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CHARGE TRANSFER IN A MULTIPERIPHERAL PICTURE*

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Abstract

The transfer of charge from one c. m. hemisphere to the other is discussed in terms of schematic multiperipheral models. The resultant expectations for experimental quantities contrast markedly with predictions based on the fragmentation picture. Particularly sensitive tests between the rival viewpoints are indicated.

I. INTRODUCTION

In a recent paper[1], Chou and Yang have generalized the notion of charge-exchange reactions to the realm of multiparticle final states, and introduced the concept of the net charge transferred from one c.m. hemisphere to the other in high-energy collisions. They identify as an essential aspect of the fragmentation picture the absence of charge transfer in infinite energy hadron-hadron collisions, and discuss in terms of a simple model the manner in which the characteristic limiting behavior is attained. Past experience with quasi-two body reactions, among which typical charge-exchange cross sections decrease at least as rapidly as s^{-1}, leads one to suspect that the qualitative limiting behavior is not peculiar to the fragmentation philosophy, but must be shared by any "reasonable" model for particle production. Distinctions among various viewpoints are therefore to be drawn from attention to quantitative differences. It is for example now well known[2] that apparently useful distinctions between the fragmentation and multiperipheral philosophies can be made on the basis of contrasting predictions for multiplicity fluctuations and multiparticle distribution functions. In this note, by studying the consequences of a simple multiperipheral model, we address the question of whether similarly useful distinctions are to be perceived through the study of charge transfer reactions. We find that while a multiperipheral picture also embodies declining charge transfer cross sections at high energies, there indeed are quantitative differences between the fragmentation- and multiperipheral-model predictions which may be sufficient to permit experiments to rule in favor of one scheme or the other.

II. A MULTIPERIPHERAL PICTURE

We consider "typical" events to be of the form

$$p + p \rightarrow p + p + N\pi^{+} + N\pi^{-} + N\pi^{o} , \tag{1}$$

and we imagine pions to be produced in isoscalar triplets $(\pi^{+}\pi^{-}\pi^{o})^{3}$. For brevity we shall refer to this basic cluster of the multiperipheral ladder as an ω meson. We define the c. m. rapidity variable

$$y = \tfrac{1}{2} \log \frac{E^{*} + p_{||}^{*}}{E^{*} - p_{||}^{*}} \tag{2}$$

where E^{*} and $p_{||}^{*}$ are respectively the c. m. energy and longitudinal momentum of the particle under discussion. Thus y lies in the interval $[-Y/2, Y/2]$, where $Y \propto \log s$. For an event in which N clusters are produced, we assign to the protons the rapidities $\pm Y/2$, and space the clusters at $y_1, \ldots, y_N$ according to

$$\frac{1}{\sigma_N} \frac{d\sigma}{dy_1 \ldots dy_N} = Y^{-N} . \tag{3}$$

We are assuming the clusters have an equal chance to be produced anywhere in the rapidity interval $[-\frac{Y}{2}, \frac{Y}{2}]$. In the simplest version of the multiperipheral model, the multiplicity cross sections

$$\sigma_N \equiv \int dy_1 \ldots dy_N \frac{d\sigma}{dy_1 \ldots dy_N}$$

follow a Poisson distribution

$$\sigma_N = \sigma_{total} \, e^{-Z} \frac{Z^N}{N!} \tag{4}$$

with

$$Z = a + bY \tag{5}$$

representing the mean number of clusters.

The simple model described in Eqs. (1) - (5) is all we require to explore the essential quantitative differences between the multiperipheral and fragmentation pictures of multiparticle production.

A. Charge Transfer

Pions which emerge from a cluster will be characterized by a mobility parameter Δ (in rapidity) which can be estimated from the cluster mass and the mean transverse momentum of pions. [The mobility parameter does not depend on the primary energy s.] An ω produced with rapidity y_i therefore yields pions with rapidities $y_i - \Delta$, y_i, $y_i + \Delta$. This means that only those clusters with rapidities lying in $(-\Delta, \Delta)$ have the potential to transfer charge from one hemisphere to the other. For every cluster, the correspondence between rapidities $(y_i - \Delta,\ y_i,\ y_i + \Delta)$ and pion charges $(-1,\ 0,\ +1)$ can be made in six ways, which we assume equiprobable. Therefore each cluster in the active region $(-\Delta, \Delta)$ has a 1/3 probability to contribute $(1, 0, -1)$ to the <u>net charge transfer</u>

$$
\begin{aligned}
u \equiv\ & \tfrac{1}{2}\ (\text{Total charge in the forward hemisphere}) \\
& -\tfrac{1}{2}\ (\text{Total charge in the backward hemisphere}), \qquad (6) \\
\equiv\ & \tfrac{1}{2}\,(Q_R - Q_L)
\end{aligned}
$$

independent of the behavior of the other clusters. It is easy to verify that for M active clusters the net charge transfer $u = k$ occurs with a probability given by the coefficient $g_{M:k}$ of x^k in the generating function

$$
G_M(x) = 3^{-M}(x + 1 + x^{-1})^M = \sum_{k=-M}^{M} g_{M:k}\, x^k \; . \qquad (7)
$$

