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Multiplicity Distribution in Diffractive Dissociation at High Energies

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ABSTRACT

The assumption that diffractive production of particles at high energies proceeds through the exchange of a factorizable Pomeron is shown to lead directly to predictions about the multiplicity of particles produced. A simple model is constructed which exhibits these predictions.

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Several analyses[1,2,3] of high-energy data on multiplicity of hadrons produced in proton-proton collisions have found the data consistent with the dominant production mechanism being of the multiperipheral type. In addition a relatively small diffractive component seems to be present, although its magnitude is not yet unambiguously determined.[4] In this paper we consider some consequences of the assumption that the diffractive component is itself understandable in terms of the multiperipheral picture. (By "diffractive component" we mean not only diffractive production of resonances but also dissociation into large missing mass above the resonance region; in fact, it is this second part which we consider here.) We find that some properties of the multiplicity distribution of diffractively-produced hadrons are predictable in this picture, and provide clear tests of the validity of the idea, advanced by Chew and collaborators,[5] that diffractive production proceeds through the exchange of a Pomeron whose properties are similar to those of other Reggeons.

The standard Regge pole picture of single diffraction dissociation says that it proceeds through Pomeron exchange, as shown in Fig. 1. If "Pomeron" signifies only a J-plane singularity not far from $J=1$, it is not very controversial to believe that it exists. The picture in Fig. 1 develops greater predictive power, however, when the hypothesis is made that the Pomeron is dominantly a factorizable Regge pole.[6] Only then can one regard the blob in Fig. 1 as representing a Pomeron-hadron scattering cross section which behaves similarly to ordinary hadronic cross sections. We shall show that this factorizable-Pomeron hypothesis leads directly to predictions about the multiplicity of diffractively-produced hadrons over a finite range of energies, including NAL and possible ISR energies.

With the assumption of a factorizable Pomeron with intercept $\alpha_P(0) = 1$ we write the diffraction dissociation cross section corresponding to Fig. 1 as

$$\frac{d^2\sigma_{ab}}{dM^2dt} = \frac{|\beta_{bP}(t)|^2}{16\pi s^2}\left(\frac{s}{M^2}\right)^{2(\alpha_P(t))} M^2\sigma_{aP}(M^2,t), \tag{1}$$

where the notation is the same as that of Ref. 7 and 8, except that we have written the formula in terms of a Pomeron-hadron cross section $\sigma_{aP}(M^2,t)$. Some support for the validity of this formula comes from the successful fits of Ellis and Sanda and related work (although the triple-Pomeron term, on which our conclusions are based, does not play an important role in these fits).[8,9] A formula like Eq. (1) should be equally valid for the cross section $d^2\sigma_{ab}{}^n/dM^2dt$ for single diffractive dissociation into a definite number of particles n,

$$\frac{d^2\sigma_{ab}{}^n}{dM^2dt} = \frac{|\beta_{bP}(t)|^2}{16\pi s^2}\left(\frac{s}{M^2}\right)^{2(\alpha_P(t))} M^2\sigma_{aP}{}^n(M^2,t) \tag{2}$$

where $\sigma_{aP}{}^n(M^2,t)$ is the cross section for the production of n particles by a hadron and a Pomeron.

Consider now the average multiplicity of hadrons of type i produced in diffractive dissociation of hadron "a" into a state of mass M at a momentum transfer t (see Fig. 1)

$$\langle n_a^i(M^2,t)\rangle \equiv \sum_{n_i} n_i \frac{d^2\sigma_{ab}^{n_i}}{dM^2dt} \Big/ \frac{d^2\sigma_{ab}}{dM^2dt}\,,$$
$$\equiv \sum_{n_i} n_i\, \sigma_{aP}^{\ n_i}(M^2,t) \Big/ \sigma_{aP}(M^2,t). \tag{3}$$

