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Inclusive Decay Amplitudes

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ABSTRACT

An inclusive decay amplitude is defined and discussed. It can be understood as a continuation of an absorptive part of 2→2 scattering which is related in a different kinematical region to a total cross section. In exotic channels we employ an assumption of smooth behaviour of this amplitude to obtain a relation between the width of a resonance to total cross sections. Some examples are discussed.

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I. THE INCLUSIVE FOUR-POINT FUNCTION

We investigate here decay reactions of the type $R \rightarrow c$ + anything, which we denote by (R, c). Let us use the definitions $P = p_R - p_c$, $P^2 = s$ and E = energy of particle c in the rest frame of R. It is then easy to see that

$$s = M^2 + \mu^2 - 2ME \qquad (1)$$

where M and μ are the masses of R and c respectively. We can now define an inclusive decay rate by

$$\Gamma(s) = \frac{\sqrt{E^2-\mu^2}}{16\pi^2 M^2} \sum_n \int |M|^2 \frac{d^3k_1}{2\omega_1(2\pi)^3} \cdots \frac{d^3k_n}{2\omega_n(2\pi)^3} (2\pi)^4 \delta^{(4)}(P - \sum_{i=1}^{n} k_i) \qquad (2)$$

where the sum is over all allowable decays that contribute to (R, c). The integral over $\Gamma(s)$ is

$$\int ds\ \Gamma(s) = \langle n_c \rangle\ \Gamma \qquad (3)$$

where Γ is the width of the resonance R and $\langle n_c \rangle$ is the average number of particles of the type c observed in its decays. These results follow along the same lines of reasoning that leads to inclusive distributions in high energy production processes. As a matter of fact it can be viewed as a continuation of the high energy production processes down to a resonance pole in the incoming channel. It is therefore also easily identifiable with a discontinuity of a two-body reaction amplitude $\bar{c} + R \rightarrow \bar{c} + R$.

To investigate this last point let us look at the various cuts in the $\bar{c}R$ forward scattering amplitude using the narrow resonance approximation. We show some characteristic cuts in Fig. 1. As an example we consider here the case of a pionic decay of $K^*(1420)$. We encounter both s and u-channel cuts. One should note that $s = (M \pm \mu)^2$ corresponds to $u = (M \mp \mu)^2$. Thus it turns out that the elastic u-channel opens up at the end of the observable s-channel decay region. Nevertheless the inclusive decay rate of Eq. (2) represents only the discontinuity across the s-channel cuts. We will consider in the following the discontinuity across all the s-channel cuts only and designate it by A(s). It can be represented by

$$A(s) = \frac{1}{2} \sum_n \int |M|^2 \frac{d^3k_1}{2\omega_1(2\pi)^3} \cdots \frac{d^3k_n}{2\omega_n(2\pi)^3} (2\pi)^4 \delta^{(4)}(P - \sum_{i=1}^{n} k_i) \qquad (4)$$

where P is either $p_R + p_{\bar{c}}$ or $p_R - p_c$ according to whether we consider scattering or decay processes. Above the elastic threshold one can use the optical theorem to write

$$A(s) = \sqrt{(s - (M+\mu)^2)(s - (M-\mu)^2)}\ \sigma_T(s) \qquad s \geq (M+\mu)^2 \qquad (5)$$

thus relating A(s) to the total cross section of $\bar{c}R$ scattering. In the decay region one can use Eq. (2) to relate the same function to the inclusive decay rate:

$$A(s) = \frac{8\pi^2 M^2}{\sqrt{E^2 - \mu^2}}\ \Gamma(s) \qquad m^2 \leq s \leq (M-\mu)^2 \qquad (6)$$

Here we used m^2 to designate the lowest threshold of the decay region. The upper limit is determined by $E=\mu$. Thus we see that the optical theorem can be generalized in an obvious way in the narrow resonance approximation to include both the total cross section and the inclusive decay rate as manifestations of the same absorptive part in different kinematical regions. The function $A(s)$ represents all the interesting physical features such as resonances in the low energy region. They can be observed if they fall into the decay region. Since our initial particle R is unstable we may expect to obtain a direct information in the scattering region only through model dependent evaluations of cross sections in nuclei. The measureable inclusive decay rate represents therefore the best available information about the function $A(s)$.

