



Dynamical Issues in Hadron Physics

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Invited talk prepared for the New York Academy of Sciences'
"Conference on Recent Advances in Particle Physics", March 1973.



Today I wish to discuss two topics of central interest in hadron dynamics. The first is diffraction scattering and the view we have of the Pomeron singularity in the complex angular momentum plane. The study of inclusive reactions and the ability to discuss, theoretically at least, Reggeon-particle amplitudes has provided a number of rather strong constraints on the structure that the Pomeron may have. In brief we learn that unitarity in the t-channel forbids the vacuum singularity from being an isolated simple pole with $\alpha(0) = 1$ and finite slope at $t = 0$. This nice uncomplicated picture is one we relinquish with reluctance, but since we must, sorting out the correct solution from among the myriads of complicated alternatives now becomes imperative.

The second topic I will bring here today concerns the attempts to find a space-time description of a spatially extended or composite hadron. This has been carried out in the context of dual resonance models for two reasons: (1) to build a composite hadron with the mass spectrum of dual models (linearly rising Regge trajectories) is a tantalizing beginning point for any hadron theory, (2) it is a natural development in the attempt to formulate a ghost-free version of dual models. A few word description of the status of such theories is that, as far as physics goes, they are, until now, failures. Only in the extraordinarily unphysical world of one time and twenty-five space dimensions along with at least one state of negative mass squared is

one able to give a space-time picture of a hadron with the dual, harmonic oscillator, spectrum. In reviewing these failures, brilliant failures I hasten to add, we learn a great deal on how one might salvage the ideas and bring them into the world we happen to live in.

I. Constraints on Diffraction Phenomena

The observation of nearly constant total cross sections for hadronic collisions long ago focused attention on the structure of partial wave amplitudes in the vicinity of angular momentum $l = 1$ and momentum transfer $t \approx 0$. The particular partial wave amplitude we want to consider is the positive signature amplitude $F(l, t)$ which is gotten from the absorptive part $A(s, t)$ of a spinless two-to-two amplitude by a Mellin transform,

$$F(l, t) = \int_0^\infty ds s^{-l-1} A(s, t), \quad (1)$$

which is a form of the usual Froissart-Gribov formula accurate to leading order in energy s .

The data from $p_{\text{lab}} = 30 \text{ GeV}/c$ up to $p_{\text{lab}} \approx 1500 \text{ GeV}/c$ on proton-proton collisions reveals a remarkably constant total cross section $\sigma_{\text{T}}(s)$, certainly any variation is only of a $\log s$ or $(\log s)^2$ nature. If $\sigma_{\text{T}}(s) \sim \text{const.}$ for large s , then since the optical theorem relates $\sigma_{\text{T}}(s)$ and $A(s, t = 0)$ by

$$\sigma_T(s) = 1/4 A(s, t=0), \quad (2)$$

to $O(1/s)$, we would conclude that $F(\ell, t=0)$ has a simple pole at $\ell = 1$. This "Pomeron" object enters in any channel with vacuum quantum numbers and is by assumption the appropriate description of s -independent behavior of cross sections. The actual evidence on constancy of total cross sections, which I presume will be reviewed by Dr. Sens this afternoon, is very much up in the air. Three experiments have been done at the highest CERN-Intersecting Storage Ring energies. One yields constant σ_T ; two, rising cross sections by powers of $(\log s)$! The determination of these cross sections involves measuring the elastic differential cross section, estimating the ratio of real to imaginary parts of $T_{\text{elastic}}(s, t)$ and extrapolating back to $t = 0$ where (2) is used to extract $\sigma_T(s)$. This is, at best, a procedure which can be very sensitive to systematic mistakes and although σ_T may indeed rise in s , the honest skeptic must suspend his judgement or do his own experiment.

By the way, while waiting about for the experimental dust to settle it is useful to remember that in the ISR experiments the slope parameter $b(s)$ defined by

$$d\sigma_{el}/dt = \left. d\sigma_{el}/dt \right|_{t=0} e^{-b(s)t} \quad (3)$$

is a very precisely determined quantity and remembering that $\sigma_{el}(s) \leq \sigma_T(s)$ the optical theorem provides us with the inequality¹

$$\sigma_T(s) \leq 16\pi b(s). \quad (4)$$

It will be recalled that $b(s)$ at most rises linearly in $\log s$ and, indeed, turns over and becomes rather s independent at ISR energies. $\sigma_T(s)$ cannot rise forever unless $b(s)$ starts up too.