The factorizable-Pomeron hypothesis permits one to apply the standard multiperipheral or Mueller-Regge arguments[10] to infer the asymptotic behavior for large M^2,

$$\sigma_{aP}(M^2,t)\,\langle n_a^i(M^2,t)\rangle \sim \left[\beta_{aP}^{\ (0)} g_P(t)\, A^i \ln M^2 + B_a^i(t)\right] + O\left[(M^2)^{\alpha_M(0)-1}\right] \tag{4}$$

where $g_P(t)$ is the triple-Pomeron coupling defined in Ref. 4, and to infer that the Pomeron-hadron cross section should behave as

$$\sigma_{aP}(M^2,t) \sim \beta_{aP}(0)\, g_P(t) + O\left[(M^2)^{\alpha_M(0)-1}\right], \tag{5}$$

where $\alpha_M(t)$ represents the highest-ranking secondary trajectory. For any desired level of accuracy there exists a value M_0 such that for $M > M_0$ the second term in Eqs. (4) and (5) representing the contribution of secondary trajectories, can be neglected. We then obtain the simple result:

The average multiplicity of hadrons of type i produced in diffraction dissociation into a state of mass M, where $M > M_0$ rises linearly with M^2,

$$<n_a^i(M^2, s, t) = A^i \ln M^2 + B_a^i(t) \qquad (6)$$

with the same coefficient A^i (independent of s, t, and incident particle type) found in the average multiplicity measured in hadronic reactions.

To obtain better statistics it may be desirable to measure the cross section integrated over M^2. The lower limit M_0^2 is chosen as above such that the contribution of the secondary trajectories in σ_{aP} can be neglected, whereas the upper limit is chosen to be $M^2 = rs$, where r is a fraction sufficiently small to permit Eq. (1) to apply without inclusion of secondary terms.[11] In the Feynman x-variable, this range is $1-r \le x \le 1-\frac{M_0^2}{s}$. Performing the integration using Eq. (1) and (5) one finds

$$\int_{M_0^2}^{rs} \frac{d^2\sigma}{dM^2 dt}\, dM^2 \approx \frac{G_P^{ab}(t)}{16\,\pi} \ln\left(\frac{rs}{M_0^2}\right) + 0(1) \qquad (7)$$

where

$$G_P^{ab}(t) \equiv \left|\beta_{bP}(t)\right|^2 \beta_{aP}(0)\, g_P(t). \qquad (8)$$

We have done the integration in the approximation, valid for small t, that $\epsilon(t) \simeq 0$, where

$$\epsilon(t) \equiv 2 - 2\alpha_P(t). \qquad (9)$$

Note that the cross section given by Eq. (7) rises logarithmically with energy. Such a rise should, we believe, be observed at NAL-ISR energies, but a self-consistent multiperipheral picture eventually damps this rise by repeated Pomeron exchange.

One can also integrate Eq. (9) over t to obtain the total cross section for diffractive dissociation into states of mass $M_0^2 \le M^2 \le rs$,

$$\sigma_D(M_0^2 \le M^2 \le rs) = \frac{\overline{G}_P^{ab}}{16\pi} \ln \frac{rs}{M_0^2} + 0(1), \tag{10a}$$

where

$$\overline{G}_P^{ab} \equiv \int_{-\infty}^{0} dt\, G_P^{ab}(t). \tag{10b}$$

The above expression assumes the validity of the approximation $\epsilon(t) \approx 0$ over the range of the t-integration. The adequacy of this approximation depends on how fast the residue function $G_P^{ab}(t)$ falls with t. When particle b is a nucleus, the approximation should be very good, but when b is a proton, it is rather crude. Corrections are discussed in Comment (iii).

The average multiplicity for diffraction dissociation into states in a range of $M_0^2 \le M^2 \le rs$ at a given value of t can be calculated similarly, with the result

$$\langle n_a^i\ (M_0^2 \le M^2 \le rs, t)\rangle \sim A_i \frac{\int d(\ln M^2)(M^2)^{\epsilon t} \ln M^2}{\int d(\ln M^2)(M^2)^{\epsilon t}} + \ldots \tag{11a}$$

$$\approx \frac{1}{2} A^i \ln \frac{rs}{M_0^2} + C_a^i(t) + \ldots \tag{11b}$$

Note the coefficient $A^i/2$, <u>half</u> the coefficient found in the average multiplicity measured in hadronic reactions. Again, the t-integration can be performed under the assumption that $\epsilon(t)$ can be neglected over the entire range of t, with the result

$$<n_a^i\ (M_0^{\ 2} \le M^2 \le rs)> \sim \frac{1}{2} A^i \ln \frac{rs}{M_0^{\ 2}} + \overline{C}_a^i + \ldots \qquad (12)$$

Corrections for finite Pomeron trajectory slope are discussed in Comment (iii).

