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SATURATION OF THE ADLER SUM RULE

(A Comment)

E. A. PASCHOS
National Accelerator Laboratory
P. O. Box 500, Batavia, Illinois 60510

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One of the most remarkable theoretical results to be tested in the upcoming experiments is the Adler sum rule. In the past months we have witnessed renewed interest[1] on the saturation of the sum rule, with several of the authors arriving at the conclusion that the sum rule will not be saturated even in the multi-GeV region. It is the purpose of this note to make a critical analysis of the situtation. In the standard notation the sum rule reads

$$\int_0^\infty \left[W_2^{\nu n}(Q^2,\nu) - W_2^{\nu p}(Q^2,\nu) \right] d\nu = 2\cos^2\theta_c \approx 1.86 \tag{1}$$

or in the Bjorken Scaling region (keeping only the vector contribution)

$$\int_0^1 \left[F_2^{\nu n}(x) - F_2^{\nu p} \right] \frac{dx}{x} = \cos^2\theta_c \approx .93 \tag{2}$$

The sum rule has been tested (via PCAC) in the $Q^2 \ll M^2$ region by Adler and Gilman.[2] There it was found that it saturates to more that 95% of its value for $\nu \sim 5$ GeV.

Under zero dynanical assumptions, the sum rule for the vector contribution alone can be bounded from above by using the Bjorken inequality

$$2.8 \pm .2 \geq 2\int_{0.05}^{1} \left[F_2^{ep} + F_2^{en} \right] \frac{dx}{x} \geq \int_{0.05}^{1} \left[F_2^{\nu p} + F_2^{\nu n} \right] \frac{dx}{x} \tag{3}$$

The numerical value was evaluated in terms of the SLAC-MIT data. The bound is obviously very crude. However, it does not indicate anything very alarming. In fact, it does not exclude the possibility that the sum rule could be saturated (to within 90%) at $x \sim .05$, provided that the structure functions are quite different. Table 1 illustrates the situtation. Define

$$r = \frac{F_2^{\nu n}}{F_2^{\nu p}} \tag{4}$$

Then the sum rule reads

$$2 \left\langle \frac{r-1}{r+1} \right\rangle \int \left[F_2^{ep} + F_2^{en} \right] \frac{dx}{x} \geq \int \left(\frac{r-1}{r+1} \right) \left[F_2^{\nu n} + F_2^{\nu p} \right] \frac{dx}{x} \tag{5}$$

Assuming that the isoscalar contribution to the sum of the electroproduction functions is small ($\sim$ 10%) we obtain the average value of r required in the non-Regge and the intermediate region. Table 1 gives the average $\bar{r}$ and the corresponding contribution to the sum rule is denoted by Σ. The sum rule can be saturated easily provided that the ratio $F_2^{\nu n}/F_2^{\nu p} \sim 2.5 - 4$ in the non-Regge region.

Such a value is not unreasonably large. Nachtman's[3] positivity relations give the bound

$$F_2^{\nu n} / F_2^{\nu p} \geq \frac{1}{2} \tag{6}$$

Whenever the ratio $y(x) = F_2^{\gamma n}/F_2^{\gamma p}$ is known, as is the case with the MIT experiment, we can improve the bound on the neutrino structure functions

$$z(x) \equiv \frac{F_2^{\nu n}}{F_2^{\nu p}} \geq \frac{1}{2} + \frac{1}{2}\,\frac{6-9y}{4y-1} \quad . \qquad (7)$$

This bound is better than (6) for $y < 2/3$. The results shown in Fig. 1 indicate that for small values of y, z should be quite large.

The conclusion is that there is still enough freedom for the sum rule to be saturated for a value of x > .050 - 0.025. This, however, leads to other observable results at larger values of x.[5] The early saturation requires a contribution to the cross section which is large at intermediate values of x but goes away faster than $\sqrt{x}$ as $x \to 0$. Such a contribution can arise from a $J \leq 1/2$ singularity.

REFERENCES

[1] Slow convergence of the integral has been discussed by:

a. H. Pagels, Phys. Rev. D3, 1217 (1971);

b. J. D. Bjorken and S. F. Tuan, Comments on Nuclear and Particle Physics, (1969);

c. J. J. Sakurai, H. B. Thacker and S. F. Tuan, UCLA/72/TEP/58 (1972).

In these papers the rate of convergence is determined essentially by the ρ-trajectory, whose residue is adjusted to fullfil the sum rule. Lower trajectories could in general be present and change the rate of saturation drastically.

