national accelerator laboratory

NAL-THY-93
October 1972
(Not for Publication)

On a Two-Component Interpretation of Multiplicity Distributions

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ABSTRACT

A simple model combining the features of the fragmentation and multiperipheral pictures is fitted to the NAL data on topological cross sections. With diffractive and multiperipheral contributions in the ratio of 1:3 we obtain a satisfactory fit. The possible significance of such a two-component description is discussed, and extrapolations to higher energies are presented. The dip in the multiplicity distribution which is the signature of such a superposition may appear at ISR energies.

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Operated by Universities Research Association Inc. Under Contract with the United States Atomic Energy Commission

The study of high-energy collisions has taken a dialectical turn of late, as we have sought to determine whether the multiperipheral language or the fragmentation language is the more convenient for discussing multiparticle phenomena. In order to expose the differences between these two points of view, one has represented each by a caricatured model, and contrasted the predictions of the models for various observable distributions. The bare-bones caricatures are on the one hand the so-called Feynman gas analogy,[1] corresponding to a Poisson distribution of topological cross sections, and on the other hand the prototype fragmentation model,[2] corresponding to a $1/n^2$ multiplicity distribution. While such simplified mathematical models are of value for understanding the (often conflicting) qualitative expectations of the philosophies they represent, it has come as no great surprise to proponents of either school to find that neither caricature accounts quantitatively for the multiplicity distributions observed in the recent bubble chamber experiments at NAL.[3] Thus multiperipheralists can justifiably point to "end effects" which at NAL (even ISR?) camouflage the simple gas analogy predictions, whereas fragmenters can with equal justification appeal to finite energy or threshold effects to postpone their day of reckoning.

In this note we explore the possibility that an accurate description of high-energy collisions may lie between the antipodes mentioned above.[4,5] To do so we fabricate a particular and arbitrary caricature of a two-component theory for which we can supply no "fundamental" justification but which embodies features of the simplest multiperipheral and fragmentation models. Specifically, we envisage a diffractive (fragmentation) component of the inelastic cross section which contributes an energy-independent amount to each topological cross section σ_n plus a multiperipheral component which contributes an energy-independent amount to the total inelastic cross section, with an energy-dependent division among charged-particle multiplicities. We have therefore fitted the NAL data on topological cross sections with the form

$$\sigma_n(s) = A(e^{-Bn} - Ce^{-Dn})/n^2 + E\,\frac{e^{-(\frac{1}{2}\lambda(s)-1)}(\frac{1}{2}\lambda(s) - 1)^{(\frac{1}{2}n-1)}}{(\frac{1}{2}n-1)!} \qquad (1)$$

in which the first term represents the fragmentation component and the second term the multiperipheral component.[6] A best fit[7] to the 102, 205, and 303 GeV/c data for $n \geq 4$ results in the expression

$$\sigma_n(s) = \{37.8/n^2 + 24.9e^{-(\frac{1}{2}\lambda(s)-1)}(\tfrac{1}{2}\lambda(s)-1)^{(\frac{1}{2}n-1)}/(\tfrac{1}{2}n-1)!\}\,\text{mb}, \qquad (2)$$

with

$$\lambda(s) = 6.6 + 2.4 \log (P_{lab}/100\ \text{GeV}/c). \qquad (3)$$

[The logarithmic parametrization (3) is chosen only for simplicity, as we have values at only three points. A power-law or more complicated function is by no means excluded.] The results are compared with the data in Figs. 1-3. Although the n = 2 cross section was not included in the fit, we also show in Figs. 1-3 the values of σ_2 given by (2).

They are in remarkable (if purely fortuitous) agreement with the total (elastic plus inelastic) two-prong cross section. It is tempting to interpret the diffractive term ex post facto as a prescription for both elastic and inelastic events.[8]

Wilson[1] has pointed out that the signature of a two-component theory is the evolution of a dip in the multiplicity distribution as the primary energy is increased. Extrapolating the Poisson mean multiplicity parameter $\lambda(s)$ according to (3), we arrive at the predictions shown in Figs. 4-7 for beam momenta of 400, 500, 1000, 1500 GeV/c. By 1500 GeV/c such a dip has appeared: σ_6 is less than σ_4 and σ_8. Even granting the arbitrariness of our parametrization and the extremely speculative nature of the extrapolation, this result gives added impetus to the precise measurement of low-multiplicity cross sections at the CERN ISR.[9]

Although the fit given by (2) contains all possible information on the caricature multiplicity distribution, we nevertheless make some explicit observations. Firstly, with the interpretation that σ_2 includes the elastic cross section, Eq. (2) sums to a total cross section of 40.5 mb. The "multiperipheral" part is 25 mb, leaving 15.5 mb for the diffractive "fragmentation" component. Since the elastic cross section is approximately 7 mb,[3] the "fragmentation" part of the inelastic cross section is roughly 8.5 mb, only 1/3 of the size of the "multiperipheral" contribution.[10]

The relative smallness of the "fragmentation" component persists in other features of the multiplicity at NAL energies, although its effects can become dominant at higher energies depending on how one handles the n^{-2} tail. For all but the zeroth order moment it is necessary to specify a maximum value of n for the "fragmentation" component of σ_n. One relatively uninteresting possibility is that the "fragmentation" cross section is cut off at some finite, energy-independent value of n [or equivalently, $B \neq 0$ in Eq. (1)].[11] Then the asymptotic properties of the two-component caricature become those of the "multiperipheral" component, apart from an energy-independent structure at small multiplicities. A more interesting possibility that we pursue here is that the "fragmentation" component competes at all energies. We thus let the maximum value of n be proportional to $\sqrt{s}$.[12] The absolute maximum number of pions (of all charges) that can occur is $(\sqrt{s} - 2m_p)/m_\pi$. Making allowances for an average energy ω for each pion and assuming that 2/3 of the pions are charged, we obtain a plausible maximum value of n:

