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Strangeness Exchange Resonances

HARRY J. LIPKIN
National Accelerator Laboratory, Batavia, Illinois 60510

Argonne National Laboratory, Argonne, Illinois 60439

and

Weizmann Institute of Science, Rehovot, Israel

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Harry J. Lipkin

I. INTRODUCTION

1.1 Strangeness Exchange and Strangeness Analog Resonances

When a kaon strikes a nucleus and one or two fast pions are emitted a unit of strangeness has been transferred to the nucleus to make a hypernuclear state. Some examples of such reactions are

$$K^- + (Z, A) \to \pi^- + {}_{\Lambda}(Z, A) \tag{1a}$$

$$K^- + (Z, A) \to \pi^0 + {}_{\Lambda}(Z - 1, A) \tag{1b}$$

$$K^- + (Z, A) \to \pi^+ + {}_{\Sigma}(Z - 2, A) \tag{1c}$$

and

$$K^- + (Z, A) \to \pi^+ + \pi^- + {}_{\Lambda}(Z - 1, A). \tag{2}$$

If the momentum transfer to the nuclear target is very small there is a high probability that the strangeness has been transferred to a single nucleon inside the nucleus, transforming it into a hyperon without changing the wave function of the nucleon. The probability of producing hypernuclear resonant continuum states by such strangeness exchange processes was first pointed out in 1964. However this paper was ignored as it was published in an obscure journal which is not widely read.[1] Additional theoretical predictions of such resonant states[2] were also forgotten by the time the first experimental indications for continuum excited states of hypernuclei were observed.[3] At that time a more detailed theoretical discussion was given which included some quantitative predictions for excitation energies of these states.[4] Now that new experimental evidence has been presented for the existence of these states,[5] it is time to review the theoretical background.

Strangeness exchange can occur on any nucleon in a complex nucleus and can thus produce many different states. There is also the possibility of coherent excitation of a particular linear combination of states, each of which has a different nucleon in the nucleus changed into a hyperon. One such linear combination has been called the strangeness analog state, by analogy with isobaric analog states[6, 7] in nuclei. Strangeness exchange processes are similar to charge exchange processes in nuclei where a neutron is changed into a proton without changing the wave function. The isobaric analog state is a coherent linear combination of such states. Symmetries like isospin can enhance production of coherent analog states and can give them interesting properties.

The analog state is characterized by its permutation symmetry. The wave function for the target nucleus is required by the Pauli principle to be antisymmetric with respect to all the neutrons and all the protons in the nucleus. No such symmetry principle exists between the hyperon and the nucleon, and all possible permutation states are allowed. The strangeness analog state for the reaction (1a) is one in which a neutron in the target is changed into a hyperon and the exact wave function is unchanged, including permutation symmetry; i. e., the wave function is also antisymmetric with respect to interchange of any neutron and the hyperon. Similarly for reactions (1b) and (2), which involve charge exchange in addition to strangeness exchange, the strangeness analog state has a proton changed into a hyperon with the wave function antisymmetric with respect to interchange of any proton with the hyperon. In this paper we use the term "strangeness analog" state or resonance for the coherent excitation which preserves permutation symmetry. We use the more general term "strangeness exchange resonance" for any excitation produced by single nucleon strangeness exchange.

1.2 The Role of the Nucleon Hole

When a nucleon has been changed into a hyperon without changing its wave function, the state originally occupied by the nucleon is vacant and appears as a nucleon hole. This hole is crucial to the understanding of the produced continuum state. There are two approaches to the analysis of the nucleon hole. In standard hypernuclear spectroscopy both the experimentalists and the theorists wait for this situation to settle down. The strong nuclear interactions produce transitions which fill the nucleon hole, the hyperon descends to a low-lying level, excess energy is given off by evaporating particles, and an equilibrium state is reached which is stable against further decay by strong interactions.

It is also possible to ask what happens immediately after the kaon comes in and the pion goes out, before the strong interactions have had a chance to settle down. The hypernucleus is then in an unstable excited state, with a nucleon hole somewhere and a hyperon in some level which may be far from its ground state. Such "particle-hole" excitations are common in nuclear physics, and are treated by the methods of nuclear reaction theory. Experiments in which the energy spectrum of the decay pions is measured are studying just this short time behavior of the excited hypernucleus. To understand these experiments, we must change our approach from the long-time-scale approach of classical hypernuclear spectroscopy to the short-time-scale approach of reaction theory.

1.3 Digression—Be Prepared for Surprises

We should also be prepared for surprises. The states which are produced in specific reactions may have very different properties from those of familiar low-lying bound states. The example of nuclear fission should be remembered as a state with a peculiar collective oscillation completely unexpected at the time of its discovery. There may also be peculiar excitation modes of hypernuclei which are unexpected on the basis of our present knowledge of nuclear and hypernuclear structure. Such new phenomena may well be much more exciting than the phenomena presently predicted by theorists. Experimentalists should be on the lookout for them.

One possible indication of new hypernuclear effects induced by mesons is the apparent tendency of nuclei to emit alpha particles under the influence of mesons. Experiments with stopped kaons in nickel and copper by a Carnegie-Mellon-Argonne collaboration[8] at the ZGS showed an anomalously large number of nuclear gamma rays identified as coming from nuclei which differed from the target by one, two and three alpha particles, as compared to very weak evidence for the production of nuclei with intermediate masses and charges. A recent Argonne experiment[9] examined the gamma rays produced after a ^{60}Ni target was bombarded with 500 MeV/c pions. The cross sections for the production of the first excited states of ^{56}Fe and ^{52}Cr (formed by removing one and two alpha particles, respectively, from ^{60}Ni) were about 7 mb, half of the cross section for producing the corresponding excited state in the target nucleus ^{60}Ni itself. The cross section for producing the first excited state of ^{48}Ti (three alpha particles removed) was about 3.5 mb or only 1/4 of the ^{60}Ni cross section. No lines were seen with a limit of 2 mb for ^{59}Ni or ^{59}Co which would be produced by the removal of one nucleon. The large production of nuclei which would be formed by the emission of two and three alpha particles from the target is very mysterious and has no counterpart in other nuclear reactions. So far there has been no satisfactory explanation for this effect. While one can imagine mechanisms which would knock out one alpha particle from a nucleus, the removal of two or three alpha particles without any additional nucleons and the comparative absence of final states in which unequal even numbers of protons and neutrons are emitted defies any conventional explanation. The true explanation could be very exciting, and this effect warrants further investigation. More details of these experiments are given elsewhere in these proceedings.[8,9] They are not directly relevant to strangeness exchange resonances and are presented only as an example of new unpredicted phenomena which may be found.

1.4 Spectroscopy vs. Reactions

There is a considerable difference in approach between the descriptions of the long-lived states of the discrete spectrum and those of the short-lived continuum states in nuclei and hypernuclei. Nuclear physics investigations are conveniently divided into (1) nuclear spectroscopy, which studies low-lying discrete states which are stable against breakup and (2) nuclear reactions, which studies the continuum. So far experimental techniques for studying hypernuclei have emphasized the discrete long-lived states. Thus hypernuclear physics has developed primarily in the direction of spectroscopy while only a small amount of work on continuum states has been done with stopped kaons. The availability of high-intensity kaon beams now makes possible detailed investigations of hypernuclear reactions, as we have seen in the experiments reported by Bressani.[5] To understand these new data, we need to develop the reaction approach.

In nuclear and hypernuclear spectroscopy the aim is to provide approximate descriptions of the observed discrete states which are exact eigenfunctions of the nuclear Hamiltonian. In nuclear and hypernuclear reactions the eigenfunctions of the Hamiltonian lie in a continuum. This infinite set of states is of no interest to anybody. The states of interest are those produced in particular scattering experiments and which give rise to nontrivial structure in the dependence of the scattering cross section on kinematic variables like energy and scattering angle. Although it is tempting to describe resonant continuum states as if they were stable, using the standard techniques of spectroscopy, and to imagine that their coupling to decay channels can somehow be neglected, this approach is misleading and not strictly correct. It is impossible to separate the excited continuum states produced in a given experiment from the reaction which produced them; the two must be studied together.

