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RATIO OF REAL TO IMAGINARY PARTS FOR pp, $\bar{p}p$, $\pi^{\pm}p$, AND $K^{\pm}p$ FORWARD ELASTIC SCATTERING AMPLITUDES FROM 50 TO 5000 GeV

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ABSTRACT

Without introducing any additional parameter or assumption, we use our previous model for hadron scattering based on the impact picture to predict the ratio of the real to imaginary parts of various forward elastic scattering processes at high energies.

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Three years ago we predicted, on the basis of quantum field theory, many features of the limiting behavior of cross sections at extremely high energies.[1] For example, all total cross sections must increase without bound, and the total elastic cross section approaches half of the total cross section. Since these limiting behavior cannot be expected to hold at energies available at the present time, it is interesting to construct models that have all the desired properties at infinite energy. A first attempt in this direction was carried through last year.[2] The major difficulty that we encountered is that some assumption must be made to fix, roughly speaking, the energy scale for increasing cross sections. We chose the condition that the background term is absent in the case of K^+p elastic scattering which is the only process observed to have an increasing cross section at Serpukhov.[3] This was the most reasonable assumption at that time, but we certainly had no cogent reason to argue this must be so.

Recently, measurements of proton-proton scattering give dramatic qualitative confirmation of our prediction of increasing total cross sections.[4] By using one of the measured points, we can remove the above-mentioned assumption of the absence of background in the K^+p case. In this way, we have obtained a number of quantitative predictions, not only about the proton-proton and proton-antiproton scattering, but also for K^+p, K^-p, π^+p and π^-p scattering.[5]

Once the total cross sections are known as functions of the energy, the real parts of the forward scattering amplitudes are uniquely determined because of dispersion relations. No additional parameters or assumptions are needed. It is the purpose of this note to report the results of such a calculation for laboratory energy between 50 and 5000 GeV. The formulas are remarkably simple, and no integration is involved.

We recall that, in the model based on the impact picture we reported earlier[5], the total cross section for a channel j such as pp or $\bar{p}p$ is given by

$$\sigma_{total}(j) = A_j s^{-\frac{1}{2}} + \frac{4.8932}{s} \text{ Im } M_j(s, 0) \qquad (1)$$

where

$$M_j(s, 0) = \frac{is}{2\pi} \int d\vec{x}_\perp \, D_j(s, \vec{x}_\perp) \qquad (2)$$

with

$$D_j(s,\vec{x}_\perp) = 1 - \exp\{-f_j (E e^{-i\pi/2})^c \exp\left[-\lambda (x_\perp^2 + x_{oj}^2)^{\frac{1}{2}}\right]\}. \qquad (3)$$

Here E is the laboratory energy of the incident particle, c and λ are universal constants (independent of j), and f_j and x_{oj} are parameters dependent on j but are, however, the same if the incident particle is replaced by its anti-particle.

Since crossing symmetry is properly taken into account in this model, $M_j(s, 0)$ is analytic in the upper half plane. It is thus simply a matter of extending the background term $A_j s^{-\frac{1}{2}}$ to complex values of s. If we neglect the masses compared with E, then the ratio α of the real and imaginary parts of the forward elastic amplitude is given by

$$\alpha = \frac{-A_j' s^{\frac{1}{2}} + 4.8932 \text{ Re } M_j(s, 0)}{A_j s^{\frac{1}{2}} + 4.8932 \text{ Im } M_j(s, 0)}. \qquad (4)$$

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Here j' denotes the corresponding u channel. For example, if j is $K^{+}p$, the j' is $K^{-}p$.

In Figs. 1-3 we plot α from (4) in the range $10^2 < s < 10^4$ GeV^2 for pp, $\bar{p}p$, $\pi^{\pm}p$, and $K^{\pm}p$. For the solid lines, the parameters previously found[5] are used. To test the sensitivity to parameterization, we also used an alternative form for D_j:

$$D_j(s,\vec{x}_\perp) = 1 - \exp\{-f_j(Ee^{-i\pi/2})^c \left[\ln(Ee^{-i\pi/2})\right]^{-1} \exp\left[-\lambda(x_\perp{}^2 + x_{oj}{}^2)^{1/2}\right]\} . \tag{5}$$

The new parameters are again determined from the set of fourteen pieces of experimental data listed in Reference 5, and the resulting α are plotted as the dashed lines in Figs. 1-3. The difference between the two sets of curves is quite small. Of course, both sets of curves depend sensitively on the values of the recently measured total pp cross section at ISR.[4] In Fig. 1 we also show the experimental result for pp from Serpukhov[6] and ISR[7]. The agreement is quite satisfactory.

