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The Parton Model and e^+e^- Colliding Beam Experiments*

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One of the most intreaging questions that have been around ever since the success of nuclear physics is whether the proton and neutron are made out of yet more elementary particles. From the meassurement of proton electromagnetic form factors, we now know that the proton indeed has structure. The recent meassurement[1] of the inelastic form factor of the proton gave us yet another hint in understanding the proton structure. The variables for the inelastic proton electron scattering is shown in Fig. 1.

Fig. 1

The lepton part of the matrix element can be calculated exactly and the matrix element for the hadronic part is given by

$$(2\pi)^6 \sum_n \delta^4(p + q - p_n) \langle p | J_\mu(0) | n \rangle \langle n | J_\nu(0) | p \rangle \tag{1}$$

$$= - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2, \nu) + \frac{1}{M^2}\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) W_2(q^2, \nu) .$$

To interpret the experimental results, we suppose that the proton can be seen as a collection of constituents called partons[2]. The virtical photon interacts with a parton which carries a fraction x of the total momentum. If we neglect

Fig. 2

the final state interaction and therefore require that the struck parton is on the mass shell[3],

$$m^2 = (q + xp)^2 \tag{2}$$

then $x = Q^2/2M\nu$. For fixed $Q^2/2M\nu$, meassurement of the inelastic proton electron scattering gives the form factor of the parton which carries fraction $x = Q^2/2M\nu$ of the intial proton momentum. Experimentally, νW_2 is shown in Fig. 3.

Fig. 3

The constancy of $\nu W_2(q\,,\,\nu)$ at fixed x can be interpreted in terms of parton constituents which are point like. The general idea of the parton model, that the proton has a composite structure made up of point like particles to explain scaling of νW_2 may be correct and it is quite exciting. The assumption of neglecting the final state interactions altogether to give physical interpretation for scaling of νW_2, Eq.2, however, may be too simplifying and may contradict future experiments. Such an example may already be happening. According to the parton model[4], the process $e^+e^- \longrightarrow$ hadrons is given by the diagram shown in Fig. 4.

Fig. 4

Again the final state interaction is neglected. Then

$$R = \frac{\sigma(e^+e^- \longrightarrow \text{hadrons})}{\sigma(e^+e^- \longrightarrow \mu^+\mu^-)} = \sum_i e_i^2 \tag{3}$$

$= 2/3$ if partons are quarks

$= 2$ if partons are collored quarks.

e_i is the charge of parton "i" in the unit of e and i runs over different types of partons. Experimental result[5] is shown in Fig. 5. If experiments show that R continues to

Fig. 5

rise, do we abondon the parton model? To understand this problem, we like to obtain a minimal set of most reasonable and general assumptions to be used in the parton model and give experimental predictions which critically test such assumptions.

We start out by supposing that there is some underlying field theory which governs the hadronic physics. We call the bare particles in such a theory partons. Then we ask (a) what can we say about such a theory if we demand that partons influence experimentally meassurable quantities? (b) What additional assumptions are needed to prove scaling of $\nu W_2(q^2, \nu)$? The answer to (a) is that the requirement gives a very strong restriction on the underlying field theory of the hadronic physics. The answer to (b) is that we need very little additional assumptions to obtain scaling for the structure functions. These assumptions, however give very strong prediction on the multiplicity in the process $e^+e^- \longrightarrow$ hadrons and experimental check on this prediction will be a crucial test for our hypothesis.

Thinking along the line of (a), we note that in nuclear physics, the wave function for a nucleus to be in a definite state of protons and neutrons play a crucial role. Similarly, if the bare fields, the partons, were to influence experimentally meassurable quantities, the wave function for a proton to be in a certain state of partons must be meassurable.