II. COMMON NUCLEAR EXCITATIONS

To introduce the reaction approach it is instructive to examine four common nuclear excitations and discuss their relevance to hypernuclear strangeness exchange resonances: (1) Single-particle and single-hole excitations produced by stripping and pickup reactions, (2) collective particle-hole excitations like the giant dipole resonance produced by photons, (3) deep-lying hole excitations like those produced by (p, 2p) reactions, and (4) isobaric analog resonances. In all discussions of these excitations the nuclear shell model plays a dominant role and the states are described either as containing particles in well-defined shell model orbits or as simple coherent linear combinations of such shell model states. However, the particular states considered depend strongly upon the reaction mechanism.

2.1 Stripping and Pickup

Single-particle and single-hole excitations are produced by stripping or pickup reactions on a nuclear target, which either add a particle with well-defined quantum numbers to the nucleus or remove one from it. These states are not eigenfunctions of the Hamiltonian; they eventually decay into the continuum. The decay widths depend upon the residual interactions between the excited particle or hole and the remaining nucleons in the nucleus. The residual interactions may also split the state into several components to give a fine structure to the resonance. These continuum states are described by giving the distribution in energy of the "single particle strength," rather than giving energy levels and their properties as in nuclear spectroscopy.

In hypernuclear reactions, such single-particle excitations are not of interest, since there is no simple way to "strip" a hyperon into a nucleus. Although hypernuclear physicists tend to think of a hypernucleus as a nucleus to which a hyperon has been added, this is not how hypernuclei are made in the laboratory. The reaction producing the hypernuclear state is always one of strangeness exchange, in which a nucleon in the nucleus is changed into a hyperon. A nucleon hole is therefore always produced in any reaction where a hypernucleus is produced. The presence of the nucleon hole is crucial to the understanding of the hypernuclear reaction.

2.2 Collective Particle-Hole Excitations

Collective particle-hole excitations are produced as electromagnetic giant resonances by photon absorption which moves one nucleon in a nucleus from one single-particle state to another. Since the electromagnetic current is a single nucleon operator and since the electromagnetic interaction is weak and acts only in first order, only a single particle-hole pair is excited by photon absorption. However, the photon can be absorbed by any nucleon in the nucleus. Thus the state produced is a coherent linear combination of states, each of which has only one excited particle-hole pair. The relative magnitude and phase of each state is determined by the structure of the nuclear wave function and by the electromagnetic interaction. The degree with which this coherence persists in time after the state is formed depends upon the interactions of the individual nucleons in the nucleus, and in particular on the scattering of the excited particle-hole pair into other particle-hole states. These interactions give rise to the observed widths and structures for the giant electromagnetic resonances having definite quantum numbers, e.g., E1, E3 and M1. Here again the states are described in terms of the distribution in energy of a given strength function, rather than as well-defined approximate eigenfunctions of the Hamiltonian.

2.3 Deep-Lying Hole States

Deep-lying hole excitations are produced by reactions[10] like (p, 2p) in which the kinematics of the initial and final states are carefully chosen to correspond to the knockout of a nucleon from the nucleus with minimum momentum transfer to the rest of the nucleons. The state originally produced thus has one particle removed but all the other particles in their original states. If the shell model were exact, this state would be an exact eigenfunction of the nuclear Hamiltonian and would live forever. The residual interactions which are not taken into account in the simplest single-particle shell model allow other nucleons to jump into the hole left by the knocked-out particle. These interactions thus determine the lifetime and width of the hole state. It was one of the surprises of nuclear physics that these deep-lying hole states were found experimentally to live long enough to be observable as peaks in the spectra of the outgoing nucleons.

Such states have been observed at excitation energies as high as 60 MeV with widths of only a few MeV.[11] This suggests the possibility of observing states with 60 MeV excitation also in hypernuclei. Since the Σ-Λ mass difference is only 80 MeV there might be a possibility of observing Σ hypernuclear states as resonances with widths of a few MeV.

2.4 Isobaric Analog States

Isobaric analog states are produced by charge-exchange reactions which change a neutron in a nucleus into a proton with small momentum transfer, thus leaving the nucleon wave function unchanged. Such states are of particular interest because of isospin symmetry. Operating on a nuclear ground state with an isospin step operator produces a state which is a coherent linear combination of states in which one neutron has been changed into a proton or vice versa and which preserves the permutation symmetry of the target nucleus. This "isobaric analog state" would be an exact eigenfunction of the nuclear Hamiltonian if isospin were an exact symmetry. Because isospin symmetry is broken by the Coulomb interaction, converting a neutron into a proton raises the Coulomb energy and the analog of a discrete ground state is usually found in the continuum. The state decays with a finite lifetime and has a finite width because of the Coulomb interaction which breaks isospin symmetry. Because the residual Coulomb interaction between nucleons is much smaller than the strong nuclear interactions, these isobaric analog states have a much smaller width than other continuum excitations at the same energy which can decay by isospin-conserving strong interactions.

Isobaric analog resonances are of great interest to nuclear physicists because they combine a simple theoretical structure with a source of rich and easily analyzed data. Their structure is much more easily analyzed than that of neighboring continuum states because they have a simple theoretical description obtained by isospin symmetry operations on nuclear ground states. But unlike nuclear ground states, which do not decay, these high-lying continuum states also have many decay channels open and provide rich experimental data to indicate their composition. Thus they give the nuclear structure physicist the "best of both worlds"— the easily analyzed structure of a nuclear ground state together with the rich data of continuum states.

2.5 Comparison of Strangeness Exchange with Common Nuclear Excitations

Strangeness analog states were first proposed because of the similarity with isobaric analog states. Instead of a charge exchange reaction in which a neutron is changed into a proton, we have a strangeness exchange reaction in which a nucleon is changed into a hyperon. A group-theoretical formalism analogous to isospin can describe such strangeness exchanges in the nucleus. However, it is somewhat misleading to take the analogy too seriously, because there is no strong interaction symmetry relating nucleons and hyperons which is anywhere nearly as good as isospin. Thus if the strangeness analog resonances are found with reasonable widths, the dynamical reason for the narrow width must be something other than symmetry.

The particle-hole and deep-lying-hole excitations in nuclei may be more relevant to the physics of hypernuclear strangeness exchange reactions than the isobaric analog states. As mentioned above, a nucleon hole is always produced whenever a nucleon in a nucleus is changed into a hyperon by a strangeness exchange reaction. If the active nucleon is in the lowest s orbit and the hyperon is produced in the same lowest orbit, the resulting hypernucleus has the hyperon in its ground state, but has a deep-lying nucleon hole. The subsequent decay of this state should be governed by interactions which are qualitatively similar to those which determine the widths of nuclear hole states. Since such hole states with a width of several MeV have been reported at energies up to 60 MeV it is reasonable to expect to see excitations at these energies in hypernuclei.

If the active nucleon in a strangeness exchange reaction is not in the lowest s orbit, and the hyperon produced is left in the same orbit as the initial nucleon, the resultant hypernucleus has a hyperon in an excited orbit as well as a nucleon hole. The excitation is thus a "particle-hole" excitation and can be compared with nuclear particle-hole excitations. If there are many such particle-hole states with very nearly the same energy, then a comparatively small residual particle-hole interaction can mix these states to produce giant resonances.

In the case of nuclear isobaric analog resonances, the different individual particle-hole excitations produced by charge exchange have nearly the same energy because it costs about the same energy to change a neutron into a proton without changing the orbit of the nucleon, regardless of which orbit the nucleon is in. The energy is just the difference between the depths of the nuclear wells for neutrons and protons. In the hypernuclear case, a similar situation obtains in a simple shell model description with nucleons and hyperons moving in a potential which comes from the average interaction with all other particles. The range of the potential is determined by the size of the nucleus and is independent of whether the baryon considered is a nucleon or a hyperon. The difference between nucleon-nucleon and nucleon-hyperon forces appears only in the depth of the potential, not in the range. The nucleon and hyperon wave functions inside the well should be very similar if the principal difference between the Hamiltonians of nucleons and hyperons is only the depth of the well. The energy required to change a nucleon into a hyperon in the same orbit should only be the difference in well depths and should be independent of the orbit of the active baryon. There are therefore many particle-hole excitations produced by strangeness exchange which are very nearly degenerate. These will therefore mix as a result of the particle-hole interaction and coherent giant resonances can be produced. The isobaric analog state is the particular linear combination of nuclear charge exchange particle-hole excitations involving a neutron hole and an excited proton which has the permutation symmetry of the target. The strangeness analog state is the particular linear combination of hypernuclear strangeness exchange particle-hole excitations involving a nucleon hole and an excited hyperon which has the permutation symmetry of the target.

