national accelerator laboratory

NAL-THY-75

August 1972

Finite-Energy Sum Rule for Inclusive Reactions II

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ABSTRACT

The finite-energy sum rule for inclusive reaction with fixed s and t was previously shown to be consistent with experimental data. In the present paper, we discuss fixed M^2/s and t sum rules. The data obtained from resonance search experiments using Jacobian peak method is particularly suited for testing the sum rule.

Operated by Universities Research Association Inc. Under Contract with the United States Atomic Energy Commission

I INTRODUCTION

By now it is well known that an inclusive reaction cross section for $a + b \to c + X$,(X stands for all possible unobserved states.) is related to the absorptive part of a scattering amplitude for $a + b + \bar{c} \to a + b + \bar{c}$ when the latter is analytically continued to the proper kinematical region. In this paper, we will call this a consequence of generalized unitarity. In a previous paper,[1] we pointed out that the analyticity of the scattering amplitude for $a + b + \bar{c} \to a + b + \bar{c}$ allowed us to write a finite-energy sum rule for inclusive reactions. (This result has also been discussed by Ref. 2.)

$$\int_{-t}^{\bar{M}_0^2} (\bar{M}^2)^n \frac{d\sigma}{dt\, d\bar{M}^2}(a+b\to c+X)\, d\bar{M}^2 - (-1)^n \int_{-t}^{\bar{M}_0^2} (\bar{M}^2)^n \frac{d\sigma}{dt\, d\bar{M}^2}(c+b\to a+X)\, dM^2$$
$$= \sum_{ijk} \left(1-(-1)^n \tau_i \tau_j \tau_k\right) \frac{G_{ijk}(t)}{16\pi\; s^2} \left(\frac{s}{\bar{M}_0^2}\right)^{\alpha_i(t)+\alpha_j(t)} \frac{(\bar{M}_0^2)^{\alpha_k(0)+n+1}}{\alpha_k(0)-\alpha_i(t)-\alpha_j(t)+n+1} \qquad (1)$$

where s is fixed and $M_0^2 << s$.

The momenta are defined in Fig. 1,

$$M^2 = (p_a + p_b - q)^2, \quad s = (p_a + p_b)^2, \quad t = (p_a - q)^2,$$
$$\bar{M}^2 = M^2 - m_b^2 - t$$

i, j, k are Regge trajectories shown in Fig. 2 and $\alpha_i(t)$, $\alpha_j(t)$ and $\alpha_k(0)$ are their trajectory functions. τ_i, τ_j, and τ_k are signatures of the trajectories i, j, and k respectively. $G_{ijk}(t)$ are triple-Regge residue functions for the Reggeons i, j and k. n is a positive integer. Eq. (1) is nothing but a consequence of the Cauchy's theorem if the scattering amplitude $T(s, t, M^2)$ satisfies following conditions. (a) $T(s, t, M^2)$ has the triple-Regge behavior

$$T(s,t,M^2) \xrightarrow[\substack{\lim s/M^2 \to \infty \\ M^2 \to \infty}]{} \frac{\pi}{4m^2} \sum_{ijk} \frac{\left(1 + \tau_i \tau_j \tau_k e^{-i\pi(\alpha_k(0) - \alpha_i(t) - \alpha_j(t))}\right)}{\sin \pi\,(\alpha_k(0) - \alpha_i(t) - \alpha_j(t))} \left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)} (M^2)^{\alpha_k(0)} \tag{2}$$

(b) When s is large, $t < 0$ and both are fixed, $T(s, t, M^2)$ is analytic in M^2 everywhere except along the real axis, where the generalized unitarity requires $T(s, t, M^2)$ to have singularities. The discontinuity across the cut is related to the inclusive cross section for $a + b \to c + X$.