The probability for a proton to be in a parton state which includes n partons plus anything is given by[6, 7]

$$|{}_{\pm}\langle p|k_1,\dots,k_n\rangle'|^2 = \sum_m \int \prod_{j=n+1}^{m} d^3k_j |{}_{\pm}\langle p|k_1,\dots,k_n,k_{n+1},\dots,k_m\rangle|^2 . \quad (4$$

Fig. 6

We have integrated over the unobserved partons and sumed over number of unobserved partons. This complication, in comparesion to nuclear physics wave functions, arise from the relativistic nature of the problem. If the binding energy is large, compared to the rest mass of hadrons, we expect many pairs of parton-antiparton to be created. The probability for a given parton state to be found in a hadron state which contains n hadrons is defined by

$$\left|\langle k_1,\dots,k_l|\,p_1,\dots,p_n\rangle'_{\pm}\right|^2 = \sum_m \int \prod_{j=n+1}^{m} d^3p_j \left|\langle k_1,\dots,k_l|\,p_1,\dots,p_n,p_{n+1},\dots,p_m\rangle_{\pm}\right|^2 \tag{5}$$

Fig. 7

(i) In order for (4) and (5) to be well defined, in the integrals

$$\int \left|{}_{\pm}\langle p|k_1,\dots,k_n\rangle'\right|^2 \prod_{i=1}^{n} d^3k_i = 1, \text{ and}$$

$$\int \left|\langle k_1,\dots,k_l|\,p_1,\dots,p_n\rangle'_{\pm}\right|^2 \prod_{i=1}^{n} dp_i = 1, \tag{6}$$

$\prod_i d^3k_i$ and $\prod_i d^3p_i$ must be convergent.

It is the condition that must be satisfied in order to talk about partons within the context of a field theory. This condition, trivial as it may seem, is very strong condition on the field theory of the strong interaction. The perturbative approach to quantum electro dynamics, for example, does not satisfy this condition.

What additional assumptions do we need ? Since integrands for (6) are positive definite, only a finite region out of the entire phase space is important. If we expect the parton model to be useful, we must be able to say something about the parton configurations by observing the hadronic configurations. So, we assume that

(ii) The character of the strong interaction is such that the contribution from d^3n_i integration in (6) comes from the part of phase space about the directions $\vec{k}_1, \dots, \vec{k}_l$.

Fig. 8

(iii) Consider $\langle k_1, k_2 | p_1, \dots, p_l \rangle_{\pm}$
We expect that the overlap between a state $| k_1, k_2 \rangle$ and hadron state $| p_1, \dots, p_l \rangle_{\pm}$ contained in a finite phase space η to be small so that

$$\int \langle k_1, k_2 | p_1, \dots, p_l \rangle_{\pm} \, d^3k_1 \quad (7)$$

is convergent. For example, the configuration shown in Fig. 9 is assumed to be sufficiently small so that

Fig. 9

(7) converges.

These are our hypothesis. The concept of partons within a context of a field theory yields (i). The physical nature of partons gives (ii) and (iii).

Using these hypothesis it has been shown that

(A) (i), and (ii) $\longrightarrow$ scaling for νW_2.[8]

(B) (i) ~ (iii) $\longrightarrow$ scaling for $\nu\bar{W}_2$ and $M\bar{W}_1$.[9]

$\nu\bar{W}_2$ and $M\bar{W}_1$ are structure function for the process $e^+e^- \longrightarrow$ hadron + anything. It is defined by

$$(2\pi)^6 \sum_n \delta^4(k_1 + k_2 - p - p_n) \ \langle 0|J_\mu(0)|n,p\rangle\langle n,p|J_\nu(0)|0\rangle$$

$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\bar{W}_1 + \frac{1}{M^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\bar{W}_2. \qquad (8)$$

The variables are defined in Fig. 10.

Fig. 10

Let us give a rough idea how the argument goes for (A). First, when Q^2 is large, it can be shown that (i) and (ii) $\longrightarrow$ two jet structure for hadrons in the final state. See Fig. 11.

Fig. 11

We observe the reaction in the frame where the proton in the initial state is moving very fast. In such a frame,

$$p = (P + \frac{M^2}{2P},\ 0,\ 0,\ P)$$

$$q = (\frac{M\nu}{2P},\ Q,\ 0,\ -\frac{M\nu}{2P}) \qquad P \longrightarrow \infty \tag{9}$$

$$k = (xP + \frac{m^2 + k_\perp^2}{2xP},\ 0,\ 0,\ xp)$$

$p_1, \ldots, p_l$ denote hadron momenta whose z component $p_{iz} = y_i P$ satisfies

