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Induced Neutral Current Effects In Unified Models of Weak and Electromagnetic Interactions

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C. Bouchiat, J. Iliopoulos and Ph. Meyer pointed out correctly that the contributions of diagrams (c) and (d) contain a factor

$$-i\left(\frac{G_F \alpha}{\sqrt{2}\,\pi}\right) r [\bar{\nu}\gamma^{\alpha}(1+\gamma_5)\nu]\,[\bar{e}\gamma_{\alpha} e] \qquad (7)$$

where r is of order $\ell n\ [m_{\mu}/m(Y^+)]$. More precisely, for q^2, $m^2(Y^+) << m_W^2$, r is given by

$$r = 2\int_0^1 d\alpha\,\alpha(1-\alpha)\,\ell n\,\frac{m_{\mu}^2 - q^2\alpha(1-\alpha)}{m^2(Y^+) - q^2\alpha(1-\alpha)} + 0\left(\left(\frac{m(Y^+)}{m_W}\right)^2, \left(\frac{m_{\mu}}{m_W}\right)^2, \left(\frac{q^2}{m_W^2}\right)\right)$$

$$\simeq (1/3)\,\ell n\left[\frac{m_{\mu}}{m(Y^+)}\right]^2 - 1/15\,\frac{q^2}{m_{\mu}^2} \quad \text{for } q^2 << m_{\mu}^2,$$

$$(1/3)\,\ell n\,\frac{q^2}{m(Y^+)^2} \qquad \text{for } m_{\mu}^2 << q^2 << m^2(Y^+).$$

Numerically r is of order unity for $m(Y^+) \simeq 1$ GeV. Equation (7) should be added to the right hand sides of Eqs. (3) and (4).

We thank the above authors for communicating their results to us prior to publication.

It has often been remarked,[4] in the context of "conventional" weak interaction theory, that the reactions (1) and (2) must in any event take place via the intervention of weak and electromagnetic interactions,[5] at the level of $G_F\alpha \ln (\Lambda/m_\mu)^2$; also as second order weak processes with strength of order $G_F^2 \Lambda^2$, where Λ is some cutoff mass, $G_F = 10^{-5} m_p^{-2}$, and $\alpha = (137)^{-1}$. In a unified gauge theory of weak and electromagnetic interactions there is no intrinsic distinction between the two mechanisms, and, indeed, gauge independent results are obtained only if the two effects, which are formally of the same order, are taken into account together. In the renormalizable models we shall discuss the amplitudes for (1) and (2) are finite, that is to say, independent of cutoff. The magnitude of these amplitudes is precisely of the order of $G_F\alpha$, as one would most naively guess: they are neither enhanced nor suppressed.

We shall outline the calculational procedure for the reaction (1) in the GG model in some detail. In order to avoid any ambiguities associated with divergences, we perform the calculation in one of the renormalizable gauges, the so-called R_ξ-gauge with $\xi = 1$.[6] In this gauge, which was first used by 't Hooft[7], the vector boson propagators are proportional to $g_{\mu\nu}$, and the propagators for unphysical scalar mesons (would-be Goldstone bosons) are $i(k^2-m_W^2)^{-1}$ where m_W is the mass of the weak vector bosons. In this gauge all the relevant diagrams are finite and well-defined. In order to simplify our discussion, we shall assume that the mass ratio $m(Y^+)/m_W$ is small, where $m(Y^+)$ is the mass of the charged heavy lepton.[2] This assumption is certainly compatible with the requirement that the weak interaction correction to the anomalous magnetic moment of the muon[8] be kept within the bounds set by theoretical and experimental uncertainties. Accordingly we shall set equal to zero the mass ratios $m(Y^+)/m_W$, m_e/m_W, m_μ/m_W.[9] In this limit the unphysical scalars decouple from leptons and we need not consider their contributions to the amplitude. It can be shown that the contributions of the physical Higgs scalar boson ϕ vanish at threshold, irrespective of the mass ratios $m(Y^+)/m_W$, m_ϕ/m_W. This leaves six diagrams of Fig. 1 to be considered.