$$n_{max} = 2 + \frac{2}{3}\,\frac{(\sqrt{s} - 2m_p)}{\omega}\,, \tag{4}$$

where $\omega = 0.4$ GeV is a reasonable value.[13] With the assumption that $\sigma_{el} = 7.0$ mb at all relevant energies, the first few moments $\langle n^k \rangle$ of the inelastic prong distribution following from the parameterization (2) are

$$\langle n \rangle = -0.417 + 0.564\, S\left(\frac{n_{max}}{2}\right) + 0.744\lambda \tag{5}$$

$$\langle n^2 \rangle = -0.835 + 0.564\, n_{max} + 0.744\,[\lambda^2 + 2\lambda - 4] \tag{6}$$

$$\langle n^3 \rangle = -1.67 + 0.564\, n_{max}\left(\frac{n_{max}}{2} + 1\right) + 0.744\,[\lambda^3 + 6\lambda^2 - 8\lambda - 8] \tag{7}$$

where $S(n) = \sum_{1}^{n} (1/k) \simeq \ln(n + \frac{1}{2}) + 0.5772$ for large n, and $\lambda(s)$ is given by (3). With n_{max} given by (4) and $\omega = 0.4$ GeV, the mean multiplicities at 100, 200, and 300 GeV/c are in agreement with experiment, as is expected from the fits shown in Figs. 1-3. Similarly, the values of $\langle n^2 \rangle$ agree with the data within errors, while the calculated values of $\langle n^3 \rangle$ are somewhat high.[14] The increasing sensitivity of the higher moments to the exact value of n_{max} makes it senseless to push such comparisons of the caricature and experiment beyond $\langle n \rangle$ or $\langle n^2 \rangle$. The range of values for $\langle n^2 \rangle$, $\langle n^3 \rangle$ which emerge from variations in ω are indicated in Fig. 8. We note that if n_{max} grows as a power of the energy as in (4), the "fragmentation" component will ultimately govern all the moments beyond $\langle n \rangle$, even though in the 100 - 300 GeV/c range it contributes relatively only of the order of 1/4 to $\langle n^2 \rangle$ and 1/2 to $\langle n^3 \rangle$. Had we taken seriously the mild exponential damping of our alternative fit,[7] our results would be essentially unchanged in the NAL energy range, but the asymptotic behavior would be that of a general multiperipheral model, i.e., $\langle n^q \rangle \propto \log^q s$. Abarbanel and Kane,[11] whose parameterization is equivalent to $n_{max} \propto s^{1/4}$, obtain $\langle n^q \rangle \propto s^{(q-1)/4}$.

Of some interest, perhaps, are the relative sizes of the contributions to the asymptotic multiplicity. With $n_{max} \propto \sqrt{s}$ in (5), it can be inferred that the coefficient A in the representation, $\langle n \rangle = A \ln s + B +$ lower order, is built up as follows:

$A = 0.282 + 0.744(2.4) = 2.07$, with $0.28/2.07 \simeq 0.14$ and $1.79/2.07 \simeq 0.86$ as the relative contributions of the "fragmentation" and the "multiperipheral" components, respectively. We note that the effective coefficient of ln s for the "fragmentation" term alone is $A_{frag} = 0.282/0.266 \simeq 1.06$, less than half of the "multiperipheral" value from (3), $A_{multiper} = 2.4$. It is suggestive that the two-component value, $A = 2.07$, is significantly smaller than the "multiperipheral" value of 2.4. It is well-known that if correlations are of finite range, A is related to the integral over $p_{\perp}^2$ of the invariant cross section evaluated at $\Theta_{cms} = 90^o$ ($y_{cms} = 0$). If we conjecture that the multiperipheral mechanism populates the central region of rapidity relatively more than the fragmentation mechanism, we have a tentative explanation of the somewhat puzzling fact that data from the ISR on charged particle production[15, 16] at $y_{cms} \simeq 0$ have a magnitude and energy dependence implying $A \simeq 2.4 - 3.1$,[17] whereas the trend of the charged multiplicities from 50 GeV/c to ISR energies indicate $a \simeq 1.6 - 2.0$. This explanation demands taking the n^{-2} tail in (2) seriously, right out to the kinematic limit, and implies some long-range correlation effects. If our solution had a non-zero value of B in (1), then the "fragmentation" component would not contribute asymptotically to the coefficient of ln s.

A final comment concerns a plausible connection of the "fragmentation" component of σ_n with the PPR contribution in the triple-Regge description of the missing-mass cross section for pp → p + anything.[18] Duality arguments connect the Regge (R) exchange in PPR with resonance production in the missing mass spectrum. If fragmentation occurs via decay of fireballs or novae, the PPR cross section should correspond to our "fragmentation" component in σ_n. Now, at small momentum transfers, the PPR cross section has a variation in missing mass as $d\sigma/dtdM \propto M^{-2}$. If we assume that a fireball or nova of mass M emits a number of particles that is proportional (well above threshold) to M, we infer that the PPR cross section corresponds to $\sigma_n \propto n^{-k}$, with $k \sim 2$, consistent with our caricature of fragmentation. The magnitude (8.5 mb) of the inelastic fragmentation cross section found from (2) is in agreement with the rough estimates of the twice the integrated PPR cross section of the ISR data on pp → p + anything.[19] This may be further taken as an indication that the energy-independent aspect of our "fragmentation" component is not just a consequence of fitting data over a limited energy range.[20]

We conclude with several observations on the significance of the fit presented here. These are offered not as conclusions but as food for thought.

(1) A single diffraction plus multiperipheral model can account for the multiplicity distributions observed at NAL. It does not adequately describe lower energy data, nor is it by any means unique.[5, 11, 21]

(2) It may be unnecessary to give elastic events a distinguished role in fragmentation models.

(3) If a two-component interpretation is realistic, the dip in multiplicity distributions foreseen by Wilson may be observable in ISR experiments.