$$y_i \gg \frac{M}{Q} .$$

Then hadrons in a jet (1) satisfies

$$\sum_{i=1}^{v} p_i \simeq p - k \tag{10}$$

and hadrons in jet (2) satisfies

$$\sum_{i=v+1}^{l} p_i \simeq q + k . \tag{11}$$

With these condition, it is just a kinematics to obtain

$$\sum_{i=v+1}^{l} p_i^o = xP + \frac{Q^2}{2xP} + O(\frac{m^2}{P}) \tag{12}$$

$$\sum_{i=1}^{v} p_i^o = (1 - x)P + O(\frac{M^2}{P}) .$$

By energy conservation

$$E_i = E_f \tag{13}$$

or

$$P + \frac{M^2}{2P} + \frac{M\nu}{P} = xP + \frac{Q^2}{2xP} + (1-x)P + O(\frac{M^2}{P})$$

$$= P + \frac{Q^2}{2xP} + O(\frac{M^2}{P}) \tag{14}$$

and thus

$$x \cong \frac{Q^2}{2M\nu} .$$

This is the same result as the one previously obtained from (2). The assumption of neglecting the final state interaction, or (2), is much too stronger assumption than what is actually needed.

In the similar way, the scaling of the structure function for $e^+e^- \longrightarrow$ hadron + anything can be obtained. This gives crucial test of the hypothesis.

Note that the cross section for $a + b \longrightarrow c + x$, $\frac{d\sigma}{d^3p_e}$ is related to the multiplicity and the total cross section by[10]

$$E\sigma_T = \int E_c \frac{d\sigma}{d^3p_c} d^3p_c$$

$$\langle n\rangle\sigma_T = \int \frac{d\sigma}{d^3p_c} d^3p_c \tag{15}$$

For $e^+e^- \longrightarrow$ hadron + x, we have

$$\frac{d\sigma}{d\omega d\cos\theta} = \frac{3}{2}\sigma_1 \omega \left[M\overline{W}_1(\omega) + \frac{\cos^2\theta}{4} \nu\overline{W}_2(\omega)\right] \tag{16}$$

where $\omega = \frac{2M\nu}{q^2}$, σ_1 is the total cross section for $e^+e^- \longrightarrow \mu^+\mu^-$. (14) can be rewritten as

$$\langle n\rangle R = 3 \int_{\frac{M}{Q}}^{1} \omega\left[M\overline{W}_1(\omega) + \frac{1}{6}\nu\overline{W}_2(\omega)\right] d\omega . \tag{17}$$

Similarly

$$R = \frac{3}{2} \int_{\frac{M}{Q}}^{1} \omega^2\left[M\overline{W}_1(\omega) + \frac{1}{6}\nu\overline{W}_2(\omega)\right] d\omega . \tag{18}$$

Since it have been shown that, under our hypothesis, $\overline{W}_1$, and $\nu\overline{W}_2$ are function only of ω, any energy dependence of $R(Q^2)$ must come from the lower limit of the integration. Suppose $R(Q^2) \sim \log Q^2$. Then the integrand of (18) must behave as $\frac{1}{\omega}$ and the integrand of (17) behave as $\frac{1}{\omega^2}$. This yields

$$\langle n\rangle \sim Q/\log Q. \tag{19}$$

To summarize, our hypothesis for the parton model implies scaling for νW_2, MW_1, $\nu\overline{W}_2$, $M\overline{W}_1$. This implies that if R(Q) increases faster than log Q, than <n> must increase faster than Q/log Q. Physically, the increase in R(Q) must come from very slow hadrons in the laboratory frame.

In evitably, it is hard to detect very slow hadrons. A careful meassurement of the multiplicity is needed. If R(Q) continues to rise and the multiplicity does not increase linearly in Q, our hypothesis is violated in nature. Our hypothesis is weaker than any of the assumptions commonly used in the parton model. Therefore, if R(Q) continues to rise and the multiplicity of hadrons in the final state does not increase appropriately, the parton model requires a major overhaul.