2.6 Three Aspects of Continuum Excitations

All of these excitations involve a decaying continuum state produced by a particular reaction mechanism on a given nuclear target. There are thus three aspects of the excitation which must be studied: (1) the properties of the target nucleus, (2) the reaction mechanism and (3) the decay of the continuum state. The same Hamiltonian interactions and dynamical approximations used in treating nuclear and hypernuclear spectroscopy are relevant to the study of the target nucleus and the decay of the continuum state. However the continuum state and the target in a strangeness exchange process are not of the same nucleus and the ground state of the hypernucleus is not necessarily simply related to either. The continuum state has a simple relation with the nuclear target, sometimes called the parent state, while the hypernuclear ground state may not be produced at all in this reaction.

An important difference between the treatment of the parent state and the continuum state is in the role of models and symmetries like the shell model and isospin. The parent state is treated by the standard methods of nuclear spectroscopy and is described by zero order wave functions which are calculated with the assumption that such models and symmetries are exact. For the decay of the continuum state the violations of the model and of the symmetry are crucial, since if these were exact such continuum states might never decay at all. For example, if the shell model and isospin symmetry were exact the isobaric analog state would be a stable stationary state.

In the treatment of the reaction mechanism the interaction forces and Hamiltonian are completely unimportant. In the first approximation the continuum state is produced on the wave function of the parent state by an operator which is completely determined by the reaction mechanism; e.g., an operator which changes a nucleon into a hyperon without changing its wave function for the case of strangeness exchange processes. In the next approximation the dynamics of the reaction process enter in determining the probability or cross section for the reaction and the relative probability of exciting different nucleons in the nucleus. In the reactions (1), for example, the dynamics determine the probability that the incident kaon is absorbed before penetrating into the interior of the nucleus and the probability that the outgoing pions may produce secondary interactions.

III. SYMMETRIES AND MODELS IN STRANGENESS EXCHANGE REACTIONS

3.1 Exact and Approximate Symmetries and Models

In strangeness exchange reactions the models and symmetries used in spectroscopy play an important role but have a quite different interpretation. We now consider in detail the role of exact symmetries like rotational and space-inversion invariance, approximate symmetries like isospin and approximate models like the nuclear shell model.

The exact symmetries provide classification schemes for the initial and final states, which must be eigenstates of angular momentum and parity. They also impose selection rules on the changes in angular momentum and parity produced by the reaction, depending upon the kinematics. This is discussed in detail in Section IV.

There is never a simple answer to the question of the validity of approximate models and symmetries. The same model or symmetry may be good in one context and very bad in another. The only way to decide whether a model or symmetry is good in a particular case is to use your head. In strangeness exchange reactions models and symmetries appear in ways which are unfamiliar to classical spectroscopy and it is dangerous to jump to conclusions.

3.2 The Nuclear Shell Model

The shell model is relevant to nuclear and hypernuclear reactions because the target nucleus on which the experiment is performed is generally described to a good approximation by the shell model and because the reaction mechanism generally changes the state of only one particle and leaves the remaining particles in their original states. The state produced in the reaction is therefore described simply in terms of one or a few shell model configurations. An excitation in which the state of a single particle having well-defined quantum numbers is changed is called a single particle excitation. Excitations which are coherent linear combinations of many single particle excitations are often called collective excitations. Both types of excitations occur.

The shell model states produced by the reaction mechanism are not eigenfunctions of the Hamiltonian. They therefore have a finite lifetime and a decay width. If the nuclear shell model were exact these shell model states would be exact eigenfunctions and would not decay. Their decay is therefore determined by the residual interactions which break the shell model.

3.3 Approximate Symmetries and Transition Operators

The example of the nuclear shell model illustrates the three ways in which an approximate symmetry or model affects a reaction process. The initial state is described to a good approximation by the model because the interactions which are neglected in the model are small in some sense. The reaction mechanism is simply described in the framework of the model because the operator which formally describes the transition induced by the reaction mechanism has a simple form, in this case one which changes the state of only one particle. For the decay of the continuum state it is the interactions which are left out of the model which play the dominant role. Similar considerations apply to approximate symmetries like isospin. The different roles of the symmetry algebra are conveniently expressed by the notation of Dothan, Gell-Mann and Ne'eman[13] of approximate symmetry algebras (ASA) and transition operator algebras (TOA). The role of isospin in isobaric analog states is a very instructive example.

Isospin symmetry is relevant to isobaric analog states because (1) the target nucleus is an isospin eigenstate to a good approximation and (2) the reaction mechanism which changes a neutron into a proton without changing the wave function is described to a good approximation by an operator which is an isospin generator. Note that isospin appears in defining the state of the target as an approximate symmetry operator of an ASA but in the reaction mechanism it is a transition operator of a TOA. These two aspects of isospin are completely different from one another. That an experiment can be chosen in which the transition acts like an isospin generator is not related to the charge independence of nuclear forces. "Symmetry" says that neutrons and protons have the same interactions. "Transition operator" says that the experiment changes a neutron into a proton without changing the wave function appreciably, whether the interactions are the same or not. This difference is crucial, and <u>both</u> aspects of isospin are essential to the proper understanding of isobaric analog states.

3.4 Symmetries in Strangeness Exchange

Strangeness analog states are very similar to isobaric analog states. The relevant transition operator changes a nucleon into a hyperon and the relevant approximate symmetry assumes that nucleon-nucleon and nucleon-hyperon interactions are the same. At first sight the assumption of equal hyperon-nucleon and nucleon-nucleon interactions seems much worse than the isospin assumption of charge independence of nuclear forces. But it is here that one must use one's head. In considering the properties of the target nucleus the situation is the reverse.

All target nuclei are very peculiar extreme cases of hypernuclei which contain only nucleons and no hyperons. Their wave functions are therefore determined completely by the nucleon-nucleon interaction and are completely independent of the hyperon-nucleon interaction. We introduce no error in the nuclear wave function if we assume a fictitious world in which hyperon-nucleon interactions are exactly equal to the known nucleon-nucleon interactions. This situation does not occur for isospin because all stable complex nuclei contain <u>both</u> neutrons <u>and</u> protons. If a nucleus existed which contained 7 neutrons and no protons we would know that it was a pure isospin eigenstate with isospin 7/2 and a wave function completely independent of the Coulomb interaction, which breaks isospin but has no effect on a system containing only neutrons.

We thus find the paradoxical situation where charge independence of nuclear forces seems to be a much poorer approximation than the strangeness independence of baryon-baryon forces in describing the states of target nuclei in experiments, whereas charge independence is much better than strangeness independence in discussing the properties of the two-body forces in the baryon-baryon system. This is because the target nuclei contain approximately equal numbers of neutrons and protons and no hyperons. The small differences between proton-proton, proton-neutron and neutron-neutron forces affect the wave functions for these nuclei while the comparatively large differences between nucleon-nucleon, nucleon-hyperon and hyperon-hyperon forces have no effect because the nuclei contain no hyperons.

The question remains whether the transition operator describing the reaction mechanism is in some approximation a generator of the strangeness-changing symmetry algebra analogous to isospin. This question is considered in terms of the Sakata SU(3) symmetry[13] and is discussed in detail in Section VI. There is also the question of how the difference between hyperon-nucleon and nucleon-nucleon interactions which breaks the symmetry affects the decay of the continuum state. These questions are considered in more detail below.

IV. KINEMATICS, IMPULSE APPROXIMATION AND SELECTION RULES IN THE REACTION MECHANISM

4.1 The Role of Quasi-Two-Body Kinematics

Kinematics plays a crucial role in strangeness exchange reactions. To excite strangeness exchange states the kinematics must be chosen for minimum momentum transfer so that one nucleon can undergo the transition while the rest of the nucleus remains undisturbed. As an analogous example consider the (p, 2p) experiment in which an incident high-energy proton knocks out a proton from a nucleus in a quasi-elastic two-body collision. The outgoing protons are measured at energies and angles corresponding to the kinematics of such a quasi-free collision.