The results above follow directly from the factorizable-Pomeron hypothesis, and therefore provide definitive model-independent tests of that hypothesis (subject to Comments (i) and (ii) below). In order to clarify the origin of these results it is helpful to examine a simple model, which is offered in the spirit of exhibiting the simplest possible multi-peripheral-type model of diffractive production. Like the Chew-Pignotti model, of which it is a simple extension, it should have illustrative value, but it must be recognized as an oversimplification rather than as a definitive prediction of the multiperipheral picture.

We construct the model by taking $\sigma_{aP}^{\ \ n}(M^2)$ to be the simplest multi-peripheral type of distribution, a Poisson distribution,

$$\sigma_{aP}^{\ \ n}(M^2, t) = \beta_{aP}(0) g_P(t) \ \frac{[A \ln (M^2/s_a)]^n}{n\,!} \ e^{-A \ln (M^2/s_a)} \qquad (13)$$

Such a distribution has been found by Frazer, Peccei, Pinsky, and Tan[1] to be a good fit to ~ 80% of the inelastic production in the ragne 100 - 300 GeV. The remainder of the production was taken, in that fit, to be diffractive. We ignore the diffractive component in Eq. (13), in the spirit of a one-Pomeron exchange approximation (see Comment (i) and footnote 6).

The scale factor s_a is a parameter of the model, and will not necessarily have the same value as that needed to fit the multiplicity distribution in p-p collisions. This reflects the fact that the parameter B_a^i in Eq. (4) depends on the nature of the incident particles.

Substituting Eq. (13) in Eq. (1) one finds

$$\frac{d\sigma_{ab}^{n}}{dM^2 dt} = \frac{G_P^{ab}(t)}{16\pi s^2}\left(\frac{s}{M^2}\right)^{2\alpha_P(t)}\left(\frac{M^2}{s_a}\right)^{1-A} \frac{[A \ln (M^2/s_a)]^n}{n!} \tag{15}$$

That is, the multiplicity distribution of hadrons produced diffractively into a state of high mass M is Poisson-distributed in this model. Again, we can integrate over a range of M, $M_0^2 \le M^2 \le rs$, to obtain, for $\epsilon(t) \approx 0$,[12]

$$\int_{M_0^2}^{rs} \frac{d^2\sigma_{ab}^{n}}{dM^2 dt}\, dM^2 \approx \frac{G_P^{ab}(t)}{16\pi A}\left[P\left(n+1,\ A \ln \frac{rs}{s_a}\right) - P\left(n+1,\ A \ln \frac{M_0^2}{s_a}\right)\right] \tag{16}$$

where $P(n, x)$ is an incomplete gamma function.[13] We can make the simplifications of dropping the factor r by absorbing it into a redefinition of the parameter s_a,[14] and dropping the second term in the braket, which affects only small values of n. Integrating over t, we then obtain the following model for the cross section for the production of n particles by single diffraction dissociation,[15]

$$\sigma_D^{n}(s) \approx \frac{\overline{G}_P^{ab}}{16\pi A}\ P(n+1,\ A \ln\frac{s}{s_a}) + (a \to b). \tag{17}$$

The two terms represent the dissociation of particle a and particle b.

respectively. Double dissociation is ignored because the Pomeron coupling to elastic channels seems to be much larger than to inelastic channels;[16] or, stated more formally, the coupling $g_P(t)$ is relatively weak. In addition, the contribution of low-mass resonances, which are not included in Eq. (13), should be added to Eq. (17) to form the complete diffraction dissociation cross section. Although this contribution is not well known, it will affect only the low-multiplicity cross sections. For this reason, and because of the approximations which led to Eq. (17), we propose it as a reasonable model only for higher values of n.

The distribution given by Eq. (17) is well known to physicists; it is just the χ^2 distribution with the "number of degrees of freedom" $\nu/2 = n+1$ and with $\chi^2/2 = A \ln (s/s_a)$. It is shown in Fig. 2 for the case appropriate to E_L = 1500 GeV. As is characteristic of χ^2 distribution, it falls to half its maximum at $n \approx A \ln (s/s_a)$, which is just the average multiplicity of the non-diffractive component, shifted by a constant amount.[14] Qualitatively, then, we see in Fig. 2 a distribution which is flat at small n, cutting off at a value near the peak of the distribution of the non-diffractive component.