(4) The fragmentation component with $\sigma_n \propto n^{-2}$ may plausibly be identified with the PPR contribution to the triple Regge description of $pp \rightarrow p$ + anything even though the energy dependences do not quite match; our total fragmentation contribution to the inelastic cross section is in agreement with estimates of the PPR cross section at ISR energies.

ACKNOWLEDGEMENT

We thank Myron Bander and S. D. Ellis for useful discussions.

FOOTNOTES AND REFERENCES

[1] K.G. Wilson, Cornell preprint CLNS-131 (1970, unpublished).

[2] C. Quigg, J.-M.Wang, and C.N. Yang, Phys. Rev. Letters 28, 1290 (1972); R.C. Hwa, ibid. 28, 1487 (1972). More elaborate versions were reviewed by R. Slansky, Washington APS Meeting, 1972.

[3] G. Charlton, et al. [ANL-NAL-Iowa State-Michigan State-Maryland Collaboration], Phys. Rev. Letters 29, 515 (1972); J. Chapman, et al. [Mighican-Rochester Collaboration], University of Rochester Report No. UR-395; F. T. Dao, et al. [NAL-UCLA Collaboration], contribution to the XVIth International Conference on High Energy Physics; E. L. Berger, Phys. Rev. Letters 29, 887 (1972).

[4] A. Białas, K. Fiałkowski, and K. Załewski, Jagellonian University preprint TPJU 5-72 (Nucl. Phys., to be published).

[5] W.R. Frazer, R.D. Peccei, S.S. Pinsky, and C.-I Tan, UCSD-10P10-113 (to be published).

[6] The use of a Poisson distribution in $\frac{1}{2}(n-2)$ instead of n was first suggested by C.P. Wang, Phys. Rev. 180, 1463 (1969).

[7] Each datum was weighted by the number of events from which it was derived. Weighting instead by the inverse squared error we obtain an alternative (and nearly identical) fit,

$$\sigma_n(s) = \{43.0\, e^{-0.016n}/n^2 + 25.1\, e^{-(\frac{1}{2}\lambda(s)-1)}(\tfrac{1}{2}\lambda(s) - 1)^{(\frac{1}{2}n-1)}/(\tfrac{1}{2}n - 1)!\,\}\,\mathrm{mb},$$

with $\lambda(s) = 6.7 + 2.4 \log (P_{lab}/100\ \mathrm{GeV/c})$.

[8] Wilson, Ref. 1, treats elastic scattering and diffraction dissociation into low multiplicities as a single entity. M. Bender raised this possibility to us.

[9] This conclusion is at least slightly more general than our specific parametrization, because it is shared by Frazer, et al., Ref. 5, who restrict their diffractive component to low multiplicities.

[10] A very similar apportionment between diffractive and multiperipheral components has been found in a related context by K. Fiałkowski, Phys. Letters 41B, 379 (1972).

[11] Fiałkowski, Ref. 10, makes this choice, confining his diffractive component to $n \leq 12$.

[12] H. D. I. Abarbanel and G. L. Kane, NAL-THY-84, advocate a $t_{minimum}$ effect which results in $n_{max} \propto s^{1/4}$. R. C. Hwa, University of Oregon preprint, August, 1972 has also discussed the influence of a momentum transfer cutoff. In both papers the authors fit with no multiperipheral component.

[13] In a fragmentation picture we imagine the very highest multiplicity events from a fireball or nova at rest in the center of mass and emitting pions with a mean energy corresponding to the observed transverse momenta.

[14]At 200 GeV/c, for example, Eq. (7) with $\omega = 0.4$ gives $\langle n^3 \rangle = 961$, while experiment is 827 ± 31. At 300 GeV/c, the comparison is 1344 vs. 1261 ± 62. To evaluate quantities such as $f_2 \equiv \langle n(n-1)\rangle - \langle n\rangle^2$, which are derived by subtracting two large numbers, we must exercise additional caution in fixing $\sigma_{2,\ \mathrm{inelastic}}$ and n_{max}. If we choose $\sigma_{2,\ \mathrm{inelastic}}$ equal to the experimental value and replace n_{max} by the greatest even integer $\leq n_{max}$ given by (4), we find $\langle n\rangle$, $\langle n^2\rangle$, $\langle n^3\rangle$, f_2 in agreement with the data, for ω=0.5 GeV. But because of the sensitivity of these quantities to details outside the realm of our fit - intended only as an example - it does not seem useful to discuss them in detail.

[15]G. Barbiellini et al., Phys. Letters 39B, 294 (1972).

[16]M. Breidenbach et al., Phys. Letter 39B, 654 (1972).

[17]R.N. Cahn, Ph.D. thesis, University of California, Berkeley, Lawrence Berkeley Laboratory Report LBL-1007, July 18, 1972. See also R.N. Cahn, SLAC-PUB-1087, August, 1972.

[18]See, for example, S.D. Ellis and A.I. Sanda, Phys. Rev. D6, 1347 (1972).

[19]Private communication from J.C. Sens and S.D. Ellis. A correspondence is made here between the large peak in the data at $0.85 < x < 1.0$ and the PPR contribution. Independent of triple-Regge theory it is plausible for us to identify the area of this peak with one half of our 8.5 mb.

[20] The discussion of the PPR cross section in this paragraph is somewhat casual. In more detail, the cross section reads $\frac{d\sigma^{(PPR)}}{dt dM} \simeq \frac{1}{M^2} \beta(t) \exp\left\{2[\alpha_P(t) - 1] \ln(s/M^2)\right\}$, where $\beta(t)$ is a residue function and $\alpha_P(t)$ is the Pomeranchuk trajectory. With a linear trajectory, $\alpha_P(t) = 1 + \alpha_P'(0)t$, and $\beta(t) = \beta(0) e^{bt}$, the integration over t gives a missing mass distribution, $\frac{d\sigma^{(PPR)}}{dM} \simeq \frac{1}{M^2}[b + 2\alpha_P'(0) \ln(s/M^2)]^{-1} \cdot \exp\left\{t_{min}[b + 2\alpha_P'(0) \ln(s/M^2)]\right\}$, where t_{min} for a single fireball of not too large mass is given by $t_{min} \simeq -m_p^2 (M^2 - m_p^2)^2/s^2$. The presence of the logarithms and t_{min} causes some departure from the M^{-2} variation and gives at high energies an integrated cross section of the form, $\sigma^{(PPR)} \simeq c_1 [b + 2\alpha_P'(0) \ln(s/m_p^2)]^{-1}$. Consequently the PPR cross section is not strictly energy independent unless the Pomeranchuk trajectory has zero slope.