At first sight the use of simple quasi-two-body kinematics appears to be an assumption of the impulse approximation. This seems unreasonable since protons interact strongly with nucleons in the nucleus and it seems unlikely that a knock-out process can occur without any additional interaction of the rest of the nucleus with the incident proton or one of the outgoing protons. The crucial feature of this type of experiment is the concentration of desired events in a very tiny portion of phase space where the background is very nearly zero. In any event where additional interactions occur between the incoming or outgoing protons and the rest of the nucleus there will no longer be two outgoing protons with energies and angles nearly satisfying the kinematic constraints for quasi-two-body scattering. These additional interactions thus remove the event from consideration but do not increase background in the desired region of phase space. The relevant background is so low that an observable effect can be obtained even if the signal is reduced by absorption by an order of magnitude. Thus kinematics allows the experiment to select just those events for which the impulse approximation is apparently valid, and to separate these events from a background in which additional interactions take place.

The direct analog of the (p, 2p) reaction for the case of strangeness exchange is the reaction (2) with the energy and angle of the outgoing pions chosen to correspond to zero momentum transfer to the nucleus. Feshbach and Kerman[2] have pointed out that zero momentum transfer is also obtainable for the exothermic reactions (1) with single pion emission if the incident kaon has the proper momentum. If the incident kaon and the outgoing pion have the same momentum, there is no momentum transfer to the nucleus, while the energy transfer can be anything from zero to the K-π mass difference, depending upon the momentum of the incident kaon. The beautiful experiments reported[5] at this meeting have made use of the Feshbach-Kerman kinematics.

4.2 Angular Momentum, Parity and Symmetry Selection Rules

The kinematical constraints from angular momentum and parity conservation are also of interest. If the kinematics are chosen for zero momentum transfer, there is also zero angular momentum transfer. Thus for an even-even target nucleus, with $J^P = 0^+$, only 0^+ excited states can be reached

$$\Delta p = 0 \rightarrow \Delta J = 0; \text{ only } 0^+ \rightarrow 0^+ \text{ allowed.} \tag{3a}$$

For finite momentum transfer, with the pion emitted in the direction of the momentum transfer (i.e., forward) only natural parity exchange ($P = (-1)^J$) is allowed. Thus for an even-even target, only 0^+, 1^-, 2^+, etc. states can be reached.

$$\Delta p \text{ forward} \rightarrow P = (-1)^{\Delta J}; \text{ only } 0^+ \rightarrow 0^+, 1^-, 2^+, \ldots \text{ allowed.} \tag{3b}$$

Note that this selection rule applies also to experiments with stopped kaons, where the pion is always emitted in the direction of the momentum transfer.

Other symmetries can provide additional selection rules. For example, if the transition operator transforms in a definite manner under isospin, the change in isospin is restricted. The strangeness exchange transition operator which annihilates a nucleon and creates a Λ hyperon transforms under isospin like a $T = \frac{1}{2}$ isospin or, since the annihilated nucleon carries isospin $\frac{1}{2}$ and the Λ has zero isospin. This leads to the following selection rule for strangeness exchange states,

$$|\Delta T| = \tfrac{1}{2} \text{ for strangeness exchange transitions.} \tag{3c}$$

If the transition operator is a generator of a higher symmetry, the excited state produced must be a member of the same symmetry multiplet as the initial state. This applies to isospin symmetry in the excitation of isobaric analog states, and to the Sakata SU(3) symmetry in the excitation of strangeness analog states.

4.3 Detailed Analysis of the Case of ^{16}O

Let us now examine in detail how symmetries restrict the relative magnitudes and phases of the different single-particle strangeness exchange transitions in a given experiment and determine the conditions for the production of an analog state. As an example we consider the case of ^{16}O for which the new experimental data are available. The ground state of ^{16}O has the $s_{1/2}$, $p_{3/2}$ and $p_{1/2}$ levels filled. A strangeness exchange reaction produces a nucleon hole in one of these shells and a Λ particle in the same shell. The excited states which can appear in these Λ particle-nucleon hole configurations are listed in Table I.

Table I

Strangeness exchange excitations in ^{16}O

Configuration	Spin and parity of excitation			
$s_{1/2}, s_{1/2}^{-1}$	0^+	1^+		
$p_{3/2}, p_{3/2}^{-1}$	0^+	1^+	2^+	3^+
$p_{1/2}, p_{1/2}^{-1}$	0^+	1^+		
$p_{1/2}, p_{3/2}^{-1}$		1^+	2^+	
$p_{3/2}, p_{1/2}^{-1}$		1^+	2^+	

We have included the configurations where the baryon makes a transition between the $p_{1/2}$ and $p_{3/2}$ orbits during the strangeness exchange process. These can be excited at finite momentum transfer since the added momentum can change orbital angular momentum but not spin and can change the couplings of the spins and orbital angular momenta. It is instructive to compare the shell-model configurations for the ground states of ^{16}O, the ground state of the hypernucleus ${}^{16}_{\Lambda}O$, and the excited state ${}^{16}_{\Lambda}O^*$ produced by strangeness exchange, say in the s shell. These are listed in Table II, assuming the jj-coupling shell model.

Table II

Shell Model Configurations for ${}^{16}O$, ${}^{16}_{\Lambda}O$ and ${}^{16}_{\Lambda}O^*$

Nucleus	${}^{16}O$	${}^{16}_{\Lambda}O$	${}^{16}_{\Lambda}O^*$
$p_{1/2}$ shell occupancy	nn pp	n pp	nn pp
$p_{3/2}$ shell occupancy	nnnn pppp	nnnn pppp	nnnn pppp
$s_{1/2}$ shell occupancy	nn pp	Λ nn pp	Λ n pp
J^P	0^+	0^-, 1^-	0^+

Table II illustrates the similarity between the configurations of the parent state and the strangeness exchange state, and the difference between both of these and the hypernuclear ground state. In this case the hypernuclear ground state has opposite parity and cannot be excited at all at zero momentum transfer because of the selection rule (3a). The strangeness exchange state in Table I differs from the ground state of the hypernucleus by having a neutron excited from the $s_{1/2}$ shell to the $p_{1/2}$ shell; it is thus a nucleon particle-hole excitation of the hypernucleus with parity change.

The simplest symmetry rotational invariance plays a very important role in restricting the allowed linear combinations of single-particle excitations produced by a particular reaction. Consider the reaction (1a) on a ${}^{16}O$ target, in which any one of the eight neutrons in the nucleus can be converted into a Λ. Rotational symmetry tells us not to consider the eight individual states in which a neutron in a well-defined single-particle orbit has been transformed into a Λ. Such states are not eigenfunctions of the total angular momentum. Instead we must consider the states listed in Table I which are linear combinations of single-particle-single-hole excitations with the angular momenta of the particle and the hole coupled to a definite total angular momentum. The eight states thus become grouped into three 0^+ states, three 1^+ states, and one each of 2^+ and 3^+. If no momentum is transferred in the process, there is no preferred direction in space defined by the transition, and no angular momentum is transferred. Thus only the three 0^+ states are allowed and all the others forbidden. With finite momentum transfer in the forward direction, only the 0^+ and 2^+ states are allowed and the others forbidden. In this case the two additional 2^+ states from $p_{1/2} \rightarrow p_{3/2}$ transitions can be present, since the momentum transfer which acts only in the orbital space can decouple spin and orbit.

At zero momentum transfer, the particular linear combination of the three allowed 0^+ states which is produced in the reaction is not determined by rotational symmetry alone. A higher symmetry could choose one particular combination. If the strangeness exchange process acts equally on all neutrons in the nucleus it does not change the permutation symmetry of the nuclear wave function and the analog state is produced. In the creation of the analog state wave function $|A\rangle$ all neutrons have an equal probability of being excited.

$$|A\rangle = \frac{1}{2} \{\sqrt{2}\, |p_{3/2}, p_{3/2}{}^{-1}\rangle + |p_{1/2}, 1_{1/2}{}^{-1}\rangle + |s_{1/2}, s_{1/2}{}^{-1}\rangle\} \qquad (4)$$

The statistical factor $\sqrt{2}$ appears because the statistical weight of the $p_{3/2}, p_{3/2}{}^{-1}$ configuration is double that of the others.

If the reaction mechanism does not act equally on all nucleons the state produced is not the analog state (4). However, even with the extreme assumption that the kaon is completely absorbed at the surface of the nucleus and can only excite p-shell nucleons without entering the s shell, the resulting state still has a large overlap with the analog state (4). The state with the $s_{1/2}$ component removed from (4) has an overlap of 87% with the analog state.