Phenomenologically, we have verified that the data $p + p \to p + X$ taken at BNL and $\pi^{\mp} + p \to p + X$ taken at Serpukov are consistent with only two triple-Regge terms[3] G_{PPf} and G_{ffP}. Furthermore, we have verified that the values obtained for the triple-Regge residue functions are consistent with Eq. (1) for both sets of data.[4]

With this initial success we may ask whether there are other results which may follow from the analyticity of $T(s, t, M^2)$. In this paper we concentrate our efforts on writing a finite-energy sum rule with a much larger contour. That is to say we relax the condition $M^2 < M_0^2 << s$ and consider the case M^2/s finite and fixed.[5] We suppose that

(a) $T(M^2/s, t, M^2)$ for fixed M^2/s and t has the Regge behavior in the fragmentation region. The precise Regge behavior required will be given below. (b) $T(M^2/s, t, M^2)$, for fixed M^2/s and t, is analytic in M^2 except for the unitarity cut.

These assumptions will enable us to derive a sum rule which relates the integral of the inclusive cross section over the missing mass M^2 (with M^2/s and t fixed) to the residue functions $g_k(M^2/s, t)$. We stress that the analyticity assumption for $T(M^2/s, t, M^2)$ is by no means trivial. In Ref. 1 we gave an example of a diagram in the ϕ^3 theory which produces complex cuts in the region of our consideration. We will, for now, take the view point that these complex cuts contribute very little to the sum rule.

II FIXED $M^2/s, t$ SUM RULE

In Section II of Ref. 1 we have given a discussion which shows that the absorptive part of $T(s, t, M^2)$ when taken properly is indeed the cross section for $a + b \to c + X$. It was important to state exactly how we take the absorptive part. To restate the procedure, let us for the moment, consider a non-forward scattering amplitude shown in Fig. 3. Define

$$s = (p_a + p_b)^2, \quad s' = (p_a' + p_b')^2, \quad t = (p_a - q)^2, \quad t' = (p_a' - q')^2$$

$$M_1^2 = (p_a - p_b' - q)^2, \quad M^2 = (p_a + p_b - q)^2.$$

(For the rest of the variables we refer the reader to Ref. 1) Then

$$2i\,\frac{d\sigma}{dt\,dM_0^2} = \frac{m^2}{(2\pi)^2 s_0^2} \lim_{\substack{\epsilon_1\to 0\\ \epsilon_2\to 0}} \lim_{\epsilon_3\to 0} \times$$

$$\Big\{ T(s=s_0+i\epsilon_1,\; s'=s_0-i\epsilon_2,\; t,\; M^2=M_0^2+i\epsilon_3)$$

$$- T(s=s_0+i\epsilon_1,\; s'=s_0-i\epsilon_2,\; t,\; M^2=M_0^2-i\epsilon_3)\Big\} \qquad (3)$$

where $\epsilon_1, \epsilon_2, \epsilon_3 > 0$. The order of limits in Eq. (3) is important to assure that discontinuities due to other channels are absent. Keeping $s = s_o + i\,\epsilon_1$, $s' = s_o - i\,\epsilon_2$ fixed, ϵ_1, ϵ_2 small but finite, we can isolate a M^2 plane on which there are right and left hand cuts. The left hand cut corresponds to the M_1^2 channel. (Note that $M_1^2 = 2t + 2m^2 - M^2$.) The absorptive part for these two channels are shown in Fig. 4.

So far we considered the case of fixed $s >> M_0^2$ and $t < 0$. We now consider the problem of writing a finite-energy sum rule where the cut off in the M^2 integral becomes the same order as s with t fixed. This is the fragmentation region of particle a. We start by defining the variables which treat both channels shown in Fig. 4 symmetrically.[2]

$$\eta = 2\,p_b\cdot(p_a+q)\,, \quad \eta' = 2\,p_b'(p_a'-q')$$

$$\nu = 2\,p_b\cdot(p_a-q) = 2\,p_a'\cdot(p_a'-q')\,.$$

They are related to previously defined variables.

$$
\begin{aligned}
&s = m_a^2 + m_b^2 + \tfrac{1}{2}(\eta + \nu) \ , \quad s' = m_a^2 + m_b^2 + \tfrac{1}{2}(\eta' + \nu) \\
&M^2 = m_b^2 + t + \nu \\
&M_1^2 = m_b^2 + t - \nu \\
&\bar{M}^2 = \nu
\end{aligned}
\tag{5}
$$