[21] J. Lach and E. Malamud have several other examples of two-component fits based on two Poisson distributions. (Private communication.)

FIGURE CAPTIONS

Fig. 1 Fit to the 103 GeV/c topological cross sections. Solid line: two-component model; long dashes: fragmentation component; short dashes: multiperipheral component. The two-prong datum includes elastic and inelastic events. The dashed point is the inelastic two-prong cross section.

Fig. 2 Same as Fig. 1, at 205 GeV/c.

Fig. 3 Same as Fig. 1, at 303 GeV/c.

Fig. 4 Extrapolation of the fit to 400 GeV/c.

Fig. 5 Extrapolation of the fit to 505 GeV/c.

Fig. 6 Extrapolation of the fit to 1000 GeV/c.

Fig. 7 Extrapolation of the fit to 1500 GeV/c.

Fig. 8 Experimental results for $\langle n^2 \rangle$ and $\langle n^3 \rangle$, together with our calculations, for several choices of the parameter ω in Eq. (4).

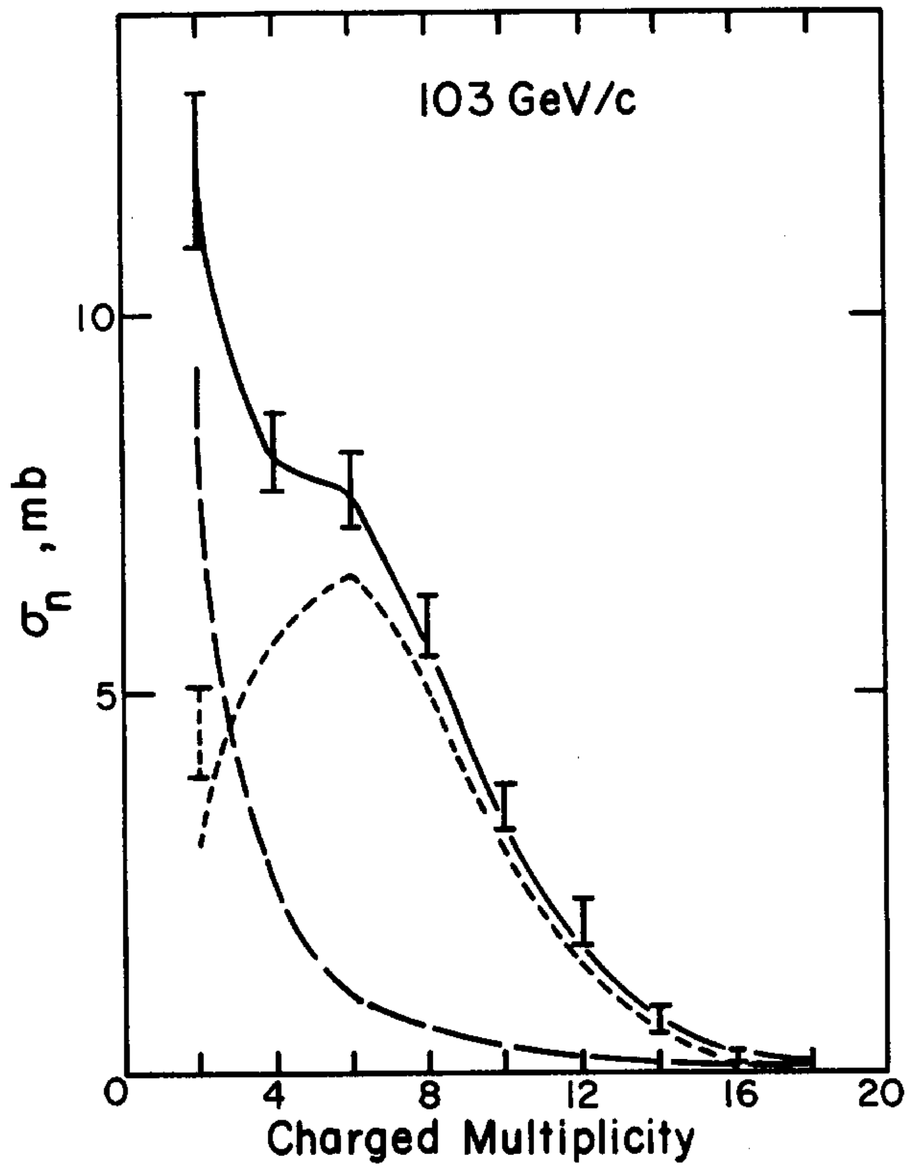


Figure 1

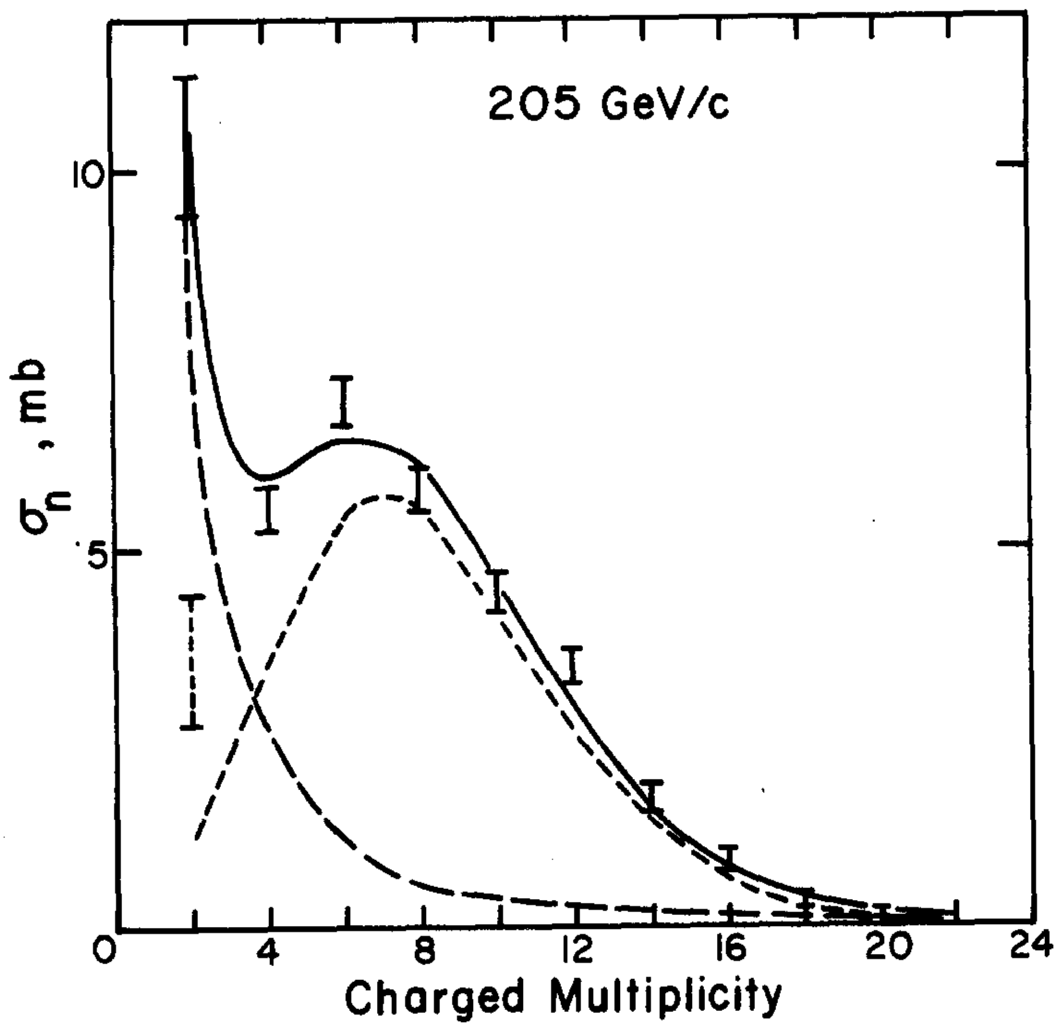


Figure 2

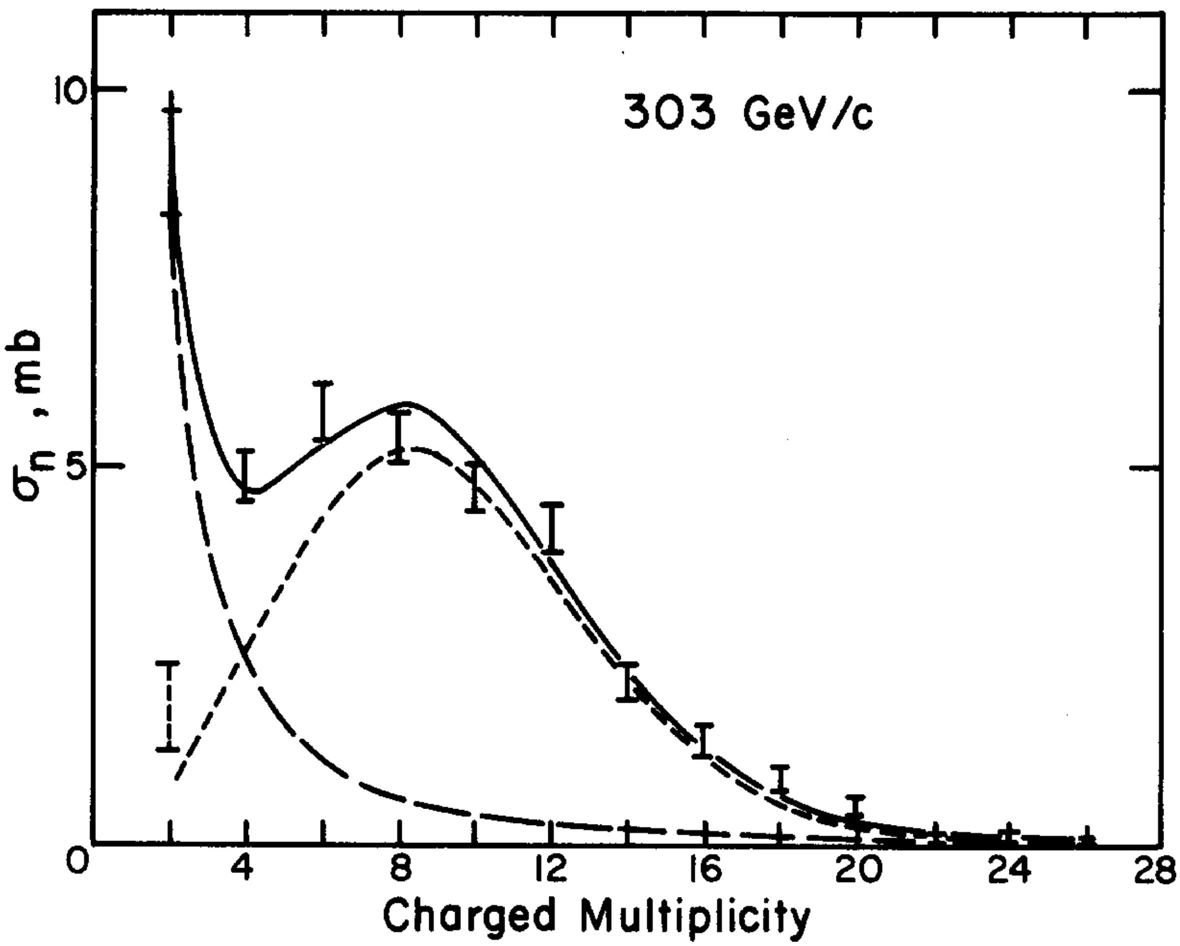
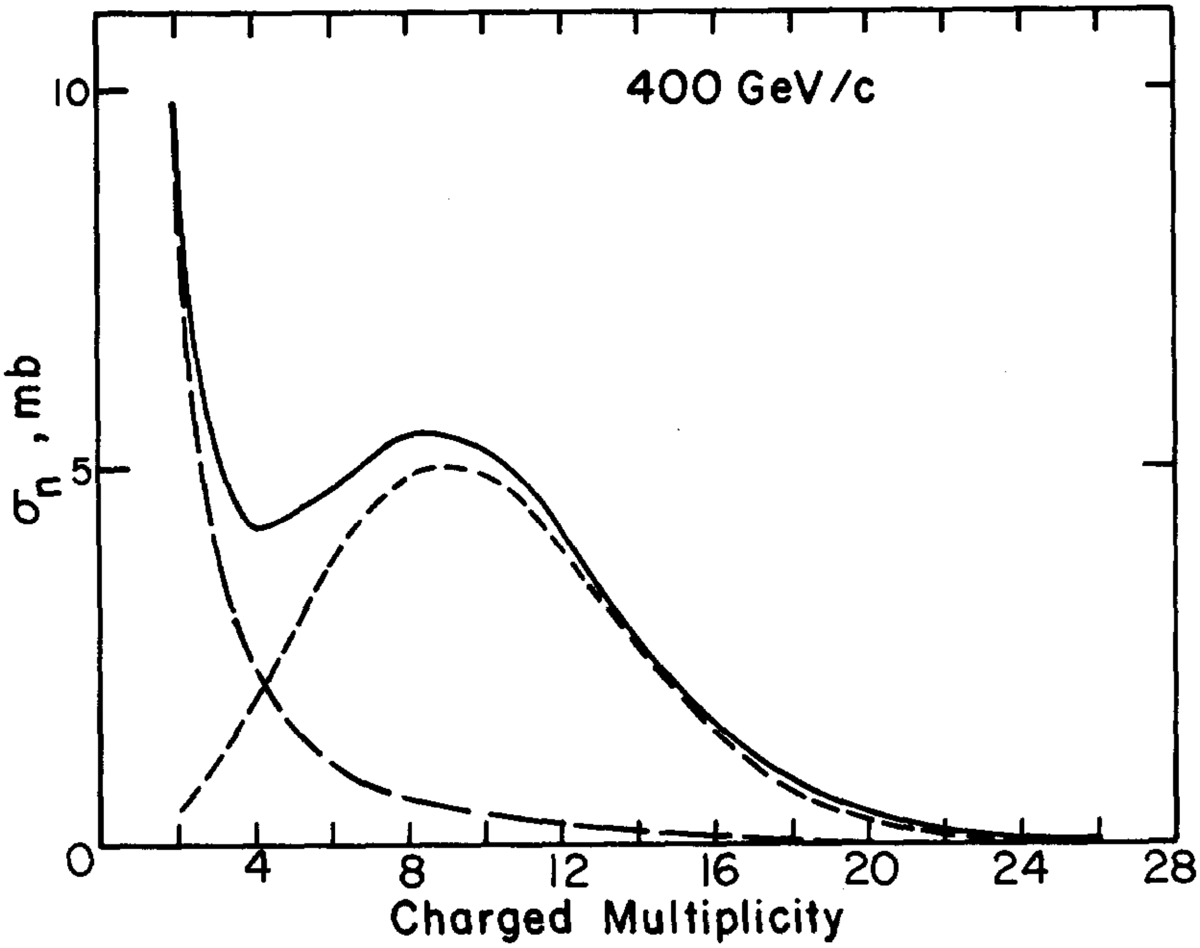
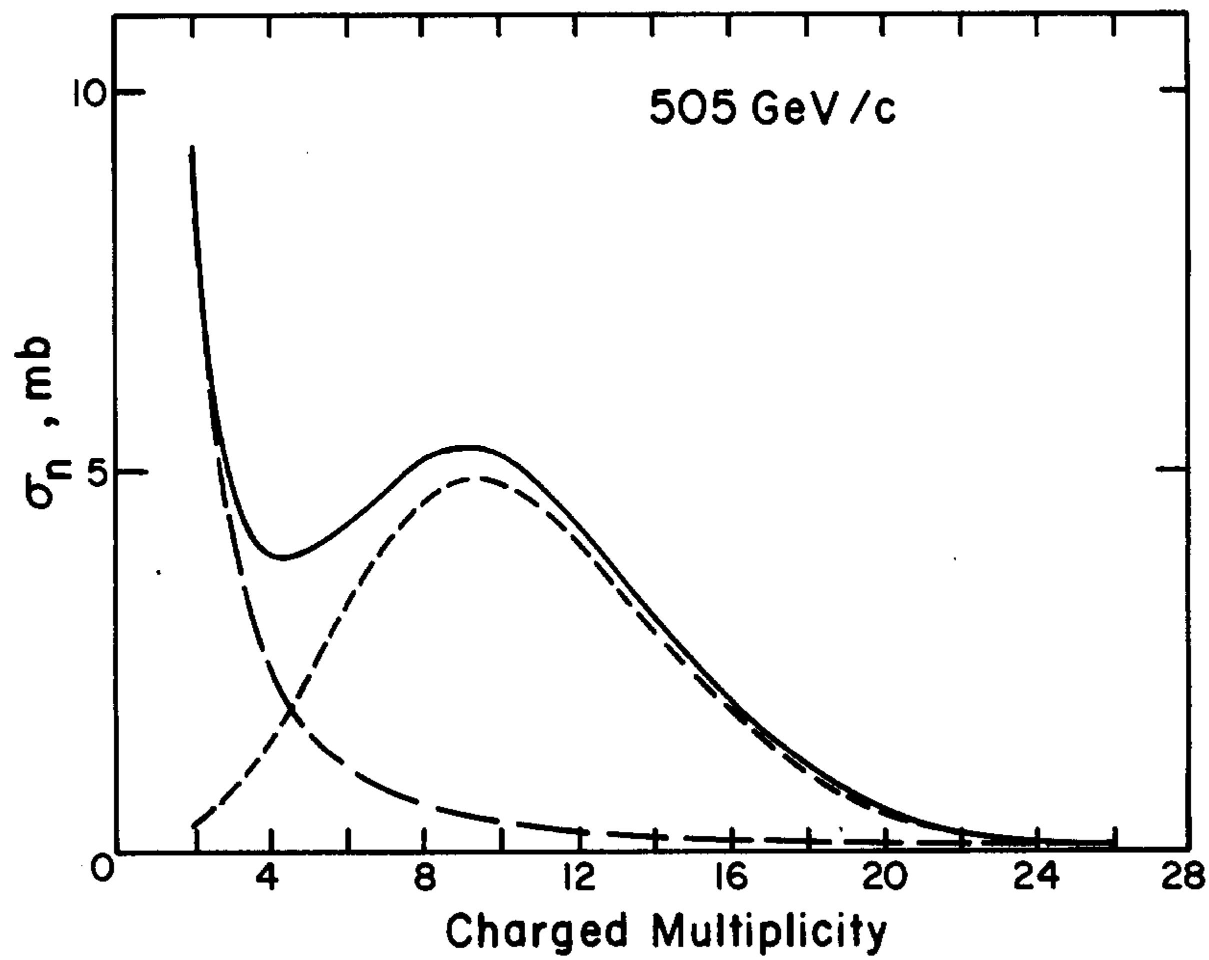