4.4 Transitions between $s_{1/2}$ Orbits

There is one case where hypernuclear ground states might be easily excited by strangeness exchange. Suppose the highest orbit for the nuclei in Table II had been a radially excited $s_{1/2}$ orbit instead of a $p_{1/2}$ orbit. Then strangeness exchange in this orbit would produce a state with the same quantum numbers as the hypernuclear ground state, differing only by radial excitation of the Λ. In such a case there might be an appreciable probability for producing the hypernuclear ground state in a strangeness exchange reaction. Because of the differences between the nucleon and Λ wells, the wave functions for the radially excited $s_{1/2}$ nucleon and the ground state $s_{1/2}$ Λ need not be orthogonal and there might be an appreciable overlap. This point might be of particular interest for the production of Σ analog states, as discussed in Section VII.

Strangeness exchange experiments on nuclei with high-lying radially excited $s_{1/2}$ orbits should show this effect as peaks in the outgoing pion spectrum with higher energies than those expected from strangeness analog states. If the $s_{1/2}$ orbit is exactly the last valence orbit, the ground state of the hypernucleus would be produced. If the $s_{1/2}$ orbit is near the top but is not the very last orbit, a low-lying excited state should be produced. Nuclear targets for observing this effect can be selected either by looking at tables of shell model configurations or by looking at experimental data on nuclear pickup reactions to find cases with a weakly bound $s_{1/2}$ nucleon. Since the cross section for exciting the low-lying hypernuclear state depends on the overlap of the $s_{1/2}$ hyperon ground state and $s_{1/2}$ nuclear excited state wave functions which should vanish if the wells are the same, these excitations might provide interesting information on the differences between nucleon and hyperon wells in a nucleus. The relevant overlap integrals could be calculated from theoretical models. Because the Coulomb potential can appreciably affect these overlaps, significant differences can exist between neutron and proton excitations and between Λ, Σ^+ and Σ^- production.

The study of $s_{1/2}$ transitions thus appears to be a promising line of strangeness exchange research.

V. DYNAMICS OF THE DECAY PROCESS

The continuum state produced by strangeness exchange decays into the continuum of two-particle and multiparticle states as a result of the residual interaction between the hyperon and the nucleon. It is always instructive to look at the decay both from the point of view of time development and the complementary energy spectrum. If this state has a relatively long lifetime it appears as a narrow peak in the energy spectrum of the outgoing pions in the reaction (1). If the state decays immediately after it is formed its width in energy is so large that it will not be observable as a peak in the pion spectrum but only contribute a continuous background. In experiments one generally expects to find an intermediate case between these two extremes; the state produced consists of several components one of which decays immediately and the others live long enough to appear as one or more resonant peaks against the continuous background provided by the short-lived component.

5.1 Particle-Hole Interactions and Symmetries

The strangeness exchange state produced by the reaction mechanism is a linear combination of states involving a single hyperon and a nucleon hole. It is convenient to divide the residual interaction into two parts. The first acts only in the space of the Λ-particle-nucleon-hole states, the second connects these states to the other states which are the final states for the decay. The first interaction determines which linear combinations of these particle hole excitations appear as isolated peaks in the energy spectrum and determines the excitation energies and the splittings between the components. The second part of the residual interaction determines the decays and widths of these peaks. If the particle-hole interaction is known exactly, the energies and splittings are obtained by diagonalizing the interaction in the subspace of Λ-particle-nucleon hole states. Any symmetries present simplify this diagonalization.

In the case of ^{16}O shown in Table I rotational symmetry separates the 8 possible Λ-particle-neutron-hole states into the 8 eigenfunctions of total angular momentum. If the reaction mechanism insures that there is no angular momentum transfer, only 0^+ states are produced and the particle-hole interaction matrix relevant to our discussion is the 3×3 matrix in the subspace of 0^+ states. If there are no further symmetries the eigenvectors and eigenvalues of this 3×3 interaction matrix defines 3 states appearing at 3 different excitation energies. These states are defined completely independently of the state produced by the reaction mechanism. The experimental spectrum of the outgoing pions in the reaction (1) then has 3 components having strengths determined by the overlap of the state produced by the reaction mechanism with each of the 3 states which are eigenvectors of the particle-hole interaction matrix. The excitation energies and splittings of these states are determined by the eigenvalues of the interaction matrix.

If an additional symmetry is present, as in the case of isobaric analog resonances, then the analog state will be one of the eigenvectors of the interaction matrix. If, furthermore, the transition operator defined by the reaction mechanism is to a good approximation also a symmetry generator as in the case of isobaric analog resonances, a single peak will be dominant corresponding to the excitation of the analog state. In the case of the strangeness analog state there is no good symmetry like isospin, but other factors like the validity of the nuclear shell model and permutation symmetry of the interaction may still favor the excitation of the analog state.

It is not surprising that the experimental results quoted for ^{16}O show possible evidence for an additional state.[5] Even if the analog transition is dominant, it would not be expected to carry 100% of the reaction strength function. With sufficient sensitivity and high resolution, all three states should be seen. Note that the analog state (4) is a linear combination of the state obtained by adding a $p_{1/2}$ hyperon to the ground state of the hypernucleus ^{15}O and states obtained by adding a hyperon in a different orbit to "excited core" states. All three states obtained by diagonalizing the particle-hole interaction matrix can be expected to be linear combinations of the form (4) with different coefficients, but containing all three components. It is therefore misleading to characterize these excitations as having either a ground state core or an excited core; the two kinds of states will certainly be mixed by the particle-hole interaction.

5.2 Nucleon and Λ Shell Model Potentials

We first consider the diagonal components of the energy in the different shell-model configurations such as the 3 listed in Table I. If the shell model potential for a Λ differs from that for a nucleon moving in the same nucleus only by a different well depth, the diagonal energies are the same for all configurations and represent simply the difference between the well depths for the Λ and the nucleon. This implies that it costs the same energy to change a nucleon into a Λ regardless of the particular shell that the nucleon is in. However, one can also ask whether the well depth is really shell independent; i.e., whether there are nonlocal components in the potential which can make the energy required to turn a nucleon into a Λ in the p shell quite different from the energy required in the s shell. This is still an open question and determines the degree of coherence of the different components of the strangeness exchange state.

If in a case like ^{16}O very different excitation energies are required in the s and p shells, this only destroys the coherence between the s and p components of the wave function (4) for the analog state. The p wave component which is still coherent has an overlap of 87% with the analog state. There are thus two completely different physical effects which can remove the s-wave component from the coherent state produced by strangeness exchange. In the reaction mechanism the kaon can be absorbed before reaching the central s orbit. In the decay a difference in excitation energy of the s and p states can destroy coherence. But both effects leave intact 87% of the analog state.

5.3 Permutation Symmetry and the Residual Interaction

For the nondiagonal part of the interaction, permutation symmetry may favor the analog state. The strangeness analog state is characterized by having the same permutation symmetry as the parent state; i.e., the hyperon is in an antisymmetric state with respect to all of the neutrons in the hypernucleus. If this antisymmetrization maximizes or minimizes the energy, then the analog state is an eigenvector of the interaction matrix by the variational principle. If the most important part of the residual interaction is a two-body force between particles in a relative s-state, permutation symmetry is preserved. The ^3S and ^1S states have different angular momenta as well as permutation symmetry; thus angular momentum conservation requires conservation of permutation symmetry in two-body interactions. The totally antisymmetric state of the many-particle system will thus be an eigenstate of the interaction and will either have the maximum or the minimum eigenvalue depending upon whether the antisymmetric ^1S two-body interaction is greater or smaller than the symmetric ^3S interaction. In the nucleon-nucleon case the symmetric state has the lower energy; thus the antisymmetrized isobaric analog state is pushed <u>up</u> by the residual interaction. For the ΛN interaction the antisymmetric singlet state is lower for s-wave interactions and higher partial waves seem to have considerably smaller interactions although not very much information is available.[14] This would indicate that the antisymmetric analog state can be expected to be an eigenstate of the interaction matrix with the <u>minimum</u> energy. In contrast to the isobaric analog state, the strangeness analog state is pushed <u>down</u> in energy by the interactions.[4]

5.4 Estimates of Excitation Energies and Decay Widths*

With the aid of available data on properties of nuclei and hypernuclei, the excitation energies of the strangeness analog states has

*The treatment in this section follows that of reference 4, where additional details can be found.

been estimated[4] to be about 10 MeV in light nuclei like ^{12}C and about 30 MeV in heavy nuclei. The difference in energy between the analog state and the parent state is just the difference between the Λ and nucleon well depths, since the permutation symmetries of the two states are the same. However, the excitation energy of the analog state is specified with respect to the ground state of that hypernucleus. We must therefore add to the well depth the energy difference between the parent and hypernuclear ground states; namely the energy required to remove a Λ from the s-orbit and to add a nucleon into the highest orbit. This process costs the Λ binding energy B_Λ for the hypernucleus and gains the nucleon separation energy B_n for the parent nucleus. Adding the difference $V_{n\Lambda}$ between the depths of the Λ and nucleon wells then gives the excitation energy of the analog state as shown in Fig. 1, taken from ref. 4.