The M^2 and M_1^2 channel singularities on the ν plane is shown in Fig. 5. We have also drawn the contours used to obtain the finite-energy sum rule. It will be seen below that the Regge expansion for the fragmentation region involves a residue functions which depends on M^2/s or equivalently ν/η. Thus we are forced to keep the variable fixed throughout the contour of integration. For complex ν, we must yet specify how they are fixed. For example there are at least two choices ν/η fixed or ν/η^* fixed. As the phase of ν changes, the phase of η changes differently in two cases. However, in order to avoid going into the second sheet on both η and η' planes we must keep ν/η and $\nu/(\eta')^*$ fixed. To see this, we will trace the scattering amplitude $T(\eta, \eta', t, M^2)$ with fixed ν/η and $\nu/(\eta')^*$ around the contour shown in Fig. 5

Between a and b	$T\ (\eta + i\,\epsilon_1,\ \eta - i\,\epsilon_2,\ t,\ \nu + i\epsilon_3)$
Between b and c	$T\ (\eta\, e^{i\theta},\ \eta\, e^{i(2\pi-\theta)},\ t,\ \nu\, e^{i\theta})\quad 0<\theta<\pi$
Between c and d	$T\ (-\eta + i\,\epsilon_1,\ -\eta - i\epsilon_2,\ t,\ -\nu + i\,\epsilon_3)$
Between d and e	$T\ (-\eta - i\epsilon_1,\ -\eta + i\epsilon_2,\ t,\ -\nu - i\,\epsilon_3)$

Between e and f $\quad T\ (\eta e^{i\theta},\ \eta e^{i(2\pi-\theta)}, t, \nu e^{i\theta})\ \pi<\theta<2\pi$

Between f and a $\quad T\ (\eta - i\epsilon_1,\ \eta + i\epsilon_2,\ t,\ \nu - i\epsilon_3)$

where η, ν are taken to be real and positive.

The Cauchy's theorem

$$\oint \nu^n\, T\left(\frac{\nu}{\eta}, \frac{\nu}{(\eta')^*}, t, \nu\right) d\nu = 0 \tag{6}$$

can be used to obtain the sum rule.[6]

To relate the integrand to physical quantities we note that

$$\begin{aligned} &T(\eta - i\epsilon_1, \eta + i\epsilon_2, t, \nu - i\epsilon_3) \\ &\quad = T(\eta + i\epsilon_1, \eta - i\epsilon_2, t, \nu - i\epsilon_3)\ . \end{aligned} \tag{7}$$

This will be proven in the Appendix.

Using Eqs. (3)and (7), we obtain

$$\begin{aligned} &\Big[T(\eta+i\epsilon_1, \eta-i\epsilon_2, t, \nu+i\epsilon_3) - T(\eta-i\epsilon_1, \eta+i\epsilon_2, \nu-i\epsilon_3)\Big]_{\substack{\nu>0\\ \eta>0\\ t<0}} \\ &= 2i\,\frac{(2\pi)^2\eta^2}{m^2}\,\frac{d\sigma}{dt\,d\nu}\,(a+b\rightarrow c+X) \end{aligned} \tag{8}$$

$$\left[T(\eta - i\epsilon_1, \eta + i\epsilon_2, t, \nu - i\epsilon_3) - T(\eta + i\epsilon_1, \eta - i\epsilon_2, t, \nu + i\epsilon_3) \right]_{\substack{\nu<0 \\ \eta<0 \\ t<0}}$$

$$= 2i \frac{(2\pi)^2 \eta^2}{m^2} \frac{d\sigma}{dt\, d\nu} (\bar{a} + b \to \bar{c} + X) \tag{9}$$