We conclude with three remarks.

1. For s below 100 GeV^2, it is necessary to use the experimental data at lower energies and perform the dispersion integral. The unphysical region in the $\bar{p}p$ case can be handled in a way entirely similar to that of Söding[8].

2. For s above 10^4 GeV^2, (4) still applies but the two sets of curves are not in good agreement.

Qualitatively, all α's approach zero slowly from positive values, as previously discussed.[1]

3. For the two sets of curves, the value of c is quite different: with (3), c = 0.082925; with (5) c = 0.20225. Since we expect c to be less than 1 from the theorem of Jin and Martin[9], these values, particularly the second one, are perhaps reasonable. Another difference between the two sets of parameters is that the value of A(pp) is significantly smaller when (5) is used.

REFERENCES AND FOOTNOTES

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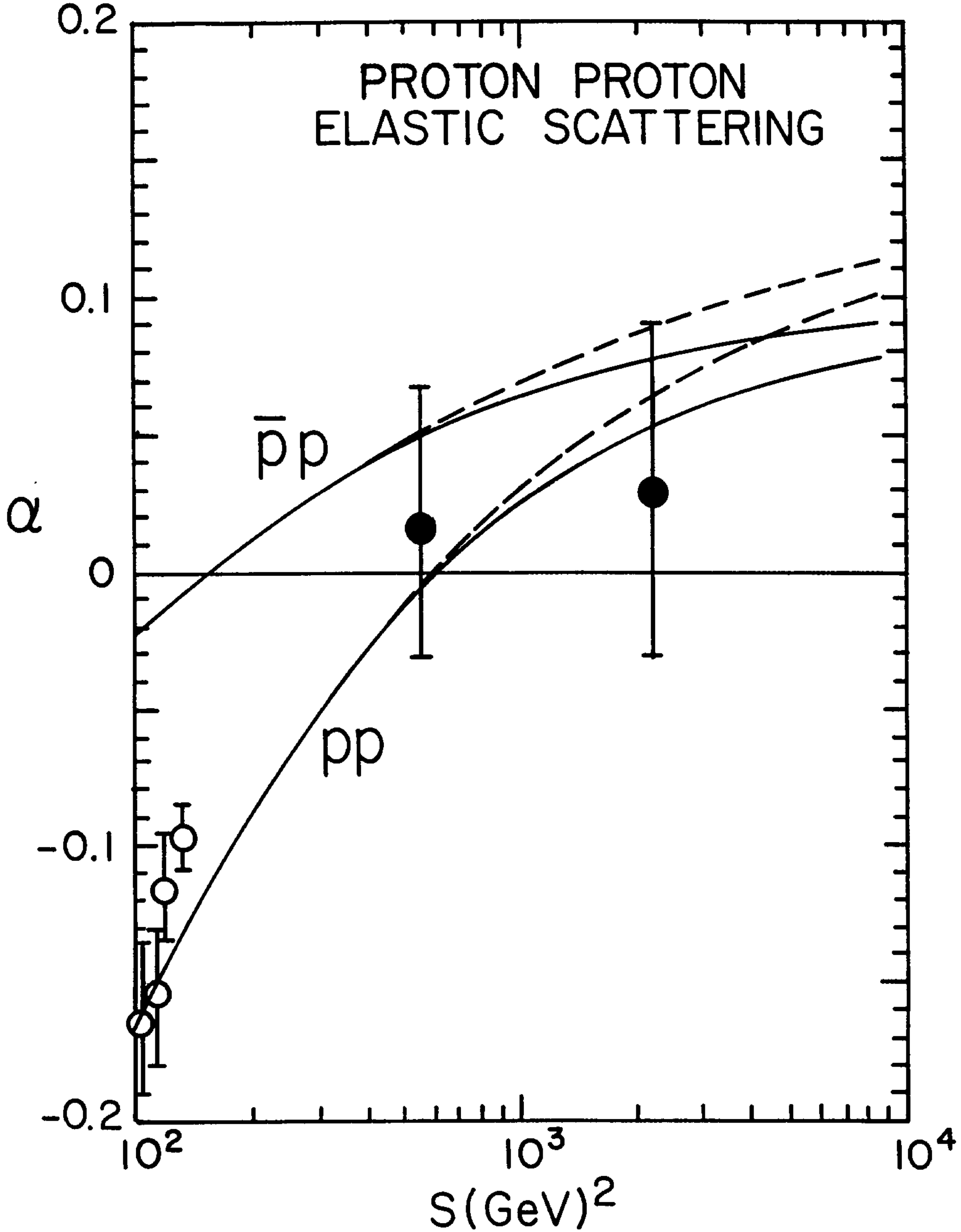
CAPTIONS FOR FIGURES