Figure 3



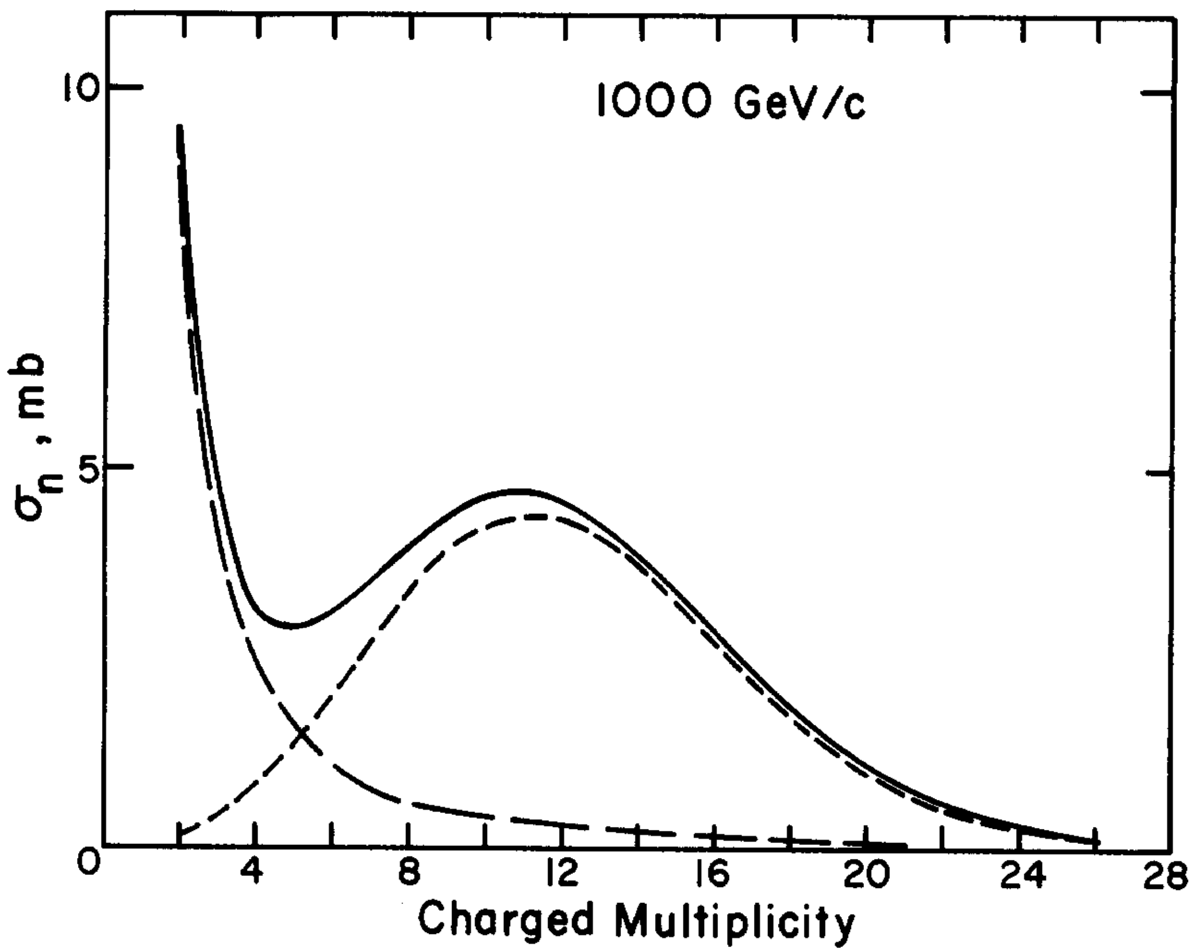
400 GeV/c
10
5
0
σ_n , mb
4
8
12
16
20
24
28
Charged Multiplicity