It is easy to see that the sum of the first four diagrams (a) - (d) in Fig. 1 vanishes in the limit where μ^- and Y^+ are degenerate, so on dimensional grounds the amplitude corresponding to this sum must be of the form

$$\sim i G_F \alpha \frac{m(Y^+)^2 - m_\mu^2}{m_W^2} [\bar{\nu} \gamma^\alpha (1+\gamma_5) \nu][\bar{e} \gamma_\alpha e],$$

which is negligible. The remaining two diagrams (e) and (f) may be computed in a straightforward manner. Neglecting terms of order p/m_W where p is a typical external momentum, we obtain

$$T^{GG}(\nu_\mu + e \to \nu_\mu + e) = i\,\frac{G_F}{\sqrt{2}}\left(\frac{3\alpha}{2\pi}\right)\left[\bar{\nu}\gamma^\alpha(1+\gamma_5)\nu\right] \tag{3}$$
$$\times\left[\bar{e}\gamma_\alpha\gamma_5 e + \cos^2\beta\,\bar{e}\gamma_\alpha(1+\gamma_5)e\,F\left(\left(\frac{m(X^0)}{m_W}\right)^2\right)\right],$$

where

$$F(x) = \int_0^1 \frac{\lambda\, d\lambda}{(\lambda+x)(\lambda+1)^2} - 1 ,$$

and β is the mixing angle of the GG model: $\sin\beta = m_W/53$ GeV. Notice that $F(x)$ vanishes like $x \ln x$ in the limit $x \to 0$, so that the second term in Eq. (3) can be ignored if $m(X^0) \ll m_W$.

The calculation of $T(\nu_\mu + e \to \nu_\mu + e)$ in the LPZ model proceeds in much the same way. We will simply record the result in the same approximation as above:

$$T^{LPZ}(\nu_\mu + e \to \nu_\mu + e) = i\,\frac{G_F}{\sqrt{2}}\left(\frac{3\alpha}{4\pi}\right)\left(\frac{m_W}{53\,\mathrm{GeV}}\right)^2 \times [\bar{\nu}\gamma^\alpha(1+\gamma_5)\nu][\bar{e}\gamma_\alpha(1+\gamma_5)e]. \tag{4}$$

For the process (2) our estimates are less certain, both because the extentions of the models to the hadronic system are tentative at best, and because we cannot reliably take into account strong interaction effects. In any case, we shall base our estimates on the observation that, when external momenta are not too large on a hadronic scale, the dominant contribution to $T(\nu + p \to \nu + p)$ comes from large internal momenta, and, under such circumstances, fundamental fermions behave perhaps like free fields. Assuming the masses of fundamental fermions to be small compared to m_W, we estimate the <u>effective</u> interactions responsible for reactions such as (2) to be

$$\mathcal{L}^{GG}_{eff} = \frac{G_F}{\sqrt{2}}\left(\frac{3\alpha}{2\pi}\right)[\bar{\nu}\gamma^\alpha(1+\gamma_5)\nu] \times [\bar{p}\gamma_\alpha\gamma_5 p + \bar{n}\gamma_\alpha(1+\gamma_5)n\sin^2\beta], \tag{5}$$

and

$$\mathcal{L}^{LPZ}_{eff} = \frac{G_F}{\sqrt{2}}\left(\frac{3\alpha}{4\pi}\right)\left(\frac{m_W}{53\,\mathrm{GeV}}\right)^2[\bar{\nu}\gamma^\alpha(1+\gamma_5)\nu] \times [\bar{p}\gamma_\alpha(1+\gamma_5)p + 2\bar{n}\gamma_\alpha(1+\gamma_5)n], \tag{6}$$

where we have taken the Cabibbo angle $\theta_c = 0$, and $\mathcal{P}$, $\mathcal{N}$ are integrally charged, isospin doublet quark fields. Equations (5) and (6) are to be interpreted as phenomenological Lagrangians which generate low energy limits of semileptonic processes such as (2), wherein expressions such as $\bar{\mathcal{P}}\gamma_\rho(1 + \gamma_5)\mathcal{P}$ are mnemonics for the corresponding $U(2) \times U(2)$ hadronic currents. Despite uncertainties attendant in these kinds of estimates, it seems to us safe to conclude that there is no obvious mechanism within weak interactions either to enhance or to suppress the process (2) in these models. That is, at least for energies which are modest on a hadronic scale we expect the amplitude for process (2) to have roughly the structure and strength given by Eqs. (3) or (4) respectively for the two models under discussion, with electron replaced by proton.

One of the authors (BWL) wishes to thank J. D. Bjorken for discussions on this and related subjects.

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[6] See K. Fujikawa, B. W. Lee and A. I. Sanda, to be published, for a detailed discussion on this subject. We have verified the ξ-independence of our results which is also a check on the unitarity of the S-matrix.