The right hand side of Eq. (9) can be seen as follows: $\nu < 0$, $\eta < 0$, $t < 0$ implies that $p_a \cdot p_b < 0$, $p_a \cdot q > 0$ and $(p_a - p_b')^2 > 0$. In the rest frame of a, $p_b^0 < 0$, $q^0 > 0$. Therefore the discontinuity of the amplitude is proportional to the cross section for the inclusive reaction $a + \bar{b} \to c + X$ or equivalently $\bar{a} + b \to \bar{c} + X$. In order to evaluate the contribution from the circular part of the contour, let us state precisely the Regge behavior of the amplitude. In the fragmentation region, we have

$$\frac{d\sigma}{dt\, dM^2} \xrightarrow[\substack{t, \frac{s}{M^2} \text{ fixed} \\ M^2 \to \infty}]{} \frac{1}{16\pi \eta^2} \sum_k g_k\left(\frac{s}{M^2}, t\right) (M^2)^{\alpha_k(0)} \tag{10}$$

where the trajectory k is shown in Fig. 6 and $g_k(\frac{s}{M^2}, t)$ is the Regge residue function. We must find the form of the amplitude whose discontinuity matches Eq. (10).

$$T\left(\frac{\nu}{\eta}, \frac{\nu}{(\eta')^*}, t, \nu\right) \xrightarrow[\substack{t,\ \frac{\nu}{\eta},\ \frac{\nu}{(\eta')^*}\ \text{fixed} \\ \nu \to \infty}]{}$$

$$\frac{\pi}{4m^2} \sum_K g_K\left(\frac{\nu}{\eta}, \frac{\nu}{(\eta')^*}, t\right) \frac{(\eta_K + e^{-i\pi\alpha_K(0)})}{\sin \pi\alpha_K(0)} \nu^{\alpha_K(0)} . \tag{11}$$

satisfies this condition. To obtain this form, in addition to the analyticity assumption stated earlier, we require that the amplitude satisfies the conditions for the Phragmen-Lindelöff theorem.

Eqs. (6), (8), (9), (11) gives our result

$$\begin{aligned} &\int_t^{\nu_0} \nu^{n+2} \frac{d\sigma}{dt\,dM^2} (a+b \to c+X)\, dM^2 \\ &- (-1)^n \int_t^{\nu_0} \nu^{n+2} \frac{d\sigma}{dt\,dM^2} (\bar{a}+b \to \bar{c}+X)\, dM^2 \\ &= \left(\frac{\nu}{\eta}\right)^2 \sum_K \frac{g_K\left(\frac{\nu}{\eta}, \frac{\nu}{(\eta')^*}, t\right)}{16\pi} \frac{\nu_0^{\alpha_K(0)+n+1}}{\alpha_K + n + 1} \left(1 - (-1)^n \eta_K\right) \end{aligned} \tag{13}$$

III APPLICATIONS

The phenomenological implication of the fixed M^2/s, t sum rule should be quite interesting. Let us make a duality assumption. That is, suppose that the Regge expansion, Eq. (10), with only one or two terms will describe the ν dependence of the inclusive cross section. In the resonance region, the Regge form is supposed to be good in the average sense. Then the sum rule should work with small ν_0. Say large enough to include the well established resonances. But since ν/η is fixed, missing mass data required for the sum rule will come from low energy experiments. For example, take $a = c = \pi^+$, $b = p$. Then X will contain baryon resonances. The missing mass spectrum is smooth when $M^2 \gtrsim 6\ \mathrm{GeV}^2$. Therefore, take $\nu_0 \approx 8\ \mathrm{GeV}^2$ and for example take $\eta/\nu = 2$. Then for the left hand side of Eq. (13), only inclusive reaction cross sections taken at $2\ \mathrm{GeV}^2 \lesssim s \lesssim 14\ \mathrm{GeV}^2$ is necessary. The Regge residue function $g_k(\nu/\eta,\ t)$ thus obtained can then be used to predict cross sections which will be taken at NAL or ISR.