Several comments are in order: (i) All the discussion is based on the narrow resonance approximation. This means that while looking for the location of cuts one considers R to have zero width. Eventually one calculates of course the width Γ and the hope is that as long as it is small with respect to M the narrow width approximation is justified. In reality the fact that Γ can be of order μ ruins the careful distinction between the boundaries of the different physical regions in Fig. 1.
(ii) In our discussion we ignored the effects of spin. The results should therefore be regarded as averaged over the spin states of R and summed over those of c. One can of course formulate the same problem for each

individual helicity component. In particular the spin components of R may be of interest in future applications. (iii) One may also extend this discussion to non-forward four point functions ($t \neq 0$). Over a finite t-range one finds contributions from inclusive decays. They form then a continuation of the absorptive part of the corresponding elastic scattering amplitude.

II. RELATION BETWEEN WIDTH AND CROSS SECTION

Let us discuss now the case in which the $\bar{c}R$ quantum numbers are exotic. It is of particular interest because in this case one may safely assume that the absorptive part A(s) is a smooth function in the entire s range. From our experience with exotic meson-baryon channels we know that this is the case and we try to generalize this property to the present problem.

As an example let us investigate the consequences of a structure like A(s) = cs in the case μ, m << M. The inclusive decay rate becomes then

$$\Gamma(s) \approx \frac{Ecs}{8\pi^2 M^2} = \frac{cs(M^2-s)}{16\pi^2 M^3} \qquad 0 \lesssim s \lesssim M^2 \qquad (7)$$

whereas the asymptotic total cross section is $\sigma_T = c$. Using Eq. (3) we find then

$$\langle n_c \rangle \Gamma \approx \frac{cM^3}{96\pi^2} \qquad (8)$$

which gives an interesting relation between the width of (R, c) and the asymptotic cross section of $\bar{c}R$.

The assumption that A(s) = cs is, of course, arbitrary. Nevertheless if we assume that no significant structure exists at low s values we can view it as an upper limit on the order of magnitude of Γ. In other words we allow the real A to be smaller than cs but not much larger than that.

Alternatively, if $<n>$ is given, we can use the resulting c of Eq. (8) as a lower bound on the order of magnitude of σ_T:

$$<n_c>\Gamma \lesssim \frac{M^3}{96\pi^2}\sigma_T \qquad (9)$$

Let us investigate the consequences of this relation for several interesting cases. As a first example we will look at the decay properties of the $K^*(1420)$. The kinematics of this problem lead to the structure of cuts shown in Fig. 1. Using the data[1] we find that for (K^{*+}, π^-) one obtains $<n_{\pi^-}> \approx 0.2$ and therefore $<n_{\pi^-}>\Gamma \approx 20$ MeV. Inserting this value into Eq. (9) we obtain $\sigma_T(\pi^+K^{*+}) \gtrsim 2.6$ mb.

Alternatively one can work back from cross sections to widths. Nuclear measurements[2] show that $\sigma_T(A_1p) = 23 \pm 3$ mb and $\sigma_T(Qp) = 21 \pm 7$ mb in the ranges of 10 - 15 GeV. Let us assume that the corresponding meson-meson reactions are reduced by at least a factor of $\frac{2}{3}$ and set $\sigma_T(A_1\pi) \lesssim 15$ mb and $\sigma_T(Q\pi) \lesssim 14$ mb. If the main decay mode of A_1 is $A_1 \to \rho\pi$ we find for (A_1^+, π^-) that $<n_{\pi^-}> = 0.5$ and Eq. (9) leads to $\Gamma_{A_1} \lesssim 110$ MeV (assuming $M_{A_1} = 1.1$ BeV). Similarly if the Q decays are dominated by $Q \to K^*\pi$ one can conclude for (Q^+, π^-) that $<n_{\pi^-}> = \frac{4}{9}$ and Eq. (9) leads to $\Gamma \lesssim 150$ MeV and $\Gamma \lesssim 190$ MeV for the choices $M_Q = 1.2$ BeV and $M_Q = 1.3$ BeV respectively. Note however that for M = 1.2 BeV one finds $m^2 \approx .4$ BeV^2, $(M-\mu)^2 = 1.12$ BeV^2 and $M^2 = 1.44$ BeV^2. Hence the approximations needed for the derivation of