Actually since $b(s) \approx 12/(\text{GeV})^2$ the bound (4) is not remarkably useful since 16π is a big number. Some improvement can be made by noting that $T_{el}(s, t)$ is known to have a very small real part and that $\sigma_{el} \approx \frac{1}{5} \sigma_T$ at ISR or NAL energies. This yields

$$\sigma_T(s) \leq \left[b(s) \text{ in } (\text{GeV})^{-2} \right] 4 \text{ mb}, \quad (5)$$

a relation which should be quite accurate since $\text{Re } T_{el} / \text{Im } T_{el} \leq 0.1$ and is clearly on the verge of being violated unless $b(s)$ starts growing.

Whatever the detailed behavior of $\sigma_T(s)$ it appears it is going to differ from a constant by some $\log s$ factors only and it makes sense to see whether a simple pole in l at $\alpha(0) = 1$ which would give a strictly constant σ_T can be an intelligent starting point for discussion of the detailed l -plane structure in the vicinity of $l = 1$.

In inclusive reactions near the edge of phase space ($|x| \approx 1$ or $|y| \approx \log s$) one observes a very striking leading particle effect. That is taking pp collisions as our universal example, in each hemisphere of the center of mass one observes a very fast proton. Let's concentrate

on such events and recall how one describes them in l-plane language. The inclusive cross section $d\sigma(pp \rightarrow p + x)$ depends on three variables which we'll choose to be s , the momentum transfer t between the observed fast proton and the incident proton moving in the same direction, and M^2 the mass squared of x the unobserved stuff. As s becomes large with t and M^2 held finite, our presumed Pomeron exchange ought to govern the s behavior as²

$$s^2 \frac{d\sigma}{dt dM^2} \underset{t, M^2 \text{ fixed}}{\underset{s \rightarrow \infty}{\sim}} \left(\frac{s}{M^2}\right)^{2\alpha(t)} \frac{|\beta(t)|^2}{16\pi} F(t, M^2), \quad (6)$$

where $\beta(t)$ is the two-proton-Pomeron coupling which appears in elastic scattering and $F(t, M^2)$ describes the absorption of a "Pomeron beam" of mass squared equal $-t$ on a proton at energy M^2 .

There is an experimental way to examine whether this boldness of thinking of $F(t, M^2)$ as a Pomeron-proton absorption makes any sense. Namely, look simultaneously at one of the particles in the x (so do a two arm experiment), then one is doing single particle inclusive experiment on Pomeron + proton \rightarrow detected hadron + x . If the Pomeron is acting like one of the hadrons we ordinarily induce inclusive reactions with, the distribution in rapidity (or x) and p_T of the detected hadron ought to be pretty much the same as in ordinary inclusive experiments with M^2 playing precisely the usual role of s . Unusually preliminary data from the CERN-ISR³ in fact shows that Pomeron beams do produce

single particle distributions essentially like what we have so often seen at accelerator energies. That's nice to know and bolsters our resolve to continue.

If we take this Pomeron exchange formula, (6), and consider it for large M^2 , but still $M^2 \ll s$ and t finite, then one more Pomeron exchange may be expected to be governing the behavior. We now encounter the coupling of three Pomerons $g_P(t)$ in the regime just described

$$s^2 d\sigma_{TR}/dt dM^2 = \frac{|\beta(t)|^2 \beta(0)}{16\pi} \left(\frac{A}{M^2}\right)^{2\alpha(t)} (M^2)^{\alpha(0)} g_P(t), \quad (7)$$

where the subscript TR is for triple-Reggeon. The integral of this cross section is proportional to the cross section $\sigma_{TR}(s)$ for this piece of phase space times the multiplicity of fast enough protons in the same piece of phase space. The latter is finite by our choice of cross sections.