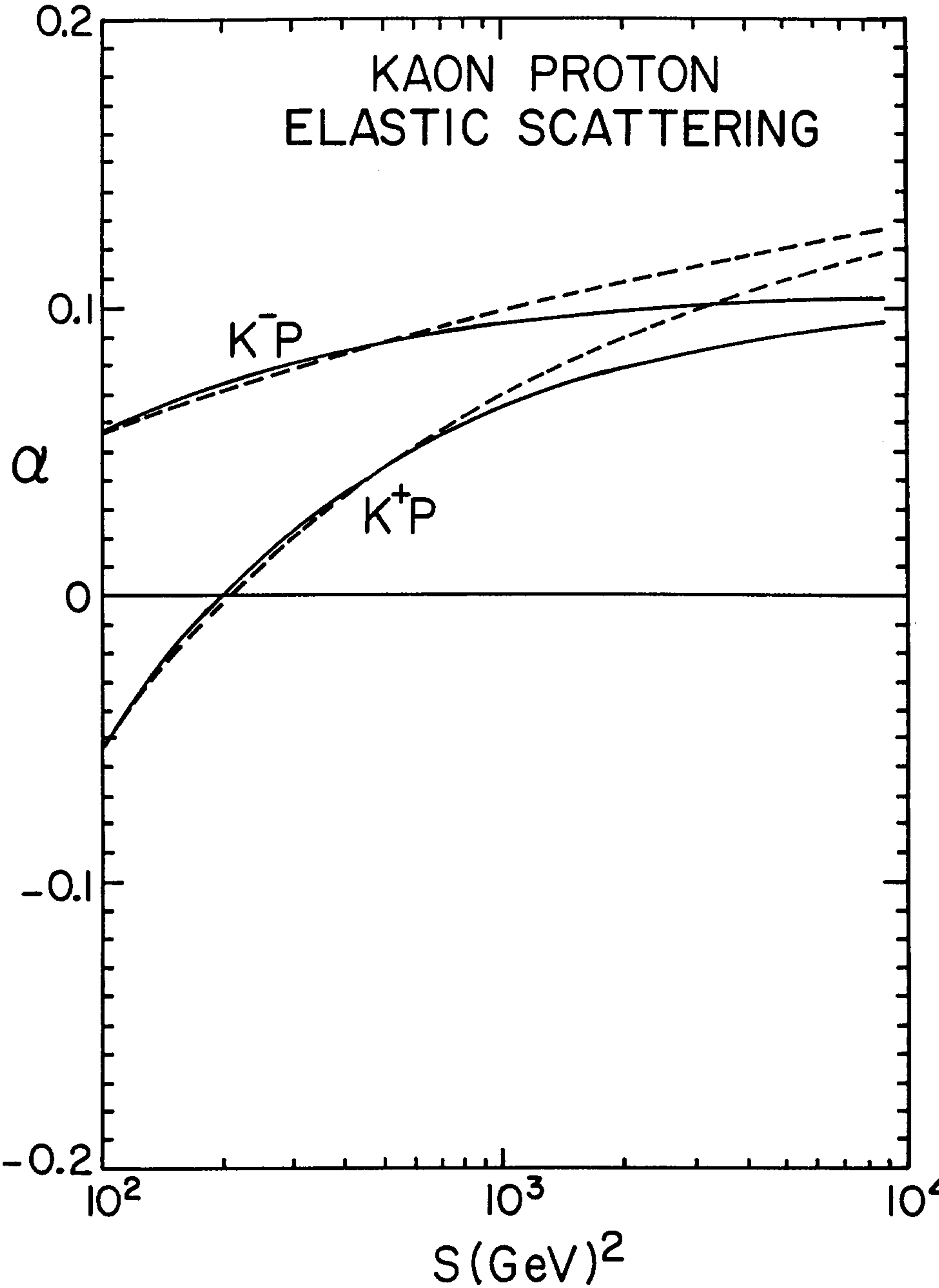
Fig. 1. Theoretical values of α for pp and $\bar{p}p$ forward elastic amplitudes together with experimental results for pp. The solid lines are obtained from Eq. (3) while the dashed lines are from Eq. (5).

Fig. 2. Theoretical values of α for K^+p and K^-p forward elastic amplitudes. The solid lines are obtained from Eq. (3) while the dashed lines are from Eq. (5).

Fig. 3. Theoretical values of α for π^+p and π^-p forward elastic amplitudes. The solid lines are obtained from Eq. (3) while the dashed lines are from Eq. (5).