The sum rule can be used to discuss the relation between triple and single Regge expansions. Since physics is smooth, we expect the triple-Regge and single-Regge regions to be connected smoothly. That is

$$g_k\left(\frac{s}{M^2}, t\right) = \sum_{ij} G_{ijk}(t) \left(\frac{s}{M^2}\right)^{\alpha_i(t) + \alpha_j(t)} \tag{14}$$

Eq. (14) is not useful if we have to sum over a large number of trajectories. But we rewrite the sum rule

$$\int_{-t}^{\nu_0} \nu^2 \frac{d\sigma}{dt\,dM^2}(a+b\to c+X)\,dM^2 - \int_{-t}^{\nu_0} \nu^2 \frac{d\sigma}{dt\,dM^2}(\bar{a}+b\to \bar{c}+X)\,dM^2$$

$$= \sum_{ijk} (1-\tau_k)\,\frac{G_{ijk}}{16\pi}\left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)+2} \frac{(M^2)^{\alpha_k(0)+1}}{\alpha_k(0)+1} \tag{15}$$

and note that on the left hand we integrate over various resonance contributions in ab and $\bar{a}$b channels. So again we expect very few trajectories i and j are sufficient on the right hand side. If this expectation is valid, we are lead to

$$g_k\left(\frac{s}{M^2}, t\right) \approx \sum_{\text{few } i,j} G_{ijk}(t)\left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)} \tag{16}$$

This is varified experimentally. Only two triple-Regge terms G_{PPf} and G_{ffP} were sufficient to describe the data at s/M^2 as small as 4. On the basis of Eq. (16) it will be interesting to analyze the data for the inclusive cross section in the fragmentation region in terms of the triple-Regge formula.

IV. SUMMARY

We have derived a fixed M^2/s, t sum rule, Eq. (13). The major assumption is the analyticity of the scattering amplitude stated in footnote 6. This assumption, at least in a ϕ^3 theory, may not be valid. Among many things, complex singularity may be present. If the sum rule works, then nature chooses the discontinuity across such singularities to be weak. Also, if the sum rule works, this is probably the strongest test of the generalized unitarity. For now we have accepted the stated assumptions to be true and we have discussed some applications of the sum rule. There is a possibility that only low energy missing mass data is required to obtain the residue function in the fragmentation region, $g_k(M^2/s, t)$. Finally we note that the resonance search experiments using the Jacobian peak method measures $s \frac{d\sigma}{dtdM^2}(M^2/s, t)$. Such experiments are already underway at NAL and the sum rule can soon be tested.[7]

ACKNOWLEDGEMENTS

We thank Aspen Institute for Physics for their hospitality where most of present work was completed. We also thank S. D. Ellis and A. Pagnamenta for discussions.

APPENDIX

In this appendix we prove Eq. (7). Let us start from S matrix

$$S(\eta+i\epsilon_1, \eta+i\epsilon_2, t, \nu-i\epsilon_3)$$

$$= \sqrt{\frac{E_a E_b E_a' E_b'}{m_a^2 m_b^2}}\, i^2 \int d^4x\, d^4y\, e^{+iqx} e^{-iq'y} K_x K_y \langle p_a' p_b' \text{ out} | T(\phi_c^+(x)\, \phi_c(y)) | p_a p_b \text{ in} \rangle$$

$$= i\,(2\pi)^4 \delta^4(p_a+p_b-q-p_a'-p_b'+q') \sqrt{\frac{E_a E_b E_a' E_b'}{m_a^2 m_b^2}}\, (2\pi)^3 (q^2-m_c^2)(q'^2-m_c^2)$$

$$\times \sum_n \Bigg[\delta^3(p_a+p_b+q'-p_n) \frac{\langle p_a' p_b' \text{ out} | \phi_c^+(0) | n \rangle \langle n | \phi_c(0) | p_a p_b \text{ in} \rangle}{p_n^0 - p_a^0 - p_b^0 - q_0' + i\epsilon_3} \qquad (18)$$

$$+ \delta^3(p_a+p_b-q-p_n) \frac{\langle p_a' p_b' \text{ out} | \phi_c(0) | n \rangle \langle n | \phi_c^+(0) | p_a p_b \text{ in} \rangle}{p_n^0 - p_a^0 - p_b^0 + q_0 + i\epsilon_3} \Bigg]$$