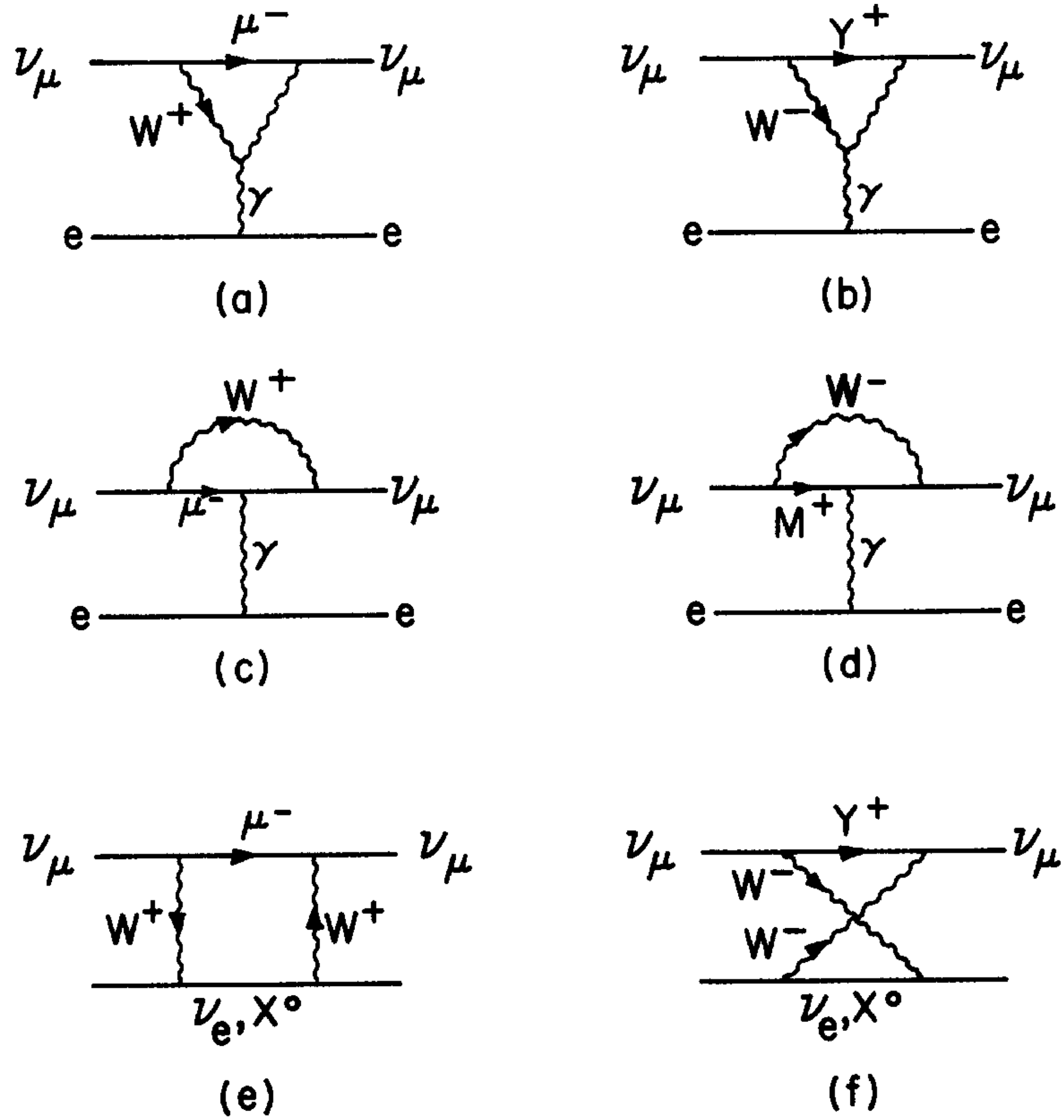
[7] G. 't Hooft, Nuclear Physics B35, 167 (1971).

[8] See K. Fugikawa et.al., Ref. 6, and J. Primack and H. R. Quinn, to be published, for details.

[9] Here and in the following we adopt the notations of Ref. 2 as much as possible.

FIGURE CAPTION

Figure 1: Six diagrams which contribute to the process $\nu_\mu + e \rightarrow \nu_\mu + e$ in the Georgi-Glashow model.



ν_μ
μ⁻
ν_μ
W⁺
γ
e
e
(a)
ν_μ
Y⁺
ν_μ
W⁻
γ
e
e
(b)
W⁺
ν_μ
μ⁻
ν_μ
γ
e
e
(c)
W⁻
ν_μ
M⁺
ν_μ
γ
e
e
(d)
ν_μ
μ⁻
ν_μ
W⁺
W⁺
ν_e,X°
(e)
ν_μ
Y⁺
ν_μ
W⁻
W⁻
ν_e,X°
(f)