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OF THE PION MASS

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ABSTRACT

The measurement of the negative pion mass by crystal diffraction of pionic x-rays is adjusted for the revised fine structure constant, improved radiative energy-level calculations, and more precise values of the calibration lines. We find that the result quoted earlier should be reduced by 80 ppm (parts per million) to 139.566 ± 0.010 MeV (± 73 ppm).

I. Introduction

In 1965 we reported^{1,2} an estimate of the negative pion mass by measuring the Ca and Ti 4f-3d pionic x-ray energies with a 7.7m bent crystal spectrometer.³ Since that time there has been considerable improvement in the accuracy of the fine structure constant and the energies of the calibration γ rays. Also, recent interest in the radiative corrections to pionic and muonic atom energy levels, stimulated by discrepancies between additional precision x-ray measurements and theory, have allowed more precise energy level calculations for pionic transitions. We therefore present a re-evaluation of the pion mass measurement.

II. Energy Level Calculations

The original energy-level calculations for the Ca and Ti 4f-3d transitions were carried out by this author^{1,2} in 1965 using perturbation theory corrections to the unperturbed Klein-Gordon wave equation. In recent years calculational techniques have improved considerably, namely by numerically integrating the Klein-Gordon equation after adding the finite nuclear size and vacuum polarization corrections to the Coulomb field for a point nucleus. In 1969 Fricke⁴ published an extensive theoretical evaluation of radiative corrections in pionic and muonic atoms. Based on this paper, Backenstoss et al.⁵ observed that the original perturbation theory calculations did not include a finite-size correction to the lowest order vacuum polarization term, and estimated the effect to cause a 200 ppm shift in the energy level calculation. As perturbation theory indicated the effect to be approximately 5 ppm,⁶ the discrepancy was quickly traced to Fricke's paper. Also, the discrepancies between muonic atom measurements and theory^{7,8} lead to finding additional errors in the same reference.^{9,10}

Using numerical integration techniques and Blomqvist's results,¹⁰ Tauscher¹¹ has recently re-evaluated the original perturbation theory calculation, and finds that the scale factors (the ratio of the pion mass to the

transition energies) for the Ca and Ti 4f-3d transitions should be reduced by approximately 46 ppm (41 ppm of this is due to the 1969 re-evaluation¹² of the fine structure constant). Tauscher's results, which reduce the theoretical uncertainties by about a factor of 4, are used in this re-evaluation, and are presented in Table I.

III. Bent Crystal Spectrometer Calibration

The bent crystal spectrometer was calibrated with the 84 keV γ rays and the 52 keV Yb $K\alpha_1$ x-rays emitted in the decay of Tm.¹⁷⁰ Since the original calibration, additional measurements have been made on the energy of the 84 keV γ ray,¹³⁻¹⁷ resulting in a more accurate value. As these measurements were referenced to the energies of either the W or Yb $K\alpha$ x-rays, which depend on the 1969 evaluation of the voltage-wavelength conversion product,¹² the adjusted values are shown. We include a measurement of the γ ray energy carried out on the spectrometer used for the pion mass measurement.¹⁷ The weighted mean is found to be 84.258 ± 0.003 keV, the error representing external rather than internal consistency, as the distribution of measured values is somewhat larger than the quoted internal errors would seem to indicate.

Using a wavelength of 0.147147 \AA^* (± 39 ppm) for the γ ray, a wavelength of 0.236655 \AA^* (± 13 ppm) for the Yb $K\alpha_1$ x-ray,¹⁸ and the measured diffraction angles

for these lines, we find a quartz (310) lattice spacing of $d = 1.180004 \text{ \AA}^*$ ($\pm 15 \text{ ppm}$) for the spectrometer calibration. In addition, each measured wavelength should include an error of $\pm 2 \times 10^{-6} \text{ \AA}^*$ to allow for possible nonlinearities in the sine screw mechanism.

IV. Results

Table I shows the revised evaluation of the pion mass measurement. The error matrix for the two measurements is

$$V_{\text{tot}} = \begin{pmatrix} (122 \text{ ppm})^2 & (25 \text{ ppm})^2 \\ (25 \text{ ppm})^2 & (87 \text{ ppm})^2 \end{pmatrix}$$

where V_{11} and V_{22} represent the variance of the Ca and Ti measurements respectively. The covariances are due entirely to spectrometer calibration and energy level calculations.

The resulting revised estimate for the π^- mass is

$$M_{\pi} c^2 = 139.566 \pm 0.010 \text{ MeV } (\pm 73 \text{ ppm}),$$

in agreement with the recent revised results of Backenstoss et al.¹⁹

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Table I. Revised evaluation of the pion mass from measurement of the 4f-3d transitions in pionic Ca and Ti. All errors are shown in parts per million.

Quantity	$\pi\text{Ca } 4f-3d$	$\pi\text{Ti } 4f-3d$	units
diffraction angle a) ($\sin\theta$)	0.0726120 (119 ppm)	0.0599388 (84 ppm)	---
Wavelength	0.171365 (121 ppm)	0.141456 (86 ppm)	\AA^*
Energy	72.350 (121 ppm)	87.648 (86 ppm)	keV
Scale factor b)	1929.130 (14 ppm)	1592.299 (11 ppm)	---
$M_\pi c^2$	139.573 (122 ppm)	139.562 (87 ppm)	MeV

a) Table II of ref 2.

b) Ref 11