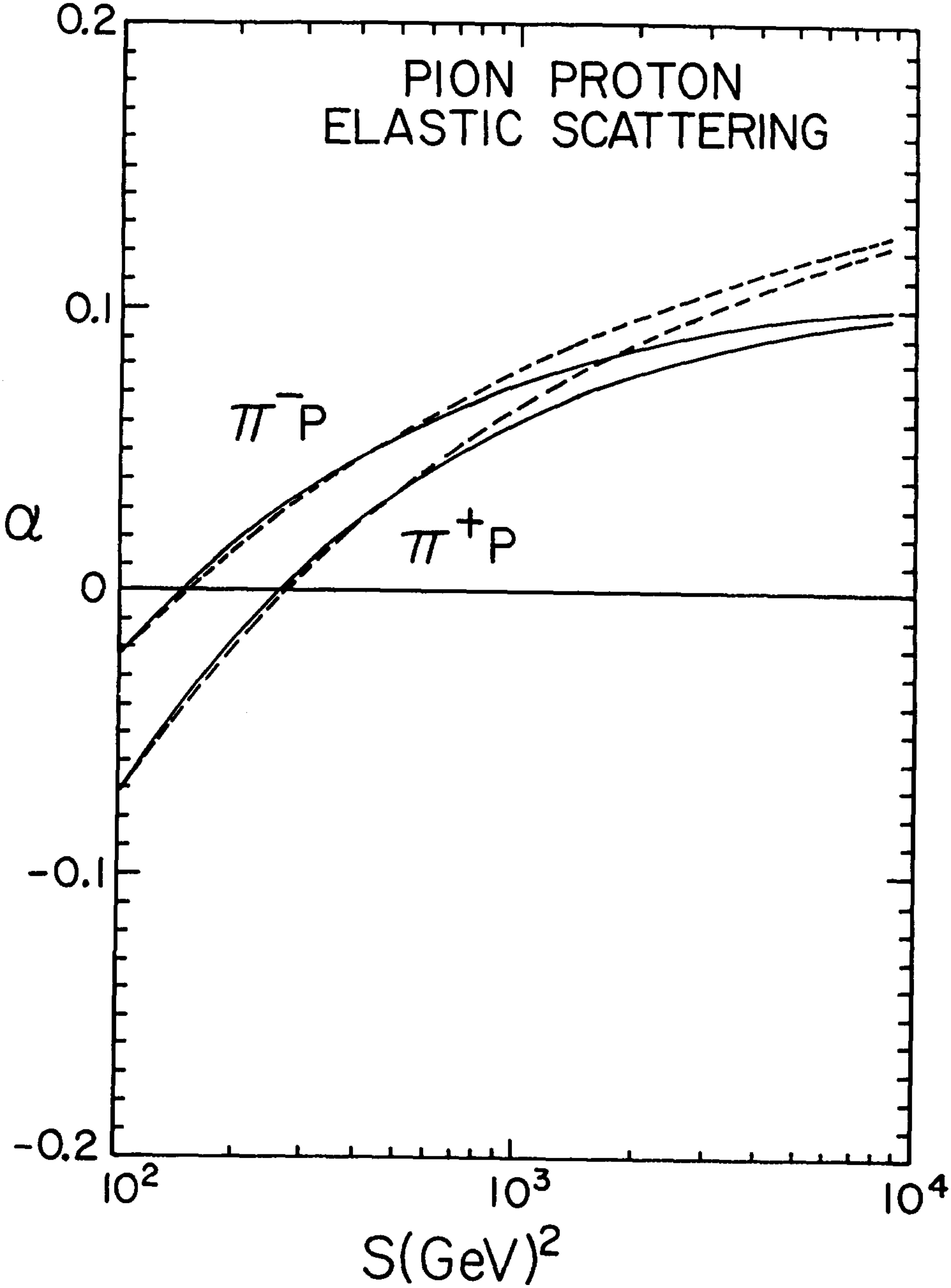


PROTON PROTON
ELASTIC SCATTERING
0.2
0.1
0
-0.1
-0.2
α
$\bar{p}p$
pp
10^2
10^3
10^4
S(GeV)^2



KAON PROTON
ELASTIC SCATTERING
K⁻P
K⁺P
α
0.2
0.1
0
-0.1
-0.2
10²
10³
10⁴
S(GeV)²

Fig 7



PION PROTON
ELASTIC SCATTERING
0.2
0.1
0
-0.1
-0.2
α
$\pi^- P$
$\pi^+ P$
10^2
10^3
10^4
S(GeV)^2