A similar result is obtained by considering the analog state as a particle-hole excitation from the hypernuclear ground state and correcting for the symmetry energy due to antisymmetrization of the hyperon with the neutrons. In the shell model approximation where all particle-hole states involving the same shells are degenerate, estimates of the particle-hole excitation energy are obtained from the experimental single particle energies. The analog state is thus lower than this unperturbed particle-hole energy by the symmetry energy.

The matrix elements of the residual interaction between the analog state and the continuum determine the lifetime and width of the analog state. Without detailed knowledge of the residual interaction it is difficult to calculate these widths quantitatively. However, qualitative estimates are obtainable by comparison with comparable nuclear excitations. When a nucleon in a stable nucleus is changed into a Λ without changing its wave function, the resulting wave function would be an exact eigenfunction of the nuclear Hamiltonian if the nucleon-nucleon and Λ-nucleon interactions were equal. Thus it is not the full residual interaction which contributes to the decay of strangeness exchange states, but only the difference between hyperon-nucleon and nucleon-nucleon residual interactions. This is much larger than the differences between proton-proton, neutron-proton and neutron-neutron interactions produced by the isospin-symmetry-breaking Coulomb interaction which determines the decay of isobaric analog states. However, it is also considerably smaller than the entire nucleon-nucleon residual interaction which determines the decay of those nuclear continuum excitations which are not isobaric analog states. Thus experimental data on these two types of nuclear excitations can give lower and upper limits on the expected widths for hypernuclear strangeness exchange resonances.

There are two kinds of decay processes, one in which the Λ escapes from the nucleus and one in which a nucleon escapes. The Λ-escape probability is determined by the residual interaction of the Λ with all other nucleons in the nucleus. These add together to give an average field in which the Λ moves. The escape of the Λ is therefore due to the change in this average field when the baryon changes from a nucleon to a Λ. The diagonal matrix element of this change in average field is just the well depth differences which gives the energy shift, as discussed above. The off-diagonal elements describe the escape of the Λ from the nucleus. This average field can be compared with the Coulomb field which determines the energy shifts and widths of isobaric analog states. The average symmetry-breaking field in the hypernuclear case is of the same order of magnitude as the Coulomb field in the isobaric case, even though the Coulomb interaction is apparently much weaker. Because of its long range, the average Coulomb interaction seen by a nucleon is due to all other nucleons while the average symmetry-breaking nuclear interaction is short range and is due only to nearest neighbors. This can be checked quantitatively by noting that the Coulomb energy for a proton in a heavy nucleus and the difference between nuclear and Λ well depths are comparable in magnitude. Thus Λ-escape widths should be comparable to escape widths of isobaric analog states in heavy nuclei.

Nucleon escapes involve single two-body matrix elements which are not summed over particles to give an average field. The only matrix elements relevant to a particular nucleon escape is that involving the Λ and the nucleon that escapes. Since these two-body matrix elements are not summed, they should be less important than Λ escape and should give a smaller contribution to the width. For an independent check on this estimate, the experimental escape widths of comparable excitations in nuclei can be used as upper limits on hypernuclear nucleon escape widths, as discussed above.

VI. SAKATA SU(3) SYMMETRY*

6.1 Introduction

The octet model of unitary symmetry, which has been successful in particle physics, is not useful for nuclear and hypernuclear physics. The principal reason is the ΛΣ mass difference of 80 MeV, which is small on the energy scale of particle physics but very large on the energy scale of nuclear binding energies. Hypernuclei observed in nature are known to contain a Λ and not a Σ since a Σ could decay into a Λ and provide an excitation energy of 80 MeV to the nucleus. The octet-model description of hypernuclei would classify them in states containing linear combinations of Λ's and Σ's.

The old Sakata triplet model[13] attempted to create all hadron states out of the basic npΛ triplet. This model is no longer relevant to hadron physics and has been superseded by the octet. However, the complex nuclei and hypernuclei having strangeness 0 and -1 are indeed composed only of members of the basic Sakata triplet. It is therefore useful to consider hypernuclear spectroscopy from the point of view of the Sakata model by using the triplet rather than the octet model of unitary symmetry.

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*The treatment in this section follows that of reference 4, where additional details can be found.

Triplet unitary symmetry assumes that the Λ-nucleon interaction is the same as the nucleon-nucleon interaction. Although this symmetry is a very poor approximation for the two-body system (in contrast to charge independence, which is very good and violated only by small electromagnetic contributions), the Sakata symmetry is useful for the many-particle states having strangeness 0 and -1 (the states that so far have been found to be relevant to hypernuclear spectroscopy). As we have seen in Section 3.4, all strangeness zero nuclei are automatically good eigenstates of Sakata SU(3), and the transition operator describing strangeness exchange at zero momentum transfer is a generator of Sakata SU(3). In the discussion of the decay of strangeness exchange resonances, we have seen that the primary breaking of Sakata SU(3) symmetry is due to the difference between the nucleon-nucleus and the Λ-nucleus interactions rather than the two-body interactions. The validity of the shell-model description of complex nuclei allows us to replace the major portion of the sum over two-body interaction by an average field in which the single nucleon or hyperon moves. The radius of the baryon-nucleus interaction potential is determined by the size of the nucleus and is independent of whether the baryon considered is a nucleon or a Λ. The latter affects only the depth of the well. Thus the major breaking of the Sakata symmetry for these states results from the differences in the well depths seen by the nucleon and the Λ. This gives a large diagonal contribution to the energy of a nucleus or hypernucleus, but may not change the wave function too much from that given under the assumption of Sakata SU(3) symmetry. The symmetry-breaking effects that would mix in other SU(3) representations and destroy the purity of the Sakata state are smaller effects depending on differences in the shape of the neutron and Λ wells and differences in the residual two-body interaction.

6.2 U-Spin and V-Spin Analog States

It is convenient to describe Sakata SU(3) in terms of the three SU(3) subgroups commonly called isospin, U spin and V spin which operate respectively in the n-p, Λ-n and p-Λ spaces, as shown in Fig. 2. The isospin, U-spin and V-spin operators satisfy commutation rules like angular momenta among themselves. We define the raising and lowering operators with the conventions of particle physics, so that

$$T_+ |n\rangle = |p\rangle \tag{5a}$$

$$T_- |p\rangle = |n\rangle \tag{5b}$$

$$U_+ |\Lambda\rangle = |n\rangle \tag{5c}$$

$$U_- |n\rangle = |\Lambda\rangle \tag{5d}$$

$$V_+ |p\rangle = |\Lambda\rangle \tag{5e}$$

$$V_- |\Lambda\rangle = |p\rangle \tag{5f}$$

The stable nuclei used as targets and available for the parent state $|\pi\rangle$ in a strangeness exchange reaction have no Λ's. Thus

$$U_+ |\pi\rangle = V_- |\pi\rangle = 0. \tag{6a}$$

For nuclei which are on the neutron-rich end of an isospin multiplet,

$$T_- |\pi\rangle = 0 \tag{6b}$$

and

$$T \ |\pi\rangle = -T_\pi |\pi\rangle \tag{6c}$$

where T_π is the isospin of the state $|\pi\rangle$. Note that the U-spin and V-spin relations (6a) are satisfied exactly while the isospin relation (6b) is only approximate because of the isospin symmetry breaking by the Coulomb interaction. This expresses the peculiar symmetry properties of target nuclei discussed in Section 3.3, where U spin and V spin are better than isospin.

