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Status of Gauge Theories of Weak Interactions

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I.

We have heard from Dr. Morrison[1] about the CERN Gargamelle experiment on $\nu_\mu + e \rightarrow \nu_\mu + e$ and from Dr. Nagashima[2] on NAL neutrino experiments, especially on the bound on heavy lepton mass they can now deduce. The situation is not yet completely clear-cut, but we hope that experimental searches along these lines will eventually shed some light on the mechanism of weak interactions and, in particular, on the correctness of various models based on gauge principles for combined weak and electromagnetic interactions.[3] In this talk I would like to review various attempts to incorporate hadrons into these models and problems that arise, and other developments.

II.

There have been proposed several models which are satisfactory and aesthetically appealing with respect to leptonic weak interactions.[4] It is when we attempt to incorporate hadrons into these schemes that we encounter certain uneasiness. The canon one must keep in mind is that in order to construct a renormalizable gauge model of weak and electromagnetic interactions, the gauge symmetry of these interactions should not be broken by strong interactions in the Lagrangian. Thus the observed violation of the weak symmetry must arise entirely from a spontaneous breaking. When the strangeness changing semileptonic decays are ignored,(i. e., if the Cabibbo angle were zero), this does not present any difficulty: one can treat the proton-like and neutron-like constituents of hadrons as a doublet with respect to the weak gauge symmetry, for example; then strong interactions preserve this isospin symmetry.

Cabibbo taught us how to allow strangeness changing semileptonic decays: the basic objects that participate in these reactions are the pront-like constituents (p) and the combination (n_c) of the neutron-like (n), and Λ-like (λ) constituents

$$n_c = n \cos\theta + \lambda \sin\theta$$

which results from an SU(3) rotation from n. This means that, with respect to the weak symmetry, p and n_c must belong to the same multiplet; but surely strong interactions do not respect such a symmetry. In most existing models, this clash between the weak (which treats p and n_c as members of a multiplet) and the strong (which treats p and n as a doublet) symmetries is accommodated by the intervention of Higgs scalar mesons, so that the Lagrangian is invariant with respect to the weak symmetry, but the existence of nonvanishing vacuum expectation values of scalar fields makes n and λ eigenstates of the constituent mass matrix.

The inclusion of strangeness at this level does not present much of a difficulty, even though one is apt to be lead to a model in which strong interaction symmetries are artificial. The real difficulty is that the nature does not show any sign of strangeness changing neutral current most of these models which naively incorporate Cabibbo's idea would predict. Of course, in models such as Georgi and Glashow's where the only neutral current is the electromagnetic current this problem does not arise, but second order induced effects which mimic neutral current may still be a problem. We shall deal with this topic later. In any case, the charged weak hadronic current is of the form

$$j_\mu = \bar{p}\gamma_\mu(1-\gamma_5)(n\cos\Theta + \lambda\sin\Theta) + \ldots \tag{1}$$

where ... indicates contributions to this current from other constituents of hadrons. In a gauge theory, the charges $Q = \int d^3x\, j_0(\underset{\sim}{x}, 0)$ must form a Lie algebra. The commutation of Q and $Q^\dagger$ is one of the form

$$\begin{aligned}[Q, Q^\dagger] = {} & p^\dagger(1-\gamma_5)p - n^\dagger(1-\gamma_5)n\cos\Theta \\ & - \lambda^\dagger(1-\gamma_5)\lambda\sin\Theta \\ & - [n^\dagger(1-\gamma_5)\lambda + \lambda^\dagger(1-\gamma_5)n]\cos\Theta\sin\Theta \\ & + \ldots\end{aligned} \tag{2}$$

Now the right hand side of Eq. 2 must be a linear combination of the electromagnetic current and neutral currents which couple to massive neutral vector bosons. Thus, if the strangeness changing term $n^{\dagger}(1-\gamma_5)\lambda$ + h.c. is not cancelled on the right hand side of Eq. 2, there will be strangeness-changing neutral decays comparable in strength to charged decays. Experimental upper bounds on the former are very stringent. For example,

$$\left[\frac{\Gamma(K^+ \to \pi^+ \nu\bar{\nu})}{\Gamma(K^+ \to \pi^0 \bar{e}\nu)}\right]^{\frac{1}{2}} < 5.0 \times 10^{-3},$$

$$\left[\frac{\Gamma(K_L^0 \to \mu\bar{\mu})}{\Gamma(K^+ \to \bar{\mu}\nu)}\right]^{\frac{1}{2}} < 2.6 \times 10^{-5} \qquad (3)$$