This $\sigma_{TR}(s)$ gives a contribution to $F(l, t=0)$ of the form

$$F_{TR}(l, t=0) = \frac{1}{l-\alpha(0)} \int_0^1 dt' g_P(t') / l - 2\alpha(t') + 1 \quad (8)$$

If $\alpha(t) = 1 + \alpha't$, the simplest possibility, then

$$F_{TR}(l, t=0) = g_P(0) \frac{1}{l-1} \log(l-1), \quad (9)$$

but we began by assuming that the analytic structure of $F(l, t=0)$ was just a simple pole $1/l-1$. Other contributions to $F(l, t=0)$ cannot cancel (9) since $A(s, 0) \geq 0$, so we must eliminate this singularity for consistency.

The only way to do that is to have $g_P(0) = 0$.

Making $g_P(0) = 0$ seems harmless enough in itself, but inquiring into how $g_P(0)$ is built up out of the contributions to the missing mass M^2 reveals that a large number of other Pomeron couplings must vanish.⁴ To see this consider the inclusive process Pomeron + proton \rightarrow hadron of momentum p_h + anything. Energy momentum conservation relates the integral of this cross section weighted by E_h to the Pomeron proton cross section we have called $F(M^2, t)$, see Fig. 3

$$\int \frac{d^3 p_h}{E_h} E_h \frac{d\sigma(\text{Pomeron} + p \rightarrow h + X)}{d^3 p_h / E_h} = M F(t, M^2), \quad (10)$$

Since the leading contribution in M^2 to $F(t, M^2)$ must vanish at $t = 0$, so must the equivalent quantity on the left hand side of (10). Utilizing then the positivity of an inclusive cross section, we learn that the "vertex function" for three zero mass Pomerons and any hadron h must vanish. That is, every contribution from individual hadron states to $g_P(0)$ must itself vanish if $g_P(0)$ is to be zero, (see Fig. 5). In particular that piece of this complicated vertex function which involves a Reggeon with the quantum numbers of the hadron h coupling to h and the Pomeron at $t = 0$ must vanish.

Now comes the clincher.⁵ Once one has the Pomeron ($t = 0$) - Reggeon(t) - particle vertex (Fig. 7) vanishing, it is a short step to continue t to M_h^2 and conclude that the Pomeron ($t = 0$) - two particle

vertex also must vanish. This demonstration requires knowledge of the analyticity in t of the two Reggeon-particle vertex, but arguing from analyticity found in a variety of models, Brower and Weis,⁵ conclude that the continuation in t from $t < 0$ to $t = M_h^2$ can be made in such a way as to preserve uniformity in interchanging the two limits $t_{\text{Pomeron}} \rightarrow 0$, $t_{\text{Reggeon}} \rightarrow M_h^2$. So one is left holding a bag of contradictions: assuming that the Pomeron with $\alpha(0) = 1$ sets the scale, $\beta(0)$, of constant total cross sections, we learn that $\beta(0)$ must be zero and that the whole construction is inconsistent.

This result has set armies of function inventors out to seek ways around the analytic continuations necessary to prove the last step in the argument. Unfortunately the physical defense of these vertex functions with reduced analyticity is very slim.

I suspect that it is useful to look upon these decoupling theorems as a blessing rather than a curse. After all one of the basic assumptions made is that this Pomeron thing is a simple isolated pole at $t = 0$, $\ell = 1$. But we have known for ten years since the work of Mandelstam and Amati, et al.⁶ that moving poles in the angular momentum plane necessarily produce branch points in ℓ via unitarity in the t -channel. These branch points coincide with the pole position at $t = 0$ and lead one to wonder why he started down the primrose path of a simple pole at $t = 0$ anyway. The decoupling results then must serve to refocus our attention on the way in which branch cuts and poles must interact to produce a self-

consistent l -plane structure. It is very possible that the resulting structure will bear so little resemblance to the pole we have been offering ourselves as the Pomeron that wonderment at our naiveté will be in order.

That the latter reaction could be appropriate is suggested by several self-consistent models of diffraction studied by Zachariassen and Ball⁷ and by Sugar⁸ and collaborators. These models typically end up saturating the Froissart bound $\sigma_T \sim (\log s)^2$ or at least have $\sigma_T \sim \log s$. Often they have the elastic slope parameter $b(s)$ growing as $(\log s)^2$ as well. But the feature of importance for the minute is that the structure $F(l, t)$ near $l = 1$ and $t = 0$ is some complicated cut with no trace of a pole $\alpha(t) = 1 + \alpha't$ in sight. Of course, at this stage these models are not compelling but becoming increasingly attractive.