Three of the six step operators (5) thus annihilate the target nucleus.

For targets of isospin zero, like ^{12}C and ^{16}O, all isospin operators annihilate the state, and the remaining two U-spin and V-spin step operators, U_- and V_+, create the strangeness analog states commonly called U-spin and V-spin analog states. The U-spin and V-spin step operators transform under isospin like $T = \frac{1}{2}$ isospinors as discussed in Section 3.4, and the selection rule (3c) applies. Thus the U-spin and V-spin analog states for a $T = 0$ target have $T = \frac{1}{2}$ and are members of the same isospin doublet.

For targets with $T_z = -T \neq 0$, the isospin step operator T_+ creates the isobaric analog state, which has $T = T_\pi$, $T_z = T_\pi + 1$. The U- and V-spin analog states can have two possible isospins, $T_\pi \pm \frac{1}{2}$. Since the U-spin analog state has $T_z = -T_\pi + \frac{1}{2}$, while the V-spin analog state has $T_z = -T_\pi - \frac{1}{2}$, the V-spin analog state is an isospin eigenstate with isospin $T_\pi + \frac{1}{2}$, while the U-spin analog is a mixture of the two isospins $T_\pi \pm \frac{1}{2}$. The exact mixing of the isospins can be calculated from SU(3) algebra. If the Sakata SU(3) symmetry were exact, all the step operators (5) would commute with the Hamiltonian, all analog states would be degenerate with the parent, and the U-spin analog state would be a linear combination of two isospin eigenstates which would individually be degenerate eigenfunctions of the Hamiltonian.

Here the breaking of SU(3) becomes crucial. The eigenstates of the real hypernuclear Hamiltonian are the isospin eigenstates (we neglect the Coulomb interaction). These states are not degenerate, but are split by the symmetry energy, which arises because the two isospin eigenstates have different permutation symmetries with respect to interchanging neutrons and protons. In the SU(3) symmetry limit, this symmetry energy is exactly compensated by the difference in the Λ-nucleon interaction in the two states, which also have different permutation symmetries with respect to the Λ and the other nucleons. However, the Λ-nucleon interaction is much weaker than the nucleon-nucleon interaction, and its dependence on permutation symmetry goes in the <u>opposite direction</u> from that of the nucleon-nucleon interaction (the antisymmetric state lies lowest). The effect of the Λ-nucleon interaction on the splitting can be neglected in first order, and the conventional nucleon symmetry energy can be used to give an estimate of the splitting between the two components of the U-spin analog state.

Figure 3, taken from ref. 4, shows the energy level diagram for the parent state, the V-spin analog $|S_p\rangle$ and the two components $|S_>\rangle$ and $|S_<\rangle$ of the U-spin analog having greater and lesser isospin, respectively. The diagram is taken from ref. 4 which gives a more detailed analysis of the SU(3) Sakata algebra and its application to analog states.

The discussion of Section 5.4 and the energy level diagram of Fig. 1 applies to the V-spin analog state $|S_p\rangle$.

The higher isospin component $|S_>\rangle$ of the U-spin analog state is the isobaric analog of the V-spin analog state $|S_p\rangle$,

$$|S_>\rangle = T_+|S_p\rangle. \qquad (7)$$

The energy of $|S_>\rangle$ thus differs from that of $|S_p\rangle$ just by the Coulomb energy of the additional proton. The energy of $|S_<\rangle$ differs from that of $|S_>\rangle$ by the symmetry energy.

VII. SIGMA HYPERNUCLEAR STATES

The possibility of observing sigma hypernuclear states should be reexamined in view of the new experimental techniques available for producing hypernuclear continuum excitations. Stable nuclei containing Σ's are not expected because the Σ in a nucleus can turn into a Λ with a release of 80 MeV in reactions like

$$\Sigma^- + p \rightarrow \Lambda + n \qquad (8a)$$

$$\Sigma^+ + n \rightarrow \Lambda + p. \qquad (8b)$$

These reactions (8) are always possible, except in a system of maximal isospin; e.g., a Σ^- and one or more neutrons with no protons. So far searches for such low mass hypernuclear states with maximum isospin have been unsuccessful.

The available decay energy of Σ hypernuclei must be of the order of 80 MeV because of the Σ-Λ mass difference. The decays of these states via the reactions (8) can be compared with the decays of deep-lying nucleon-hole states via the comparable nucleon-nucleon scattering reaction. Since such nuclear excitations with energies of 60 MeV have been observed in (p, 2p) experiments, it is reasonable to expect Σ strangeness exchange resonances with excitation energies of 80 MeV to be observable.

The double charge exchange reaction (1c) provides a convenient mechanism and a unique signature for the production of Σ strangeness-exchange resonances. With Feshbach-Kerman kinematics the background should be very small and these states should be easily detected if they are produced at all and have reasonable widths.

The principal factor determining the possibility of observing Σ hypernuclei is the decay width; i.e., whether states exist whose decays are sufficiently slow to allow them to be seen. Since the width depends strongly on the available energy, the most favorable cases are those where binding and Coulomb energy effects in the transition partially compensate for the Σ-Λ mass difference. Decays via the reactions (8) involve a nucleon charge exchange, producing either a proton hole and an excited neutron or vice versa. In heavy nuclei with a neutron excess, the reaction (8b) is possible with the nucleon remaining in the same orbit after n-p charge exchange. In the reaction (8a) with p-n charge exchange the nucleon must jump to a higher orbit in making the transition since all orbits occupied by protons are already occupied by neutrons.

Thus hypernuclei containing a Σ^- should have a longer lifetime than those containing a Σ^+ as the loss in the nuclear binding energy in the reaction (8a) partially compensates for the gain in the Σ-Λ mass difference.

Coulomb effects can provide additional binding for the Σ-hypernuclear state. Although the initial and final states for the reaction (8a) have the same electric charge, the Coulomb energy is less in the initial state if the Σ^- is in the center of the nucleus and the proton is on the surface. Thus if the reaction (8a) occurs with a proton near the center of the nucleus there is a loss in binding energy in changing it into a neutron in an unoccupied state, while if the proton is near the surface there is also a loss in binding because of the increased Coulomb energy. Thus a hypernuclear ground state with a Σ^- in its lowest orbit may have less than 80 MeV available for the decay and may be observable as a resonance with reasonable width if a suitable production mechanism can be found.

Unfortunately, strangeness exchange reactions like (1c) are not very good for producing hypernuclear ground states. They produce a nucleon hole, unless the nucleon undergoing strangeness exchange happens to be in the outer valence shell. The hypernuclear ground state can be produced in a heavy nucleus by a strangeness exchange reaction in which a nucleon in a valence shell is transformed into a Σ and simultaneously jumps to the lowest shell. This seems extremely improbable, except for $s_{1/2}$ valence orbits, but cannot be ruled out completely. If such transitions occur in the reaction (1c) they will be observable in the energy spectrum of the outgoing pions.

A more probable transition in the reaction (1c) is strangeness exchange with the Σ remaining in the same orbit as the initial nucleon. If this is the lowest s orbit, a deep-lying hole state is produced in addition to the Σ; thus the energy of the hole excitation must be added to the Σ-Λ mass difference to give total excitation energies over 100 MeV. This is partially compensated by the Coulomb energy shift in replacing the positively charged proton by the negatively charged Σ^-. The magnitude of this shift can be estimated from the known Coulomb energy shifts in single-charge-exchange transitions between isobaric analog states. These are about 15 MeV in heavy nuclei. One would expect double this shift or 30 MeV from double charge exchange. The shift might be greater in deep interior shells of the nucleus, since isobaric analog transitions always involve the neutron excess orbits in the outer shells. These arguments are not intended to be precise or conclusive, but just to indicate that the numbers are in a reasonable enough ball park to encourage further investigation, both theoretical and experimental.

The decay of a Σ strangeness exchange state with a nucleon hole and a Σ in an inner shell would have to be at least a two-step process. The reaction (8a) which converts a Σ to a Λ cannot fill a proton hole and might create an additional proton hole. This might decrease the width from that estimated from what is known from deep lying nuclear hole states at comparable excitation energies, where the decay can take place in a single transition via the residual nucleon-nucleon interaction.