Figure 4



505 GeV/c
σ_n , mb
10
5
0
4
8
12
16
20
24
28
Charged Multiplicity

Figure 5



1000 GeV/c
σ_n , mb
10
5
0
4
8
12
16
20
24
28
Charged Multiplicity

Figure 6

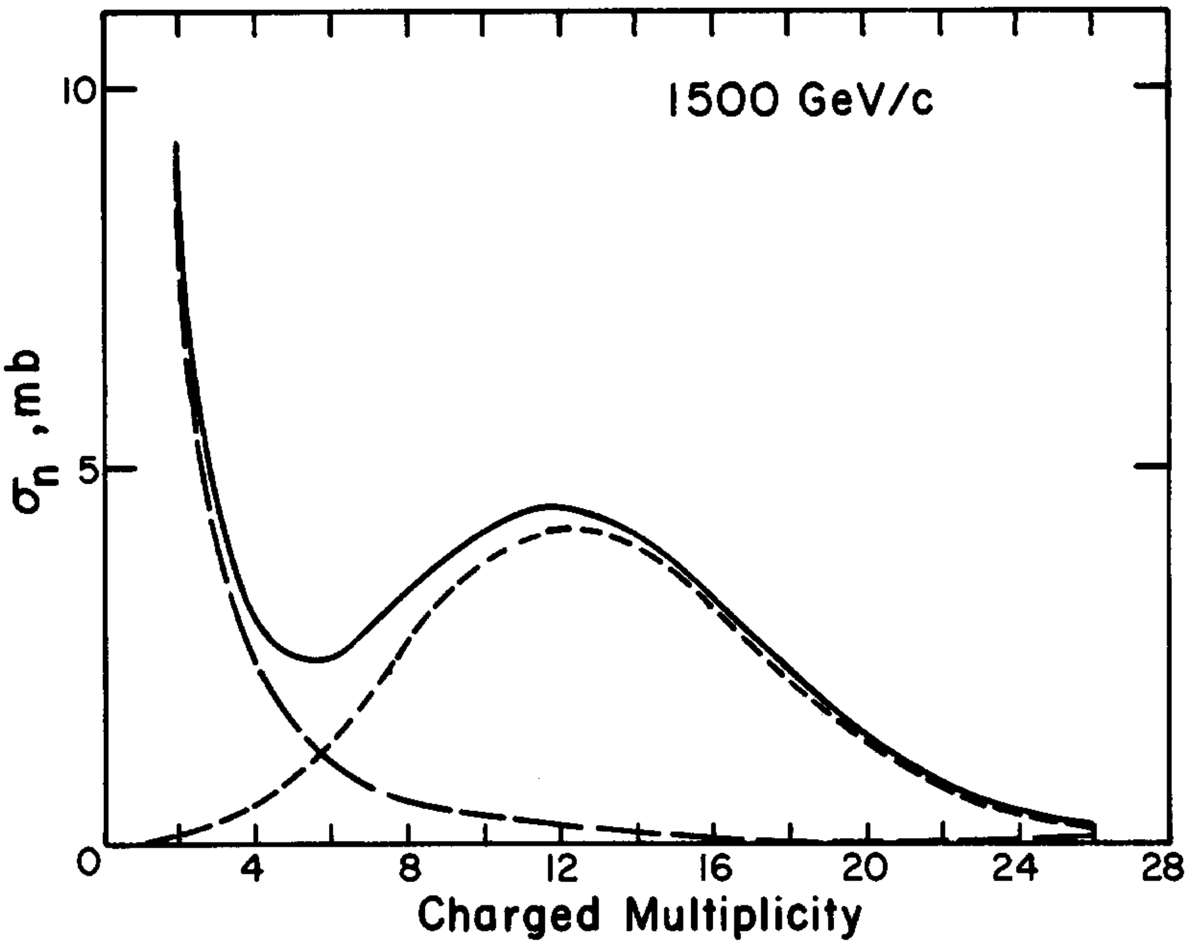
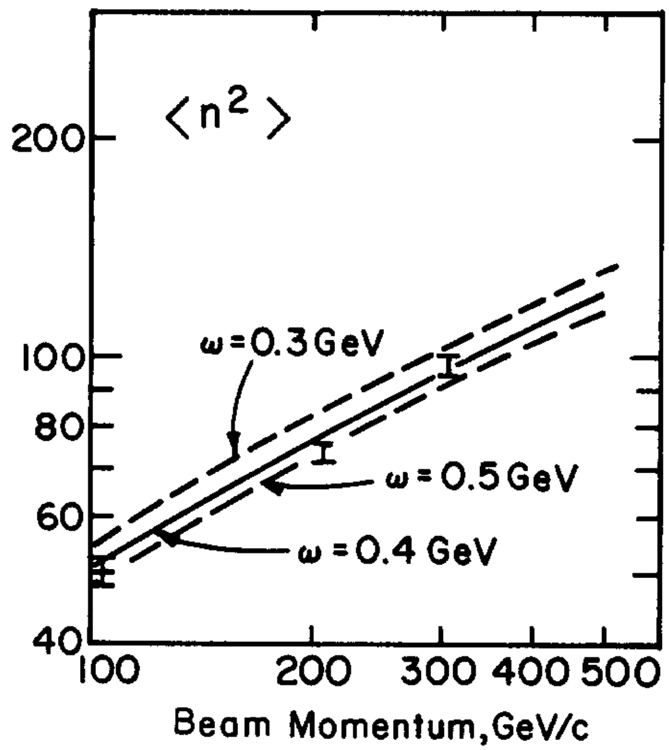


Figure 7



⟨n²⟩
200
100
80
60
40
ω=0.3 GeV
ω=0.5 GeV
ω=0.4 GeV
100
200
300
400
500
Beam Momentum, GeV/c
⟨n³⟩
2000
1000
800
600
400
ω=0.3 GeV
ω=0.5 GeV
ω=0.4 GeV
100
200
300
400
500
Beam Momentum, GeV/c

Figure 8