It is easy to understand in this model the origin of the general results obtained above. The rise of the average diffractive multiplicity, and even the factor 1/2 in Eq. (12), result from a rather flat distribution which cuts off at $n \approx A \ln (s/s_a)$. Similarly, the growth of the diffractive cross section with lns arises from the onset of higher-n cross sections.

It is interesting to note that we obtain a diffractive component whose average multiplicity rises like lns, but the mechanism which produces this logarithmic rise is not the $1/n^2$ tail assumed in some diffractive fragmentation models.[17]

We conclude with a few comments about the range of validity of our results: (i) We stated at the outset that these results are not asymptotic predictions.[6] The assumption of a factorizable Pomeron implies the possibility of repeated Pomeron exchange in the production mechanism. But our results are not valid when the energy is high enough that multiple Pomeron exchange becomes an important contribution to the total cross section. Rough estimates[16] based on the fits in Ref. 1 lead us to guess that double dissociation plus the leading two-Pomeron process will be comparable to the elastic cross section at the highest ISR energies. At this energy not only should these corrections be included, but even the utility of the approximation scheme is questionable. (ii) In order for our hypotheses, and hence our predictions, to be valid, a lower limit on the energy is necessitated by the requirements on the parameters r and M_0. We must have enough energy to diffractively produce large missing mass. In the triple-Regge language this implies that the triple-Pomeron contribution must dominate. [The practical question of how high an energy is required to isolate the triple-Pomeron contribution will have to be answered experimentally by triple-Regge fits to the s and M^2 dependence.] However, we conjecture, in accord with our basic assumption, that

Pomeron-hadron scattering will not behave very differently from hadron-hadron scattering, that NAL energies should be adequate; (iii) Our results have been derived in the approximation $\alpha_P'(0) \ll b$, where $b = d(\ln G_P^{ab}(t))/dt$, the parameter characterizing the rate of decrease of $G_P^{ab}(t)$. If we relax this approximation, but assume an exponential form $G_P^{ab}(t) = G_P^{ab}(0)e^{bt}$, the integrations leading to Eq. (10) can still be performed with the result:[13]

$$\sigma_D(M_0^2 \leq M^2 \leq rs) = \frac{G_P^{ab}(0)}{16\pi} \frac{1}{\eta b} \ln\left[\frac{1 + \eta \ln \frac{s}{M_0^2}}{1 - \eta \ln r}\right] \tag{18}$$

where $\eta = 2\alpha_P'(0)/b$. For sufficiently high energies, $\sigma_D \sim \ln(\ln s)$, but for most applications at currently available energies $\eta \ln(s/M_0^2)$ will not be large. In p-p scattering we estimate $.05 \lesssim \eta \lesssim .1$; when particle b is a nucleus, η will be much smaller. The approximation of expanding the logarithm to first order in η, which yields Eq. (1) may be adequate. Similarly, one finds

$$\sigma_D \langle n_a^i(M_0^2 \leq M^2 \leq rs)\rangle = \frac{\overline{G_P^{ab}}\, A}{16\pi\, b\, \eta^2} \left\{(1 + \eta \ln s) \times \ln\left[\frac{1 + \eta \ln \frac{s}{M_0^2}}{1 - \eta \ln r}\right] - \eta \ln s\right\} \tag{19}$$

which, in the limit of small η, reduced to Eq. (12). The average multiplicity at fixed t, Eq. (11), is valid without correction.

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FIGURE CAPTIONS

Figure 1. Diagram describing diffractive dissociation.

Figure 2. Diffractive contribution to the multiplicity distribution in p-p scattering at $E_L = 1500$ GeV, according to the model given by Eq. (17). The non-diffractive contribution is a Poisson distribution from the fit of Ref. 1. The parameters of the diffractive term are also determined by the fit in Ref. 1 to be $s_a = 44$, $A = 1.33$, and $\overline{G}/8\pi = 3.2$ mb.

FOOTNOTES

[1] W. Frazer, R. Peccei, S. Pinsky, and C.-I. Tan, University of California, San Diego preprint, October 1972.