Finally it may be that the l -plane description of diffraction is simply so complicated as to be dull and totally misleading. There are among our colleagues many who argue that diffraction is a reflection of inelastic processes feeding back into $A(s, t)$ via unitarity and that only an understanding of the s -channel production amplitudes will reveal the underlying simplicity of diffraction and that the Mellin transform (1) of that simplicity will be unspeakably ugly. Somewhere in between is the germ of an answer; at the moment both the s and the t channel advocates are muddling in the same quagmire of a very important problem.

II Hadrons as Dual Strings

We now switch gears onto a subject of somewhat less direct experimental import but which is a fascinating theoretical challenge. The origin of the subject is found in the operator formulation of multi-particle dual theories. Several good reviews of the subject are available,⁹ so I'll turn my attention only to the salient facts.

The dual model can be formulated in a space-time like picture where one has a "position" four vector $Q_\mu(\tau)$ and a "momentum" four vector $P_\mu(\tau)$ which are represented by their fourier expansions

$$Q_\mu(\tau) = X_\mu + 2\alpha' P_\mu \tau + i\sqrt{2\alpha'} \sum_{n=1}^{\infty} \frac{1}{n} (a_{n\mu} e^{-in\tau} - a_{-n\mu} e^{in\tau}), \quad (11)$$

$$P_\mu(\tau) = \frac{1}{2\alpha'} \frac{dQ_\mu}{d\tau} = P_\mu + \frac{i}{\sqrt{2\alpha'}} \sum_{n=1}^{\infty} (a_{n\mu} e^{-in\tau} + a_{-n\mu} e^{in\tau}), \quad (12)$$

where X_μ and P_μ are the "center of mass" co-ordinates satisfying

$$[X_\mu, P_\nu] = -ig_{\mu\nu} \quad (13)$$

α' is the universal Regge slope, and the operators $a_{n\mu}$ satisfy

$$[a_{m\mu}, a_{n\nu}] = -m g_{\mu\nu} \delta_{m+n,0}. \quad (14)$$

The parameter τ is like a time in which the harmonic oscillator modes $a_{n\mu}(\tau)$ evolve. Translations in τ are generated by the "Hamiltonian"

$$L_0 = \frac{\alpha'}{2\pi} \int_0^{2\pi} : P_\mu(\tau)^2 : d\tau \quad (15)$$

$$= \alpha' P_\mu^2 + \sum_{n=1}^{\infty} a_{n\mu}^+ a_n^\mu . \quad (16)$$

All the conventional dual amplitudes can be expressed in terms of the operators $a_{n\mu}$ and external momenta. The usual dual "propagator" is naturally enough given by

$$\frac{1}{(\alpha'\mu^2 - L_0 - i\epsilon)} \quad (17)$$

The question which has been raised and extensively studied¹⁰ of late is whether or not one may actually think of Q_μ as a position vector for the hadron or its constituents as observed in real space-time. The spectrum generated by L_0 is that of a center-of-mass energy $\alpha' P_\mu^2$ plus an infinite number of oscillations in internal normal modes. The fact that this is in one-to-one correspondence with the excitation spectrum of a one-dimensional uniform string has led to the idea that one may view a hadron as a one dimensional string with every point on the string having a four vector displacement $Q_\mu(\sigma, \tau)$ associated with it where σ in some way labels the "position" along the string and τ describes the time development.

If $Q_\mu(\sigma, \tau)$ satisfies the wave equation

$$\left(\frac{\partial^2}{\partial \tau^2} - v^2 \frac{\partial^2}{\partial \sigma^2} \right) Q_\mu(\sigma, \tau) = 0 \quad (18)$$

then the normal mode expansion (11) represents the displacement vector at some specified σ .