In Ref. 1, we have shown that if we define a function $F(\eta, t, \nu)$ such that

$$F(\eta+i\epsilon_2, t, \nu+i\epsilon_3) = \sqrt{\frac{E_a E_b}{m^2}} \langle p_a p_b \text{ out} | \phi^+(0) | n \rangle \qquad (19)$$

than

$$F(\eta-i\epsilon_2, t, \nu+i\epsilon_3) = \sqrt{\frac{E_a E_b}{m^2}} \langle n | \phi(0) | p_a p_b \text{ in} \rangle^* \qquad (20)$$

Using Eqs.(19) and (20), we obtain the T matrix:

$$
\begin{aligned}
& T(\eta+i\epsilon_1, \eta-i\epsilon_2, t, M^2-i\epsilon_3) \\
&= \sqrt{\frac{E_a E_b E_a' E_b'}{m_a^2 m_b^2}} (2\pi)^3 (q^2-m_c^2)(q'^2-m_c^2) \\
&\times \sum_n \Bigg[\delta^3(p_a+p_b+q'-p_n) \frac{\langle p_a' p_b' \text{ in} | \phi_c^\dagger(0) | n \text{ out}\rangle \langle n \text{ out} | \phi(0) | p_a p_b \text{ in}\rangle}{p_n^0 - p_a^0 - p_b^0 - q_0' + i\epsilon_3} \\
&\qquad + \delta^3(p_a+p_b-q-p_n) \frac{\langle p_a' p_b' \text{ in} | \phi_c(0) | n \text{ out}\rangle \langle n \text{ out} | \phi^\dagger(0) | p_a p_b \text{ in}\rangle}{p_n^0 - p_a^0 - p_b^0 + q_0 + i\epsilon_3} \Bigg]
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
& T(\eta-i\epsilon_1, \eta+i\epsilon_2, t, \nu-i\epsilon_3) \\
&= \sqrt{\frac{E_a E_b E_a' E_b'}{m_a^2 m_b^2}} (2\pi)^3 (q^2-m_c^2)(q'^2-m_c^2) \\
&\times \sum_n \Bigg[\delta^3(p_a+p_b+q'-p_n) \frac{\langle p_a' p_b' \text{ out} | \phi_c^\dagger(0) | n \text{ in}\rangle \langle n \text{ in} | \phi(0) | p_a p_b \text{ out}\rangle}{p_n^0 - p_a^0 - p_b^0 - q_0' + i\epsilon_3} \\
&\qquad + \delta^3(p_a+p_b-q-p_n) \frac{\langle p_a' p_b' \text{ out} | \phi_c(0) | n \text{ in}\rangle \langle n \text{ in} | \phi^\dagger(0) | p_a p_b \text{ out}\rangle}{p_n^0 - p_a^0 - p_b^0 + q_0 + i\epsilon_3} \Bigg]
\end{aligned} \tag{22}
$$

But the time reversal invariance implies that

$$\langle n \text{ out} | \phi_c(0) | p_a p_b \text{ in} \rangle = \langle p_a p_b \text{ out} | \phi^{\dagger}(0) | n \text{ in} \rangle \tag{23}$$

therefore

$$T(\eta - i\epsilon_1, \eta + i\epsilon_2, t, \nu - i\epsilon_3)$$
$$= \sqrt{\frac{E_a E_b E_a' E_b'}{m_a^2 m_b^2}} (2\pi)^3 (q^2 - m^2)(q'^2 - m^2)$$
$$\sum_n \Big[\delta^3(p_a + p_b + q' - p_n) \frac{\langle p_a p_b \text{ in} | \phi_c^{\dagger}(0) | n \text{ out} \rangle \langle n \text{ out} | \phi_c(0) | p_a' p_b' \text{ in} \rangle}{p_n^0 - p_a^0 - p_b^0 - q_0' + i\epsilon_3} \tag{24}$$
$$+ \delta^3(p_a + p_b - q - p_n) \frac{\langle p_a p_b \text{ in} | \phi(0) | n \text{ out} \rangle \langle n \text{ out} | \phi_c^{\dagger}(0) | p_a p_b' \text{ in} \rangle}{p_n^0 - p_a^0 - p_b^0 + q_0 + i\epsilon_3} .$$

Composing Eq. (24) and (21) in the forward direction where $p_a = p_a'$, $p_b = p_b'$, $q = q'$, Eq. (11) holds.