In all realistic models, the strangeness changing term should be banished. In the literature, there are several distinct ways proposed to this end: (1) Glashow-Iliopoulos-Maiani (GIM) scheme.[5] By this scheme we shall understand a class of strategies which postulate and make use of extra hadronic constituent(s) to cancel the strangeness changing term on the right hand side of Eq. 2. In its simplest form an extra quark p' is postulated and two doubels are formed thus:

$$\begin{pmatrix} p \\ n_c \end{pmatrix} \qquad\qquad \begin{pmatrix} p' \\ \lambda_c \end{pmatrix}$$

where $\lambda_c = -n \sin\theta + \lambda\cos\theta$. These two weak isospin doubets appear symmetrically in the expressions for currents, so that neutral currents will contain the combination

$$n_c^\dagger n_c + \lambda_c^\dagger \lambda_c = \bar{n}n + \bar{\lambda}\lambda$$

which is strangeness conserving.

(2) Bars-Halpern-Yoshimura scheme.[6] In this scheme, strong interactions are mediated by $SU(3) \times SU(3)$ gauge bosons, perhaps identifiable with the ρ- and A1-multiplets. The leptonic weak interactions are mediated by gauge bosons of the Weinberg-Salam type. The semileptonic and nonleptonic interactions are pictured as arising from the direct coupling between the two kinds of gauge bosons, caused by the nonvanishing vacuum expectation values of a new class of scalar fields which carry both hadronic and leptonic quantum numbers. To lowest order this scheme is very similar to the field algebra scheme of Lee, Weinberg and Zumino. In this scheme strangeness-changing neutral decays are banished by forbidding the conversion of the K^* and KA vector mesons into the Z (heavy neutral vectors) boson. This is accomplished by judiciously arranging the self interactions of the new class of scalar fields. This scheme, while very ingeneous, is much too complicated for a straightforward analysis.

(3) Use of the Han-Nambu,[7] or color quark[8] degrees of freedom. When one has more than one triplet of quarks, it is possible to avoid the strangeness changing current either by arranging the $\Delta S = 0$ charged currents to commute with the $|\Delta S| = 1$ charge currents [for example,[9] by taking $j_\mu(\Delta S=0) = \bar{p}_1\gamma_\mu(1-\gamma_5)n_1$ and $j_\mu(\Delta S=1) = \rho_2\gamma_\mu(1-\gamma_5)n_2$], or

by arranging them so that the $|\Delta S| = 1$ terms in neutral currents have a nontrivial Han-Nambu, or color quantum number. In the former case, the group structure underlying the gauge symmetry is $0(4) \approx 0(3) \times 0(3)$ or its extentions[10] and there have to be two sets of charged intermediate bosons, one set coupling to the $\Delta S = 0$ charged currents, the other to the $|\Delta S| = 1$ ones. In the latter case, the matrix elements of the $|\Delta S| = 1$ terms in neutral currents vanish between two observed physical states which are Han-Nambu- or color-neutral. In either case, weak interactions violate the Han-Nambu or color symmetry which may be fatal to some models of strong interactions (see below).

(4) Pseudo-Cabibbo theory. Here one assumes[11] that the $\Delta S=0$ currents are of the V-A form while the $|\Delta S| = 1$ currents are V+A. Clearly the $\Delta S = 0$ and $|\Delta S| = 1$ currents commute since the former are made of left hand constituent fermions, while the latter right handed ones. In models based on 0(4) as the one discussed here or the one discussed in the last paragraph, the Cabibbo angle is typically related to the mass ratio of the two kinds of intermediate bosons.

III.

An important fact, sometimes even embarrassing, about a renormalizable theory of weak interactions is that higher-order effects can be computed and compared with experiment. Thus, even after the

$|\Delta S| = 1$ neutral currents is eliminated by any of the schemes discussed, or even in those models where there is no neutral current (other than the electromagnetic one), it is necessary that the predicted rates for processes like $K^+ \to \pi^+ + \nu + \bar{\nu}$ or $K_L \to \mu + \bar{\mu}$ come out comfortably compared to the upper bounds cited previously. Another similar number to keep in mind is the $K_L - K_S$ mass difference. In fact, in a naive version of the Georgi-Glashow model including hadrons, these quantities turned out rather large, i.e., these are of order $G_F\alpha$ in amplitudes.[12] To suppress these effects it was found necessary to invoke an GIM scheme again, this time to suppress <u>induced</u> neutral current effects, at the price of including more fundamental constituents of hadrons.