The spectrum of L_0 is, however, too rich because it contains states with the wrong sign of norm if there are no constraints to remove the excitations created by the time oscillators a_{no} . Such constraints were discovered several years ago,¹¹ and have been shown within the last year to eliminate all the ghosts in known dual models.¹² These papers show that in the four dimensional world we live in when the intercept of the leading Regge trajectory in the theory is < 1 , there are no wrong norm states. Further, the physical states have same spectrum as that given by the spatial oscillators alone.¹²

What has now been attempted is to try to find a lagrangian for the string which will ensure the appropriate constraints and explicitly afford the space-time interpretation of $Q_\mu(\sigma, \tau)$. Such a lagrangian was proposed several years ago by Nambu.¹³ He observed that viewed in σ and τ the string swept out a two dimensional sheet (Fig. 8). He suggested that a natural choice for the action of the string be something proportional to the area of this sheet. This yields for the action for motion from τ_i to τ_f

$$A = \frac{-1}{2\pi\alpha'} \int_{\tau_i}^{\tau_f} d\tau \int_0^1 d\sigma \left\{ \left(\frac{\partial Q_\mu}{\partial \sigma} \frac{\partial Q^\mu}{\partial \tau} \right)^2 - \left(\frac{\partial Q_\mu}{\partial \sigma} \right)^2 \left(\frac{\partial Q_\mu}{\partial \tau} \right)^2 \right\}^{1/2} \quad (19)$$

where a conventional constant is put in front and the parameter σ labeling the position on the string has been chosen to lie between 0 and 1.

This action is invariant under parametrization of σ and τ ; that is the action is unchanged when

$$\sigma \rightarrow \tilde{\sigma}(\sigma, \tau) \quad (20)$$

and

$$\tau \rightarrow \tilde{\tau}(\sigma, \tau). \quad (21)$$

This invariance can be shown to explicitly yield the "gauge conditions" which are needed to eliminate the ghosts in dual theories.¹⁴

However we can see without lengthy computation that this action has too much invariance. Because we are free to choose σ and τ at will, only two of the four degrees of freedom in $Q_\mu(\sigma, \tau)$ are independent. This means that the first excited state created by the internal oscillations of the string, which arises by the application of one $a_{n\mu}^+$ to the vacuum will have only two independent modes of oscillation. This can only come about if it has spin 1/2 or is massless. We must reject spin 1/2 on physical grounds since we can only be dealing with internal orbital angular momentum out of which spin 1/2 cannot come if the angular

momentum operator is to be hermitean. If the first excited state is massless and has the lowest spin it can have, namely spin 1, then the intercept of the leading trajectory must be 1 and the ground state must be spacelike. There is even a more subtle complication, namely even if we accept this tachyon as a little burden to be considered "later", the quantization schemes¹⁰ which utilize explicitly the invariance (20) and (21) to eliminate ghost co-ordinates and are thus not explicitly co-variant do not become covariant except in a world of one time and twenty-five space dimensions. The inability of a geometric theory (orbital angular momentum) to produce spin 1/2 can again be seen to be responsible, for if spin 1/2 could be tolerated one can alter the Lorentz generators to have the correct commutation relations in four dimensional space-time and have no ghosts or tachyons.

What is clearly needed is an action which breaks the joint invariance under both σ and τ parametrizations. This broken symmetry ought to retain one parameter invariance so that out of the four $Q_{i\mu}(\sigma, \tau)$ one can be eliminated by parameter choice leaving the remaining three independent variables to generate the spectrum. We can get a hint as to what to do by looking at a set of non-interacting point particles with masses m_i and positions $Q_{i\mu}(\tau)$, τ some parameter. The action is

$$A = - \sum_i m_i \int_{\tau_i}^{\tau_f} d\tau \sqrt{\left(\frac{dQ_{i\mu}}{d\tau}\right)^2} \quad . \quad (22)$$

This action is clearly invariant under choice of the parameter τ :

$$\tau \rightarrow \tilde{\tau}(\tau) \quad (23)$$

So from each $Q_{i\mu}(\tau)$ we can eliminate one component and be left with the three degrees of freedom we expect for free point particles. If we imagine that the string is just an infinite number of these point particles which are interacting and the label i goes over into σ to identify which particle we are speaking of, then to retain contact with the non-interacting case in the limit of string's coupling goes to zero, we want to retain the independence under τ and are willing to sacrifice the σ independence.

