q^2). A violation of the last inequality would, in particular, negate the possibility of the existence of a consistent statistical picture of hadrons. It is very intriguing to ask if there is any more direct and deeper connection between the elements of the proof of the form factor bound (22) and that last observation.

VI. SPECULATIONS ON RISING $R(Q^2)$

Recent experimental data from CEA[4] suggest that over the range of $Q^2 = 10\text{-}20 \left(\frac{\text{GeV}}{c}\right)^2$; $R(Q^2)$ the ratio of $e^+e^- \to$ "γ" $\to$ hadron to the "point-like" $(e^+e^- \to \mu^+\mu^-)$ cross section is not constant, but rises significantly. In the framework of the models with scaling which we assume it means that the region where Eq. (4) applies has not been reached. One possible interpretation of this is that the bound (3) has actually not been satisfied over this range. This can happen for example if the density of states rises exponentially. This would presumably be true also in the sector of states with the quantum number of the nucleon. On the other hand, a power behaviour (the "dipole formula") can fit within $\approx 40\%$ $|F_N(Q^2)|^2$ over a very large spaces-like range of Q^2, $25\ \text{GeV}^2 \geq |Q^2| \geq 4\,\text{GeV}^2$.[18] To the extent that the same relatively slow fall off occurs in the time-like direction and is true also for all N^*s, Δ's, etc., we would expect the bound (3) to be temporarily violated. Obviously, this is hardly an explanation of the rise. In particular, it is not necessary to invoke N^* states since the π and other mesonic states could have even a slower fall off of their form factors (say, $\approx \frac{1}{Q^2}$) for a large Q^2, and have an exponentially increasing density of states. It is interesting, however, to speculate that the opening of many $N^* \overline{N^*}$ channels is the reason for the rising $R(Q^2)$.[19] It would in particular require that final states in e^+e^- collisions in this Q^2 range tend to include $N\bar{N}$ pairs.

VII. SUMMARY

In the above we have made a speculation on an asymptotic inequality between the asymptotic behaviour of form factors and density of states. We have tried to motivate this connection by considerions of e^+e^- annihilation to a pair of hadronic resonances, by using examples and intuition from potential models, and even more heuristic arguments for multi-body systems. If true the conjectured inequality will provide constraints on various theoretical models which attempt to predict the form factors.

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[8]In the case that all e^+e^- annihilations into hadrons would proceed through H^+H^- intermediate states the following inequality should become an equality. Very severe constraints are put on any dynamical quark parton model for e^+e^- annihilation, by the absence of quarks. A particular scheme for achieving it was recently suggested by A. Casher, J. Kogut, and L. Susskind, Tel-Aviv University Preprint No. 373 (1973). It is conceivable that such a mechanism would strongly damp the amplitude for producing two resonances with a large rapidity gap in which case the r.h.s. of Eqs. (5), (6), and in particular, Eq. (7), below may indeed be much larger than

the left hand side.

[9]For simplicity we have assumed $H^+ H^-$ to be spinless in which case we have only one electrical form factor normalized to $F_H(Q^2 = 0) = 1$. This reduces the density of states only by $\approx \frac{1}{\sqrt{m_{H^-}^2}}$ as can be seen from $l = kr$ considerations, by studying the Veneziano model, or the statistic model (see e.g., C. B. Chiu, R. L. Heimann, and A. Schwimmer, Phys. Rev. D4, 3177 (1971), and C. B. Chiu and R. L. Heimann, loc. cit 3186 (1971).) We can, however, also include particles with spin providing we will always use the diagonal conbination of helicity amplitudes corresponding to the electrical form factor (normalized to $F_H(0) = 1$).

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[16] This was pointed out to me originally by A. Casher.

[17] See J. D. Bjorken lecture in SLAC Summer Institute (1973).

[18] Form Factor Experiment, F. Kirk, et al., Phys. Rev. D8 63, (1973).