[2] C. Quigg and J. D. Jackson, National Accelerator Laboratory preprint NAL-THY-93, October 1972.

[3] H. Harari and E. Rabinovici, Weizmann Institute preprint, Dec. 1972.

[4] Although none of the analyses cited above claims to have determined the diffractive component unambiguously, it is interesting that they all infer about the same magnitude for the inelastic diffractive cross section, 6 - 8 mb. A recent paper by K. Fiałkowski [Physics Letters 41B, 379 (1972)] finds similar results, as does the Michigan-Rochester collaboration at 100 GeV (contribution to the Amer. Phys. Soc. meeting, January 1973).

[5] See, for example G. F. Chew, T. Rogers, and D. R. Snider, Phys. Rev. D2, 765 (1970).

[6] This hypothesis is motivated by insights gleaned from study of multi-peripheral models, which show us that although the Pomeron is probably a very complicated singularity there exists an energy region in which it can be approximated by a simple pole with intercept at $J=1$. This is the region where the energy is too low for multiple Pomeron exchange to

make a significant contribution to the total cross section. The apparent success of two-component fits to multiplicity distributions encourages us to assume that the single-Pomeron exchange approximation is valid over the NAL energy range, and possibly at ISR energies.

[7] H. D. I. Abarbanel, G. Chew, M. Goldberger, and L. Saunders, Phys. Rev. Letters 26, 937 (1971).

[8] S. D. Ellis and A. I. Sanda, Phys. Rev. D6, 1347 (1972) and Physics Letters 41B, 87 (1972).

[9] P. D. Ting and H. J. Yesian, Phys. Letters 35B, 321 (1971); J. -M. Wang and L. -L. Wang, Phys. Rev. Letters 26, 1287 (1971).

[10] R. N. Cahn, SLAC preprint 1110 (1972). B. R. Webber, Nuovo Cimento Lett. 3, 424 (1972), claims a similar but stronger result in which $B_a^i(t)$ is independent of t. We are unable to convince ourselves of the validity of this result.

[11] The fraction r must of course be sufficiently small that the upper limit M^2=rs does not extend beyond the edge of phase space.

[12] The same result was written down by Silverman, Ting, and Yesian (Phys. Rev. D5, 1971) in the context of different physical assumptions. They assumed P' dominance, so $\epsilon(t)$ was not small and the resulting distribution was geometrical rather than χ^2.

[13] M. Abramowitz and I. Segun, Handbook of Mathematical Functions,

(Dover, New York, 1968), Sec. 6.5 and 26.4.

[14]The result of the t-integration is a factor $\exp(bt_{min})$, where $t_{min} = -m_b^2(M^2/s)^2$. We choose r sufficiently small to permit expansion of the exponential, which leads to terms of order $1/s^2$, which are neglected. In our model calculation the factor $\exp(bt_{min})$ cuts off the M^2 integral. We can approximate this cutoff by integrating up to $M^2=rs$, where $r^2 \approx (bm_b^2)^{-1}$.

[15]J.S. Ball and F. Zachariasen, Phys. Letters 40B, 411 (1972) obtain the same result in the context of a self-consistent diffractive model.

[16]We estimate that $\sigma_{DD} \approx \sigma_D^2/8\ \sigma_{el}$, which is less than a millibarn. The factor 8 arises from a factor 2 from the two terms of Eq. (17), and from a factor 1/2 which multiplies σ_{DD} because of the nature of the multi-peripheral phase-space.

[17]R.C. Hwa, Phys. Rev. Lett. 26, 1163 (1971); M. Jacob and R. Slansky, Phys. Rev. D5, 1847 (1972).