FOOTNOTES AND REFERENCES

[1] A. I. Sanda, Physical Review D6, 280, (1972). This paper also contains earlier references.

[2] M. B. Einhorn, J. Ellis and J. Finkelstein, SLAC-PUB-1006 (1972).

[3] S. D. Ellis and A. I. Sanda, NAL-THY-30,1972 to be published in the Physical Review.

[4] S. D. Ellis and A. I. Sanda, NAL-THY-49,1972 to be published in Physics Letters.

[5] Note that $M^2/s = 1 - q_0^*/\sqrt{s} \approx 1 - x$ where $x = q_L^*/\sqrt{s}$. q_0^* and q_L^* are the center of mass energy and longitudinal momentum for particle c respectively.

[6] At this point we state the analyticity assumption more precisely. $T(\nu/\eta\,,\ \nu/(\eta')^*,\ t,\ \nu)$, with $\nu/\eta\,,\ \nu/(\eta')^*$, and t fixed, is assumed to be analytic in ν everywhere except where the generalized unitarity requires it to have singularity.

[7] G. Cuijanovich, B. Maglich, and F. Sannes, NAL Experiment No. 67.

FIGURE CAPTIONS

Fig. 1 $a + b \rightarrow c + X$

Fig. 2 $a = b \rightarrow c + X$ in the triple-Regge region.

Fig. 3 The amplitude for $a + b + \bar{c} \rightarrow a + b - \bar{c}$.

Fig. 4 The absorptive part of M^2 and M_1^2 channels. The discontinuities across the right and left hand cuts on the ν plane are the absorptive part of these two channels respectively.

Fig. 5. The singularities of the amplitude for $a + b + \bar{c} \rightarrow a + b + \bar{c}$ in the variable ν while ν/η, $\nu/(\eta')^*$, t are fixed. Also shown is the contour used to derive the fixed M^2/s, t sum rule.

Fig. 6. Regge diagram in the fragmentation region of a.