A related topic is the size of parity- and strangeness violation to order α. The nature of unified gauge theories demands that we treat weak and electromagnetic corrections to hadronic processes on the same footing; this is because the gauge invariance of the final results demands it, and as we shall see presently, weak corrections may be comparable to electromagnetic ones. If so, there is the danger that the theory predicts parity or strangeness violations at the level of electromagnetic effects.

To appreciate this matter at least qualitatively, let us assume that the masses of hadronic constituents are negligible compared to the mass scales of intermediate vector bosons and physical Higgs scalars.

In this approximation, we can safely ignore the couplings of Higgs scalars to quarks. In such a case weak connections to hadronic processes take the form

$$I = e^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - M^2} f(k, p, \ldots) \tag{4}$$

where f(k,...) is a matrix element of the time-ordered product of two currents between the initial and final hadronic states, and M is the mass of an intermediate boson. The coupling constant that goes with this term is $\alpha \sim e^2$ by the nature of a unified theory. Now if the integral

$$\int d^4k \, f(k, p, \ldots) \tag{5}$$

is convergent, then

$$I \sim -\frac{e^2}{M^2} \int \frac{d^4k}{(2\pi)^4} f(k, p, \ldots)$$

so that the weak effect is of order $e^2/M^2 \sim G_F$ and there will be no parity or strangeness violation of order α. If the integral in (5) is divergent, but the integral I of (4) is convergent, there will in general be a parity or strangeness violating effect of order α, and a theory which produces such an effect should be rejected. But more often one encounters a situation in which some of the integrals of the form (4) are divergent. Since we are dealing here with a renormalizable theory, there are counterterms in the Lagrangian which will eliminate the divergence in the diagram corresponding to I and/or its subdiagrams.

At first sight it may appear that in such a case one can make the size of I small enough, by a judicious choice of counterterms. Here, however, a caution is in order. In a renormalizable gauge theory, there are in general certain relations among counterterms which are dictated by gauge invariance, and therefore the number of counterterms is limited. Therefore, the question really is whether it is possible in a given theory to eliminate <u>all</u> parity and strangeness violating effects of order α by adjusting the available set of independent counterterms. To me, it seems this question can be settled only after a careful examination of individual models.[13]

As emphasized by Weinberg[14] recently there is a class of theories for which the above question can be answered in the affirmative. Theories of this class have as their symmetry the direct product of strong interaction symmetry G_{st} and weak (and electromagnetic) symmetry G_{wk} which act on two distinct indices of hadronic constituents and which are both gauge invariance of the second kind. The quark-gluon model in which the vector gluon is coupled to the conserved quark number, and the color quark model with an octet of color gauge bosons are examples of this class.[15] Let me briefly discuss the first example since the mathematics involved there is much simpler. The Lagrangian of the quark-gluon model which exhibits all the counterterms for hadrons is

$$L = \bar{q}\, Z_2 i\gamma_\mu \partial^\mu q - \bar{q}\, \bar{Z}_2^{-\frac{1}{2}}(M + \delta M) Z_2^{\frac{1}{2}}\, q$$

$$+ g\, \bar{q}\, Z_1 \gamma_\mu G^\mu q + Z_3 \left(-\tfrac{1}{4} G^{\mu\nu} G_{\mu\nu} + \tfrac{1}{2}\mu^2 G_\mu^2\right) + L_{wk}$$

where q is the renormalized quark field written as a column matrix, the renormalization constants Z_1 and Z_2 are written as matrices whose entries may involve γ_5, g is the renormalized gluon coupling constant, and $\bar{Z}_2 = \beta Z_2 \beta$. Now, by appropriate choices of δM and Z_2, the parity and strangeness violating effects due to L_{wk} in the quark propagators are made to vanish at the pole, and because $Z_2 = Z_1$ by the Ward identity, this choice makes the gluon-quark vertex parity and strangeness conserving as well when the quarks are on the mass shell and at zero momentum transfer. Parity and strangeness violating effects off mass-shell are suppressed at least by a factor of $(m_q/M)^2$ where m_q is a typical quark mass, except in the renormalized mass matrix which is of the form

$$M = \begin{pmatrix} m_p & & \\ & m_n & \\ & & m_\lambda \end{pmatrix}.$$

Since in general $m_p \neq m_n$ (i.e., there is no reason for $m_p = m_n$), the only effect of order α of the weak interaction origin which survives is an isospin breaking, the kind of non-electromagnetic isospin breaking proposed often in connection with $\Delta I = 1$ mass differences and the η-decay. Unfortunately, this model doesn't really resolve the η-decay puzzle,[16] but a detailed discussion of this topic will make us stray far afield. We have not reproduced here Weinberg's detailed analysis that the counterterms exhibited in (5) are sufficient to remove divergences from all weak connections to physical hadronic processes (not just from quark propagators and vertices), but perhaps this point is intuitively plausible.