Acknowledgement

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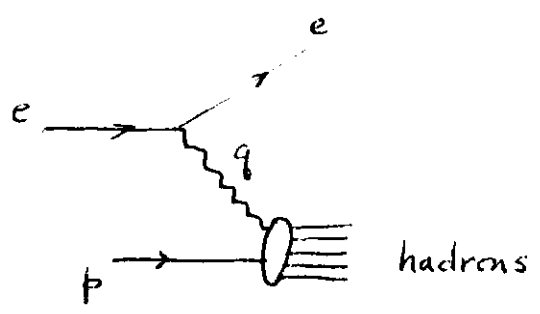
1. Miller et. al. Phys. Rev. D5, 528 (1973)
2. R. P. Feynman Photon Hadron Interaction W. A. Benjamin Inc. (1972) and references there in.
3. See ref. 2
4. See Ref. 2
5. A. Litke, et al. Phys. Rev. Letters 30, 1189 (1973)
6. ($\pm$) corresponds to $\binom{\text{in}}{\text{out}}$ hadron states. For single particles state it serves to distinguish hadron states from parton states.
7. For detail discussion on this condition see A. I. SANDA NAL preprint NAL PUB-73/36 - THY (1973)
8. See Ref. 7
9. For details see A. I. SANDA SLAC - PUB - 1279 (1973)
10. C. E. Detar, D. Z. Freedman and G. Veneziano, Phys. Rev. D4, 906 (1971)



$$M\nu = p\cdot q$$
$$Q^2 = -q^2$$

Fig 1.

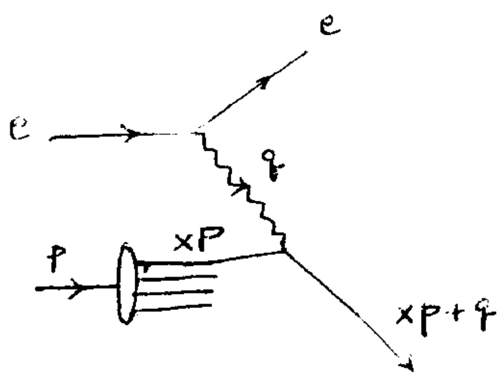


Fig 2.

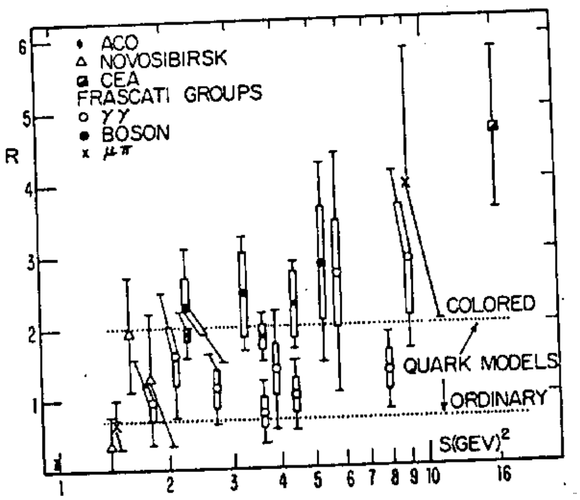

ACO
NOVOSIBIRSK
CEA
FRASCATI GROUPS
γγ
BOSON
μπ
R
COLORED
QUARK MODELS
ORDINARY
S(GEV)²

Fig. 5

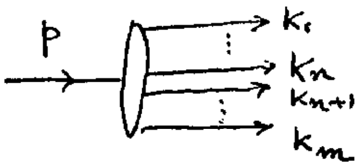

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Fig. 6

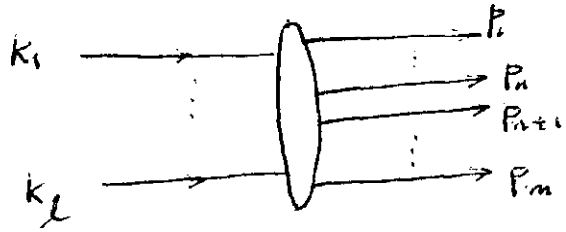


Fig. 7.