Eq. (8) do not really hold and the integration range in s in reduced by a factor of 2. The prediction for Γ should be reduced accordingly by at least a factor of 2. These results indicate that in the Q range there are presumably several resonances present.

The continuation from the decay to the scattering region is clearly a speculative step. Nevertheless the results show that it seems quite reasonable to assume that a smooth function describes both in the case of exotic channels. We would like to suggest that experimental data in these channels be presented by the function A(s) of Eq. (6). Its comparison with whatever information is available from nuclear experiments can lead to further insight into the question of the structure of resonances and their interactions.

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REFERENCES

[1]T. A. Lasinski et al., Review of Particle Properties, Rev. Mod. Phys. 45, S1 (1973).

[3]C. Bemporad, Seminar on Interactions of Elementary Particles with Nuclei, Trieste 1970. See also Introductory Paper by B. Margolis at the same seminar. A. N. Cnops et al., BNL preprint 14995 (1970).

FIGURE CAPTION

Fig. 1 The location of several s- and u-channel cuts of the inclusive (K^*, π) problem for $K^*(1420)$.

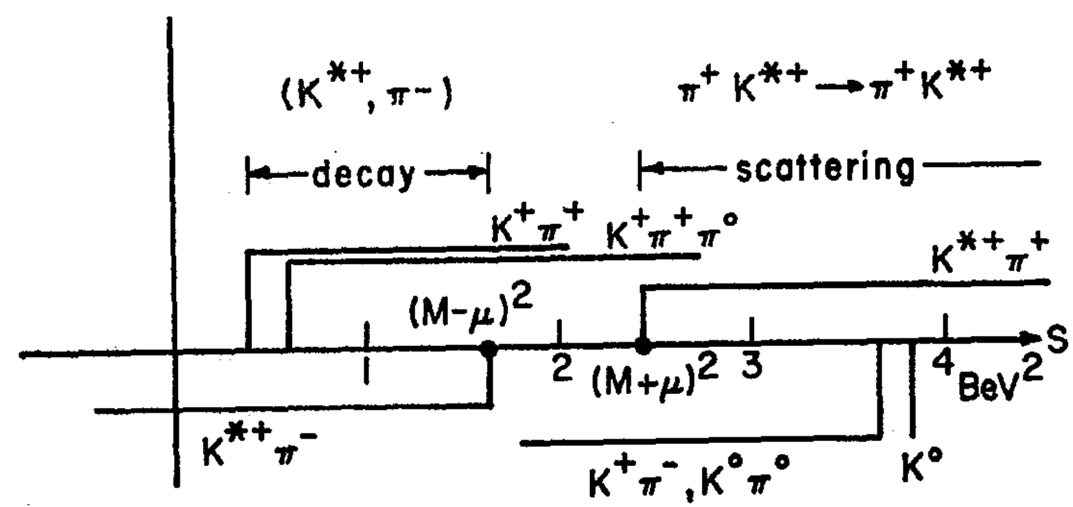
(K*+, π−)
π+ K*+ → π+ K*+
decay
scattering
K+π+
K+π+π°
K*+π+
(M−μ)2
1
2
(M+μ)2
3
4
BeV2
S
K*+π−
K+π−, K°π°
K°

Fig. 1