Indeed, this gives one another degree of freedom and brings one closer to the three dimensional internal oscillator which will generate the dual spectrum. However, exactly how to alter the action (19) to break the σ invariance is not well defined. One can reason heuristically in analogy with the electromagnetic field action.

$$A_{em} = -\frac{1}{4} \int d^4 z [\partial_\mu A_\nu(z) - \partial_\nu A_\mu(z)]^2. \quad (24)$$

This is not only gauge invariant under $A^\mu(z) \rightarrow A^\mu(z) + \partial^\mu \Lambda(z)$ but possesses a higher conformal symmetry. We also know that this action describes a massless field and only two of the components of $A_\mu(z)$ are independent variables. We can break this additional symmetry by adding a mass term to the action and still retain the gauge invariance

of the theory by the Stückelberg trick of introducing a scalar boson field $B(z)$. The Lagrangian

$$\begin{aligned} \mathcal{L}(z) = & -\frac{1}{4} (\partial_\mu A_\nu(z) - \partial_\nu A_\mu(z))^2 \\ & + (A_\mu(z) - \lambda \partial_\mu B(z))^2, \end{aligned} \quad (25)$$

is invariant under

$$A_\mu(z) \rightarrow A_\mu(z) + \partial_\mu \Lambda(z) \quad (26)$$

and

$$B(z) \rightarrow B(z) + \frac{1}{\lambda} \Lambda(z), \quad (27)$$

and describes a massive particle whose field $A_\mu(z)$ has three independent components.

The suggestion of this example is that to the action (19) somehow corresponds to the continuum generalization of a bunch of massless particles interacting. If the interacting particles had mass then before and (I presume) after they were allowed to interact, three degrees of freedom would be retained at each σ for every τ . It would be most striking if I could end this exposition with an action which broke σ invariance and gave a three dimensional harmonic oscillator spectrum and has no ghosts and avoids the subtlety of needing 25 space dimensions in which to be covariant, etc., but I cannot.

Not to leave the reader entirely disappointed I offer up the following guess for his or her entertainment. It has broken σ -invariance and thus three degrees of freedom at each σ and τ and reduces to the action for

a free point mass when the extent of the string in σ shrinks to zero

$$A = - \int_{\tau_i}^{\tau_f} d\tau \int_0^1 d\sigma \left\{ \left(\frac{\partial \phi_\mu}{\partial \sigma} \frac{\partial \phi^\mu}{\partial \tau} \right)^2 + \left[m^2 - \left(\frac{\partial \phi_\mu}{\partial \sigma} \right)^2 \right] \left(\frac{\partial \phi_\mu}{\partial \tau} \right)^2 \right\}^{1/2}. \quad (28)$$

Clearly it is invariant under τ parametrizations and when $\frac{\partial \phi_\mu}{\partial \sigma} \rightarrow 0$ becomes (22) for a point mass. Its other virtues (and faults) and its acceptability as an action for dual theories, I leave as homework for the diligent reader.

I would like to thank M. B. Einhorn for lengthy discussions on the two subject considered here.

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FIGURE CAPTIONS

Figure 1: The inclusive differential cross section for $pp \rightarrow p + x$ with Pomeron exchange. The function $F(t, M^2)$ is the cross section for absorption of a Pomeron "beam" of mass squared $-t$ on a proton at center of mass energy M^2 .

Figure 2: The large M^2 behavior of $F(t, M^2)$ revealing the triple Pomeron coupling $g_P(t)$.

Figure 3: The connection via energy momentum conservation between Pomeron + proton \rightarrow hadron + anything and Pomeron + proton \rightarrow anything.

Figure 4: The large M^2 behaviour of the energy momentum conservation relation shown in Figure 3.

Figure 5: Vanishing of $g_P(0)$ requires the terms in the integral of Figure 4 to vanish. This is shown here.

Figure 6: One contribution to Figure 5 which exhibits the Pomeron-Reggeon-particle coupling. This coupling is forced to vanish.

Figure 7: If the Pomeron ($t=0$) - Reggeon(t) - particle coupling vanishes for all $t < 0$, then analyticity arguments can be used to show that the Pomeron ($t=0$) - two particle coupling, which sets the scale for total cross sections must vanish.