This formula is sufficient for computing the average charge transfer and its fluctuation for fixed M. However, for computing averages for fixed numbers of produced clusters (or negative pions) in each c. m. hemisphere, a more detailed analysis is needed.

To carry out this analysis it is convenient first to partition N produced clusters into L clusters produced in the backward (c. m.) hemisphere and R clusters produced in the forward (c. m.) hemisphere. A further partition is made to distinguish the L_I inert clusters produced in the interval $[-\frac{Y}{2}, -\Delta]$ and the L_A clusters produced in the active region $(-\Delta, 0)$: $L_I + L_A = L$. Similarly $R = R_I + R_A$ partitions the forward hemisphere clusters into those produced in $(0, \Delta)$ (R_A) and those in $[\Delta, \frac{Y}{2}]$ (R_I). From (3) it follows at once that the relative probabilities for the regions L_I, L_A, R_A, R_I are $[\frac{1}{2}(1-p)]^{L_I}$, $(\frac{1}{2}p)^{L_A}$, $(\frac{1}{2}p)^{R_A}$, $[\frac{1}{2}(1-p)]^{R_I}$ where

$$p = \frac{2\Delta}{Y} \quad . \tag{8}$$

Finally, the clusters in the active region are partitioned according to how much charge is transferred. For the L_A clusters produced in $(-\Delta, 0)$, (L_A^+, L_A^o, L_A^-) transfer net charge (1, 0, -1) to the forward hemisphere. These possibilities exhaust L_A: $L_A = L_A^+ + L_A^o + L_A^-$. The probabilities for these cases are $(\frac{1}{2}p \cdot \frac{x}{3})^{L_A^+}$, $(\frac{1}{2}p \frac{1}{3})^{L_A^o}$, $(\frac{1}{2}p \cdot \frac{1}{3x})^{L_A^-}$ respectively. The x occurring here is a dummy parameter which appears in the numerator if the event contributes positively to u; in the denominator if the event contributes negatively to u; and not at all if there is no charge transfer (cf. Eq. (7)). This parameter is useful for carrying out calculations, and is set equal to one at the end. In an analogous manner, $R_A = R_A^+ + R_A^o + R_A^-$ partitions the forward clusters produced in $(0, \Delta)$ into those which contribute (1, 0, -1) to the charge transfer u with a probability $(\frac{1}{2} p \cdot \frac{x}{3})^{R_A^+}$, $(\frac{1}{2} p \cdot \frac{1}{3})^{R_A^o}$, $(\frac{1}{2} p \cdot \frac{1}{3x})^{R_A^-}$.

The above partition of N leads to a multinomial distribution (i. e. an expansion of $(\sum_{i=1}^{8} \lambda_i)^N$),

$$P_N(x) = \sum_{\Sigma L + \Sigma R = N} \frac{N!}{L_I!\, L_A^+!\, L_A^-!\, L_A^o!\, R_I!\, R_A^+!\, R_A^-!\, R_A^o!}$$

$$\cdot [\tfrac{1}{2}(1-p)]^{L_I} [\tfrac{px}{6}]^{L_A^+} [\tfrac{p}{6}]^{L_A^o} [\tfrac{p}{6x}]^{L_A^-}$$

$$\cdot [\tfrac{1}{2}(1-p)]^{R_I} [\tfrac{px}{6}]^{R_A^+} [\tfrac{p}{6}]^{R_A^o} [\tfrac{p}{6x}]^{R_A^-} \tag{9}$$

$$= [1 - p + \tfrac{p}{3}(x + 1 + x^{-1})]^N,$$

which is useful for computing the mean charge transfer $\bar{u}$ and its fluctuation $\overline{u^2} - \bar{u}^2$ with various quantities held fixed. [Note that $\bar{u}_N$ is simply $x \frac{\partial}{\partial x} P_N(x)|_{x=1}$, and $\overline{u_N^2}$ is $(x \frac{\partial}{\partial x})^2 P_N(x)|_{x=1}$.] As an example, Eq. (7) results from (9) if one fixes $M = L_A^+ + L_A^o + L_A^- + R_A^+ + R_A^o + R_A^-$ and $N\text{-}M = L_I + R_I$. In this case

$$P_N(x) = \sum_{M=0}^{N} \binom{N}{M} (1-p)^{N-M} p^M 3^{-M} (x + 1 + \tfrac{1}{x})^M \tag{10}$$

demonstrates the origin of the generating function in Eq. (7).