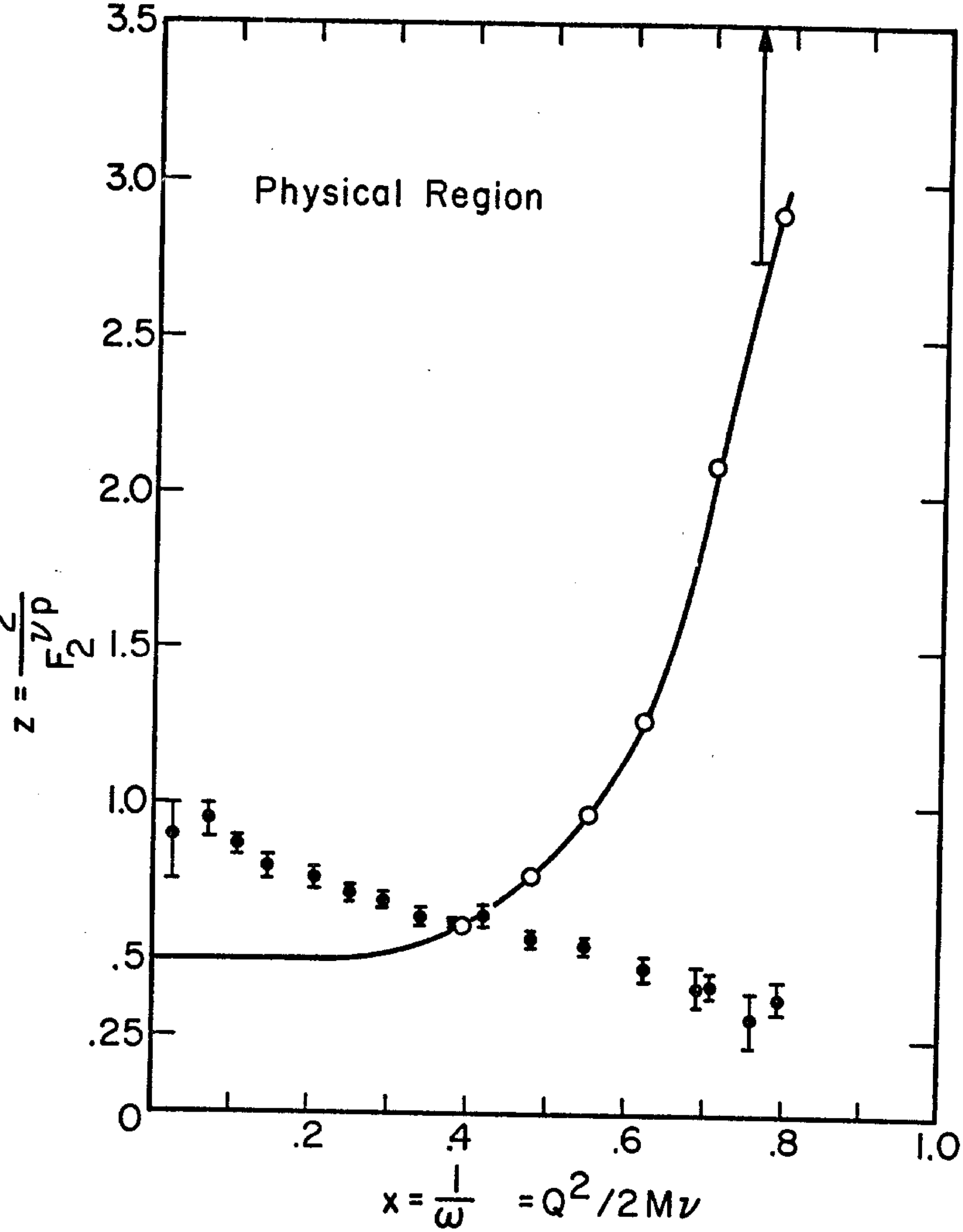
[2] S. Adler and F. Gilman, Phys. Rev. 156, 1598 (1967).

[3] O. Nachtman, Nucl. Phys. B38, 397 (1972). C. Callan, M. Gronau, A. Pais, E. Paschos, S. B. Treiman, Phys. Rev. D6, 387, (1972).

[4] Presented to this Conference by Soggard for the MIT-Harvard Group.

TABLE 1

Non-Regge Region	Intermediate Region
$.2 \le x \le 1.0$	$.1 \le x \le .2$
$\bar{r} = 2.5$ $\Sigma = .58$	$\bar{r} = 2.4$ $\Sigma = .32$
$\bar{r} = 3.0$ $\Sigma = .68$	$\bar{r} = 1.8$ $\Sigma = .22$
$\bar{r} = 4.0$ $\Sigma = .82$	$\bar{r} = 1.25$ $\Sigma = .08$



3.5
3.0
2.5
2.0
1.5
1.0
.5
.25
0
Physical Region
$z = \frac{F_2^{\nu n}}{F_2^{\nu p}}$
.2
.4
.6
.8
1.0
$x = \frac{1}{\omega} = Q^2/2M\nu$