Figure 8: The two dimensional surface swept out by the dual string which is extended in the σ parameter and develops in the "time"

variable τ . At each point on the surface one would like to associate a real space-time displacement vector $Q_{\mu}(\sigma, \tau)$.

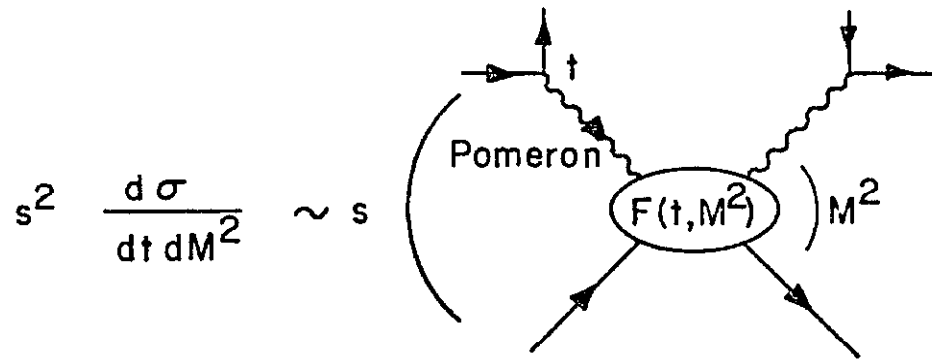


Figure 1

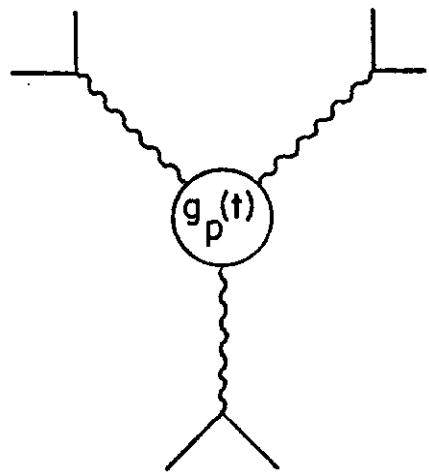


Figure 2

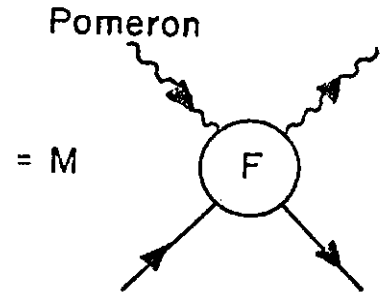
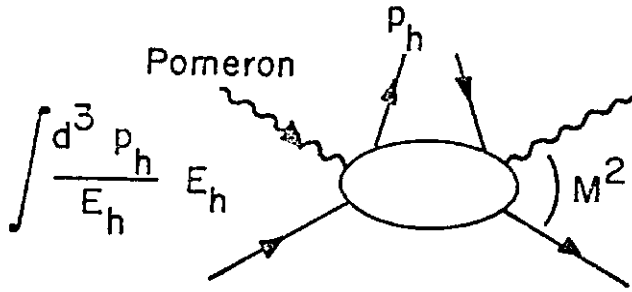


Figure 3

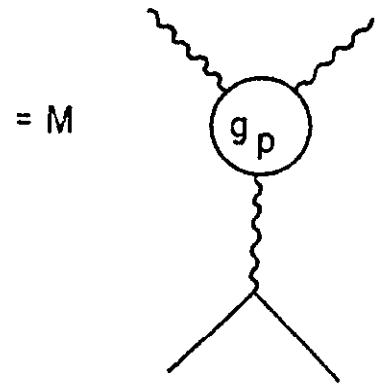
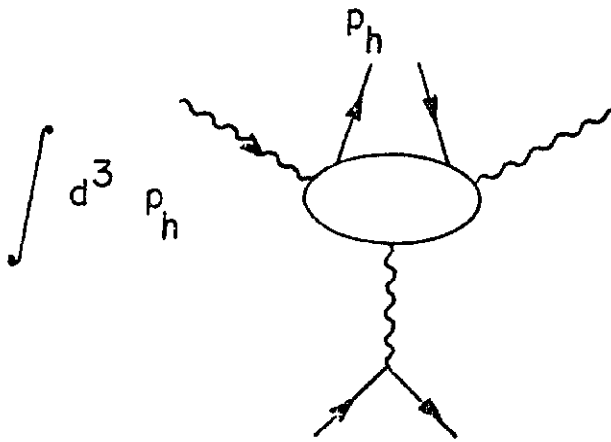