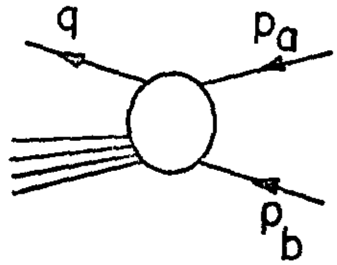

Fig. 1

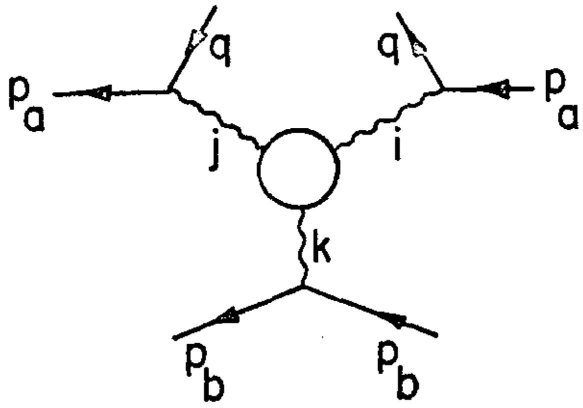

Fig. 2

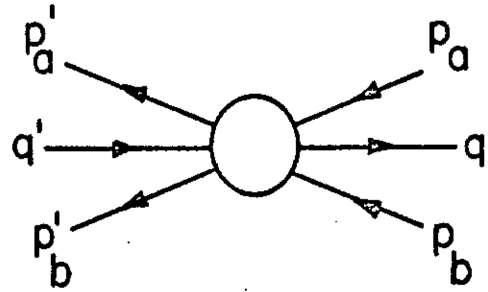

Fig. 3

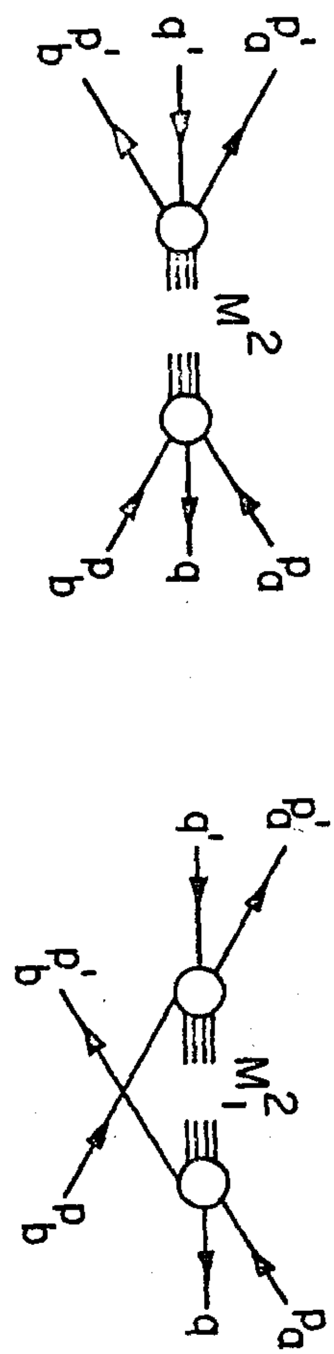

Fig. 4

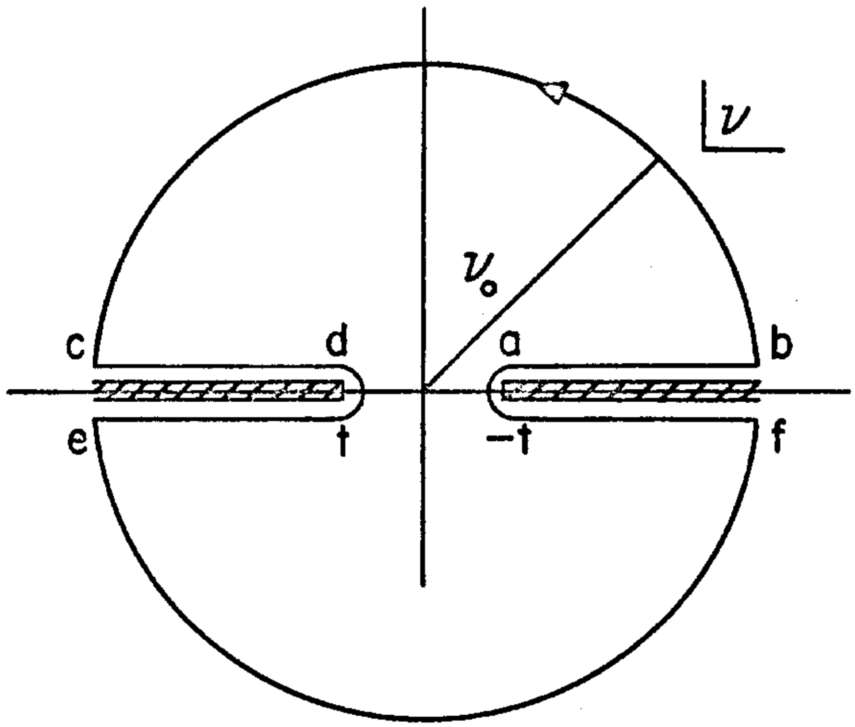

Fig. 5

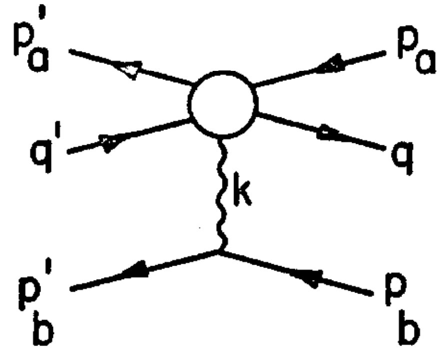

Fig. 6