[19] It is amusing in this context to note that effective $\bar{N}N$ production threshhold (which because of the peripheral dynamics occurs at much higher energies in 2 body collisons) was conjectured recently M. Suzuki, UCRL Preprint (1973), to be related to the rise in cross sections at the ISR.

FIGURE CAPTION

Fig. 1 A schematic description of the separate H^+H^- production and decay in e^+e^- annihilation into hadrons.

Fig. 2 A typical expected behaviour of the form factor of the ground state and an excited state.

Fig. 3 A two-stage scattering from a loosely bound systems A and B.

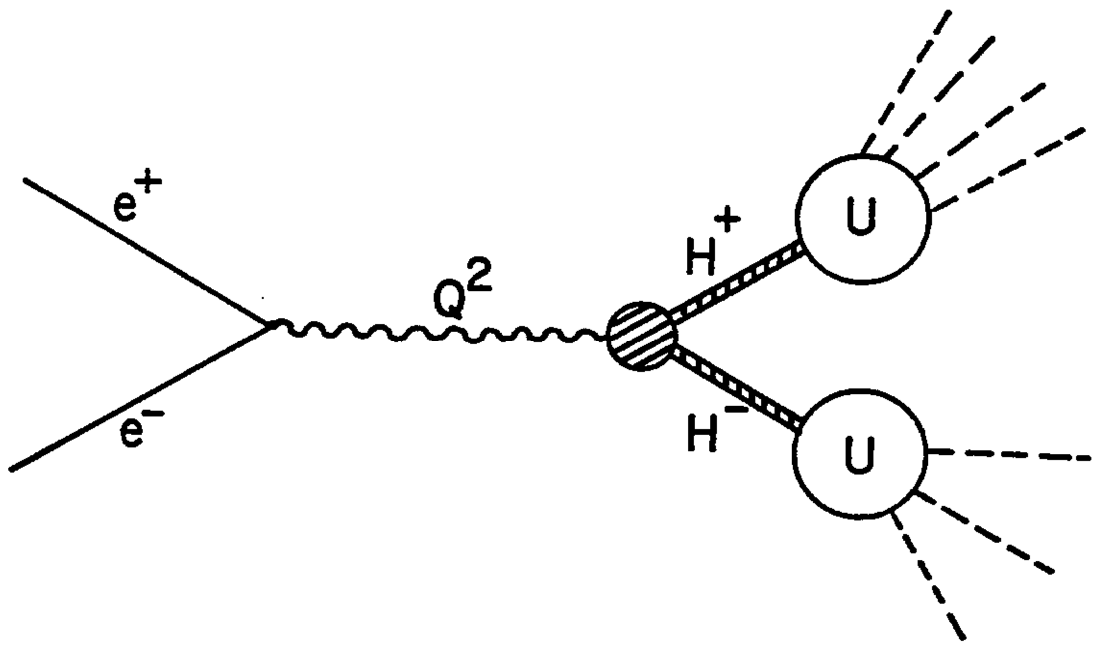

FIG. 1

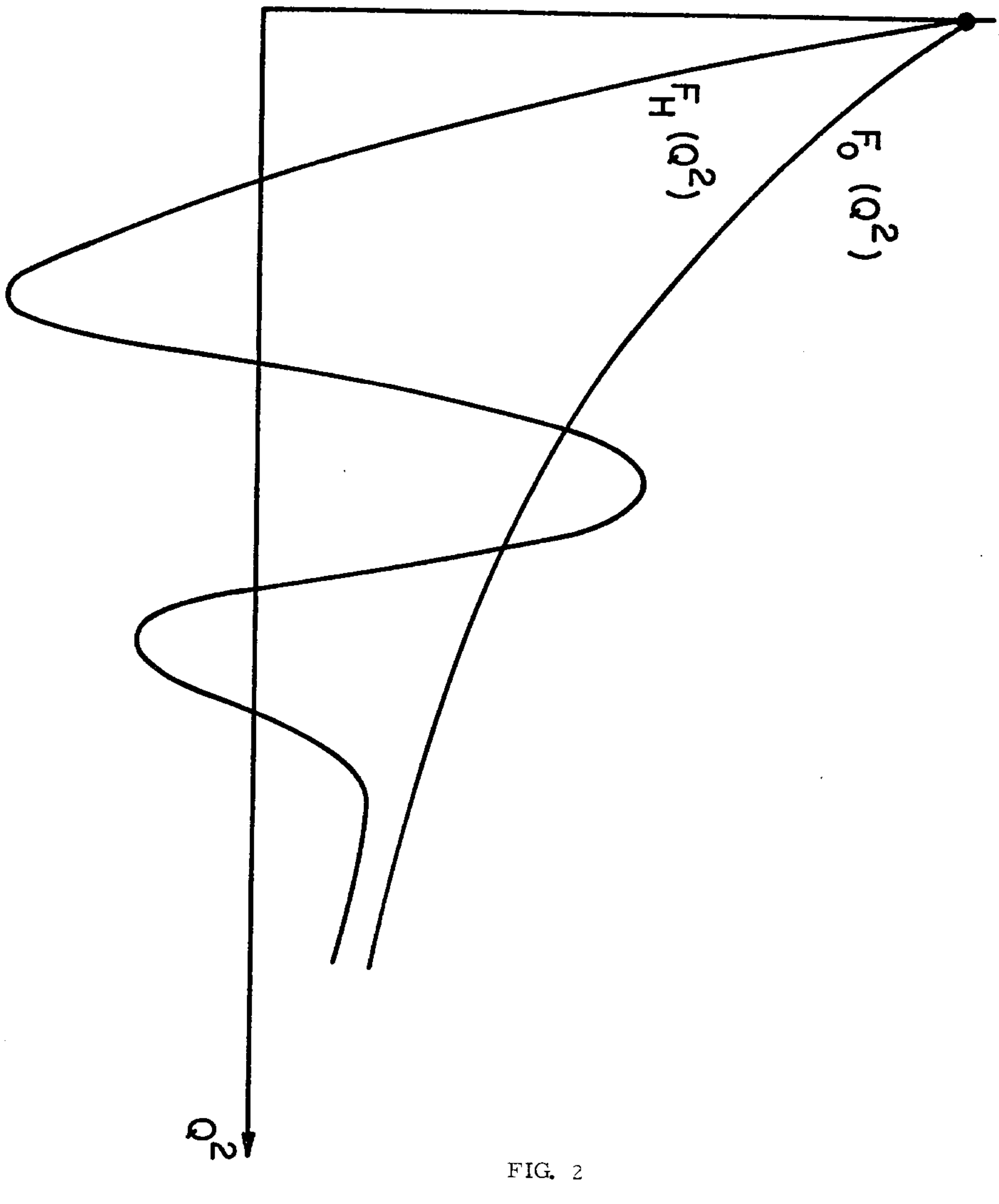

FIG. 2

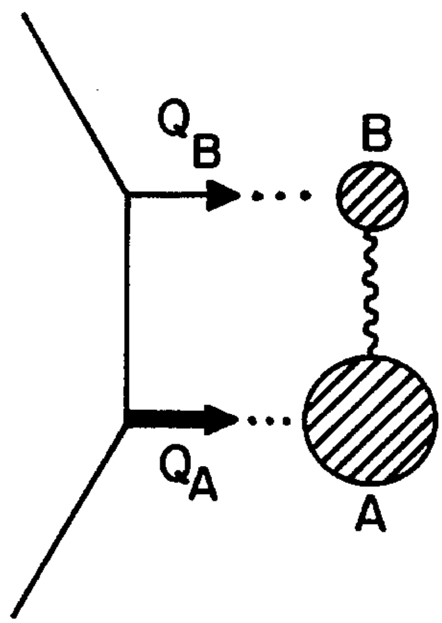

FIG. 3