The one favorable case for the production of a hypernuclear ground state or a very low-lying excited state is that of a nuclear target which has a high-lying $s_{1/2}$ valence orbit. As discussed in Section 4.4, a strangeness exchange on a nucleon in such an $s_{1/2}$ orbit produces an $s_{1/2}$ hyperon with a radially excited wave function which might have appreciable overlap with the ground state hyperon orbit. A nucleus with a weakly bound $s_{1/2}$ orbit thus seems to be the most suitable target for the first attempts to find Σ hypernuclei.

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FIGURE CAPTIONS

Fig. 1. Energy-level diagram for strangeness analog states; (a) in ^{12}C, and (b) in a heavy nucleus.

Fig. 2. Isospin, U-spin and V-spin step operators in the Sakata model.

Fig. 3. Schematic representation of the strangeness analog states and the transitions from the parent.

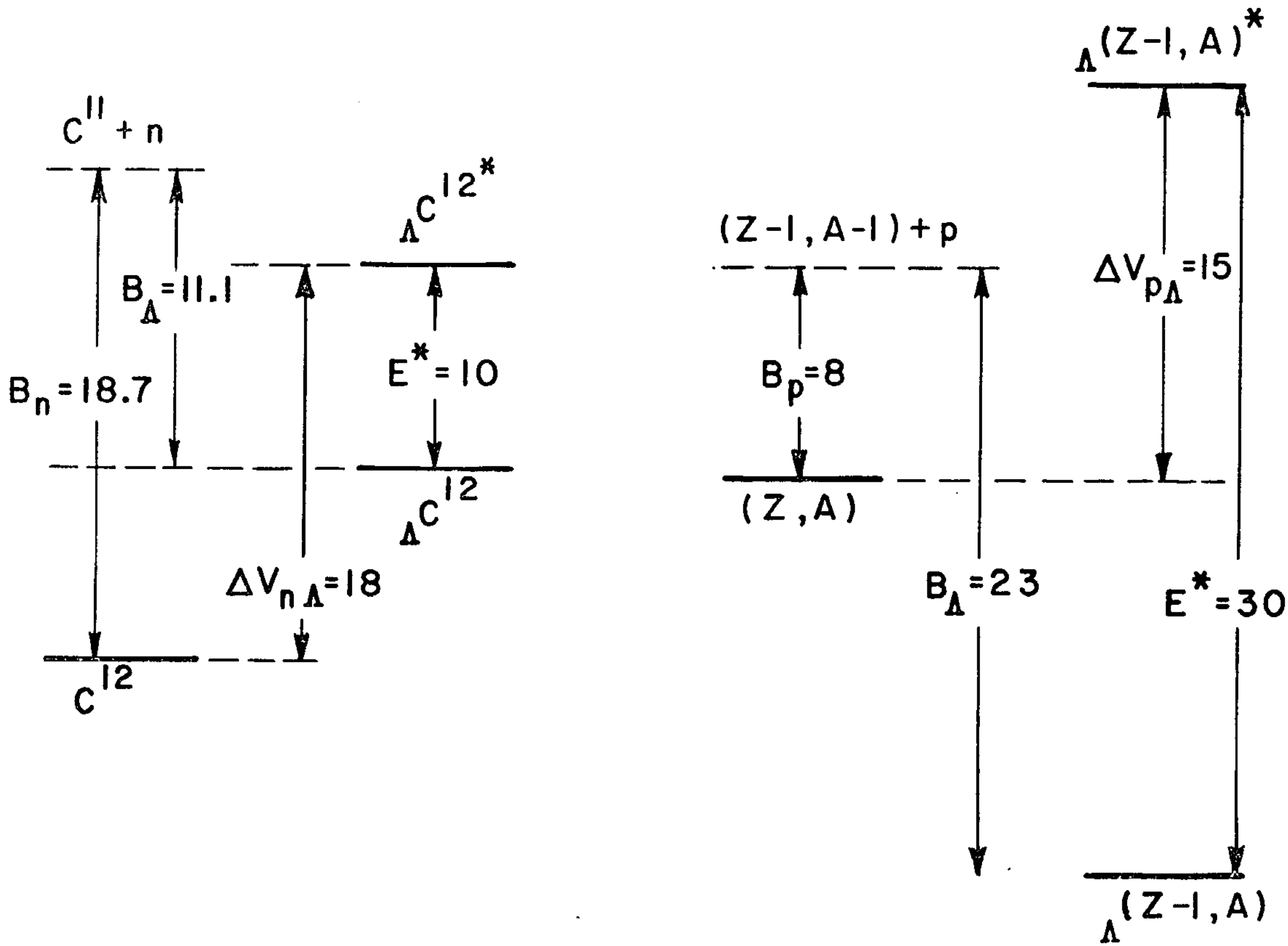

Fig. 1. (Neg No. 209-1195; PHG 9875).

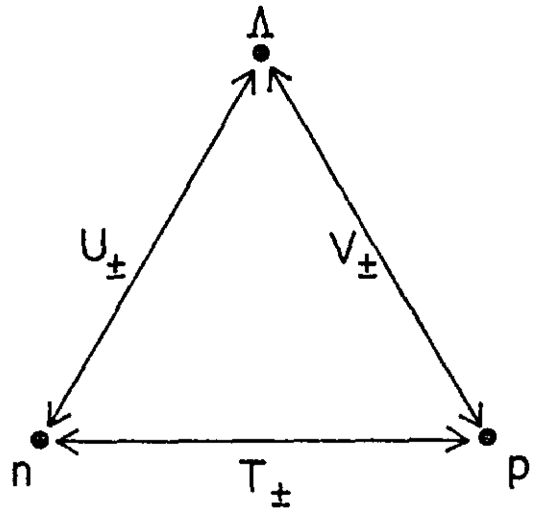

Fig. 2.

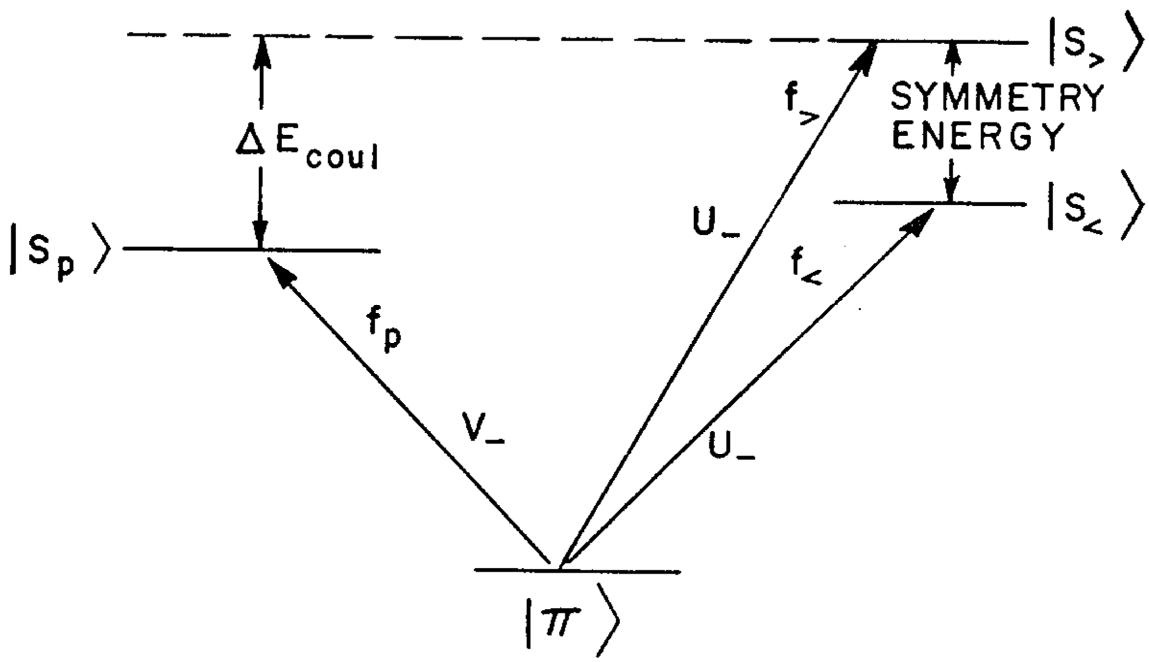

Fig. 3. (Neg. No. 209-1194; PHG 9874).