Figure 4

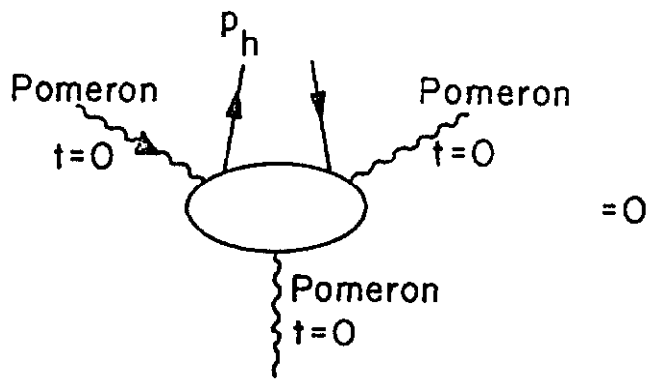


Figure 5

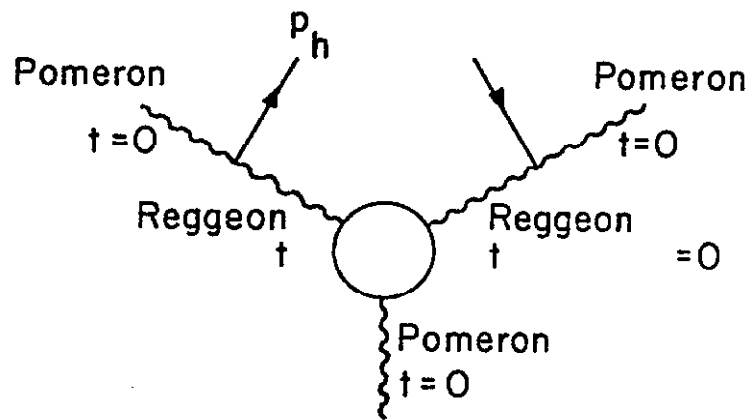


Figure 6

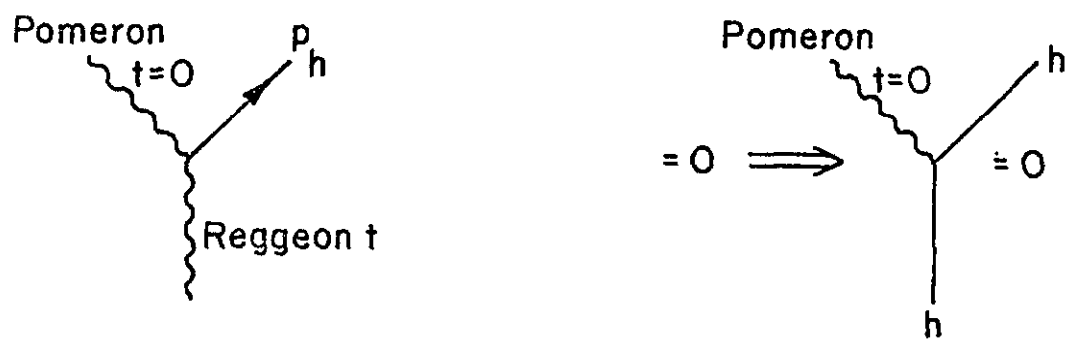


Figure 7

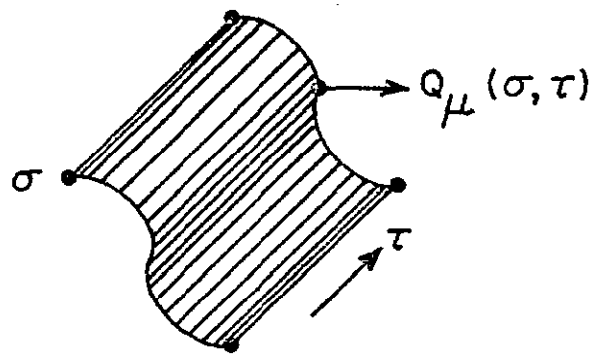


Figure 8