B. Results

The mean charge transfer for a fixed number of clusters is, from Eq. (9),

$$\bar{u}_N = x \frac{\partial}{\partial x} \quad P_N(x) \Big|_{x=1}$$

$$= x \frac{\partial}{\partial x} \left[1-p + \frac{p}{3} (x + 1 + \frac{1}{x}) \right]^N \Big|_{x=1} \tag{11}$$

$$= 0$$

while the charge fluctuation is

$$\overline{u_N^2} = \frac{2Np}{3} \tag{12}$$

$$= \frac{4\Delta}{3Y} N \quad .$$

When averaged over all possible numbers of clusters, (11) and (12) lead to

$$\bar{\bar{u}} = 0 \tag{13}$$

$$\overline{\overline{u^2}} = \sum_{N=0}^{\infty} \frac{\sigma_N}{\sigma_{total}} \overline{u_N^2} = \frac{4\Delta}{3Y} \langle N \rangle \tag{14}$$

It is noteworthy that (14) depends only on the mean multiplicity and <u>not</u> on the multiplicity distribution σ_N. If the number of clusters in the backward and in the forward hemispheres are fixed at ℓ and r respectively, where (q. v. Eq. (9))

$$\ell = L_I + L_A$$

$$r = R_I + R_A$$

then the mean net charge transfer is

$$\overline{u}_{\ell,\,r} = x\frac{\partial}{\partial x}\left\{\left[1-p+\frac{p}{3}\left(x+1+\frac{1}{x}\right)\right]^{\ell+r}\right\}\Big|_{x=1} \tag{15}$$

$$= 0 ,$$

and the net charge fluctuation is

$$\overline{u^2_{\ell,\,r}} = \frac{4\Delta}{3Y}\,(\ell + r) . \tag{16}$$

Finally the mean net charge transfer for fixed numbers of π^-'s in the backward (ℓ_-), and in the forward (r_-) hemispheres, where

$$\ell_- = L_I + L_A^+ + L_A^o + R_A^+$$

$$r_- = R_I + R_A^- + R_A^o + L_A^- ,$$

is computed from Eq. (9) to be

$$\overline{u}_{\ell_-,\,r_-} = x\frac{\partial}{\partial x}\left\{\left(1-\frac{2p}{3}+\frac{2px}{3}\right)^{\ell_-}\cdot\left(1-\frac{2p}{3}+\frac{2p}{3x}\right)^{r_-}\right\}\Big|_{x=1} \tag{17}$$

$$= \frac{4\Delta}{3Y}\,(\ell_- - r_-) .$$

Similarly, the net charge transfer fluctuation is

$$\overline{u^2_{\ell_-,\,r_-}} - (\overline{u}_{\ell_-,\,r_-})^2 = \frac{4\Delta}{3Y}\left(1-\frac{4\Delta}{3Y}\right)(\ell_- + r_-) . \tag{18}$$

The results (11) - (18) are to be compared with the corresponding results in the fragmentation model.[1]

III. DISCUSSION

Certain qualitative features of $\overline{u}$ and $\overline{u^2}$ are identical in both the multiperipheral and fragmentation models, such as the sign of $\overline{u}_{\ell_-, r_-}$ and the fact that for fixed ℓ_- and r_-, $\overline{u}_{\ell_-, r_-} \to 0$ and $\overline{u^2}_{\ell_-, r_-} \to 0$ as $s \to \infty$. An important quantitative difference to be discovered from data is whether $\overline{u}_{\ell_-, r_-}$ and $\overline{u^2}_{\ell_-, r_-}$ decrease like $s^{-\frac{1}{2}}$ or like $(\ln s)^{-1}$. The most striking contrast between the models is found in the charge transfer fluctuation, averaged over all events. Thus $\overline{\overline{u^2}}$ is expected to increase like $s^{\frac{1}{2}}$ in the fragmentation model, and tend to a constant in the multiperipheral model[4]: both models must of course have $\overline{\overline{u}} = 0$ for pp collisions.