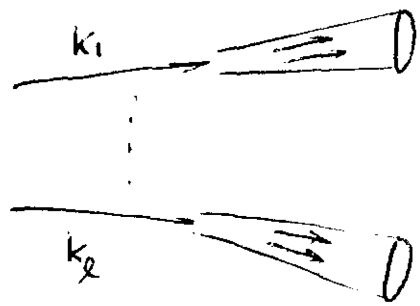


Fig. 8

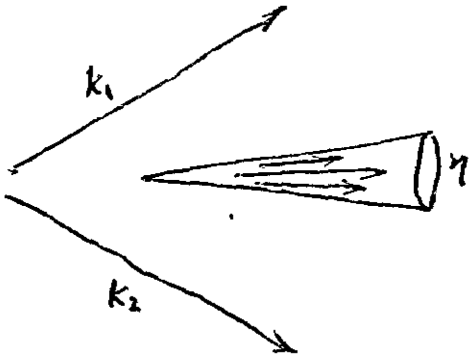


Fig 9

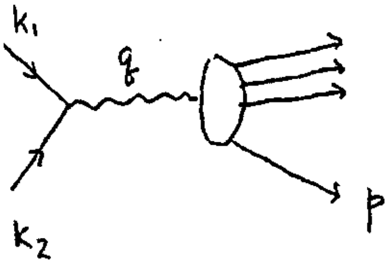
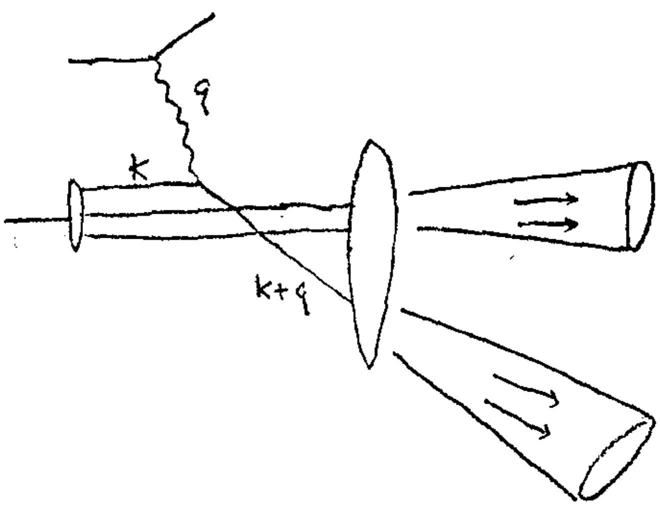


Fig 10



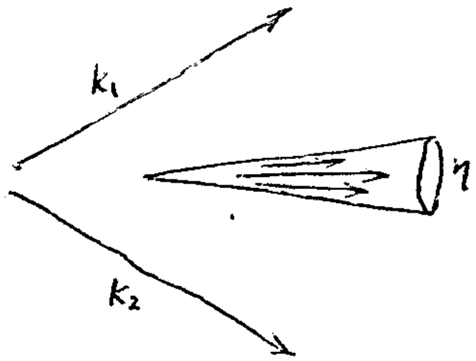


Fig 9

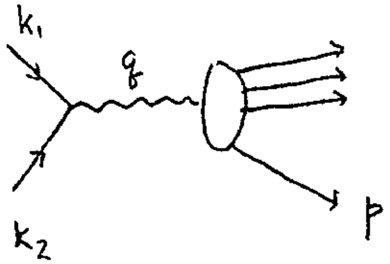


Fig 10

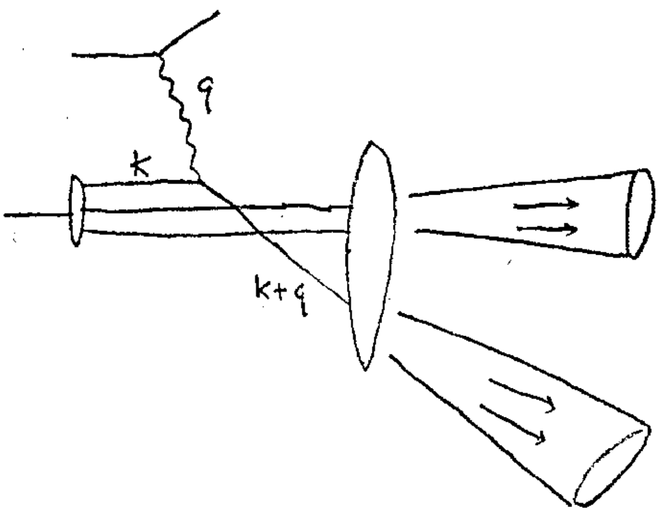


Fig 11