The moral of this rather pedantic discussion should be clear: in a model of this class, most of apparent weak effects of order α are "transformed away" by gauge transformations afforded by the gauge symmetry of strong interactions.

IV.

Because of lack of time I cannot review the current status of the proof that spontaneously broken gauge theories are both unitary and renormalizable. After long discussions with J. Zinn-Justin, G. 't Hooft and M. Veltman this summer in Europe, I feel, and I think my colleagues will agree with me, that the renormalizability of anomaly-free gauge theories, with or without spontaneous symmetry breaking, is in a very good shape, perhaps in as good a shape as that of quantum electrodynamics was in the sixties. There are, of course, a number of points in the argument which can be improved, as to rigor, at the hands of more mathematically minded field theorists. The renormalizability of gauge theories can now be discussed much more economically and generally by the use of Ward-Takahaslin identities for irreducible vertices,[17] rather than for Green's functions. 't Hooft and Veltman[18] are making progress in applying many of the techniques and insights gained from gauge theories to gravitation, a path pioneered by R. P. Feynman[19] and B. S. de Witt.[20]

V.

I will mention in passing several new developments in gauge theories.

The first is attempts to include the observed CP violation in gauge theories. There are two classes of such attempts, from the viewpoint of strategy, and not from the size of CP violation predicted. One attempt, due to Mohapatra[21] and Pais[22] separately, is to put CP violation by hand, so to speak, in the Lagrangian; this is a nontrivial task, since, in a gauge theory more so than in other theories, one must be certain that CP violation is not just illusory, i.e., it cannot be transformed away by redefinitions of fields. Another scheme is due to T. D. Lee,[23] and is perhaps more intriguing and attractive. Here CP violation arises spontaneously due to nonvanishing vacuum expectation values of two independent scalar fields which cannot be made real simultaneously.

The second is attempts to construct spontaneously broken gauge theories _senza_ Higgs, i.e., without scalar mesons. The desirability of such a theory is obvious in economy. Jackiw and Johnson,[24] and

Cornwall and Norton[25] consider an abelian model with fermions and study the possibility and consequences that the bilinear form $\bar{\psi}\,\psi$ develops a nonvanishing vacuum expectation value. This question can only be tackled in a nonperturbative framework and much of the mathematical questions involved is very similar to those that arise in the Baker-Johnson-Wiley electrodynamics. They conclude within the context of the model considered that such a mechanism is possible, and the consequences are precisely what one expects on general grounds. Extention of this approach to non-abelian cases and construction of models of particle interactions based on this mechanism are clearly some of the outstanding problems.

The last, and perhaps the most exciting new development is the realization that some class of non-abelian gauge theories is asymptotically free,[26] i.e., for large Euclidean external momenta Green's functions of such theories exhibit the power dependence of free field Green's functions modulo computable logarithmic factors. More technically, it is the observation that in these theories the Callan-Symanzik β function has a zero at $g=0$ which is ultraviolet stable. I understand that a group at Princeton, and others elsewhere are pursuing the implications of non-abelian gauge models of strong interactions on SLAC scaling. [I must say, however, that I, for one, remains unconvinced about the connection between asymptotic freedom and SLAC scaling.] The full potentiality of this observation on model building and field theoretic analysis of gauge theories is only beginning to be studied, and there may be a surprise in store for us along this path.

VI.

In conclusion, after a considerable expense of hard work, it seems that we do not yet have a unified gauge model of weak and electromagnetic interactions which is completely satisfactory from <u>all</u> angles. In the meantime, we have learned a lot about necessary ingredients of a satisfactory theory. I have resisted for a long time the suggestion of my distinguished mentor, Professor C. N. Yang, that we may not yet have all the necessary ingredients of an ultimate theory of weak interactions - whatever they are (perhaps in the sense that the Schroedinger equation and the Coulomb potential are the ultimate key to understanding the Hydrogen atom; never mind the Lamb shift for the moment). I must confess I am now more receptive to his suggestion. But I remain optimistic that the developments of the last few years taught us something useful and led us nearer to our goal, and the principle of spontaneously broken gauge symmetry is an important ingredient of that ultimate theory.

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