There are additional differences to be found between the predictions of the simple models studied, which are certain to be more general than the models themselves. For example, $\overline{u}_{\ell_-, r_-}$ is expected[1] to be a quadratic function of ℓ_- in the fragmentation model $[(\ell_- - r_-)(a\ell_- + ar_- + b)]$, whereas in the multiperipheral model $\overline{u}_{\ell_-, r_-}$ is linear $[(\ell_- - r_-)]$. Accurate data at a single energy might distinguish these possibilities. This prediction of the multiperipheral model is probably least sensitive to the makeup of the clusters, which could be single pions, $\pi^+\pi^-$ pairs, the $\pi^+\pi^-\pi^0$ triplets considered here or other groups. We note, however, that if single pions are emitted independently, it is easy to verify that

$$\overline{u}_{\ell_-, r_-} = \frac{x}{2} \frac{\partial}{\partial x} \left\{ x^{\ell_- - r_-} \left(\frac{x}{2} + \frac{1}{2x}\right)^{\ell_- + r_-} \right\} \Big|_{x=1} \tag{19}$$

$$= \tfrac{1}{2} (\ell_- - r_-)$$

$$\overline{u^2_{\ell_-, r_-}} = \left(\frac{x}{2}\frac{\partial}{\partial x}\right)^2 \left\{ x^{\ell_- - r_-}\left(\frac{x}{2} + \frac{1}{2x}\right)^{\ell_- + r_-}\right\}\Big|_{x=1} \tag{20}$$

$$= \frac{1}{4}\,(\ell_- + r_-) + \frac{1}{4}\,(\ell_- - r_-)^2$$

and hence both the mean net charge and the fluctuation are constants as a function of incident energy. Data showing these quantities decreasing to zero would rule out this possibility which, it may be remarked, seems in any case to conflict with data on correlations between charged and neutral secondaries[3].

The multiperipheral predictions for $\bar{u}$ and $\overline{u^2}$ are unchanged if one restricts the data to an angular region $\pm\,\delta\theta^*$ about 90° in the c.m. for the pions counted in ℓ_- and r_-. This is because a fixed interval $(-\Delta, \Delta)$ in rapidity, which itself is a fixed angular region $\pm\,\delta\theta^*$, determines the final results. Likewise, the fragmentation model results derived by Chou and Yang[5] continue to hold in a fixed angular range $\pm\,\delta\theta^*$.

For completeness it must be said that all of the results derived in this paper (as well as in Ref. 1) apply equally to the transfer of <u>any</u> additive quantum number, A. In the multiperipheral picture, one assumes that each cluster has A=0, but itself is made up of pieces which have $A \neq 0$. The analysis given here can then be applied directly. For the forseeable future, only charge transfer is likely to be experimentally interesting.

Footnotes and References

1. T. T. Chou and C. N. Yang, "Charge Transfer in High Energy Fragmentation," Stony Brook preprint, 1972.

2. C. Quigg, J.-M. Wang, and C. N. Yang, Phys. Rev. Letters 28, 1290 (1972); R. C. Hwa, ibid. 28, 1487 (1972); E. L. Berger, Oxford Conference, 1972, ANL/HEP 7220.

3. This is one of the simplest schematic multiperipheral models capable of reproducing the observed correlations between numbers of charged and neutral secondaries. See E. L. Berger, D. Horn, and G. H. Thomas, ANL/HEP 7240, Phys. Rev. (to be published). However, we could equally well choose $\pi^+\pi^-$ pairs to be made without changing any of the conclusions of this paper: only a few numerical coefficients are changed in the expressions for $\bar{u}$ and $\overline{u^2}$. More complicated clusters are also possible, but here again we believe our general results for functional dependences hold. See however the discussion section [and Ref. 1] for the case of pions being made singly and independently.

4. As Chou and Yang have remarked in Ref. 1, the different asymptotic behaviors of $\overline{u^2}_{\ell_- r_-}$ and of $\overline{\overline{u^2}}$ in the fragmentation picture may be traced to the high probability for asymmetrical fragmentations (many particles produced in one hemisphere, few in the other). In the multiperipheral picture, symmetrical events are strongly favored, and the asymptotic behaviors of $\overline{u^2}_{\ell_-, r_-}$ and of $\overline{\overline{u^2}}$ differ only by a logarithm. S. Nussinov, C. Quigg, and J.-M. Wang, Phys. Rev. D, Nov. 1, 1972 have stressed the tendency of the fragmentation (multiperipheral)

mechanism to generate asymmetric (symmetric) events.

5. J.-M. Wang, Stony Brook preprint, September, 1972 has noted that in such a fixed angular interval about 90° in the c.m. the predictions of the fragmentation and multiperipheral models for the second moment of the multiplicity distribution $\langle n^2 \rangle$ are respectively $\langle n^2 \rangle \propto s^{\frac{1}{2}}$ and $\langle n^2 \rangle \rightarrow$ constant. This is the same disparity in asymptotic behavior we find for $\overline{\overline{u^2}}$.