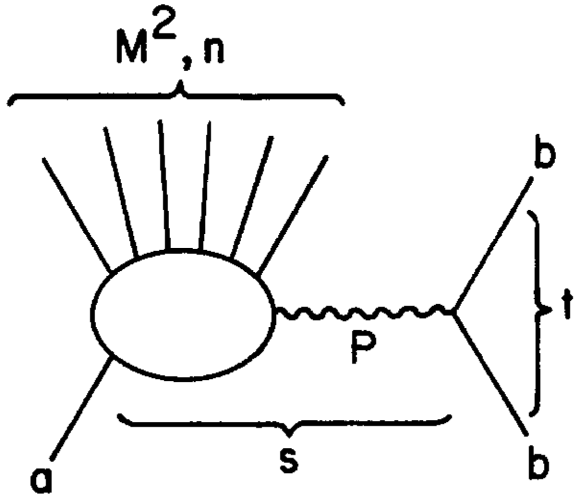

Figure 1

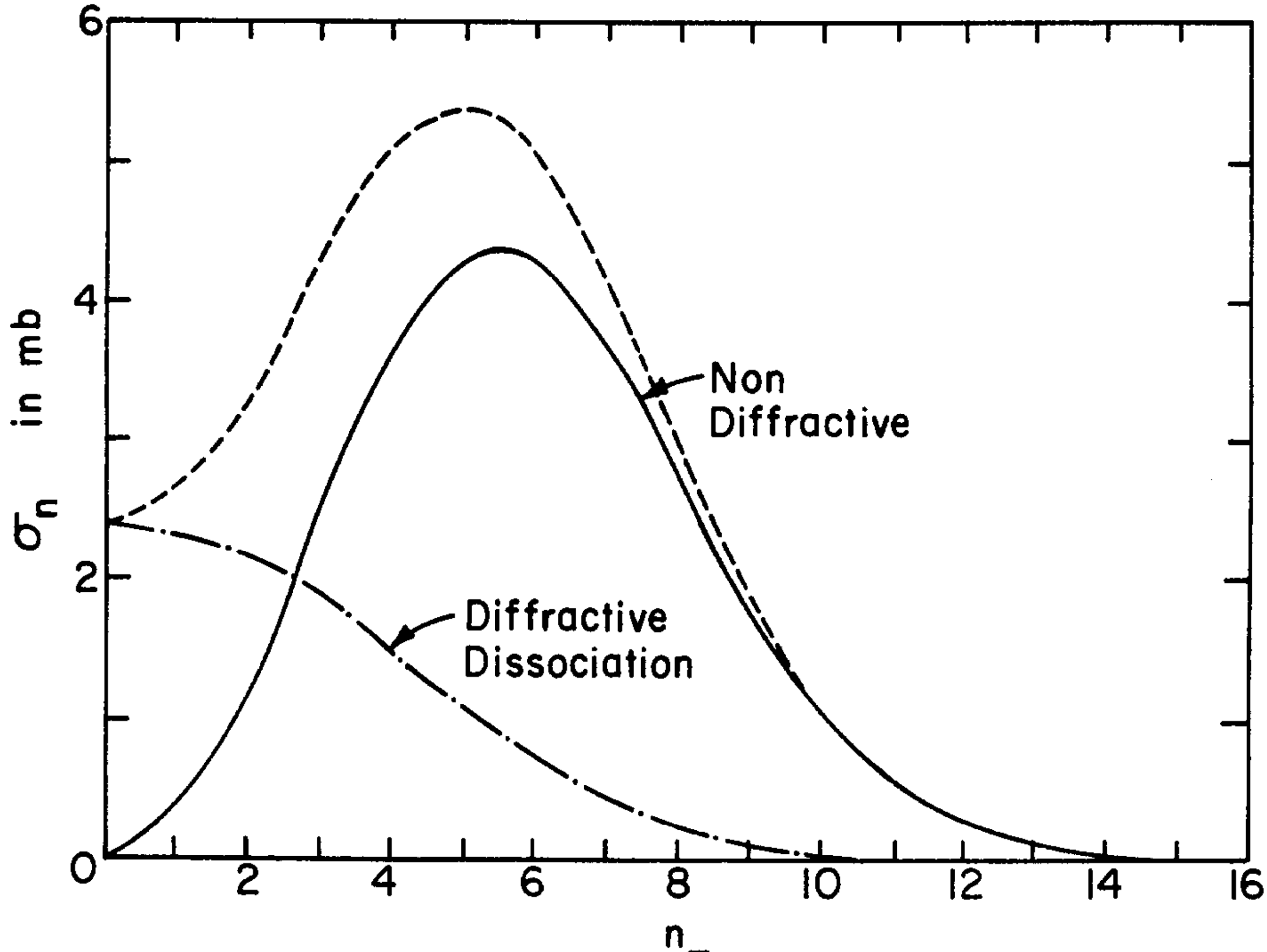

Figure 2